

MATHEMATICS SEMINAR  
of the  
UNIVERSITY OF LUXEMBOURG  
in cooperation with the  
LUXEMBOURG MATHEMATICAL SOCIETY

March 2009

3 March 2009, at 5 pm

Room 3.04 bs

Robert Coquereaux  
CNRS, Centre de Physique Théorique, Luminy

**Quantum subgroups of Lie groups and modular invariance**

Abstract

From quantum groups at roots of unity, or from affine Lie algebras at some level, one can construct a monoidal category of representations that admits, for special values of the chosen root (or of the level), module-categories, ie additive categories on which the previous one acts. In the case of quantum  $SU_2$ , those "quantum subgroups" are classified by the usual ADE Dynkin diagrams. This classification is equivalent to another problem solved long ago in the case of  $SU_2$  by theoretical physicists, in the context of conformal field theories with boundaries, namely the classification of modular-invariant sesquilinear forms, for the Hurwitz - Verlinde representations of  $SL(2, \mathbb{Z})$ . Each such quantum subgroup is associated with a weak Hopf algebra of a special kind (an Ocneanu quantum groupoid) that admits two, usually distinct, representations theories whose multiplicative structures can be encoded by graphs: the fusion graph and the graph of quantum symmetries. The purpose of the seminar is to provide a general introduction to the above ideas and to describe what happens when  $SU_2$  is replaced by more general Lie groups. This leads in particular to higher analogues of Coxeter-Dynkin diagrams (that will be presented for  $SU_3$  and  $SU_4$ ) and to higher graphs of quantum symmetries.

17 March 2009, at 5 pm

Room 3.04 bs

Janusz Grabowski  
Polish Academy of Sciences

### **Geometry of quantum systems: density states and entanglement**

Abstract

Various problems concerning the geometry of the space of Hermitian operators on a Hilbert space  $H$  are addressed. In particular, we study the canonical Poisson and Riemann-Jordan tensors and the corresponding foliations into Kähler submanifolds. It is also shown that the space  $D(H)$  of density states on an  $n$ -dimensional Hilbert space  $H$  is naturally a manifold stratified space with the stratification induced by the rank of the state. This stratification is maximal in the sense that every smooth curve in  $D(H)$ , viewed as a subset of the dual  $u^*(H)$  to the Lie algebra of the unitary group  $U(H)$ , at every point must be tangent to the strata it crosses. For a quantum composite system entangled states are defined in a geometrical way and an abstract criterion of entanglement is proved.

24 March 2009, at 5 pm

Room 3.04 bs

Dmitri Alekseevsky  
Edinburgh University and Maxwell Institute for Mathematical Sciences

### **Para-CR structures and related structures**

Abstract

A para-CR structure is a para-complex analogue of a CR structure. It is defined as a distribution  $H$  on a manifold  $M$  together with a para-complex structure  $K$  on  $H$ , i.e. a field of endomorphisms  $K$  such that  $K^2 = \text{Id}$  and the eigendistributions  $H^\pm$  of  $K$  are involutive. Many notions and results of CR geometry remain valid in para-CR case. We present a survey of basic facts of para-CR geometry. A description of maximally homogeneous para-CR manifolds of semisimple type will be given. We consider also some structures subordinated to para-CR structure, for example, quaternionic para-CR structure, which is a para-analogue of 3-Sasakian structure, and pseudo-conformal quaternionic para-CR structure and describe their relations with pseudo-hyperKähler structure and pseudo-quaternionic Kähler structure. An interesting special case of para-CR structures consists of non degenerate codimension one para-CR structures. Such structure can be defined as a decomposition  $H = H^+ + H^-$  of a contact distribution  $H$  into direct sum of two integrable Lagrangian subdistributions. We discuss relations of such structures with second order ODE discovered by P. Nurowski and G.A.J. Sparling and to parabolic Monge-Ampere equations.

**31 March 2009, at 5pm**

**room 3.04 bs**

Prof. Andreas Kollross (Universität Augsburg)

**Low cohomogeneity and polar actions on symmetric spaces.**

Abstract:

A Lie group action on a Riemannian manifold is called polar if there exists a section, i.e. a submanifold which meets all orbits orthogonally. A natural example is given by the action of a compact Lie group on itself by conjugation, where the maximal tori are sections.

Another class of examples is given by actions of cohomogeneity one. I will talk about classification results for polar actions on symmetric spaces.

MATHEMATICS SEMINAR  
of the  
UNIVERSITY OF LUXEMBOURG  
in cooperation with the  
LUXEMBOURG MATHEMATICAL SOCIETY

**April 2009**

~~7 April 2009, at 5 pm~~

**Room 3.04 bs**

please note: talk cancelled

~~Christian Pauly  
Université de Montpellier~~

**Dualities for principal  $G$ -bundles over curves**

Abstract

In this talk I will survey a couple of new results and open problems in the area of principal  $G$ -bundles over a smooth projective curve  $C$ . The space of global sections of a line bundle over the moduli space of principal  $G$ -bundles over  $C$ , also called Verlinde space or space of generalized  $G$ -theta functions, can be described as quantization space of a symplectic manifold, as well as a space of conformal blocks arising in conformal field theory. I will give a brief outline of the following dualities

1. Strange Duality or “rank-level” Duality for the Verlinde spaces associated to various classical and exceptional groups  $G$ .
2. Langland’s Duality for the cameral Prym varieties of the Hitchin system.

**21 April 2009, at 5 pm**

**Room 3.04 bs**

Fulvio Ricci  
Scuola Normale Superiore Pisa

## **An uncertainty principle for semigroups**

### Abstract

We present a general principle showing that hypercontractive estimates for a semigroup of operators on  $L^2$  imply uncertainty inequalities for its generator. We give examples in different geometric contexts, like Riemannian symmetric spaces, Carnot groups and graphs.

MATHEMATICS SEMINAR  
of the  
UNIVERSITY OF LUXEMBOURG  
in cooperation with the  
LUXEMBOURG MATHEMATICAL SOCIETY

May 2009

12 May 2009, at 5 pm

Room 3.04 bs

Erkko Lehtonen  
University of Luxembourg

**Reducts of Mal'cev's iterative algebra and generalizations of relations**

Abstract

Let  $A$  be a fixed nonempty set. A clone on  $A$  is a set of operations on  $A$  that contains all projections and is closed under composition. Clones can be defined equivalently as the universes of Mal'cev's iterative algebras that contain all projections. It is well-known that the (locally closed) clones on  $A$  can be characterized as sets of operations that preserve sets of relations on  $A$ . This preservation relation induces a Galois connection between operations and relations, known as the Pol-Inv theory. The closed classes of operations and the closed classes of relations on finite domains were first described, by explicit closure conditions, by Geiger and independently by Bodnarchuk, Kaluzhnin, Kotov and Romov, and these results were extended to infinite domains by Szabó and independently by Pöschel.

In this talk, we will survey reducts of the full iterative algebra and discuss how their subuniverses can be characterized by preservation of certain variants of relations. Corresponding Galois theories have been established by Pippenger, Hellerstein, Couceiro, Foldes, and recently by the current author.

19 May 2009, at 5 pm

Room 3.04 bs

Stephan Sturm  
TU Berlin

**A General Approach to Small-Time Large Deviations for Sample Paths of Infinite Dimensional Symmetric Dirichlet Processes with Applications to the Wasserstein Diffusion**

Abstract

Symmetric diffusions on a Polish space can be characterized via local, quasi-regular symmetric Dirichlet forms. We define a suitable pointwise intrinsic metric associated to the Dirichlet form which allows us to introduce the notion of energy (with respect to this intrinsic metric) of a sample path. Under mild assumptions – which imply the necessary exponential tightness – we prove a general small-time sample path large deviation principle for diffusions on a Polish space. As concrete application of this general approach we derive the small time large deviations for the Wasserstein diffusion on the space of probability measures on the unit interval.

26 May 2009, first talk at 3:45 pm

Room 3.04 bs

Kira Adaricheva  
Yeshiva University, New York

**On complex algebras of subalgebras**

Abstract

The modes are idempotent and entropic algebras known for the outstanding property that all their subalgebras form a mode of their own, with the same basic operations as the original algebra. Inspired by this example, we are looking at the more general picture when the algebra may bring to formation of an algebra of its subalgebras. It turns out the necessary and sufficient condition is the so-called generalized entropic property that holds in the (variety generated by) given algebra. We will discuss several open questions, in particular a hypothesis that every idempotent algebra with the generalized entropic property is a mode. The talk is based on joint research with A.Pilitowska (Warsaw University of Technology) and D. Stanovsky (Charles University, Prague).

Aklexey Sevastyanov  
University of Aberdeen

### **Algebraic group analogues of the Slodowy slices and deformed Poisson $W$ -algebras**

#### Abstract

We define algebraic group analogues of the Slodowy transversal slices to adjoint orbits in a complex semisimple Lie algebra  $\mathfrak{g}$ . The new slices are transversal to the conjugacy classes in an algebraic group with Lie algebra  $\mathfrak{g}$ . These slices are associated to the pairs  $(p, s)$ , where  $p$  is a parabolic subalgebra in  $\mathfrak{g}$  and  $s$  is an element of the Weyl group  $W$  of  $\mathfrak{g}$ . To each element  $s$  of the Weyl group we also naturally associate a parabolic subalgebra  $p$  such that one can construct the slice associated to the pair  $(p, s)$ . In the algebraic group framework simple Kleinian singularities are realized as the singularities of the fibers of the restriction of the conjugation quotient map to the slices associated to pairs  $(b, s)$ , where  $b$  is a Borel subalgebra in  $\mathfrak{g}$  and  $s$  is an element of  $W$  whose representative in  $G$  is subregular. We also define some Poisson structures on the slices associated to the pairs  $(p, s)$ . These structures are analogous to the Poisson structures introduced by DeBoer, Tjin and Premet on the Slodowy slices in complex simple Lie algebras. The quantum deformations of these Poisson structures are known as  $W$ -algebras of finite type. One of applications of our construction gives rise to new Poisson structures on the coordinate rings of simple Kleinian singularities.

MATHEMATICS SEMINAR  
of the  
UNIVERSITY OF LUXEMBOURG  
in cooperation with the  
LUXEMBOURG MATHEMATICAL SOCIETY

**June 2009**

**9 June 2009, at 5 pm**

**Room 3.04 bs**

François Drouot  
University of Nancy 1

**Crystal bases for the quantum group  $U_q(\mathfrak{gl}(m, n))$**

Abstract

Benkart, Kang and Kashiwara constructed a quantum group associated to the Lie super-algebra  $\mathfrak{gl}(m, n)$ ; they also constructed crystal bases for tensor powers of the standard representation  $V$ . However, unlike the case of the Lie algebra  $\mathfrak{gl}_{m+n}(\mathbb{C})$ , we have to take tensor powers of  $V \otimes V^*$  to obtain all simple modules of finite dimension. By weakening the definition of crystal base, I showed that these modules (they are indecomposable but not simple) have crystal bases, and under some conditions we can construct some crystal bases for sub-quotients of these modules.

MATHEMATICS SEMINAR  
of the  
UNIVERSITY OF LUXEMBOURG  
in cooperation with the  
LUXEMBOURG MATHEMATICAL SOCIETY

July 2009

14 July 2009, at 5 pm Room 3.04 bs

Professor Xiangdong Li  
(Fudan University, Shanghai)

A weighted rigidity theorem for Perelman's entropy on complete Riemannian manifolds

Abstract

Abstract: In his 2002 remarkable preprint, G. Perelman introduced the mysterious  $W$ -entropy functional for the conjugate heat equation and proved its monotonicity along the Ricci flow. Inspired by Perelman's pioneering work, we introduce the  $W$ -entropy functional associated with the heat equation for the weighted Laplacian on complete Riemannian manifolds. Under the non-negativity assumption on the Bakry-Emery Ricci curvature, we prove the monotonicity of  $W$ -entropy. Moreover, we prove a weighted rigidity theorem for the  $W$ -entropy on complete Riemannian manifolds.

MATHEMATICS SEMINAR  
of the  
UNIVERSITY OF LUXEMBOURG  
in cooperation with the  
LUXEMBOURG MATHEMATICAL SOCIETY

September 2009

14 September (Monday!) 2009, at 5 pm

Room 2.04 bs (!)

Nobutada Nakanishi  
Gifu Keizai University, Ogaki, Japan

**Systems of ODE and their associated Nambu vector fields**

Abstract

A Nambu vector field (or equivalently a Nambu system of ODE) is a kind of Hamiltonian vector field, which corresponds to  $dH_1 \wedge dH_2 \wedge \dots \wedge dH_{n-1}$ , a monomial (not finite sum) of  $n - 1$ -forms. I prove that if  $dx_1/f_1 = \dots = dx_n/f_n = dt$  is the system of ODE, there exists a function  $A$  such that  $dx_1/Af_1 = \dots = dx_n/Af_n = dt/A$  becomes a Nambu system. A function  $A$  is one of last multipliers of Jacobi for the original ODE. After this proof, I will give some applications and examples.

29 September 2009, at 5 pm

Room 3.04 bs

Pieter Moree  
Max-Planck-Institut für Mathematik, Bonn, Germany

**Cyclotomic coefficients**

## Abstract

The  $n$ -th cyclotomic polynomial has the  $n$ -th primitive roots of unity as its (simple) roots. Its coefficients are integers. The cyclotomic polynomials occur in various contexts in algebra and number theory. The tendency of the coefficients to be quite small has intrigued mathematicians since the 19th century. I will discuss recent works concerning this phenomenon, after discussing some more general aspects of cyclotomic polynomials. Little previous number theoretical exposure on part of the listener is required.

MATHEMATICS SEMINAR  
of the  
UNIVERSITY OF LUXEMBOURG  
in cooperation with the  
LUXEMBOURG MATHEMATICAL SOCIETY

**October 2009**

**6 October 2009, at 5 pm**

**Room 3.04 bs**

Pablo Ramacher  
University of Göttingen

**On the distribution of the spectrum of an invariant elliptic operator**

Abstract

The asymptotic distribution of eigenvalues of an elliptic operator has been the subject of mathematical research for many years. The first results were obtained by Weyl employing variational techniques. Later, Hörmander extended these results to elliptic pseudodifferential operators on a closed manifold  $M$  using the theory of Fourier integral operators, and showed that Weyl's law is satisfied in great generality. Let now  $G$  be a compact Lie group acting on  $M$  by isometries, and assume that  $Q$  commutes with the regular representation of  $G$  in  $L^2(M)$ . It is then natural to ask for the asymptotic distribution of the spectrum along the isotypic components in  $L^2(M)$ . While first order asymptotics can be obtained in the general case of effective group actions by using heat kernel methods, the derivation of remainder estimates within the framework of Fourier integral operators meets with serious difficulties when singular orbits are present. The reason for this is that in this case the corresponding wave front sets are no longer smooth manifolds. In this talk, we show how to circumvent this obstacle by making use of partial desingularization to obtain remainder estimates in the case of singular group actions.

**13 October 2009, at 5 pm**

**Room 3.04 bs**

Andrzej Zuk  
University Paris 7

### **Amenability**

#### Abstract

The notion of amenability was introduced by von Neumann in 1929 and became fundamental in the study of asymptotic properties of groups. We present new constructions of amenable groups.

**27 October 2009, at 5 pm**

**Room 3.04 bs**

Philippe Bonneau  
University Paul Verlaine, Metz

### **Universal deformation formulas and locally compact quantum groups**

#### Abstract

We recall the notion of (formal) Universal Deformation Formula of Giaquinto and Zhang and look at it in the framework of strict deformation quantization (“à la” Rieffel). We give examples coming from the quantization of symplectic symmetric spaces. From these examples, still by analogy with the formal setting, we try to construct locally compact quantum groups (LCQG). Then we discuss the different approaches of LCQG (Woronowicz, Vaes,...) and see how our examples fit in these frameworks.

MATHEMATICS SEMINAR  
of the  
UNIVERSITY OF LUXEMBOURG  
in cooperation with the  
LUXEMBOURG MATHEMATICAL SOCIETY

**November 2009**

**10 November 2009, at 5 pm**

**Room B02**

Gregor Fels  
University of Tübingen

**How round is the unit sphere: Tubes in the CR-Geometry**

Abstract

After a brief introduction to Cauchy-Riemann geometry we discuss the importance of tube manifolds as testing ground for various geometric phenomena, including certain types of degeneracy. We will also address the classification problem of tube manifolds and its relation to certain associative nilpotent algebras.

**17 November 2009, at 2 pm**

**Room B14**

Batu Güneysu  
University of Bonn

**The Feynman-Kac formula for Schrödinger operators on vector bundles**

Abstract

In this talk, I will explain how the Feynman-Kac formula can be generalized to a certain class of Schrödinger-type operators on vector bundles over complete Riemannian manifolds. This class includes nonnegative "potentials" which are locally square integrable.

**17 November 2009, at 5 pm**

**Room B02**

Hongxin Guo  
Wenzhou University, China

### **Geometry of gradient Ricci solitons**

#### **Abstract**

Ricci solitons play a fundamental role in the studies of the Ricci flow. They are solutions which evolve only by diffeomorphisms and scalings and occur at singularity formation and as asymptotic limits of long-time solutions. In this talk, I will report some progress towards the understanding of geometries of gradient Ricci solitons.

MATHEMATICS SEMINAR  
of the  
UNIVERSITY OF LUXEMBOURG  
in cooperation with the  
LUXEMBOURG MATHEMATICAL SOCIETY

**December 2009**

**1 December 2009, at 5 pm**

**Room B02**

C. Denson Hill  
Stony Brook University

**Einstein's Equations and Embedding of 3-dimensional CR Manifolds**

Abstract

We discuss several theorems concerning the connection between the local CR embeddability of 3-dimensional CR manifolds, and the existence of algebraically special solutions of Maxwell's and Einstein's equations, and related matters.

**8 December 2009, at 5 pm**

**Room B02**

Walter Freyn  
University of Münster

**Kac-Moody symmetric spaces: Foundations and some applications**

Abstract

Cartan introduced finite dimensional symmetric spaces as an important class of Riemann manifolds. Their geometry is governed by semisimple Lie groups. They are closely related to polar actions, buildings and isoparametric submanifolds.

In this talk we investigate the geometry and classification of Kac-Moody symmetric spaces. We explain, that Kac-Moody symmetric spaces are the natural generalization of symmetric spaces to infinite dimensions and sketch some connections to twin buildings, polar actions on Hilbert spaces and isoparametric submanifolds in Hilbert spaces.

**15 December 2009, ! at 4 pm !**

**Room B02**

Yves Le Jan  
University Paris-Sud (Orsay)

### **Markov loop measures**

#### Abstract

We explore some simple relations between Markovian path and loop measures, spanning trees, determinants, and Markov fields such as the free field.