

# Join Calculus and Algebraic Pattern Matching

*Qin*[tfin] MA 马勤

Laboratory of Advanced Software SYstems  
Faculty of Science Technology and Communication  
University of Luxembourg

5 March, 2008

# A Short Bio of Myself

- Sep. 2001:DEA, Université Paris 7 - INRIA Rocquencourt, France
- Jun. 2002:Master in Computer Engineering, Nanjing University, China
- Sep. 2005:Ph.D. in Computer Science, Université Paris 7 - INRIA Rocquencourt, France
- 2005 – 2007: research assistant in OFFIS (Institute for Information Technology), Germany
- since 2008: BFR post-doc in University of Luxembourg under the supervision of Prof. Pierre Kelsen

Research Interests: Process Calculus, Functional Programming, Object-Oriented Programming, Component Models, Model-Based System Engineering.

# Outline

- 1 The Join Calculus
- 2 Algebraic Patterns and ML Matching
- 3 Algebraic Pattern Matching in Join
- 4 Compiling Pattern Matching in Join Patterns

# Outline

- 1 The Join Calculus
- 2 Algebraic Patterns and ML Matching
- 3 Algebraic Pattern Matching in Join
- 4 Compiling Pattern Matching in Join Patterns

# The Join Calculus

Join is a name-passing calculus developed at INRIA

- Parallel processes
- Communication channels
- Message synchronization

Join has advantages w.r.t. language implementation

- Locality: at most one receptor for each channel.
- More powerful and higher level message synchronization in terms of pattern matching.
- A strong functional flavor, and can be adapted to OO settings in a straightforward manner.

# The Join Calculus

*But after all, why from process calculus to programming language?*

- formally defined syntax and semantics
- sound type systems
- and an appropriate equivalence relation which allows reasoning about systems in terms of mathematical proofs.

# Impacts of Join on Concurrent/Distributed Programming

- As language extensions:
  - JoCaml = Join + OCaml
  - JoinJava = Join + Java
  - Polyphonic C<sup>#</sup> (C $\omega$ ) = Join + C<sup>#</sup>
- As concurrency libraries:
  - Scala
  - .Net
  - C++
- Taught in courses for concurrent computing, distributed computing, mobile computing, wide area computing, global computing ...

# A One-Place Buffer in Join

```

def put(n) & Empty() ▷ Some(n)
or get(r) & Some(n) ▷ r(n) & Empty()
in Empty() ...
  
```

join definitions

reaction rules

join patterns

guarded processes

defined channels

asynchronous message sending

Note: & is overloaded to denote both channel synchronization and process parallel composition.

# A One-Place Buffer in Join

```
def put(n) & Empty() ▷ Some(n)
or get(r) & Some(n) ▷ r(n) & Empty()
in Empty() ...
```

join definitions

reaction rules

join patterns

guarded processes

defined channels

asynchronous message sending

Note: & is overloaded to denote both channel synchronization and process parallel composition.

# A One-Place Buffer in Join

```

def put(n) & Empty() ▷ Some(n)
or get(r) & Some(n) ▷ r(n) & Empty()
in Empty() ...
  
```

join definitions  
reaction rules

**join patterns**

guarded processes  
defined channels

asynchronous message sending

Note: & is overloaded to denote both channel synchronization and process parallel composition.

# A One-Place Buffer in Join

```
def put(n) & Empty() ▷ Some(n)
or get(r) & Some(n) ▷ r(n) & Empty()
in Empty() ...
```

join definitions

guarded processes

reaction rules

defined channels

join patterns

asynchronous message sending

Note: & is overloaded to denote both channel synchronization and process parallel composition.

# A One-Place Buffer in Join

```

def put(n) & Empty() ▷ Some(n)
  or get(r) & Some(n) ▷ r(n) & Empty()
in Empty() ...

```

join definitions  
 reaction rules  
 join patterns

guarded processes  
**defined channels**  
 asynchronous message sending

Note: & is overloaded to denote both channel synchronization and process parallel composition.

# A One-Place Buffer in Join

```

def put(n) & Empty() ▷ Some(n)
or get(r) & Some(n) ▷ r(n) & Empty()
in Empty() ...
  
```

join definitions  
 reaction rules  
 join patterns

guarded processes  
 defined channels  
 asynchronous message sending

Note: & is overloaded to denote both channel synchronization and process parallel composition.

# A Computation: Intuition

```
def put(n) & Empty() ▷ Some(n)
  or get(r) & Some(n) ▷ r(n) & Empty()
  . . . . .
def reply(n) ▷ print_int(n)
in Empty() & put(1) & get(reply)
```

# A Computation: Intuition

```
def put(n) & Empty() ▷ Some(n)
  or get(r) & Some(n) ▷ r(n) & Empty()
  . . . . .
def reply(n) ▷ print_int(n)
in Empty() & put(1) & get(reply)

≡ put(1) & Empty() & get(reply)
```

# A Computation: Intuition

```
def put(n) & Empty() ▷ Some(n)
  or get(r) & Some(n) ▷ r(n) & Empty()
  . . . . .
def reply(n) ▷ print_int(n)
in Empty() & put(1) & get(reply)
```

$\equiv$  put(1) & Empty() & get(reply)  
 $\rightarrow$  **Some(1)** & get(reply)

# A Computation: Intuition

```

def put(n) & Empty() ▷ Some(n)
  or get(r) & Some(n) ▷ r(n) & Empty()
  . . . . .
def reply(n) ▷ print_int(n)
in Empty() & put(1) & get(reply)

```

≡ put(1) & Empty() & get(reply)

→ Some(1) & get(reply)

≡ **get(reply) & Some(1)**

# A Computation: Intuition

```

def put(n) & Empty() ▷ Some(n)
  or get(r) & Some(n) ▷ r(n) & Empty()
  . . . . .
def reply(n) ▷ print_int(n)
in Empty() & put(1) & get(reply)

```

$\equiv$  put(1) & Empty() & get(reply)  
 $\longrightarrow$  Some(1) & get(reply)  
 $\equiv$  get(reply) & Some(1)  
 $\longrightarrow$  **reply(1) & Empty()**

# A Computation: Intuition

```

def put(n) & Empty() ▷ Some(n)
  or get(r) & Some(n) ▷ r(n) & Empty()
  . . . . .
def reply(n) ▷ print_int(n)
in Empty() & put(1) & get(reply)

```

$\equiv$  put(1) & Empty() & get(reply)  
 $\longrightarrow$  Some(1) & get(reply)  
 $\equiv$  get(reply) & Some(1)  
 $\longrightarrow$  reply(1) & Empty()  
 $\longrightarrow$  **print\_int(1)** & Empty()

# A Computation: Intuition

```

def put(n) & Empty() ▷ Some(n)
  or get(r) & Some(n) ▷ r(n) & Empty()
  . . . . .
def reply(n) ▷ print_int(n)
in Empty() & put(1) & get(reply) & put(2)

```

$\equiv$  put(1) & Empty() & get(reply)  
 $\longrightarrow$  Some(1) & get(reply)  
 $\equiv$  get(reply) & Some(1)  
 $\longrightarrow$  reply(1) & Empty()  
 $\longrightarrow$  print\_int(1) & Empty()

# Join-Pattern Matching Is Nondeterministic

```
def a() & b() ▷ P  
  or a() & c() ▷ Q  
in b() & c()
```

# Join-Pattern Matching Is Nondeterministic

```
def a() & b() ▷ P  
  or a() & c() ▷ Q  
in b() & c() & a()
```

# Join-Pattern Matching Is Nondeterministic

```
def a() & b() ▷ P  
or a() & c() ▷ Q  
in b() & c() & a()
```

## Nondeterministic choice

between launching  $P$  and  
launching  $Q$  ! May even vary  
from run to run!

# Join-Pattern Matching Is Nondeterministic

```
def a() & b() ▷ P  
or a() & c() ▷ Q  
in b() & c() & a()
```

## Nondeterministic choice

between launching  $P$  and  
launching  $Q$  ! May even vary  
from run to run!

- A join pattern is matched  $\implies$  there are messages sent to all its channels.
- All such non-determinism complies to the concurrency nature of the calculus, and should be respected when programming.

# Join-Pattern Matching Is Nondeterministic

```
def a() & b() ▷ P  
or a() & c() ▷ Q  
in b() & c() & a()
```

## Nondeterministic choice

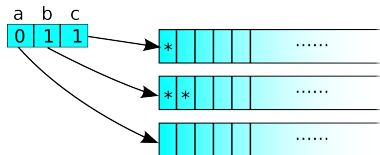
between launching  $P$  and  
launching  $Q$  ! May even vary  
from run to run!

- A join pattern is matched  $\implies$  there are messages sent to all its channels.
- All such non-determinism complies to the concurrency nature of the calculus, and should be respected when programming.

# Join Definitions Implemented as Automata in JoCaml

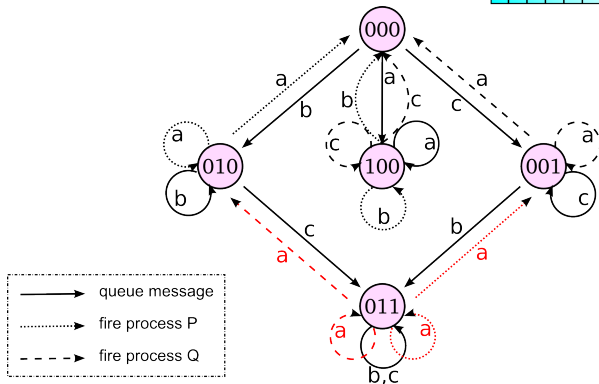
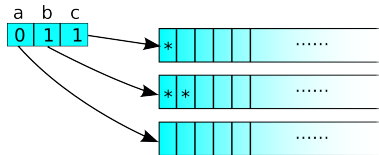
```

def a () & b () ▷ P
or a () & c () ▷ Q
  
```



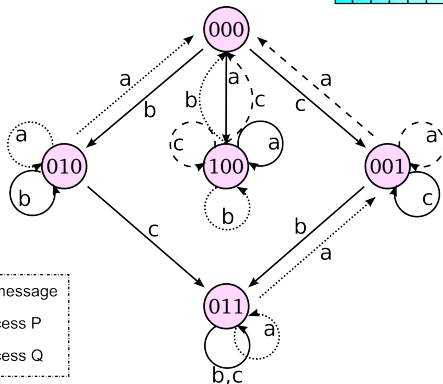
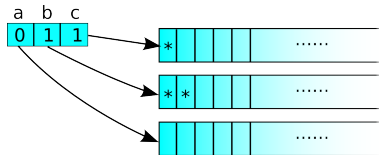
## Join Definitions Implemented as Automata in JoCaml

**def**  $a() \ \& \ b() \triangleright P$   
**or**  $a() \ \& \ c() \triangleright Q$

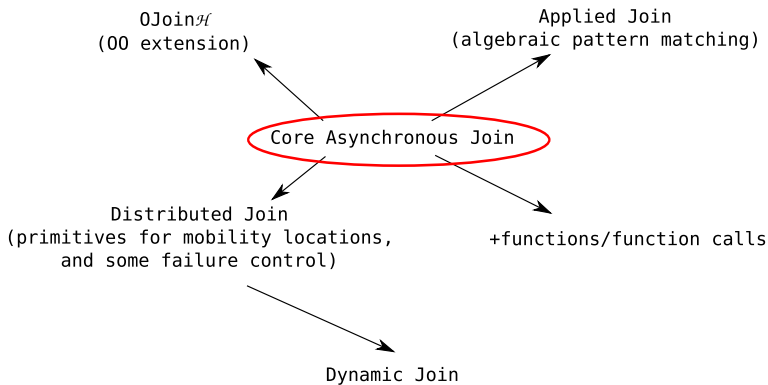


## Join Definitions Implemented as Automata in JoCaml

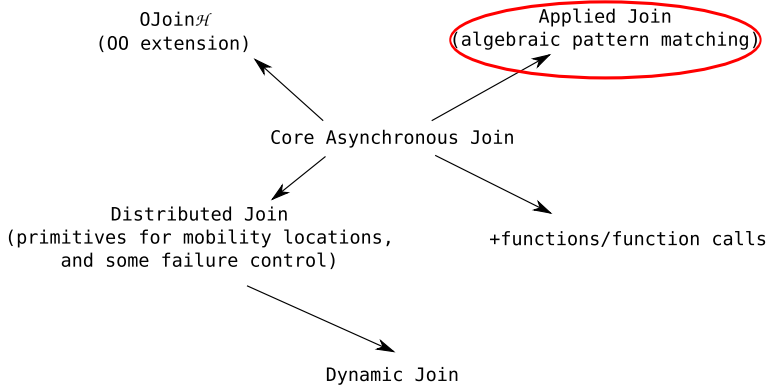
**def**  $a() \ \& \ b() \triangleright P$   
**or**  $a() \ \& \ c() \triangleright Q$



# Join Evolution Diagram



# Join Evolution Diagram



# Outline

- 1 The Join Calculus
- 2 Algebraic Patterns and ML Matching**
- 3 Algebraic Pattern Matching in Join
- 4 Compiling Pattern Matching in Join Patterns

# Algebraic Data Types and Patterns

Integer Lists and Patterns in OCaml:

```
type list = [] | int :: list
```

# Algebraic Data Types and Patterns

Integer Lists and Patterns in OCaml:

```
type list = [] | int :: list
```

values:

```
[]  
1::[]  
1::2::[]  
...
```

# Algebraic Data Types and Patterns

Integer Lists and Patterns in OCaml:

```
type list = [] | int :: list
```

values:

```
[]  
1::[]  
1::2::[]  
...
```

patterns:

```
[]  
x::[]  
1::xs  
x  
...
```

# Algebraic Data Types and Patterns

Integer Lists and Patterns in OCaml:

```
type list = [] | int :: list
```

values:

```
[]
1::[]
1::2::[]
...
```

patterns:

```
[]
x::[]
1::xs
x
...
```

- Value `1::[]` is **an instance of** both pattern `x::[]` and `1::xs`.
- Any value is **an instance of** pattern `x`.

# Algebraic Pattern Matching à la ML

```
let isEmpty ls =  
  match ls with  
  | [] -> True  
  | hd::tl -> False
```

# Algebraic Pattern Matching à la ML

```
let isEmpty ls =  
  match ls with  
  | [] -> True  
  | hd::tl -> False
```

Deterministic first match policy:

# Algebraic Pattern Matching à la ML

```

let isEmpty ls =
  match ls with
    | [] -> True
    | hd::tl -> False
      x
  
```

## Deterministic first match policy:

Although `[]` is an instance of both `[]` and `x`, the preceding pattern `[]` always has privilege to match, so only non-empty lists will be matched by the second pattern.

# Outline

- 1 The Join Calculus
- 2 Algebraic Patterns and ML Matching
- 3 Algebraic Pattern Matching in Join**
- 4 Compiling Pattern Matching in Join Patterns

## ML Pattern Matching in Processes

A stack in applied join:

```

def push(n) & Empty() ▷ Some([n])
  or push(n) & Some(l) ▷ Some(n :: l)
  or pop(r) & Some(l) ▷ match l with
    | [x] -> r(x) & Empty()
    | x :: xs -> r(x) & Some(xs)
in Empty() ...
  
```

## ML Pattern Matching in Processes

A stack in applied join:

```

                Some ([]) ?
def push (n) & Empty () ▷ Some ([n])
    or push (n) & Some (ls) ▷ Some (n :: ls)
    or pop (r) & Some (ls) ▷ match ls with
                                | [x] -> r (x) & Empty ()
                                | x :: xs -> r (x) & Some (xs)
in Empty () ...

```

## ML Pattern Matching in Processes

A stack in applied join:

Some([])? **Not yet supported!**

```

def push(n) & Empty() ▷ Some([n])
  or push(n) & Some(ls) ▷ Some(n :: ls)
  or pop(r) & Some(ls) ▷ match ls with
    | [x] -> r(x) & Empty()
    | x :: xs -> r(x) & Some(xs)
in Empty() ...
  
```

# Join Patterns with Pattern Arguments

Another stack in applied join:

```
def push(n) & State(ls) ▷ State(n :: ls)
  or pop(r) & State(x :: xs) ▷ r(x) & State(xs)
in State([]) ...
```

Observations:

- Two versions of stack express the same behavior.
- The second version requires less programming effort.

# Applied Join Calculus = Join + Algebraic Value Passing + Algebraic Pattern Matching

## Expressions and Patterns:

$$e ::= x$$

$$\kappa(e_1, e_2, \dots, e_n)$$

$$\pi ::= x$$

$$\kappa(\pi_1, \pi_2, \dots, \pi_n)$$

## Join Definitions and Join Patterns:

$$D ::= \top$$

$$J \triangleright P$$

$$D_1 \text{ or } D_2$$

$$J ::= x(\pi)$$

$$J_1 \& J_2$$

## Process:

$$P ::= 0$$

$$x(e)$$

$$P_1 \& P_2$$

$$\text{def } D \text{ in } P$$

$$\text{match } e \text{ with } \mid \pi_1 \rightarrow P_1 \mid \dots \mid \pi_m \rightarrow P_m$$

# Semantics Is for Closed Terms

- Two kinds of values:
  - channel names;
  - algebraic values.
- Two kinds of variables:
  - variables of channel type are treated as values, i.e. channel names;
  - only variables of algebraic data types are real ones, which would be bound to algebraic values.
- We define **closed terms** as terms with only free channel names i.e. all free variables are of channel types.
- We define reduction semantics only for closed terms.

# Reduction Rules for Algebraic Pattern Matching

**(match  $v$  with**  $\pi_1 \rightarrow P_1 \mid \dots \mid \pi_m \rightarrow P_m$ )  $\longrightarrow P_i\sigma$

- $v$  is an instance of  $\pi_i$ .
- $v$  is not an instance of any pattern before  $\pi_i$ .
- $P_i\sigma$  is a copy of  $P_i$ .

**def** ...

**or**  $x_1(\pi_1) \& \dots \& x_n(\pi_n) \triangleright P \longrightarrow P\sigma_1 \dots \sigma_n$

**or** ...

**in**  $x_1(v_1) \& \dots \& x_n(v_n)$

- $v_i$  is an instance of  $\pi_i$ , for all  $i$  such that  $1 < i < n$ .
- $P\sigma_1 \dots \sigma_n$  is a copy of  $P$ .

# Outline

- 1 The Join Calculus
- 2 Algebraic Patterns and ML Matching
- 3 Algebraic Pattern Matching in Join
- 4 Compiling Pattern Matching in Join Patterns**

# Refine and Dispatch

```
def a () & c ( $\pi_1$ ) ▷ ...  
or b () & c ( $\pi_2$ ) ▷ ...
```

# Refine and Dispatch

```
def a () & c ( $\pi_1$ ) ▷ ...  
or b () & c ( $\pi_2$ ) ▷ ...
```

```
def a () &  $c_{\pi_1}(x_1)$  ▷ ...  
or b () &  $c_{\pi_2}(x_2)$  ▷ ...
```

# Refine and Dispatch

```

def a() & c( $\pi_1$ ) ▷...
  or b() & c( $\pi_2$ ) ▷...

```

```

def a() & c $_{\pi_1}$ (x1) ▷...
  or b() & c $_{\pi_2}$ (x2) ▷...
  or c(x) ▷ match x with
    |  $\pi_1$  -> c $_{\pi_1}$ (x)
    |  $\pi_2$  -> c $_{\pi_2}$ (x)

```

# Refine and Dispatch

```

def a () & c ( $\pi_1$ ) ▷ ...
or b () & c ( $\pi_2$ ) ▷ ...

```

```

def a () & c $_{\pi_1}$ (x1) ▷ ...
or b () & c $_{\pi_2}$ (x2) ▷ ...
or c (x) ▷ match x with
    |  $\pi_1$  -> c $_{\pi_1}$ (x)
    |  $\pi_2$  -> c $_{\pi_2}$ (x)

```

This naive transformation does not work because of the **nondeterminism-determinism gap** between join pattern matching and ML pattern matching.

# Compatible Algebraic Patterns

## Definition

Two patterns are **compatible** if they share common instance.

## Property

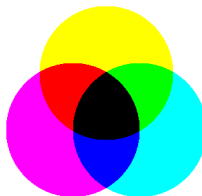
Two compatible patterns admit a **least upper bound** written  $\pi_1 \uparrow \pi_2$ , whose instance set is the intersection of the original two.

Eg.  $(x :: []) \uparrow (1 :: xs) = 1 :: []$

Note: least upper bounds can be computed at the same time when compatibility is checked. And there already are efficient algorithms for these purposes implemented in the Jocaml compiler.

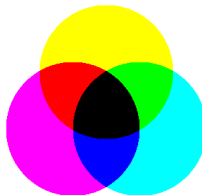
# Pattern Lattice

```
def c( $\pi_1$ ) & a()  $\triangleright P_1$   
or c( $\pi_2$ ) & b()  $\triangleright P_2$   
or c( $\pi_3$ ) & d()  $\triangleright P_3$ 
```



# Pattern Lattice

**def**  $c(\pi_1) \ \& \ a() \triangleright P_1$   
**or**  $c(\pi_2) \ \& \ b() \triangleright P_2$   
**or**  $c(\pi_3) \ \& \ d() \triangleright P_3$



$$\pi_1 \uparrow \pi_2 \uparrow \pi_3$$

$$\pi_1 \uparrow \pi_2$$

$$\pi_2 \uparrow \pi_3$$

$$\pi_1 \uparrow \pi_3$$

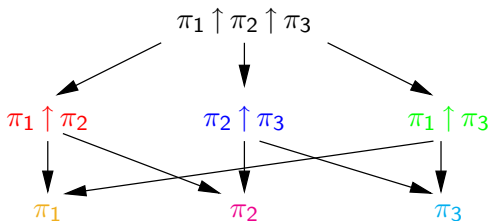
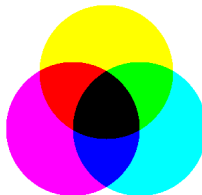
$$\pi_1$$

$$\pi_2$$

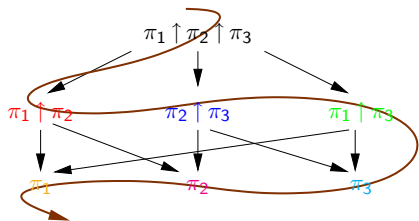
$$\pi_3$$

# Pattern Lattice

**def**  $c(\pi_1) \ \& \ a() \triangleright P_1$   
**or**  $c(\pi_2) \ \& \ b() \triangleright P_2$   
**or**  $c(\pi_3) \ \& \ d() \triangleright P_3$



# A Topological Order



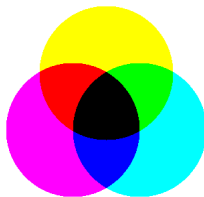
or  $c(x) \triangleright \text{match } x \text{ with}$

- |  $\pi_1 \uparrow \pi_2 \uparrow \pi_3 \rightarrow c_{\pi_1 \uparrow \pi_2 \uparrow \pi_3}(x)$
- |  $\pi_1 \uparrow \pi_2 \rightarrow c_{\pi_1 \uparrow \pi_2}(x)$
- |  $\pi_2 \uparrow \pi_3 \rightarrow c_{\pi_2 \uparrow \pi_3}(x)$
- |  $\pi_1 \uparrow \pi_3 \rightarrow c_{\pi_1 \uparrow \pi_3}(x)$
- |  $\pi_3 \rightarrow c_{\pi_3}(x)$
- |  $\pi_2 \rightarrow c_{\pi_2}(x)$
- |  $\pi_1 \rightarrow c_{\pi_1}(x)$

# Disjunction of Refined Channels

```

def c( $\pi_1$ ) & a()  $\triangleright$  P1
or c( $\pi_2$ ) & b()  $\triangleright$  P2
or c( $\pi_3$ ) & d()  $\triangleright$  P3
  
```



$c(\pi_1)$ :  $c_{\pi_1}(x_1)$  **or**  $c_{\pi_1 \uparrow \pi_2}(x_1)$  **or**  $c_{\pi_1 \uparrow \pi_3}(x_1)$  **or**  $c_{\pi_1 \uparrow \pi_2 \uparrow \pi_3}(x_1)$   
 $c(\pi_2)$ :  $c_{\pi_2}(x_2)$  **or**  $c_{\pi_1 \uparrow \pi_2}(x_2)$  **or**  $c_{\pi_2 \uparrow \pi_3}(x_2)$  **or**  $c_{\pi_1 \uparrow \pi_2 \uparrow \pi_3}(x_2)$   
 $c(\pi_3)$ :  $c_{\pi_3}(x_3)$  **or**  $c_{\pi_1 \uparrow \pi_3}(x_3)$  **or**  $c_{\pi_2 \uparrow \pi_3}(x_3)$  **or**  $c_{\pi_1 \uparrow \pi_2 \uparrow \pi_3}(x_3)$

Note:  $(J_1 \text{ or } J_2) \& J \triangleright P \stackrel{\text{def}}{=} J_1 \& J \triangleright P \text{ or } J_2 \& J \triangleright P$

# Concurrent Stack after Transformation

Before

```
def push(n) & State(ls) ▷ State(n :: ls)
  or pop(r) & State(x :: xs) ▷
                                r(x) & State(xs)
```

$x :: xs$



$ls$

After

```
def push(n) & (Statex::xs(x1) or Statels(x1)) ▷
                                ... State(n :: ls)
  or pop(r) & Statex::xs(x2) ▷
                                ... r(x) & State(xs)
  or State(x) ▷ match x with
                | x :: xs -> Statex::xs(x)
                | ls -> Statels(x)
```

## Concurrent Stack after Transformation

Before

```

def push(n) & State(ls) ▷ State(n :: ls)
  or pop(r) & State(x :: xs) ▷
                                r(x) & State(xs)

```

 $x :: xs$ 

ls

After

```

def push(n) & (Statex::xs(x1) or Statels(x1)) ▷
                                match x1 with ls -> State(n :: ls)
  or pop(r) & Statex::xs(x2) ▷
                                match x2 with x :: xs -> r(x) & State(xs)

```

---

```

or State(x) ▷ match x with
              | x :: xs -> Statex::xs(x)
              | ls -> Statels(x)

```

# Compilation Scheme $\llbracket \cdot \rrbracket$

- $Y_c$ : to transform one join definition  $D$  w.r.t. channel  $c$  defined in  $D$ .
- $\llbracket \cdot \rrbracket$ : to compile away pattern arguments in processes:

$$\begin{array}{lcl}
 \llbracket 0 \rrbracket & \stackrel{\text{def}}{=} & 0 \\
 \llbracket x(e) \rrbracket & \stackrel{\text{def}}{=} & x(e) \\
 \llbracket P_1 \ \& \ P_2 \rrbracket & \stackrel{\text{def}}{=} & \llbracket P_1 \rrbracket \ \& \ \llbracket P_2 \rrbracket \\
 \llbracket \text{def } D \text{ in } P \rrbracket & \stackrel{\text{def}}{=} & \text{def } Y_{c_n} \dots Y_{c_1} (\llbracket D \rrbracket) \text{ in } \llbracket P \rrbracket \\
 \llbracket \text{match } e \text{ with } \bigvee_{i \in I} \pi_i \rightarrow P_i \rrbracket & \stackrel{\text{def}}{=} & \text{match } e \text{ with } \bigvee_{i \in I} \pi_i \rightarrow \llbracket P_i \rrbracket \\
 \\
 \llbracket \top \rrbracket & \stackrel{\text{def}}{=} & \top \\
 \llbracket J \triangleright P \rrbracket & \stackrel{\text{def}}{=} & J \triangleright \llbracket P \rrbracket \\
 \llbracket D_1 \ \text{or} \ D_2 \rrbracket & \stackrel{\text{def}}{=} & \llbracket D_1 \rrbracket \ \text{or} \ \llbracket D_2 \rrbracket
 \end{array}$$

where we suppose  $D$  defines channels  $c_1, \dots, c_n$ .

# Equivalence relations

Weak barbed congruence ( $\approx$ ): **between closed processes**

$\approx$  is the largest equivalence relation that meets the following:

- $\approx$  is a weak reduction bisimulation;
- $\approx$  preserves weak barbs;
- $\approx$  is closed under evaluation contexts.

Static equivalence ( $\approx^s$ ): **open extension of  $\approx$**

$P \approx^s Q \iff \forall \sigma. \text{ s.t. } P\sigma \text{ and } Q\sigma \text{ closed, } P\sigma \approx Q\sigma.$

# Equivalence relations, cont.

## Theorem

*The static equivalence  $\cong$  is a full congruence.*

Reflexive, commutative, and transitive.

Evaluation contexts:

$$\begin{aligned}
 P_1 \cong P_2 &\implies P \& P_1 \cong P \& P_2 \\
 P_1 \cong P_2 &\implies P_1 \& P \cong P_2 \& P \\
 P_1 \cong P_2 &\implies \mathbf{def\ } D \mathbf{ in\ } P_1 \cong \mathbf{def\ } D \mathbf{ in\ } P_2 \\
 P_1 \cong P_2 &\implies \mathbf{match\ } e \mathbf{ with\ } \dots \mid \pi_k \rightarrow P_1 \mid \dots \\
 &\quad \cong \mathbf{match\ } e \mathbf{ with\ } \dots \mid \pi_k \rightarrow P_2 \mid \dots
 \end{aligned}$$

Guarded contexts:

$$\begin{aligned}
 P_1 \cong P_2 &\implies \mathbf{def\ } D \mathbf{ or\ } J \triangleright P_1 \mathbf{ in\ } P \cong \mathbf{def\ } D \mathbf{ or\ } J \triangleright P_2 \mathbf{ in\ } P \\
 P_1 \cong P_2 &\implies \mathbf{def\ } J \triangleright P_1 \mathbf{ or\ } D \mathbf{ in\ } P \cong \mathbf{def\ } J \triangleright P_2 \mathbf{ or\ } D \mathbf{ in\ } P
 \end{aligned}$$

# Correctness in Terms of Equivalence

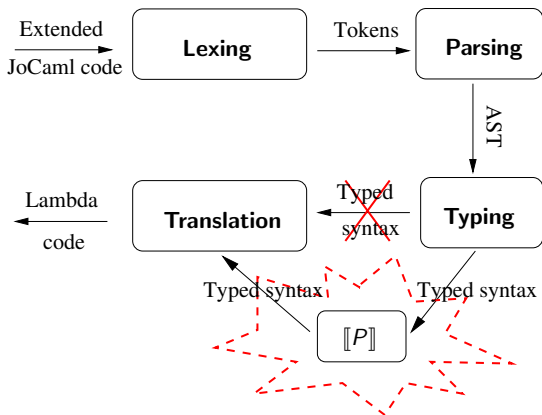
## Lemma ( $Y_c$ is correct)

*For any join definition  $D$ , channel name  $c$  defined in  $D$ , and process  $P$ ,*  
**def  $Y_c(D)$  in  $P \rightleftharpoons$  def  $D$  in  $P$ .**

## Theorem (Correctness)

*For any process  $P$ ,  $\llbracket P \rrbracket \rightleftharpoons P$ .*

# Extended JoCaml Compiler Front End



# Summary of Contributions

- General Motivation:
  - theoretical foundation for languages in concurrent settings;
  - multi-paradigm programming, language feature interaction.
- Applied Join: Join with algebraic data types.
  - Algebraic pattern arguments.
  - An equivalence theory for open terms.
- A compilation scheme from Applied Join to Join.
  - The nondeterminism problem by pattern lattice.
  - Compilation correctness in terms of process equivalence.
  - Implementation in JoCaml.

Cf.

- *Algebraic Pattern Matching in Join Calculus*, Q. Ma and L. Maranget, to appear in Journal of Logical Methods in Computer Science, 2008.
- <http://jocaml.inria.fr>

# Future Work

