MATHEMATICS SEMINAR
of the
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January 2010

12 January 2010, at 5 pm **Room B02**

Chris Rogers
University of California at Riverside

An invitation to higher symplectic geometry

Abstract

In this lecture I will describe two approaches towards generalizing symplectic geometry: "2-plectic geometry" and "Courant algebroids". Just as a symplectic manifold is equipped with a closed non-degenerate 2-form, a "2-plectic manifold" is equipped with a closed non-degenerate 3-form. The Poisson bracket makes the smooth functions on a symplectic manifold form a Lie algebra. Similarly, any 2-plectic manifold gives a "Lie 2-algebra" - the higher analogue of a Lie algebra, where the usual laws hold only up to isomorphism. Closed 3-forms and Lie 2-algebras also play a role in the theory of Courant algebroids. Roughly, Courant algebroids are vector bundles that generalize the structures found in tangent bundles and quadratic Lie algebras. Certain Courant algebroids which naturally arise in 2-dimensional variational problems are classified up to isomorphism by the third de Rham cohomology of the underlying base space. I will explain these ideas further and describe how these two different generalizations of symplectic geometry are intimately related both algebraically and geometrically.

26 January 2010, at 5 pm **Room B02**

Salem Ben Said
University Henri Poincaré, Nancy

Interpolation between two different minimal representations for two different groups

Abstract

The classical Fourier transform is one of the most basic objects in analysis; it may be understood as belonging to a one-parameter group of unitary operators on $L^2(\mathbb{R}^N)$, and this group may even be extended holomorphically to a semigroup (the *Hermite semigroup*) $I(z)$ generated by the self-adjoint operator $\Delta - \|x\|^2$. This is a holomorphic semigroup of bounded operators depending on a complex variable z in the complex right half-plane, viz. $I(z+w) = I(z)I(w)$. The structure of this semigroup and its properties may be appreciated without any reference to representation theory, whereas the link itself is rich as was revealed beautifully by R. Howe in connection with the Schrödinger model of the Weil representation.

The aim of this talk is to consider the Dunkl Laplacian Δ_k and to construct a deformation of the classical situation, namely, a generalization $\mathscr{F}_{k,a}$ of the Fourier transform, and the holomorphic semigroup $\mathscr{I}_{k,a}(z)$ with infinitesimal generator $\|x\|^{2-a}\Delta_k - \|x\|^a$, acting on a concrete Hilbert space deforming $L^2(\mathbb{R}^N)$. We analyze these operators $\mathscr{F}_{k,a}$ and $\mathscr{I}_{k,a}(z)$ in the context of integral operators as well as representation theory. Particular attention will be given to the cases $a = 1$ and $a = 2$. This is a joint work with T. Kobayashi and B. Ørsted.