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Anti-symplectic involutions on quasi-hamiltonian quotients [1]

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Abstract

In this note, we study the fixed-point set of an (anti-symplectic) involution $\hat{\beta}$ induced on a quasi-hamiltonian quotient $M//U = \mu^{-1}(\{1\})/U$ by a (form-reversing) involution β defined on the quasi-hamiltonian space $(M\omega, \mu : M \to U)$. When the action of U on $\mu^{-1}(\{1\})$ is free, we adapt the results of [2] to the quasi-hamiltonian setting. We then study the case where the action of U is not free. In that case, $M//U$ is a stratified symplectic space and $Fix(\hat{\beta})$ is a disjoint union of lagrangian submanifolds of some of the strata.

1 Introduction

The motivation underlying this note is the construction of lagrangian submanifolds of a quasi-hamiltonian quotient. One possible way to do that is to construct anti-symplectic involutions on this quasi-hamiltonian quotient, in which case we have the following lemma:

Lemma 1.1. *Let (N, ω) be a symplectic manifold and let σ be an anti-symplectic involution on N (meaning that $\sigma^*\omega = -\omega$ and $\sigma^2 = Id_N$). Denote by $N^\sigma := Fix(\sigma)$ the fixed-point set of σ. Then: if $N^\sigma \neq \emptyset$, it is a lagrangian submanifold of N.*

Observe that an anti-symplectic involution does not necessarily have fixed points. For instance, the map $(-Id_{\mathbb{R}^3})|_{S^2} : (x, y, z) \in S^2 \mapsto -(x, y, z)$ reverses orientation on S^2 (so that it is anti-symplectic with respect to the volume form $x\,dy \wedge dz - y\,dx \wedge dz - z\,dx \wedge dy$ on S^2), and has no fixed points on S^2.
In the following, let U be a compact connected Lie group. Recall that if $(M, \omega, \mu : M \to U)$ is a quasi-hamiltonian U-space, then the quasi-hamiltonian quotient $M//U = \mu^{-1}(\{1\})/U$ is a stratified symplectic space in the sense of Lerman and Sjamaar in [9]. In this note, we show how to obtain an anti-symplectic involution $\sigma = \hat{\beta}$ on $M//U$ starting from a form-reversing involution β on M. When the action of U on $\mu^{-1}(\{1\})$ of U on M is free then by a theorem of Alekseev, Malkin and Meinrenken in [1] the element $1 \in U$ is a regular value of

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μ and $M//U = \mu^{-1}(\{1\})/U$ is a symplectic manifold. In this case, which we will call the smooth case and which we will deal with in section 2, we will adapt the ideas of O'Shea and Sjamaar in [5] and of Foth in [2] in order to describe the fixed-point set of $\hat{\beta}$ in terms of the projection $p : \mu^{-1}(\{1\}) \to \mu^{-1}(\{1\})/U$. We will then study the case where the action is not free. In that case, as was recalled earlier, the quotient $M//U$ is, just as in the usual hamiltonian setting, a stratified symplectic space. To us in this note, this will simply mean that it is a disjoint union, over closed subgroups $K \subset U$ taken up to conjugacy, of symplectic manifolds (of different dimensions in general): $M//U = \sqcup_{j\in J} X_{K_j}$. As we will recall in theorem 3.1, each stratum $X_K = (\mu^{-1}(\{1\}) \cap M_K)/L_K$ is the quotient of $\mu^{-1}(\{1\}) \cap M_K$ by the *free* action of a Lie group L_K. In section 3, we will show that the involution $\hat{\beta}$ on $M//U$ restricts, under some conditions, to an involution $\hat{\beta}_K$ on X_K and that the results of section 2 apply to this $\hat{\beta}_K$.

2 The smooth case

In this section, we assume that the compact connected Lie group U acts freely on $\mu^{-1}(\{1\})$. Recall that we denote by p the projection $p : \mu^{-1}(\{1\}) \to \mu^{-1}(\{1\})/U = M//U$. Additionally, let us denote by i the inclusion $i : \mu^{-1}(\{1\}) \hookrightarrow M$. We then recall:

Theorem 2.1 ([1])**.** *If U acts freely on $\mu^{-1}(\{1\})$ then $1 \in U$ is a regular value of μ and there exists a unique 2-form ω^{red} on the manifold $M//U = \mu^{-1}(\{1\})/U$ satisfying $i^*\omega = p^*\omega^{red}$. This 2-form w^{red} is a symplectic form.*

From now on, we will assume that the compact connected Lie group U is endowed with an involutive automorphism τ and we will denote by τ^- the involution $\tau^- : u \in U \mapsto \tau(u^{-1})$. The proof of the following result is then immediate:

Theorem 2.2. *Let $\beta : M \to M$ be an involution defined on a quasi-hamiltonian space $(M, \omega, \mu : M \to U)$ satisfying the following compatibility conditions with the action of U on M and the momentum map μ :*

(i) $\beta(u.x) = \tau(u).\beta(x)$ for all $u \in U$ and all $x \in M$

(ii) $\mu \circ \beta = \tau^- \circ \mu$

Then β induces an involution $\hat{\beta}$ on the quasi-hamiltonian quotient $M//U = \mu^{-1}(\{1\})/U$, defined by $\hat{\beta}([x]) = [\beta(x)]$. If in addition $\beta^*\omega = -\omega$ then $\hat{\beta}^*\omega^{red} = -\omega^{red}$.

One may observe that, if β satisfies the compatibility conditions (i) and (ii) above, and α is a differential form on $\mu^{-1}(\{1\})$ which is basic with respect to the principal fibration $p : \mu^{-1}(\{1\}) \to \mu^{-1}(\{1\})/U$, then $\beta^*\alpha$ is also basic with respect to p.

Thus, we have seen that a compatible form-reversing involution β on M induces an anti-symplectic involution $\hat{\beta}$ on $M//U$. The fixed-point set $Fix(\hat{\beta})$ of this involution is then, if non-empty, a lagrangian submanifold of $M//U$, by lemma 1.1. As for conditions on β and (U,τ) for $Fix(\hat{\beta})$ to be non-empty, we refer to [7], and in the following we will always assume that $Fix(\hat{\beta}) \neq \emptyset$. Consider then $[x] \in Fix(\hat{\beta})$. This means that $x \in \mu^{-1}(\{1\})$ satisfies $\beta(x) = u.x$ for some $u \in U$. Since $\beta^2 = Id_M$, we obtain $x = \beta(u.x) = \tau(u).\beta(x) = \tau(u)u.x$ and therefore, as the action of U on $\mu^{-1}(\{1\})$ is free, one has $\tau(u) = u^{-1}$ or equivalently, $u \in Fix(\tau^-) \subset U$. This is the starting point of the study conducted in [2]. The first step in this study is to understand the geometry of the set $Fix(\tau^-) \subset U$. The proof of the following result may be found in [10].

Proposition 2.3. *The Lie group U acts on the fixed-point set $Fix(\tau^-) \subset U$ by $u.w = \tau^-(u)wu$ and the orbits of this action are the connected components of $Fix(\tau^-)$. In particular, the connected component of 1 in $Fix(\tau^-)$ is the set $Q_0 = \{\tau^-(u)u \ : \ u \in U\}$.*

Thus, if $Fix(\tau^-)$ is connected, we have $Fix(\tau^-) = Q_0$. If we then go back to the relation $\beta(x) = u.x$ with $u \in Fix(\tau^-)$, we have in fact $u = \tau^-(v)v$ for some $v \in U$ and therefore $\beta(v.x) = v.x$. The following proposition is then an immediate adaptation of the results of Foth to the quasi-hamiltonian setting.

Proposition 2.4. *Recall that the action of U on $\mu^{-1}(\{1\})$ is assumed to be free. If $Fix(\tau^-)$ is connected, then the projection map $p : \mu^{-1}(\{1\}) \to \mu^{-1}(\{1\})/U$ induces a surjective map $p_\beta : \mu^{-1}(\{1\}) \cap Fix(\beta) \to Fix(\hat{\beta})$. Denoting by U^τ the subgroup $U^\tau = Fix(\tau) \subset U$, one then has a bijection:*

$$\big(\mu^{-1}(\{1\}) \cap Fix(\beta)\big)/U^\tau \simeq Fix(\hat{\beta})$$

Or equivalently:

$$\big(\mu^{-1}(\{1\}) \cap U.Fix(\beta)\big)/U = Fix(\hat{\beta})$$

Proof. Let us simply prove here that

$$\big(\mu^{-1}(\{1\}) \cap Fix(\beta)\big)/U^\tau \simeq \big(\mu^{-1}(\{1\}) \cap U.Fix(\beta)\big)/U$$

as the rest of the statement is a special case of theorem 2.6. Consider the map

$$\begin{array}{rcl} \Phi : \big(\mu^{-1}(\{1\}) \cap Fix(\beta)\big)/U^\tau & \longrightarrow & \big(\mu^{-1}(\{1\}) \cap U.Fix(\beta)\big)/U \\ U^\tau.x & \longmapsto & U.x \end{array}$$

This map is well-defined because if $y = k.x$ with $k \in U^\tau$ then $U.y = U.x$. It is a surjective map because if $x \in (\mu^{-1}(\{1\}) \cap U.Fix(\beta))$, there exists $u \in U$ such that $y := u.x \in \mu^{-1}(\{1\}) \cap Fix(\beta)$, and then $U^\tau.y$ is mapped by Φ to $U.y = U.x$. Finally, let us prove that Φ is injective (this is where we will use that the action of

U is free). Assume that $x, y \in \mu^{-1}(\{1\}) \cap Fix(\beta)$ satisfy $U.x = U.y$. Then $y = u.x$ for some $u \in U$. Therefore, $y = \beta(y) = \beta(u.x) = \tau(u).\beta(x) = \tau(u).x$, hence $\tau(u).x = u.x$. Since the action of U on $\mu^{-1}(\{1\})$ is free, this implies $\tau(u) = u$, hence $U^\tau y = U^\tau x$. □

To deal with the case where $Fix(\tau^-)$ is not connected, we recall the following from [2]:

Proposition 2.5 ([2])**.** *Given an element $w \in Fix(\tau^-)$, denote by Q_w the connected component of $Fix(\tau^-)$ containing it and denote by $\tau_w : U \to U$ the map $\tau_w = Ad\, w \circ \tau$. Then τ_w is an involution on U. If $\beta : M \to M$ is a form-reversing involution on the quasi-hamiltonian space $(M, \omega, \mu : M \to U)$ and is compatible with τ and μ, denote by $\beta_w : M \to M$ the map $\beta_w : x \mapsto w.\beta(x)$. Then β_w is a form-reversing involution and is compatible with τ_w and μ.*

Observe that β and β_w induce the same involution $\hat{\beta}$ on $M//U$. One then has the following result, which we adapt from [2]:

Theorem 2.6. *Recall that the action of U on $\mu^{-1}(\{1\})$ is assumed to be free. Denote by $(Q_i)_{i\in I}$ the connected components of $Fix(\tau^-) \subset U$ and by w_i an element of Q_i. Then, in the notation of proposition 2.5, one has:*

$$Fix(\hat{\beta}) = \bigsqcup_{i\in I} \big(\mu^{-1}(\{1\}) \cap U.Fix(\beta_{w_i})\big)/U = \bigsqcup_{i\in I} \big(\mu^{-1}(\{1\}) \cap Fix(\beta_{w_i})\big)/U^{\tau_i}$$

Proof. Take $[x] \in Fix(\hat{\beta})$. We have seen that this implies the existence of $u \in Fix(\tau^-)$ such that $\beta(x) = u.x$. Let Q_i be the connected component of $Fix(\tau^-)$ containing u. Then by proposition 2.3, there exists $v \in U$ such that $u = \tau^-(v) w_i v$ and therefore $\beta(v.x) = w_i.(v.x)$. Consequently, since $w_i \in Fix(\tau^-)$, one has $\beta_{w_i}(w_i.(v.x)) = w_i.\beta(w_i.(v.x)) = w_i\tau(w_i).\beta(v.x) = \beta(v.x) = w_i.(v.x)$ hence $x \in U.Fix(\beta_{w_i})$. Therefore $Fix(\hat{\beta}) \subset \cup_{i\in I}(\mu^{-1}(\{1\}) \cap U.Fix(\beta_{w_i}))/U$. The converse implication is obvious since all the involutions β_{w_i} induce the same involution $\hat{\beta}$. To show that the union is disjoint, assume that $x \in \mu^{-1}(\{1\}) \cap Fix(\beta_{w_i})$ satisfies $(u.x) \in Fix(\beta_{w_j})$ for some $u \in U$. Then $x = \beta_{w_i}(x) = w_i w_j^{-1}.\beta_{w_j}(x) = w_i w_j^{-1} \tau^-_{w_j}(u).\beta_{w_j}(u.x) = w_i(\tau^-(u) w_j^{-1})u.x$. Since the action of U on $\mu^{-1}(\{1\})$ is free, this implies that $w_i = u^{-1} w_j \tau(u)$, which, by proposition 2.3, contradicts the fact that w_i and w_j lie in different connected components of $Fix(\tau^-)$. □

3 The stratified case

We now study the case where the action of U on $\mu^{-1}(\{1\})$ is not free. In this case, the associated quasi-hamiltonian quotient is a disjoint union of symplectic manifolds.

Theorem 3.1 ([8])**.** *Let $(M, \omega, \mu : M \to U)$ be a quasi-hamiltonian U-space. For any closed subgroup $K \subset U$, denote by M_K the isotropy manifold of type K in M:*

$$M_K = \{x \in M \mid U_x = K\}$$

Denote by $\mathcal{N}(K)$ the normalizer of K in U and by L_K the quotient group $L_K := \mathcal{N}(K)/K$. Then $\mu(M_K) \subset \mathcal{N}(K)$ and if we denote by μ_K the composed map $\mu : M_K \to \mathcal{N}(K) \to L_K = \mathcal{N}(K)/K$, then $(M_K, \omega|_{M_K}, \mu_K : M_L \to L_K)$ is a quasi-hamiltonian L_K-space. Furthermore, L_K acts freely on M_K and the orbit space

$$(\mu^{-1}(\{1_U\}) \cap M_K)/L_K$$

is a smooth symplectic manifold.
Denote by $(K_j)_{j\in J}$ a system of representatives of closed subgroups of U. Then the orbit space $M^{red} := \mu^{-1}(\{1_U\})/U$ is the disjoint union of the following symplectic manifolds:

$$\mu^{-1}(\{1_U\})/U = \bigsqcup_{j\in J} (\mu^{-1}(\{1_U\}) \cap M_{K_j})/L_{K_j}$$

A result equivalent to theorem 3.1 was proved by Hurtubise, Jeffrey and Sjamaar in [3]. Observe that each subset $X_K := (\mu^{-1}(\{1\}) \cap M_K)/L_K$ (also called a *stratum*) in the union $M//U = \sqcup_{j\in J} X_{K_j}$ is the orbit space of a *free* action of a compact Lie group. To generalize to this setting the results obtained in section 2, the first step to take is to check whether $\hat{\beta}$ induces an involution on $X_K = (\mu^{-1}(\{1\}) \cap M_K)/L_K$. To that end, we observe the following fact:

Lemma 3.2. *If β is a form-reversing involution on the quasi-hamiltonian space $(M, \omega, \mu : M \to U)$ compatible with $\tau : U \to U$ and $\mu : M \to U$, then for all $x \in M$ the stabilizer $U_{\beta(x)}$ of $\beta(x)$ in U is the subgroup $U_{\beta(x)} = \tau(U_x)$.*

Proof. If $x \in U_{\beta(x)}$ then $\beta(\tau(u).x) = u.\beta(x) = \beta(x)$, hence $\tau(u).x = x$. Therefore $\tau(u) \in U_x$ and $u = \tau(\tau(u)) \in \tau(U_x)$. Conversely, if $u \in U_x$, then $\beta(x) = \beta(u.x) = \tau(u).\beta(x)$ hence $\tau(u) \in U_{\beta(x)}$. □

Since we are ultimately interested in studying $Fix(\hat{\beta}) \subset M//U$, we ask the following question: under what conditions does the stratum $X_K = (\mu^{-1}(\{1\}) \cap M_K)/L_K \subset M//U$ contain points of $Fix(\hat{\beta})$? We then observe:

Lemma 3.3. *If $[x] \in (\mu^{-1}(\{1\}) \cap M_K)/L_K$ satisfies $\hat{\beta}([x]) = [x]$ then the subgroups K and $\tau(K)$ are conjugate in U. Moreover, $\hat{\beta}$ maps a stratum X_K to itself if and only if $\tau(K)$ is conjugate to K in U.*

Proof. If $x \in M_K$ satisfies $\hat{\beta}([x]) = [x]$, we have $U_x = K$ and $\beta(x) = u.x$ for some $u \in U$. Hence $U_{\beta(x)} = uU_xu^{-1} = uKu^{-1}$. By lemma 3.2, we also have $U_{\beta(x)} = \tau(U_x) = \tau(K)$, hence $\tau(K) = uKu^{-1}$. Finally, $\hat{\beta}$ maps X_K to itself if and only if for all $y \in \mu^{-1}(\{1\}) \cap M_K$ the stabilizer of $\beta(y)$ is conjugate to K, that is: $U_{\beta(y)} = \tau(U_y) = \tau(K)$ is conjugate to K. □

Moreover, we have:

Lemma 3.4. *Let $\beta : M \to M$ be an involution on M compatible with $\tau : U \to U$ and let $K \subset U$ be a (closed) subgroup of U. Then $\beta(M_K) \subset M_K$ if and only if $\tau(K) \subset K$.*

Proof. This is a direct consequence of the fact that $U_{\beta(x)} = \tau(U_x)$. □

Observe that since β and τ are involutions, the previous conditions are in fact equivalent to $\beta(M_K) = M_K$ and $\tau(K) = K$. From the above two lemmas, we see that the situation in the stratified case strongly relies on the properties of the involution $\tau : U \to U$. For instance, if $U = SU(n)$ and $\tau = AdI_{p,q}$, where $I_{p,q} \in U(n)$ is a diagonal matrix with p entries equal to $+1$ and q entries equal to -1, then by definition, for any subgroup $K \subset U$, one has $\tau(K)$ conjugate to K. Furthermore, $\tau(K)$ is conjugate to K by the element $I_{p,q}$, which satisfies $\tau^-(I_{p,q,}) = (I_{p,q}I_{p,q}I_{p,q}^{-1})^{-1} = I_{p,q}^{-1} = I_{p,q}$ ($I_{p,q}$ is a *symmetric* element of U *with respect to* τ). This special case is interesting because, as we shall see shortly, we can then apply the results of section 2. Moreover, the classification of involutive automorphisms of classical semi-simple compact connected Lie groups, as given for instance in [4], shows that every such group carries an involutive automorphism $\tau : U \to U$ satisfying: for any subgroup $K \subset U$, the subgroup $\tau(K)$ is conjugate to K by an element $w_K \in Fix(\tau^-) \subset U$. In fact, on groups of type B_n and D_n (for instance $SO(2n+1)$ and $SO(2n)$), every involutive automorphism is conjugate to an automorphism of this type. As for types A_n and C_n, in addition to such an involution, one also has complex conjugation $\tau(u) = \overline{u}$ on $SU(n)$ and $Sp(n)$. We do not know whether this involution satisfies the above property.
We then have:

Proposition 3.5. *Let K be a closed subgroup of U such that $\tau(K) = w_K K w_K^{-1}$ with $w_K \in Fix(\tau^-) \subset U$. Then, there exists an involution $\beta^K : M \to M$ and an involutive automorphism $\tau^K : U \to U$ such that β^K is compatible with τ^K and with the momentum map $\mu : M \to U$, and such that $\beta^K(M_K) = M_K$. Moreover, the involution $\widehat{\beta^K}$ induced by β^K on $\mu^{-1}(\{1\})/U$ coincides with $\hat{\beta}$.*

Proof. Set $\beta^K(x) := w_K^{-1}.\beta(x)$ for all $x \in M$ and $\tau^K(u) = (w_K)^{-1}\tau(u)w_K$ for all $u \in U$. Since w_K is a symmetric element of U, we have, by proposition 2.5, that β^K is an involution on M, compatible with τ^K and μ, which induces the same involution as β on $\mu^{-1}(\{1\})/U$. Further, we have $\tau^K(K) = (w_K)^{-1}\tau(K)w_K = K$. Therefore, by lemma 3.4, we have $\beta^K(M_K) = M_K$. □

We then observe:

Lemma 3.6. *If τ_0 is any involutive automorphism of U and if $K \subset U$ is a subgroup such that $\tau_0(K) \subset K$, one also has $\tau_0(\mathcal{N}(K)) \subset \mathcal{N}(K)$. Consequently, τ_0 induces an involution on $L_K = \mathcal{N}(K)/K$.*

Proof. Consider $n \in \mathcal{N}(K)$ and $k \in K$. Since $\tau_0(k) \in K$, we have $n\tau_0(k)n^{-1} \in K$, and therefore also $\tau_0(n)k(\tau_0(n))^{-1} = \tau_0(n\tau_0(k)n^{-1}) \in \tau_0(K) \subset K$, thus $\tau_0(n) \in \mathcal{N}(K)$. □

It is then immediate that the involution $(\beta^K)|_{M_K} : M_K \to M_K$ is compatible with the action of (L_K, τ_K) and with the momentum map of this action (and that $(\beta^K)^*\omega_K = -\omega_K$). Summarizing our study, we obtain the following results:

Theorem 3.7. *Let $M//U = \sqcup_{j\in J}(\mu^{-1}(\{1\}) \cap M_{K_j})/L_{K_j}$ be a quasi-hamiltonian quotient. Set $X_j = (\mu^{-1}(\{1\}) \cap M_{K_j})/L_{K_j}$. Then $\hat{\beta}(X_j) = X_j$ if and only if $\tau(K_j)$ is conjugate to K_j in U and one has:*

$$Fix(\hat{\beta}) = \bigsqcup_{j\in J \ | \ \tau(K_j)\sim K_j} Fix(\hat{\beta}|_{X_j})$$

In particular, $Fix(\hat{\beta})$ is a disjoint union of lagrangian submanifolds of the strata $X_j \subset \mu^{-1}(\{1\})/U$ for which $\tau(K_j)$ is conjugate to K_j and $Fix(\hat{\beta}|_{X_j}) \neq \emptyset$.

Observe that it is not clear which strata actually satisfy $Fix(\hat{\beta}|_{X_j}) \neq \emptyset$. Finally, since the action of L_{K_j} on M_{K_j} is free, we can, in certain cases, apply the results of section 2:

Corollary 3.8. *If in addition $\tau(K_j)$ is conjugate to K_j by a symmetric element $w_{K_j} \in Fix(\tau^-)$ of U then there exists an involution $\beta^{K_j} : M_{K_j} \to M_{K_j}$ and an involutive automorphism $\tau^{K_j} : L_{K_j} \to L_{K_j}$ such that β^{K_j} is compatible with τ^{K_j} and with the momentum map $\mu_{K_j} : M_{K_j} \to L_{K_j}$. Denoting by $(Q_i^{K_j})_{i\in I_j}$ the connected components of $Fix((\tau^{K_j})^-) \subset U$ and by w_i an element of $Q_i^{K_j}$, we then have, in the notation of proposition 2.5:*

$$Fix(\hat{\beta}|_{X_j}) = Fix(\widehat{\beta^{K_j}}) = \bigsqcup_{i\in I_j} \big((\mu^{-1}(\{1\}) \cap M_{K_j}) \cap Fix(\beta_{w_i}^{K_j})\big)/L_{K_j}$$

We refer to [6] for an example of a form-reversing involution β on a quasi-hamiltonian space as well as for comments on the condition $Fix(\hat{\beta}) \neq \emptyset$.

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