Introduction to Control Loop Observers

Presentation of High-Gain Observers

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Plan

- Introduction to observers
- Observability and the structure of observers
- Introduction to high-gain observers
- Adaptive gain extended Kalman filter
- Conclusion

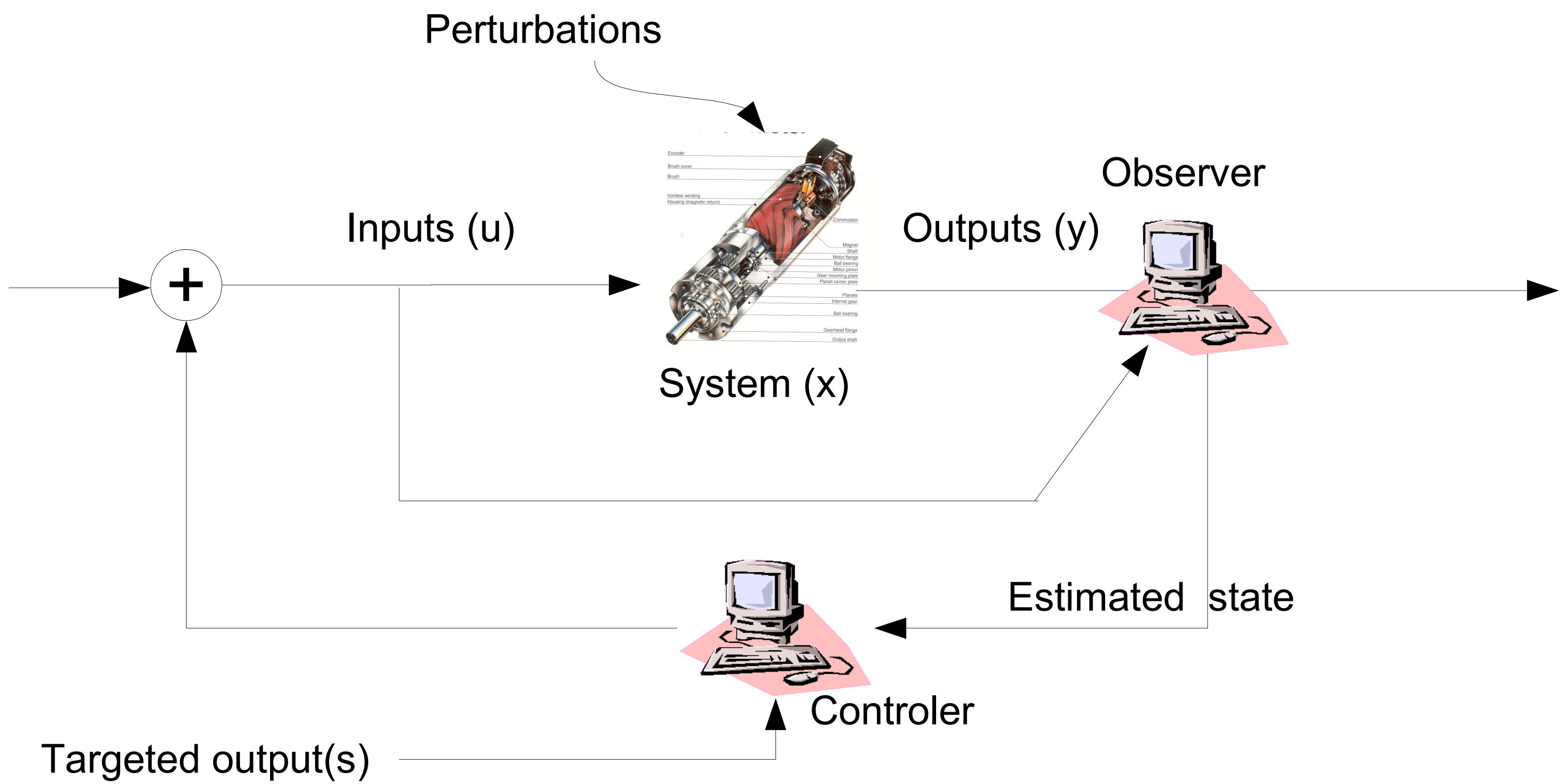
What is an Observer ?

- A soft sensor for dynamical systems
- Based on a mathematical model of the system
- The model has the following form (we consider nonlinear systems):

$$\left\{ \begin{array}{l} \dot{x}(t) = f(x(t), u(t)) \\ y(t) = h(x(t), u(t)) \end{array} \right\}$$

- Several applications of observers :
  - Process control and optimization,
  - Identification of extra parameters / nonlinear parts in a model
  - Failure détection

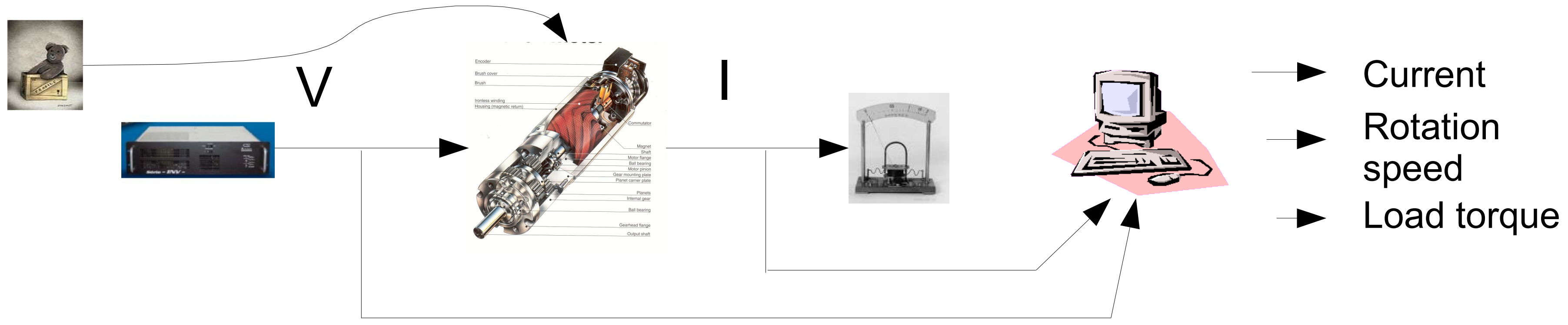
Observers in Control Loops

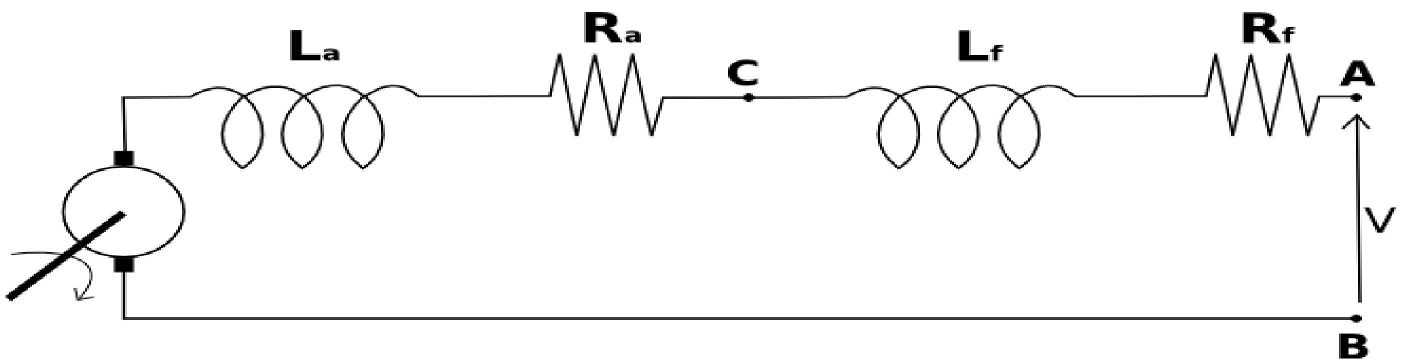


The observation problem:

reconstructing the full information about a dynamical process (its state variables) on the basis of partial measurements

Example of a series connected DC motor





$$\left\{\begin{array}{l} \dot{x}=\begin{pmatrix} \dot{I} \\ \dot{\omega}_r \end{pmatrix}=\begin{pmatrix} (V-R.I-L_{af.}\omega_r.I)/L \\ (-B_{m.}\omega_r+L_{af}.I^2+T_a)/J \end{pmatrix} \\ y=I=\begin{pmatrix} 1 & 0 \end{pmatrix}.x \end{array}\right\}$$

Observability

Is it really possible to reconstruct the full information ?

- Fisrt approach :

$$\dot{I}=(V-R.I-L_{af}.\omega_r.I)/L$$

If I and V are known and if I≠0, then the first equation gives ω_r

$$\dot{\omega}_r=(-B_m.\omega_r+T_{em}+T_a)/J$$

Once ω_r is known, the second equation gives us T_a

- Second approach : the observability canonical form

[observability]

$\Longleftrightarrow$

There exists a coordinate transformation for Single Input/Single Output (SISO) systems such that the model becomes:

$$\dot{x}=\begin{pmatrix}\dot{x}_1\\ \dot{x}_2\\ \dots\\ \dot{x}_{n-1}\\ \dot{x}_n\end{pmatrix}=\begin{pmatrix}0 & \alpha_2(u) & \dots & 0 & 0\\ 0 & 0 & \alpha_3(u) & \dots & 0\\ \dots & \dots & \dots & \dots & \dots\\ 0 & 0 & \dots & 0 & \alpha_n\\ 0 & 0 & 0 & 0 & 0\end{pmatrix}.\begin{pmatrix}x_1\\ x_2\\ \dots\\ x_{n-1}\\ x_n\end{pmatrix}+\begin{pmatrix}b_1(x_{1,}u)\\ b_2(x_{1,}x_{2,}u)\\ \dots\\ b_{n-1}(x_{1,}x_{2,}\dots,x_{n-1},u)\\ b_n(x_{1,}x_{2,}\dots,x_n,u)\end{pmatrix}$$

$$y=\begin{pmatrix}\alpha_1(u) & 0 & \dots & 0\end{pmatrix}.x$$

Structure of Observers

Principle :

- Prediction of the new state using the mathematical model
- Correction of this prediction using the error: estimation - measurement.

Model equations :

$$\begin{cases} \dot{X} = A(u).X + b(X, U) \\ Y = C.X \end{cases}$$

Equation of the observer :

$$\dot{Z} = A(u).Z + b(Z, U) - Gain.(C.Z - y)$$

Type of Observers

$$\dot{Z} = A(u).Z + b(Z, U) - Gain.(C.Z - y)$$

- **Luenberger style** observer: the *Gain matrix* is fixed and computed offline

- **Kalman style** observer: the *Gain matrix* is variable and computed online from the Ricatti equation:

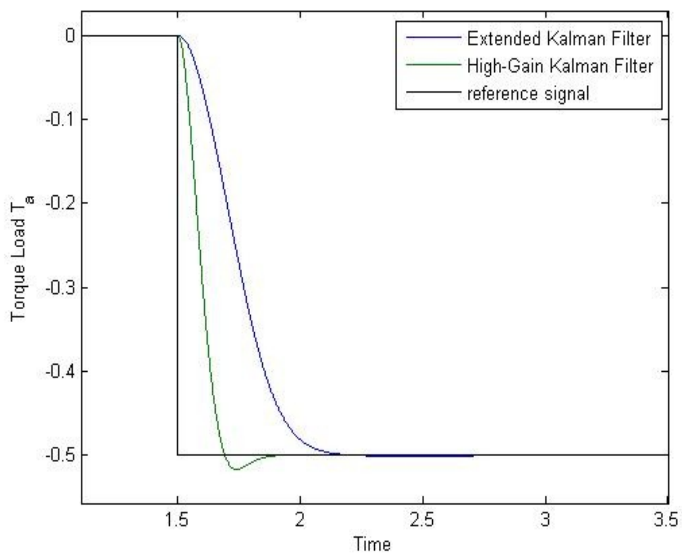
$$\left\{ \begin{array}{c} \dot{Z} = A(u).Z + b(Z, U) - S^{-1}.C'.r^{-1}.(C.Z - y) \\ \dot{S} = -(A(u) + b^{o})'.S - S.(A(u) + b^{o}) + C'.r^{-1}.C - S.Q.S \end{array} \right\}$$

High-gain Extended Kalman Filter

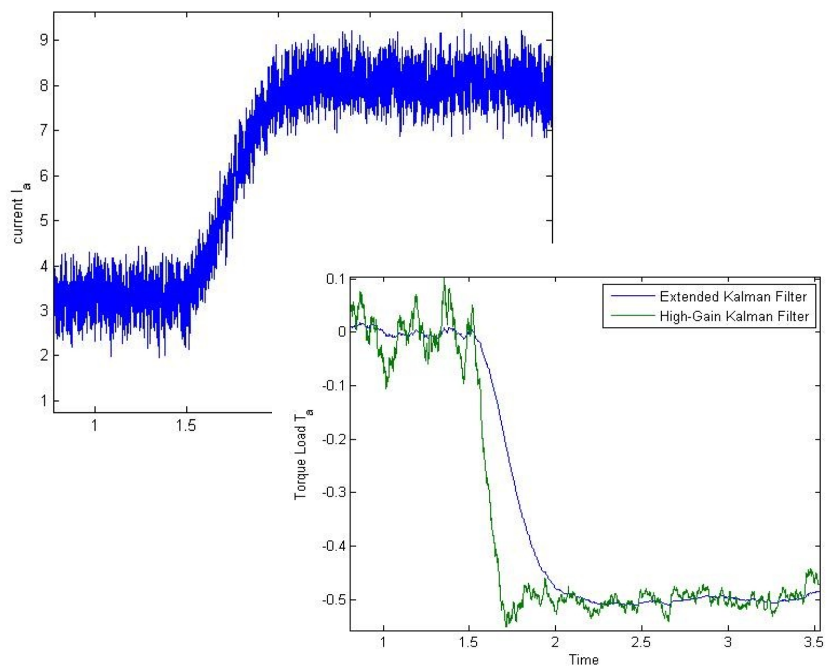
(Gauthier, Hammouri, Othman, 1992), (Gauthier, Kupka, 2001)

$$\begin{cases} \dot{Z} = A(u).Z + b(Z,U) - S^{-1}.C'.r^{-1}.(C.Z - y) \\ \dot{S} = -(A(u)+b^{o})'.S - S.(A(u)+b^{o}) + C'.r^{-1}.C - S.Q_{\theta}.S \end{cases}$$

Simulations without noise



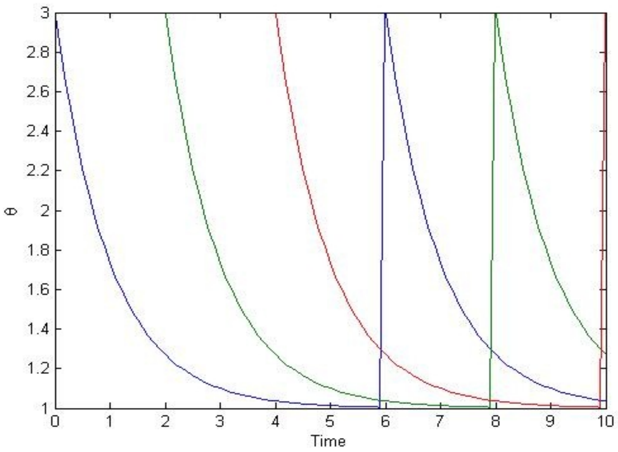
Simulations with noise



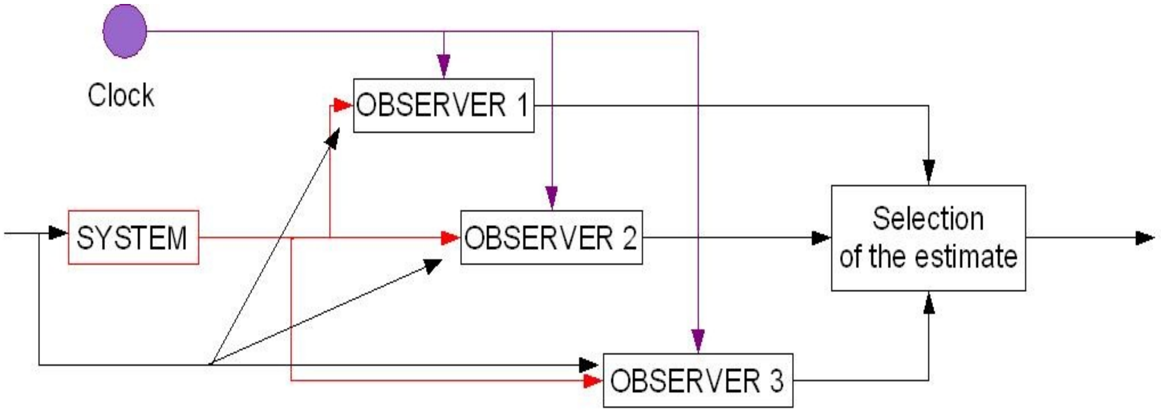
Toward a Non-constant High-gain

(Busvelle, Gauthier, 2002)

$$\left\{\begin{array}{c} \dot{Z}=A(u).Z+b(Z,U)-S^{-1}.C'.r^{-1}.(C.Z-y) \\ \dot{S}=-(A(u)+b^{o})'.S-S.(A(u)+b^{o})+C'.r^{-1}.C-S.Q_{\theta}.S \\ \dot{\theta}=\lambda.(1-\theta) \end{array}\right\}$$



Curves showing the decrease of θ vs. time



Corresponding block diagram

The Innovation Term

$$I(t) = \int_{t-d}^{t} \left\| y(v) - \hat{y}(v) \right\|^2 . dv$$

Where:

- y(v) are the outputs of the system over the time interval [t-d;t]
- $\hat{y}(v)$ is the solution of the model over the time interval [t-d;t] with initial conditions z(t-d)

Lemma :

If the model is the canonical observability form :

$$\Sigma : \left\{ \begin{array}{c} \dot{x} = A(u).x + b(x,u) \\ y = c.x \end{array} \right\}$$ u ϵ U_{adm} (bounded), x ϵ χ compact

Then for all T>0, there exist λ_T^o >0 such that :

$$\int_0^T \left\| y(v) - \hat{y}(v) \right\|^2 . \frac{1}{\lambda_T^0} > \left\| x_0 - \hat{x}_0 \right\|^2$$

Where: y(v) is the solution of the system Σ with the initial condition x_0

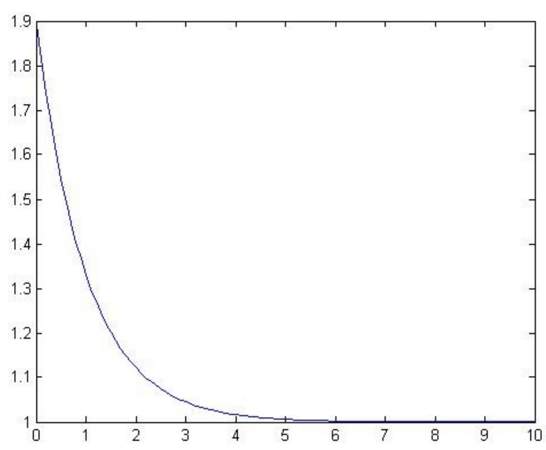
$\hat{y}(v)$ is the solution of the system Σ with the initial condition $\hat{x}_0$

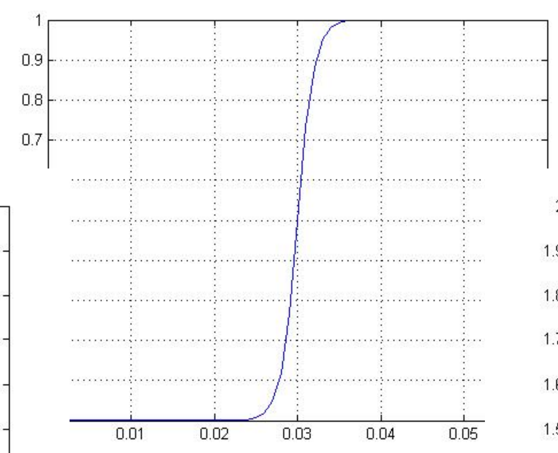
Adaptive-gain Extended Kalman Filter

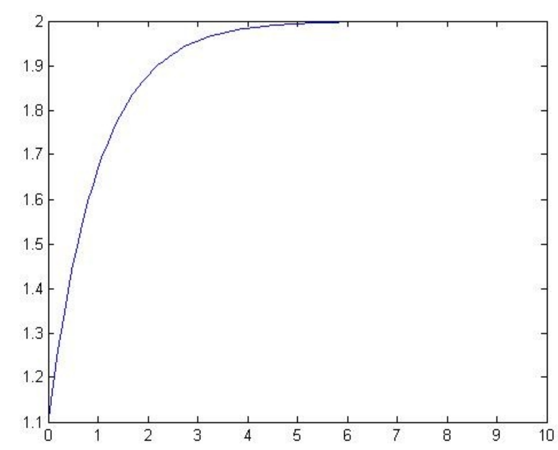
$$\begin{cases} \dot{Z} = A(u).Z + b(Z,U) - S^{-1}.C'.r_\theta^{-1}.(C.Z - y) \\ \dot{S} = -(A(u)+b^o)'.S - S.(A(u)+b^o) + C'.r_\theta^{-1}.C - S.Q_\theta.S \\ \dot{\theta} = \lambda(I).(1-\theta) + K(I)(\theta_{max} - \theta) \end{cases}$$

Small innovation term

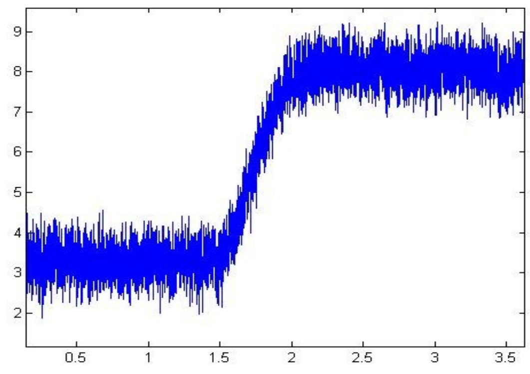
Big Innovation term



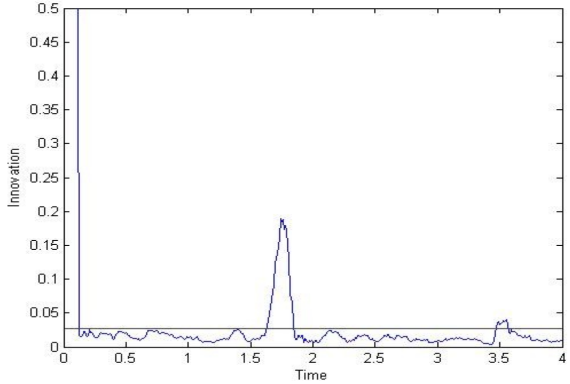




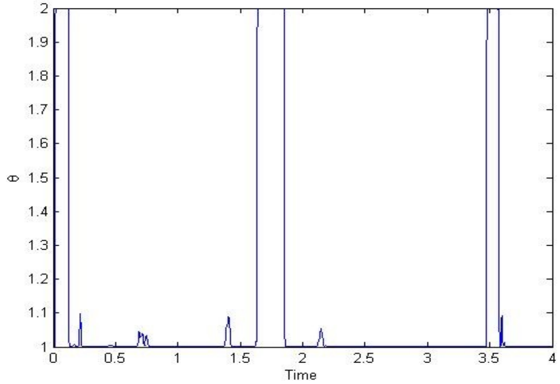
Simulations / Performance



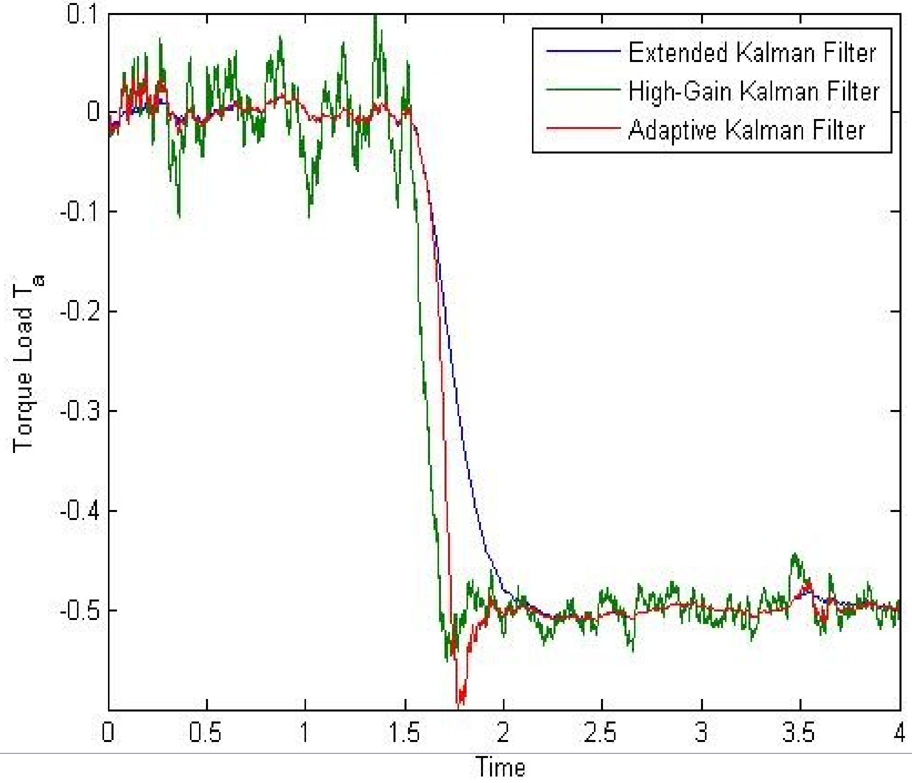
Measured output



Innovation



High-gain parameter



Estimation of the load torque

Work Perspectives

- Define a proper procedure to tune the various parameters
- Implement the observer in real time (e.g. On the DC machine)
- Study the case of MIMO systems as opposed to SISO