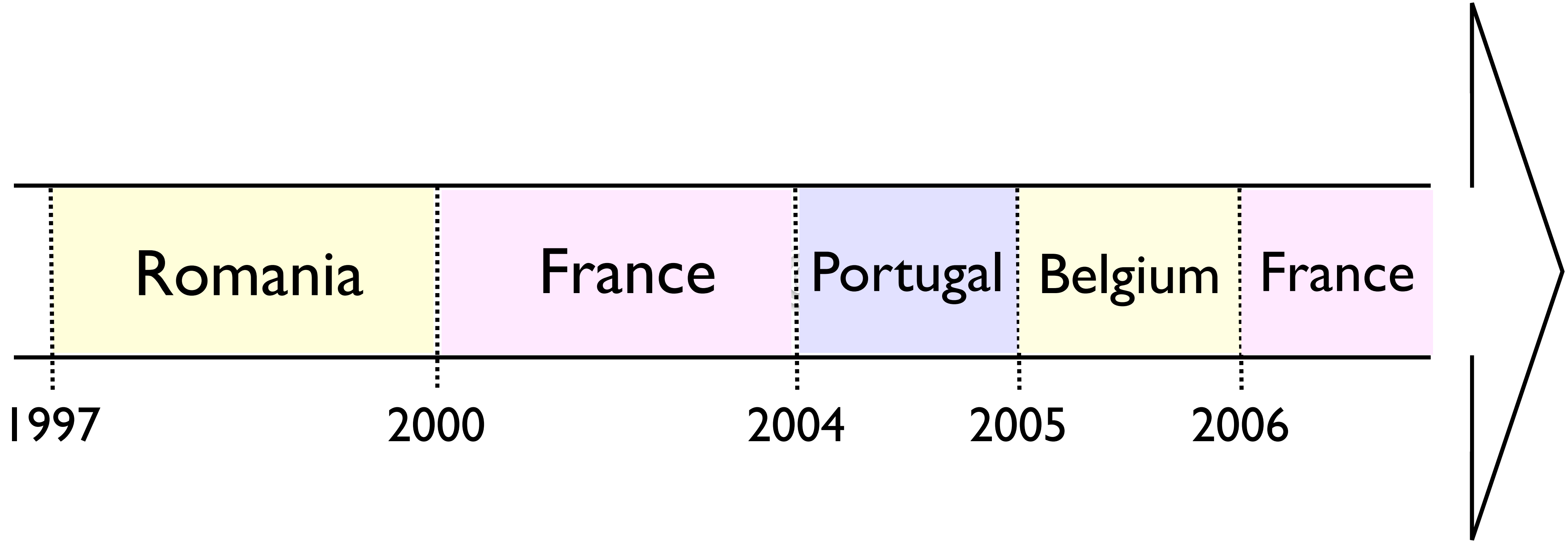
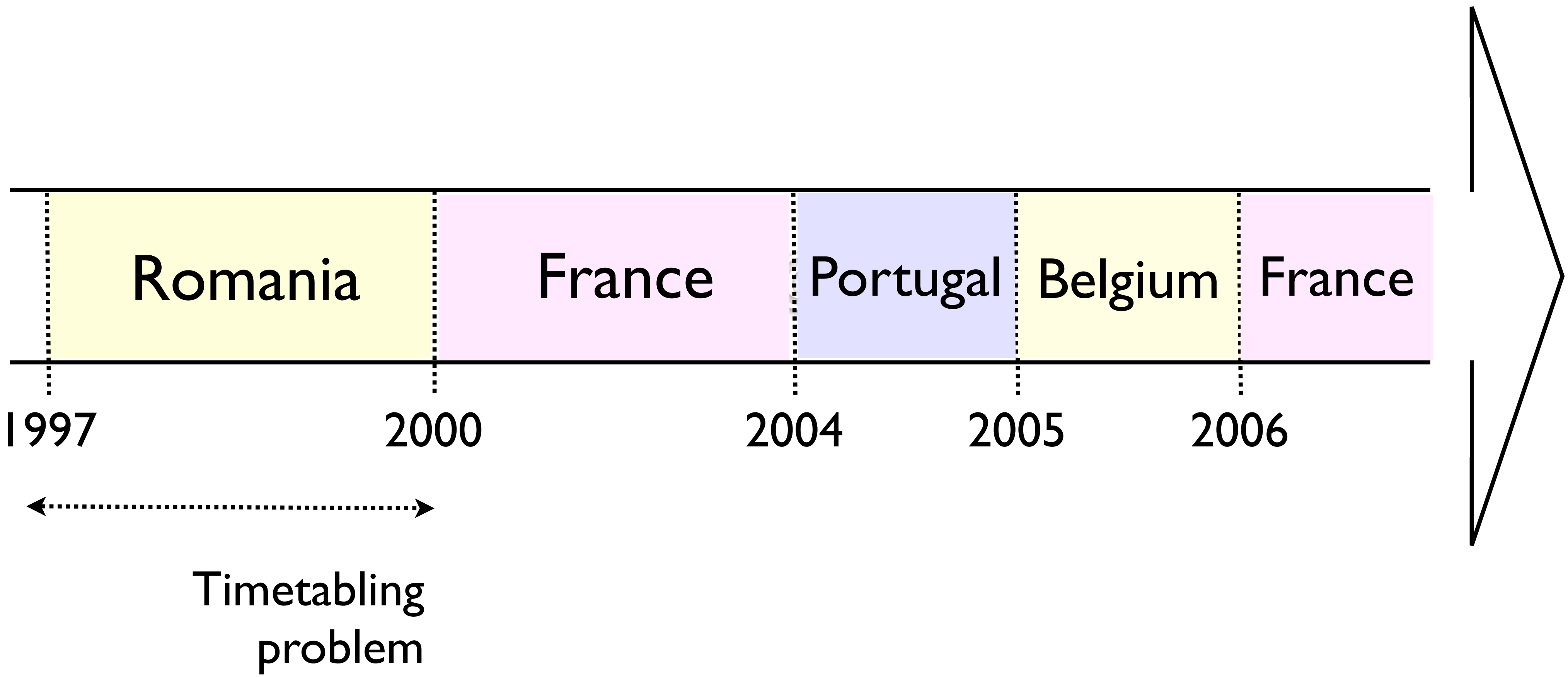
Some ideas and open problems in real-time stochastic scheduling

Liliana CUCU, Trio team, Nancy, France

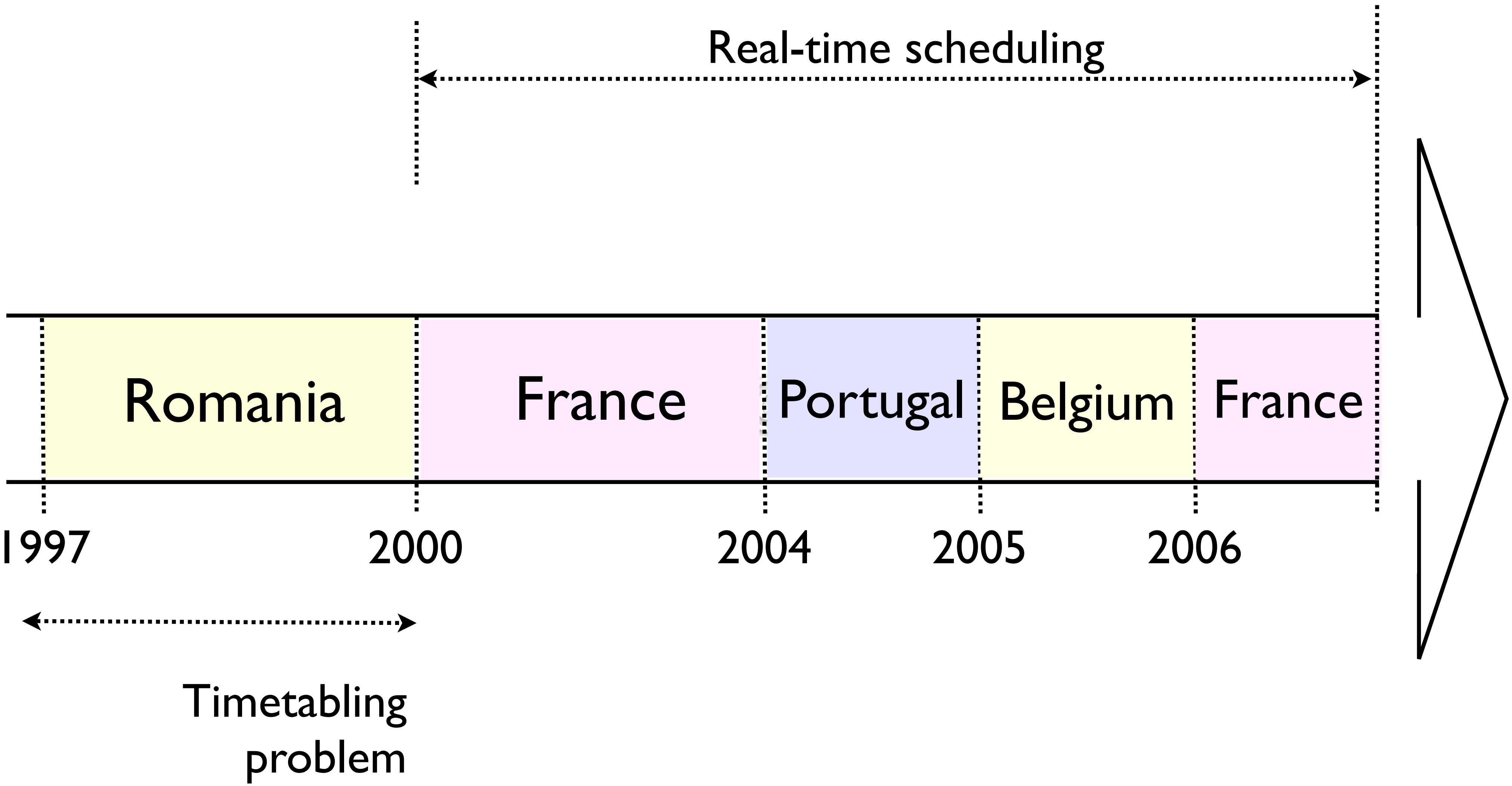
Liliana CUCU

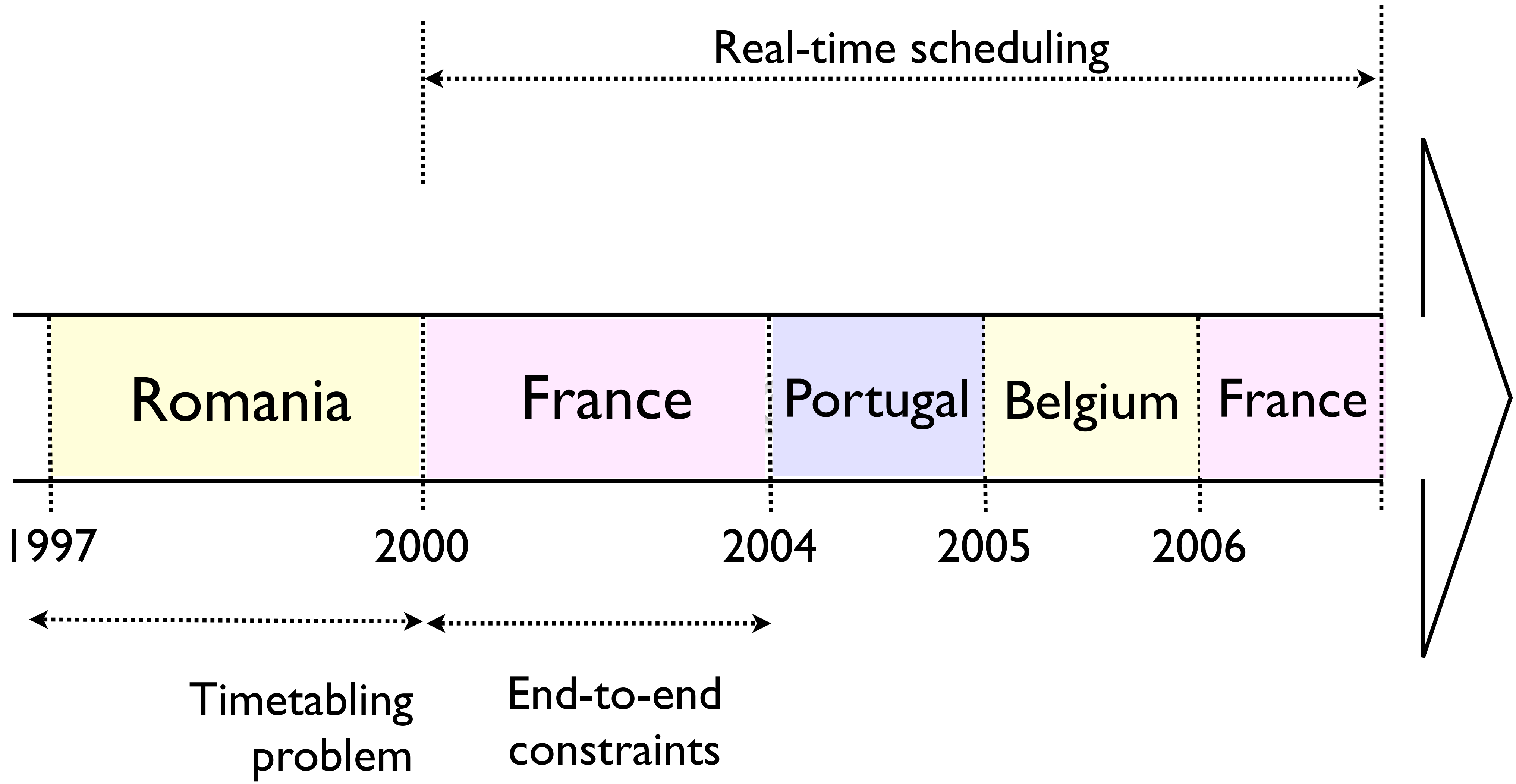


Liliana CUCU

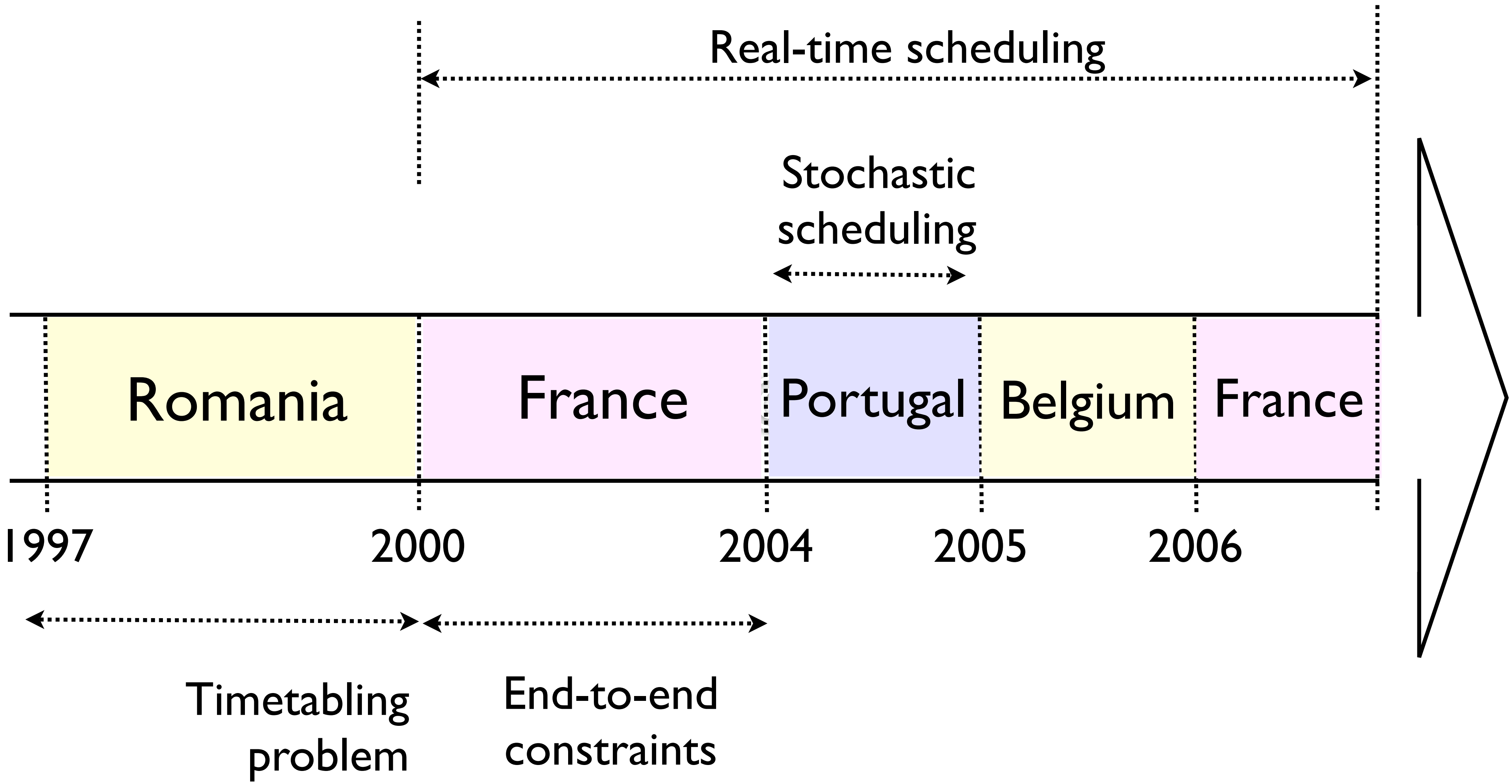


Liliana CUCU

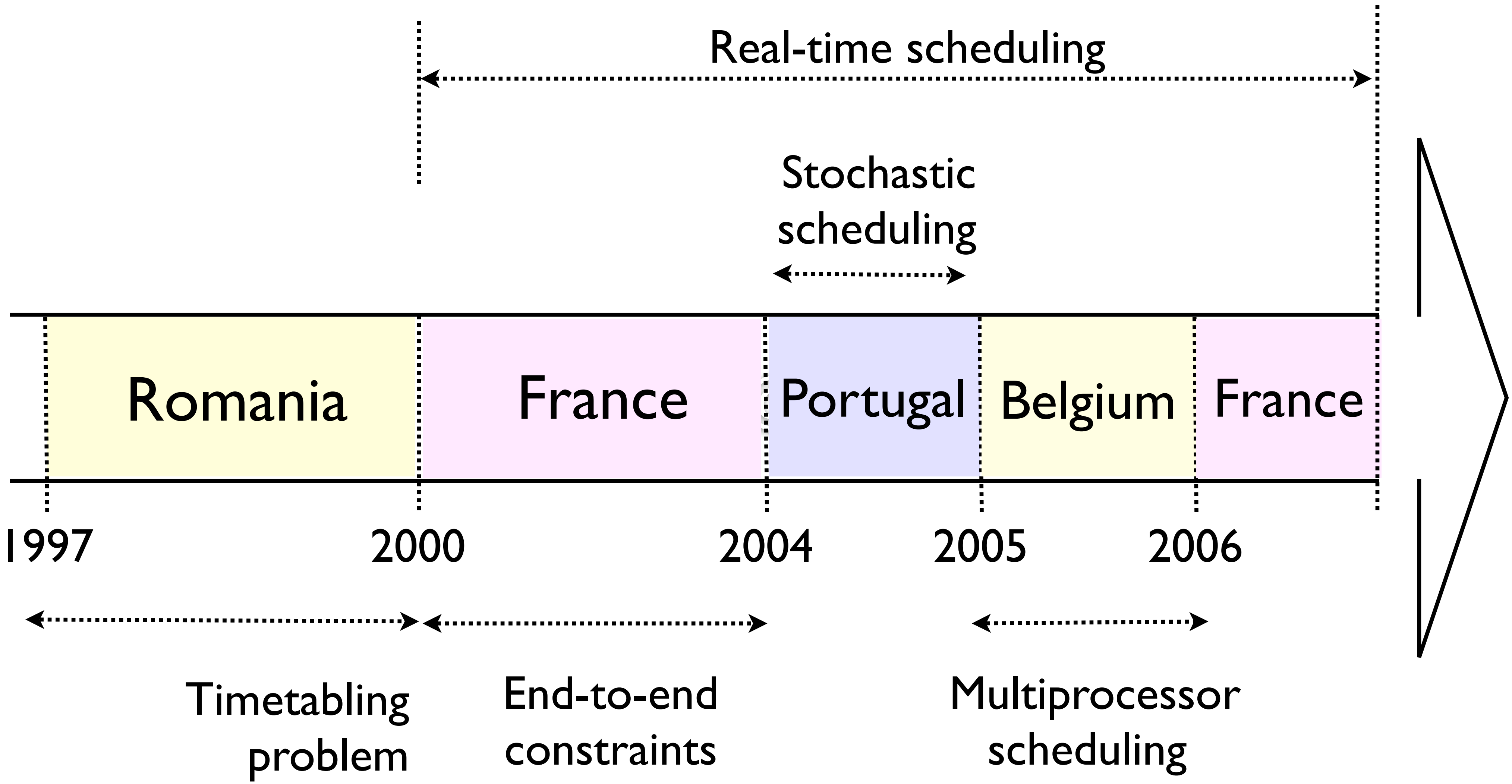



Real-time scheduling
Romania
France
Portugal
Belgium
France
1997
2000
2004
2005
2006
Timetabling problem
End-to-end constraints

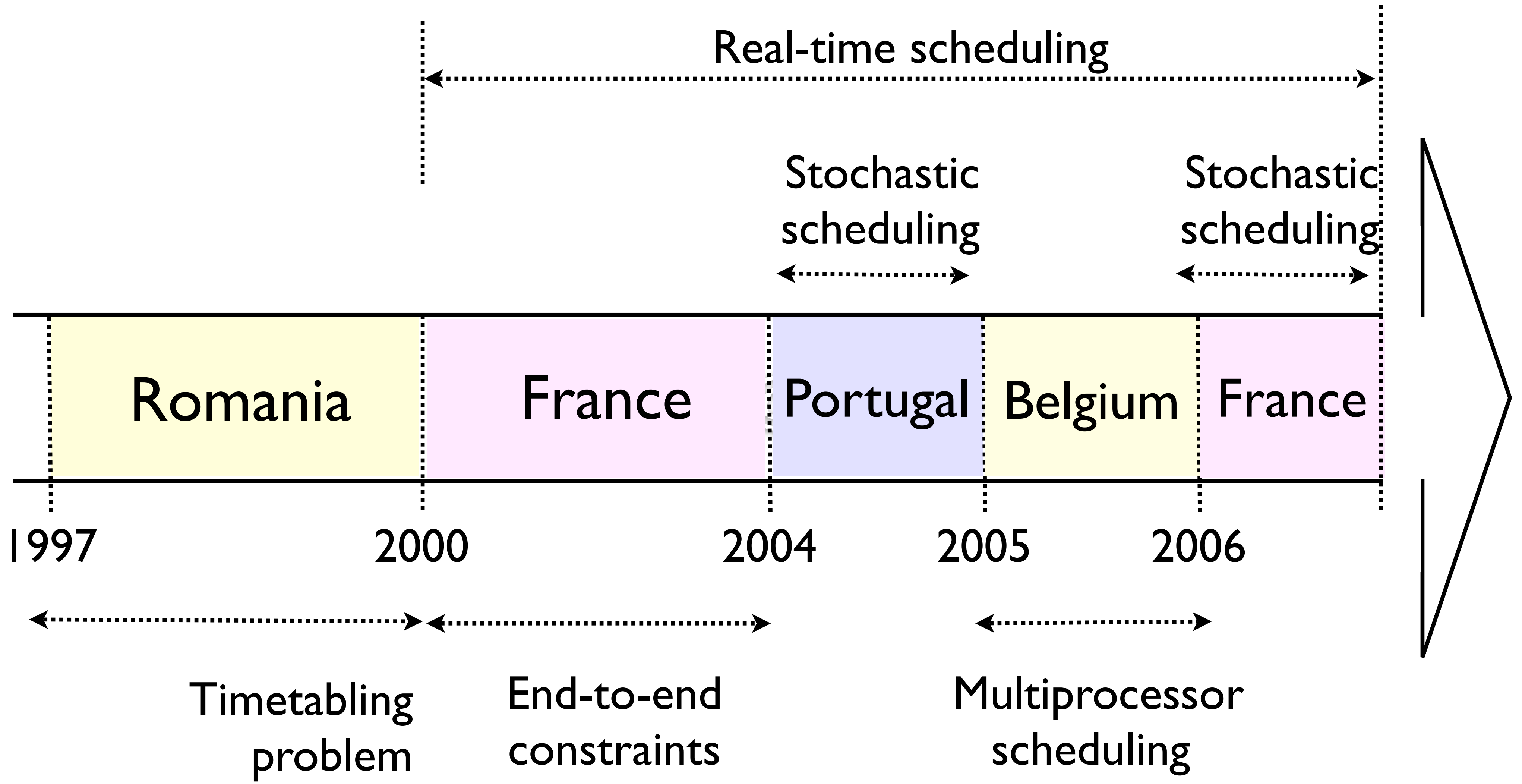
Liliana CUCU



Liliana CUCU



Liliana CUCU

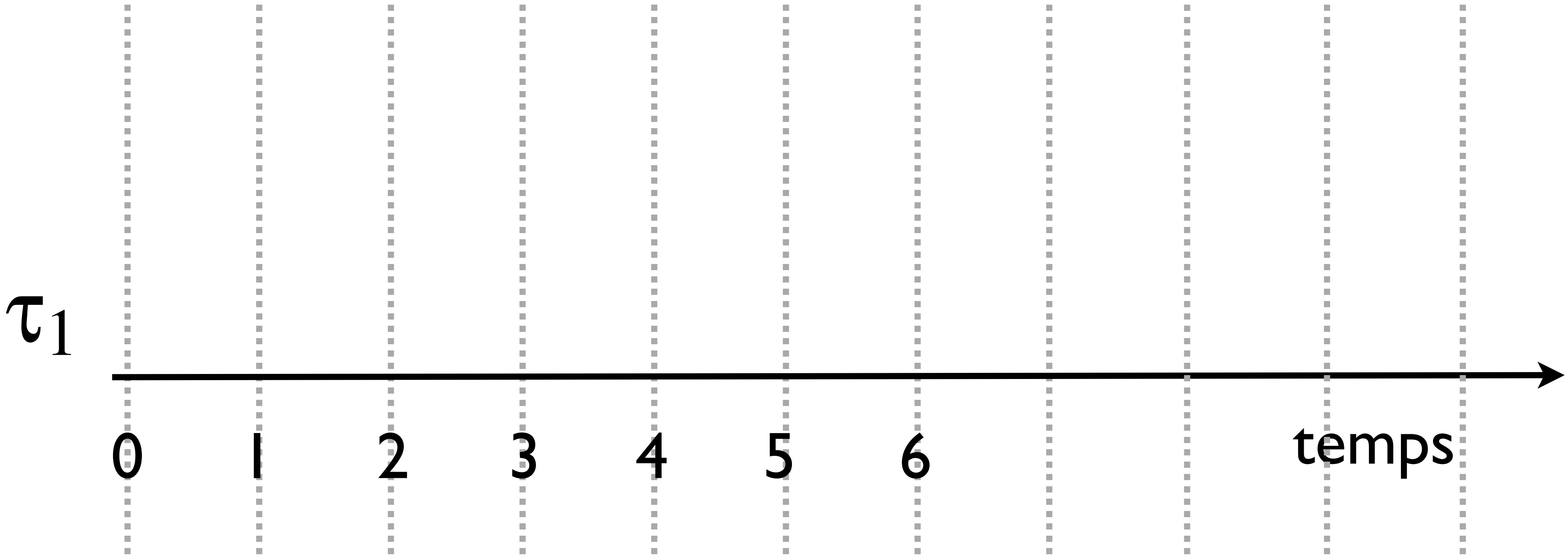


Real-time model: $\tau_i = (O_i, C_i, T_i, D_i)$

$$\tau_1 = (O_1, C_1, T_1, D_1) = (1, 2, 5, 4)$$

↑ release times

↓ deadlines

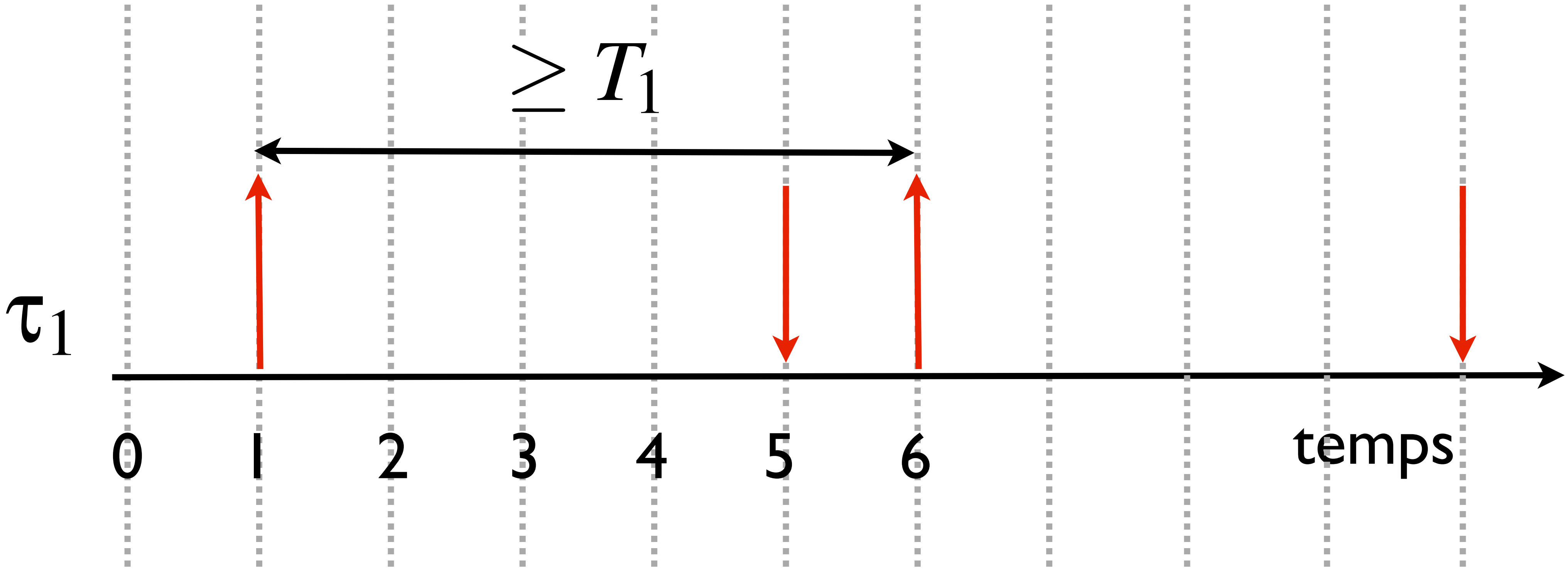


Real-time model: $\tau_i = (O_i, C_i, T_i, D_i)$

$$\tau_1 = (O_1, C_1, T_1, D_1) = (1, 2, 5, 4)$$

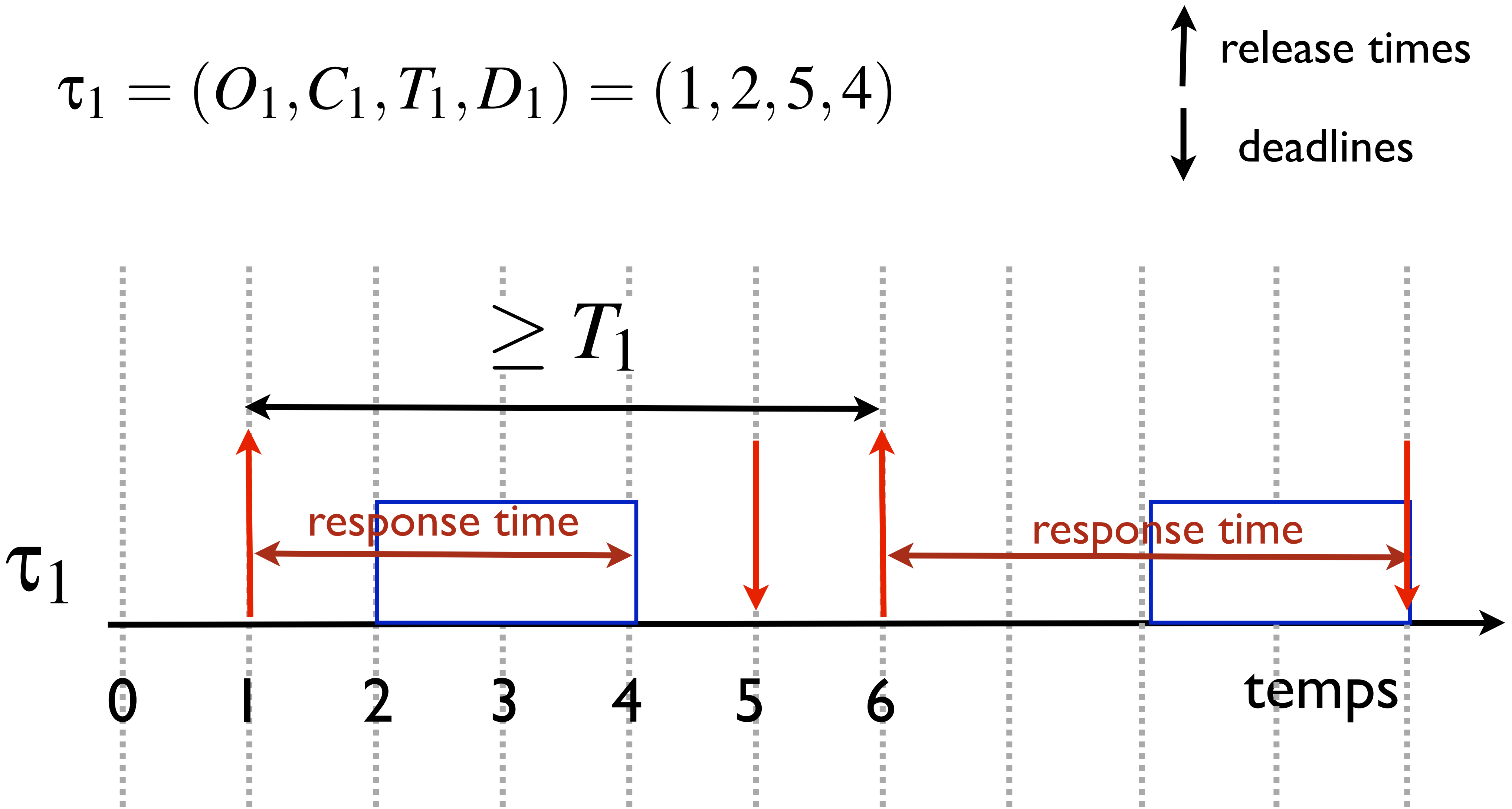
↑ release times

↓ deadlines



Real-time model: $\tau_i = (O_i, C_i, T_i, D_i)$

$$\tau_1 = (O_1, C_1, T_1, D_1) = (1, 2, 5, 4)$$



Why stochastic?

- Soft real-time constraints
- Uncertainness
- Worst-case behavior is a rare event

Why stochastic?

- ☑ Soft real-time constraints
- ☑ Uncertainness
- ☑ Worst-case behavior is a rare event

Postdocs: Porto, Nancy

Where is the “stochastic touch”?

Where is the “stochastic touch”?

Extracting quantitative information, i.e., obtaining distribution functions

Where is the "stochastic touch"?

Extracting quantitative information, i.e., obtaining distribution functions

and

Where is the “stochastic touch”?

Extracting quantitative information, i.e., obtaining distribution functions

and

Temporal analysis of systems with at least one parameter given by a random variables

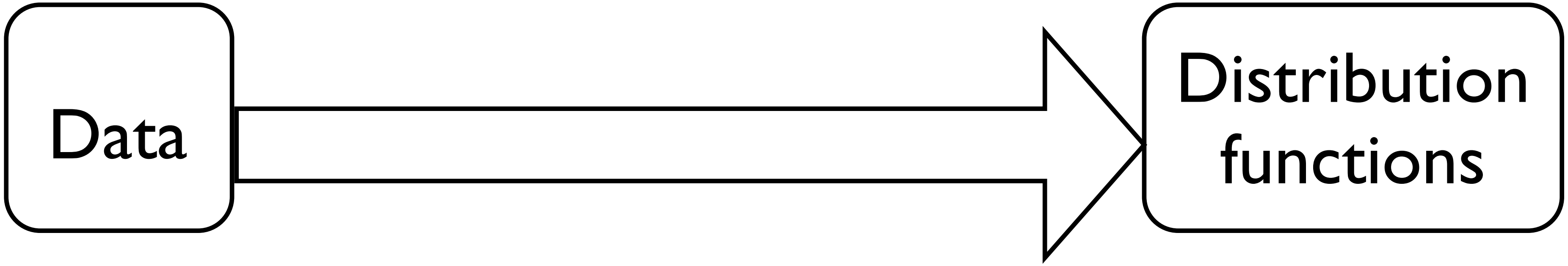
Where is the “stochastic touch”?

Extracting quantitative information, i.e., obtaining distribution functions

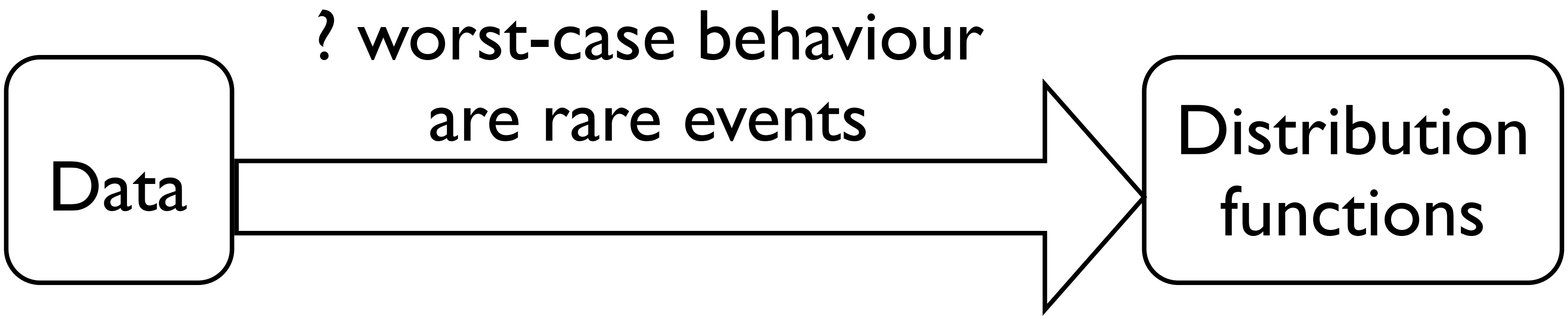
and

Temporal analysis of systems with at least one parameter given by a random variables

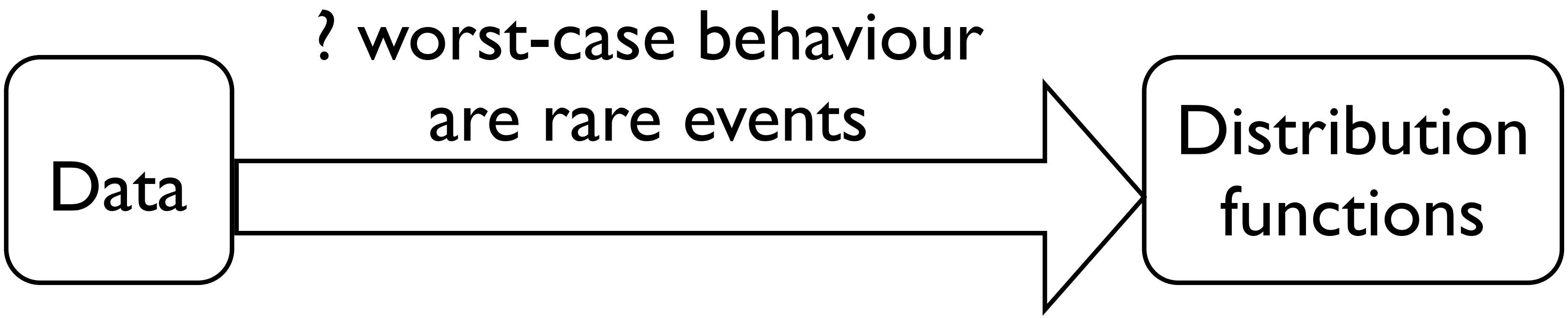
Extracting quantitative information



Extracting quantitative information



Extracting quantitative information



Joint work with N. Navet and René Schott (TRIO, Nancy)

How to estimate the average response time???

How to estimate the average response time???

Activation model of tasks not known

How to estimate the average response time???

Activation model of tasks not known

- Monte-Carlo simulation

How to estimate the average response time???

Activation model of tasks not known

- Monte-Carlo simulation
- Analytical approaches

How to estimate the average response time???

Activation model of tasks not known

- Monte-Carlo simulation
- Analytical approaches
  - Markov's, Tchebychev's, Chemoff's upper bounds

How to estimate the average response time???

Activation model of tasks not known

- ☐ Monte-Carlo simulation
- ☐ Analytical approaches
  - ☐ Markov's, Tchebychev's, Chemoff's upper bounds
  - ☑ Large deviation

How to estimate the average response time???

Activation model of tasks not known

- ☐ Monte-Carlo simulation
- ☐ Analytical approaches
  - ☐ Markov's, Tchebychev's, Chemoff's upper bounds
  - ☑ Large deviation
    - better suited than simulation to rare events
    - easily implementable
    - embedded in a broader analysis

Large deviation : main result

$M_n = \frac{1}{n}\sum_{k=1}^{n} R_{i,k}$ mean of response times over n task instances

$$P(M_n \geq value)$$

Cramer's theorem : if $R_{i,n}$ independent identically distributed random variables

$$P(M_n \in \mathbb{G}) \asymp e^{-n \inf_{x\in\mathbb{G}} I(x)}$$

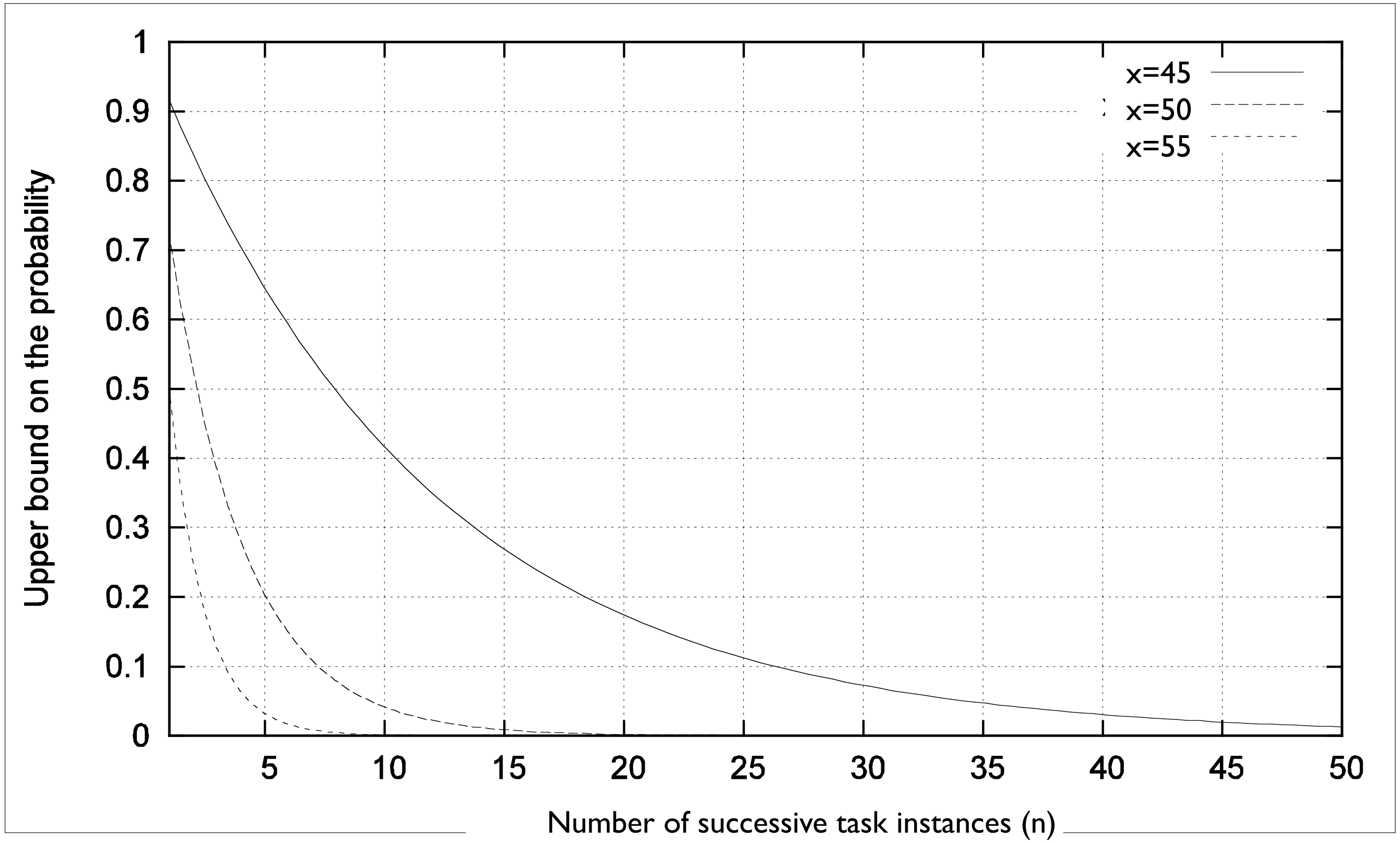
$$\mathbb{G} = [value, \infty)$$

$$I(x) = \sup_{\tau>0}[\tau x - \log E(e^{\tau x})] = \sup_{\tau>0}[\tau x - \log \sum_{k=-\infty}^{+\infty} p_k e^{k\tau}]$$

Technical contribution

Can deal with distributions given as histograms

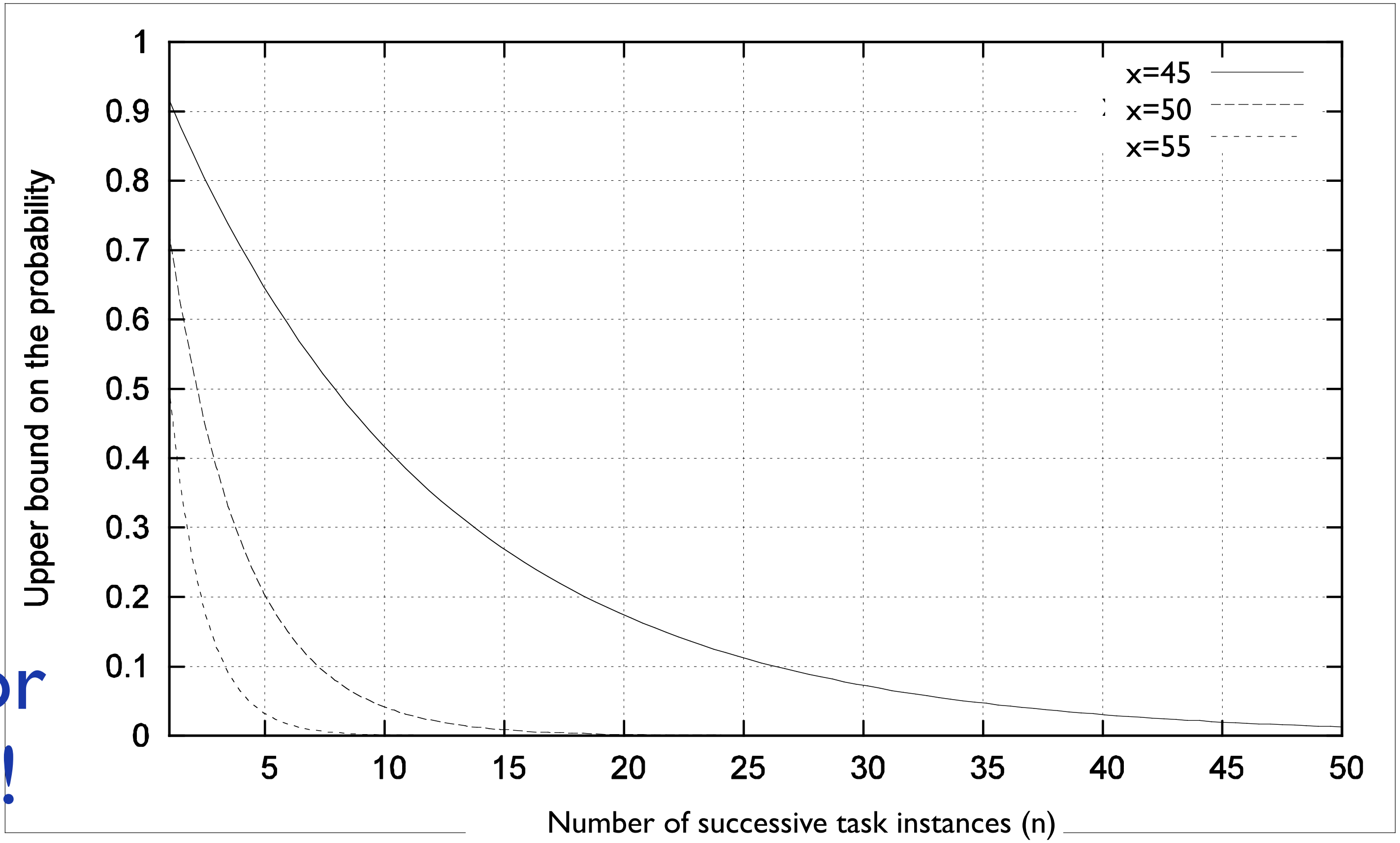
RT interval	Probability	k
$[0,10)$	$1/25$	5
$[10,20)$	$2/25$	15
$[20,30)$	$3/25$	25
$[30,40)$	$10/25$	35
$[40,50)$	$4/25$	45
$[50,60)$	$3/25$	55
$[60,70)$	$2/25$	65



Technical contribution

Can deal with distributions given as histograms

RT interval	Probability	k
$[0, 10)$	$1/25$	5
$[10, 20)$	$2/25$	15
$[20, 30)$	$3/25$	25
$[30, 40)$	$10/25$	35
$[40, 50)$	$4/25$	45
$[50, 60)$	$3/25$	55
$[60, 70)$	$2/25$	65



!! Uniprocessor or multiprocessor !!

Where is the “stochastic touch”?

Extracting quantitative information, i.e., obtaining distribution functions

and

Temporal analysis of systems with at least one parameter given by a random variables

Where is the "stochastic touch"?

Extracting quantitative information, i.e., obtaining distribution functions

and

Temporal analysis of systems with at least one parameter given by a random variables

What is the model?

$$\tau_i = (O_i, C_i, T_i, D_i)$$

What is the model?

$$\tau_i = (O_i, C_i, T_i, D_i)$$

$$\mathbb{X} = \begin{pmatrix} x_k \\ P(X = x_k) \end{pmatrix}$$

What is the model?

$$\tau_i = (O_i, C_i, T_i, D_i)$$

$$\mathbb{O}_i$$

$$\mathbb{X} = \begin{pmatrix} x_k \\ P(X = x_k) \end{pmatrix}$$

What is the model?

$$\tau_i = (O_i, C_i, T_i, D_i)$$

:)

$$\mathbb{O}_i$$

$$\mathbb{X} = \begin{pmatrix} x_k \\ P(X = x_k) \end{pmatrix}$$

What is the model?

$$\tau_i = (O_i, C_i, T_i, D_i)$$

:)

$\mathbb{O}_i$ $\mathbb{C}_i$

$$\mathbb{X} = \begin{pmatrix} x_k \\ P(X = x_k) \end{pmatrix}$$

What is the model?

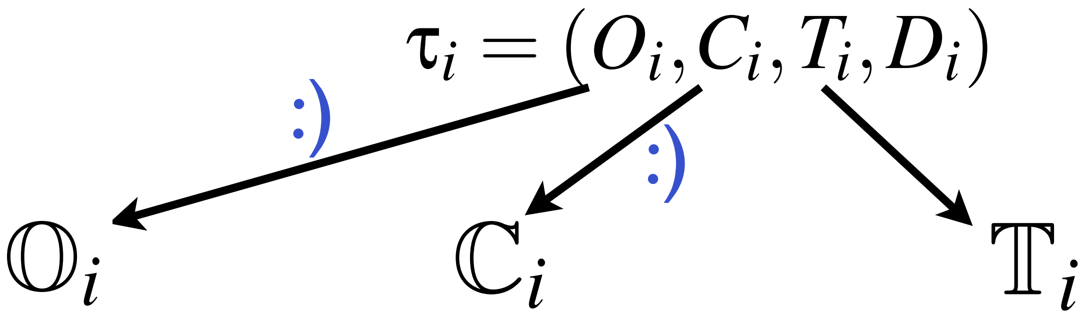
$$\tau_i = (O_i, C_i, T_i, D_i)$$

:) :)

$\mathbb{O}_i$ $\mathbb{C}_i$

$$\mathbb{X} = \begin{pmatrix} x_k \\ P(X = x_k) \end{pmatrix}$$

What is the model?



$$\mathbb{X} = \begin{pmatrix} x_k \\ P(X = x_k) \end{pmatrix}$$

What is the model?

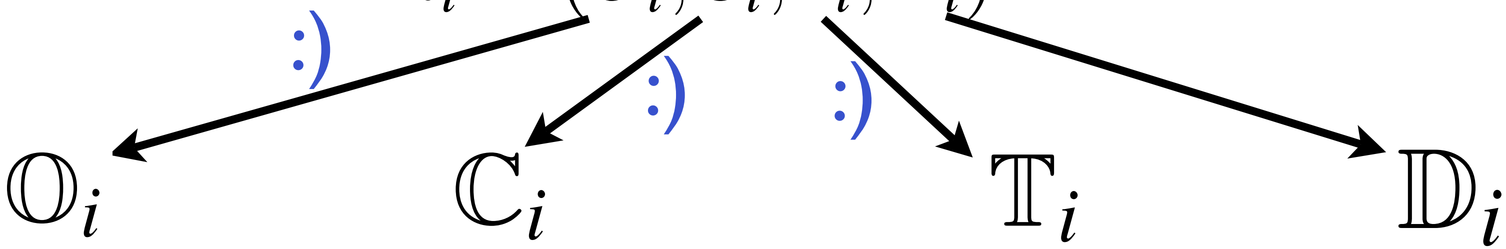
$$\tau_i = (O_i, C_i, T_i, D_i)$$

$$\mathbb{O}_i \qquad \mathbb{C}_i \qquad \mathbb{T}_i$$

$$\mathbb{X} = \begin{pmatrix} x_k \\ P(X = x_k) \end{pmatrix}$$

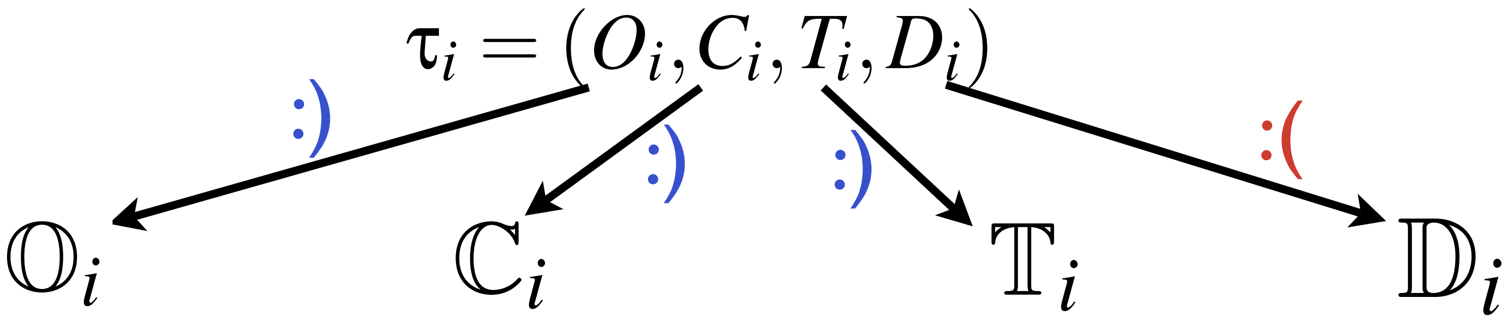
What is the model?

$$\tau_i = (O_i, C_i, T_i, D_i)$$



$$\mathbb{X} = \begin{pmatrix} x_k \\ P(X = x_k) \end{pmatrix}$$

What is the model?



$$\mathbb{X} = \begin{pmatrix} x_k \\ P(X = x_k) \end{pmatrix}$$

What do we want?

Response time $\mathbb{R}_i = \begin{pmatrix} 6 & 9 & 11 \\ 0.5 & 0.3 & 0.2 \end{pmatrix}$

What do we want?

Response time $\mathbb{R}_i = \begin{pmatrix} 6 & 9 & 11 \\ 0.5 & 0.3 & 0.2 \end{pmatrix}$

Satisfied deadline $satisfyDeadline_i = \begin{pmatrix} yes & no \\ 0.8 & 0.2 \end{pmatrix}$

What do we want?

Response time $\mathbb{R}_i = \begin{pmatrix} 6 & 9 & 11 \\ 0.5 & 0.3 & 0.2 \end{pmatrix}$

Satisfied deadline $satisfyDeadline_i = \begin{pmatrix} yes & no \\ 0.8 & 0.2 \end{pmatrix}$

Response time jitter $J_i = \begin{pmatrix} 2 & 3 & 4 \\ 0.7 & 0.1 & 0.3 \end{pmatrix}$

etc ...

What do we want?

Response time $\mathbb{R}_i = \begin{pmatrix} 6 & 9 & 11 \\ 0.5 & 0.3 & 0.2 \end{pmatrix}$

Satisfied deadline $satisfyDeadline_i = \begin{pmatrix} yes & no \\ 0.8 & 0.2 \end{pmatrix}$

Response time jitter $J_i = \begin{pmatrix} 2 & 3 & 4 \\ 0.7 & 0.1 & 0.3 \end{pmatrix}$

- Simulations? Analytical proofs?

etc ...

What do we want?

Response time $\mathbb{R}_i = \begin{pmatrix} 6 & 9 & 11 \\ 0.5 & 0.3 & 0.2 \end{pmatrix}$

Satisfied deadline $satisfyDeadline_i = \begin{pmatrix} yes & no \\ 0.8 & 0.2 \end{pmatrix}$

Response time jitter $J_i = \begin{pmatrix} 2 & 3 & 4 \\ 0.7 & 0.1 & 0.3 \end{pmatrix}$

- Simulations? Analytical proofs?

etc ...

Joint work with E. Tovar (Hurray, Portugal)

Response time of a task τ_i

When minimal inter-arrival times are considered

$$R_i = C_i + \sum_{j \in hp(i)} \lceil \frac{R_i}{T_j} \rceil C_j$$

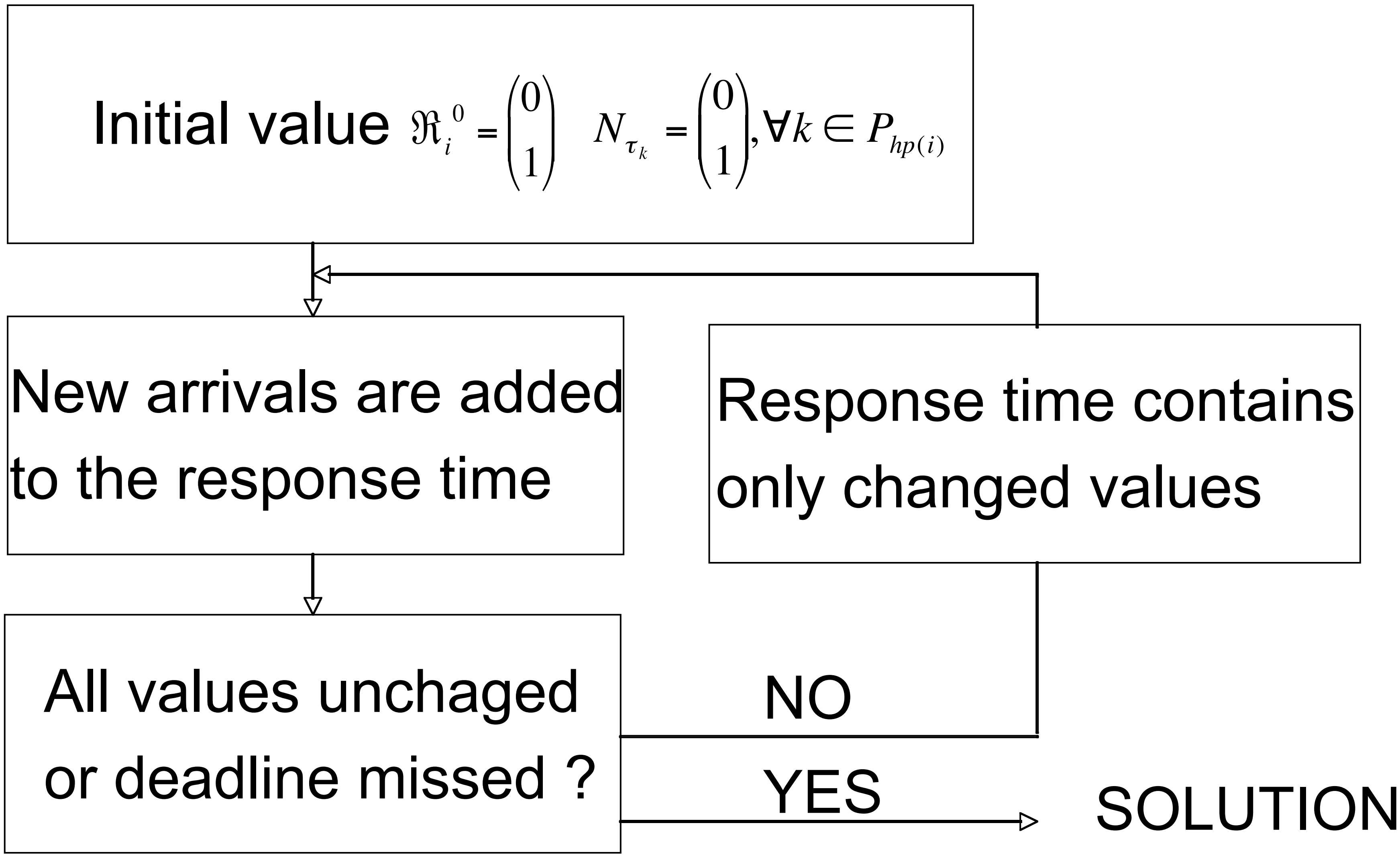
This time ...

Response time

$$\mathbb{R}_i = C_i \otimes (\otimes_{k\in P}\lceil \frac{\mathbb{R}_i}{C\ _k}) \otimes (\otimes_{k\in R} N_{\tau_k} C_k)$$

Algorithm providing a solution

$$\mathbb{R}_i = \begin{pmatrix} 6 & 9 & 11 \\ 0.5 & 0.3 & 0.2 \end{pmatrix}$$



Initial values

$$\Re_i^{\,0} = \begin{pmatrix} r_{i,1}^0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \text{ and } N_k = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \forall k \in R_{hp(i)}$$

Iteration *m* - first step

Working random variable L^m

$$L_j^m = C_i + \sum_{k \in P_{hp(i)}} \left\lceil \frac{L_j^m}{T_k} \right\rceil \cdot C_k + \sum_{k \in R_{hp(i)}} N_k(r_j^{m-1}) \cdot C_k$$

r_j^{m-1} initial value

Un example

Task	T	C
τ_1	$T_1 = \begin{pmatrix} 8 & 10 & 15 \\ 0.1 & 0.3 & 0.6 \end{pmatrix}$	3
τ_2	$T_2 = \begin{pmatrix} 10 \\ 1 \end{pmatrix}$	3
τ_3	$T_3 = \begin{pmatrix} 15 & 20 \\ 0.6 & 0.4 \end{pmatrix}$	2
τ_4	$T_4 = \begin{pmatrix} 15 \\ 1 \end{pmatrix}$	2
τ_5	$T_5 = \begin{pmatrix} 14 & 22 \\ 0.4 & 0.6 \end{pmatrix}$	2

$L_4^1 = \begin{pmatrix} l_1^1 \\ 1 \end{pmatrix} = \begin{pmatrix} 5 \\ 1 \end{pmatrix}$ where l_1^1 solution of equation $l_1^1 = 0 \cdot C_1 + \left\lceil \frac{l_1^1}{T_2} \right\rceil C_2 + 0 \cdot C_3 + 1 \cdot C_4$

with $r_1^0 = 0$ initial value

Iteration *m* - second step

$$\Re_i^m = L^m \otimes \left(\bigotimes_{k \in R_{hp(i)}} \Delta_k \cdot C_k \right)$$

Back to the example

Task	T	C
τ_1	$T_1 = \begin{pmatrix} 8 & 10 & 15 \\ 0.1 & 0.3 & 0.6 \end{pmatrix}$	3
τ_2	$T_2 = \begin{pmatrix} 10 \\ 1 \end{pmatrix}$	3
τ_3	$T_3 = \begin{pmatrix} 15 & 20 \\ 0.6 & 0.4 \end{pmatrix}$	2
τ_4	$T_4 = \begin{pmatrix} 15 \\ 1 \end{pmatrix}$	2
τ_5	$T_5 = \begin{pmatrix} 14 & 22 \\ 0.4 & 0.6 \end{pmatrix}$	2

$$\Re_4^2 = L^2 \otimes \begin{pmatrix} 1 & 2 \\ 0.6 & 0.4 \end{pmatrix} C_1 \otimes \begin{pmatrix} 0 \\ 1 \end{pmatrix} C_3$$

Iteration *m* - get ride of unchanged values

$$\begin{cases} L^m = \begin{pmatrix} 1 & 3 & 4 \\ 0.5 & 0.2 & 0.3 \end{pmatrix} \\ \Re_i^m = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 0.1 & 0.4 & 0.3 & 0.1 & 0.1 \end{pmatrix} \end{cases}$$

$$\text{New } \Re_i^m = Comp(\Re_i^m, L^m) = \begin{pmatrix} 2 & 5 \\ 0.4 & 0.1 \end{pmatrix}$$

One entire iteration (3) of our example

The periodic higher tasks are giving a response time:

$$\mathrm{I} = \begin{pmatrix} 20 \\ 0.5 \end{pmatrix}$$

The random higher tasks are giving a response time:

$$\Re^3_{n,0} = \mathrm{I} \otimes \left(F^*(20) \cdot \mathrm{C}_3\right) = \begin{pmatrix} 20 & 21 \\ 0.42 & 0.08 \end{pmatrix},$$

$$\text{where } F^*(20) = \begin{pmatrix} 2 & 3 \\ 0.84 & 0.16 \end{pmatrix}$$

One entire iteration (3) of our example

The periodic higher tasks are giving a response time:

$$I = \begin{pmatrix} 20 \\ 0.5 \end{pmatrix}$$

The random higher tasks are giving a response time:

$$\Re^3{}_{n,0} = I \otimes \left(F^*(20) \cdot C_3\right) = \begin{pmatrix} 20 & 21 \\ 0.42 & 0.08 \end{pmatrix},$$

$$\text{where } F^*(20) = \begin{pmatrix} 2 & 3 \\ 0.84 & 0.16 \end{pmatrix}$$

Some precautions when we think stochastic ...

Some precautions when we think stochastic ...

☑ Analysis able to give an answer in the deterministic case and to allow mixing hard and soft real-time constraints

Some precautions when we think stochastic ...

- ☑ Analysis able to give an answer in the deterministic case and to allow mixing hard and soft real-time constraints
- ☑ Robustness based on large deviations

Some precautions when we think stochastic ...

- Analysis able to give an answer in the deterministic case and to allow mixing hard and soft real-time constraints
- Robustness based on large deviations
- Next step?

Some references

A. Burns, G. Bernat, I. Broster - *A probabilistic framework for schedulability analysis*

M. Pinedo - *Offline deterministic scheduling, stochastic scheduling and online deterministic scheduling: a comparative overview*

S. Edgar, A. Burns - *Statistical analysis of WCET for scheduling*

N. Navet, L. Cucu, R. Schott - *Probabilistic estimation of response times through large deviations*

J.L. Diaz, D.F. Garcìa, K. Kim, C.-G. Lee, L. Lo Bello, J.M. Lopez, S.L. Min, O. Mirabella - *Stochastic analysis of real-time systems*

L. Cucu, E. Tovar - *A framework for response times analysis of fixed-priority tasks with stochastic inter-arrival times*

Some references

A. Burns, G. Bernat, I. Broster - *A probabilistic framework for schedulability analysis*

M. Pinedo - *Offline deterministic scheduling, stochastic scheduling and online deterministic scheduling: a comparative overview*

S. Edgar, A. Burns - *Statistical analysis of WCET for scheduling*

N. Navet, L. Cucu, R. Schott - *Probabilistic estimation of response times through large deviations*

J.L. Diaz, D.F. Garcìa, K. Kim, C.-G. Lee, L. Lo Bello, J.M. Lopez, S.L. Min, O. Mirabella - *Stochastic analysis of real-time systems*

L. Cucu, E. Tovar - *A framework for response times analysis of fixed-priority tasks with stochastic inter-arrival times*

Some references

A. Burns, G. Bernat, I. Broster	- *A probabilistic framework for schedulability analysis*
M. Pinedo	- *Offline deterministic scheduling, stochastic scheduling and online deterministic scheduling: a comparative overview*
S. Edgar, A. Burns	- *Statistical analysis of WCET for scheduling*
N. Navet, L. Cucu, R. Schott	- *Probabilistic estimation of response times through large deviations*
J.L. Diaz, D.F. Garcìa, K. Kim, C.-G. Lee, L. Lo Bello, J.M. Lopez, S.L. Min, O. Mirabella	- *Stochastic analysis of real-time systems*
L. Cucu, E. Tovar	- *A framework for response times analysis of fixed-priority tasks with stochastic inter-arrival times*

Some references

A. Burns, G. Bernat, I. Broster	- *A probabilistic framework for schedulability analysis*
M. Pinedo	- *Offline deterministic scheduling, stochastic scheduling and online deterministic scheduling: a comparative overview*
S. Edgar, A. Burns	- *Statistical analysis of WCET for scheduling*
N. Navet, L. Cucu, R. Schott	- *Probabilistic estimation of response times through large deviations*
J.L. Diaz, D.F. Garcìa, K. Kim, C.-G. Lee, L. Lo Bello, J.M. Lopez, S.L. Min, O. Mirabella	- *Stochastic analysis of real-time systems*
L. Cucu, E. Tovar	- *A framework for response times analysis of fixed-priority tasks with stochastic inter-arrival times*

Thank you for your attention

Open problems in
stochastic real-time scheduling :
introduction of dependent
random variables