MATHEMATICS SEMINAR
of the
UNIVERSITY OF LUXEMBOURG
in cooperation with the
LUXEMBOURG MATHEMATICAL SOCIETY

April 2010

13 April 2010, at 5 pm **Room B02**

Stéphane Guillermou
University Joseph Fourier, Grenoble

Microlocal theory of sheaves and symplectic geometry

Abstract

The microsupport of a sheaf on a manifold, introduced by Kashiwara and Schapira, is a subset of the cotangent bundle of the manifold. It is involutive with respect to the natural symplectic structure of the cotangent bundle.

We will recall some aspects of this theory and we will see that it gives a link between symplectic transformations on the cotangent bundle and operations on sheaves. Then we will explain how Tamarkin used it in a recent paper to recover some results ("non-displaceability") of symplectic geometry.

20 April 2010, at 5 pm **Room B02**

Charles-Michel Marle
University Pierre et Marie Curie, Paris

The hidden symmetries of the Kepler problem

Abstract

The *Kepler problem* is the mathematical description of all possible motions of a point mass in a central attractive Newtonian field. Solutions of that problem approximatively describe the motions of planets in the solar system.

We will first recall the equations of that problem, and how they can be solved. The total energy (kinetic plus potential) of the point mass remains constant during the motion. The orbit of the point mass is a conic section, with the attracting center as focus point: an ellipse if the total energy is negative, a parabola if it is zero and an hyperbola (or rather a connected component of an hyperbola) if it is positive (we have chosen for the zero level of the potential energy the potential energy of the point mass when it is at infinity).

We will then define the *space of motions* and, for each value e of the total energy, the *subspace of motions energy* e. After reduction (in the sense of Marsden and Weinstein) that reduced space is in one-one correspondence with the space of oriented circles in space, contained in a plane through the origin, and such that the power of the origin with respect to these circles is equal to a constant times e. For each energy level, that reduced space can be identified with an open, dense subspace of the space of oriented geodesics of an homogeneous Riemannian space with constant curvature.

That property will be used to *regularize* the problem, in order to account for singular orbits which end by a collision with (or begin by an ejection from) the attracting center. For that purpose, the problem is transformed by an inverse stereographic projection (generalized for positive energy) or by an inversion with respect to the attractive center (for energy zero).

After regularization, new *symmetries* of the Kepler problem appear. The natural symmetry group of that problem is the 3-dimensional group $\mathrm{SO}(3)$ of rotations of the 3-dimensional Euclidean space around the attractive center. After regularization, the symmetry group of the reduced space of motions with a fixed energy e is 6-dimensional: for $e < 0$, that group is the group $\mathrm{SO}(4)$ of rotations of a 4-dimensional Euclidean space around one of its points; for $e = 0$, the group of Euclidean displacements (generated by rotations and translations) of a 3-dimensional Euclidean space; and for $e > 0$, the Lorentz group $\mathrm{SO}(3,1)$. Additional symmetries so obtained explain the existence of the vector-valued first integral sometimes called the *Laplace-Runge-Lenz vector*, discovered by J. Hermann (1678–1733).

27 April 2010 **Room B02**

Paul Baum
Penn State University

What is K-theory and what is it good for?

Abstract

This talk will consist of four points:

1. The basic definition of K-theory
2. A brief history of K-theory
3. Algebraic versus topological K-theory
4. The unity of K-theory

The talk is intended for non-specialists, so the basic definitions will be carefully stated.

Notification of date and place of the next talk will be given in due time.

TBA **TBA**

Paul Baum
Penn State University

What is K-homology?

Abstract

K-homology is the dual theory to K-theory. In algebraic geometry, if X is a projective algebraic variety (which may have singularities) then the K-homology of X is the Grothendieck group of coherent algebraic sheaves on X. In topology, there are three ways to define K-homology:

1. (Homotopy theory) K-homology is the homology theory determined by the Bott spectrum.
2. (Functional analysis) The K-homology of a topological space X is the group of abstract elliptic operators on X.
3. (Geometric cycles) The K-homology of a topological space X is the group of K-cycles on X.

This talk will give the definition (following Atiyah, Brown-Douglas-Fillmore, and Kasparov) of K-homology as abstract elliptic operators. The geometric cycle approach (due to Baum-Douglas) will also be indicated. This second definition of K-homology is closely connected to the D-branes of string theory. K-homology will then be used to state the BC (Baum-Connes) conjecture.