MATHEMATICS SEMINAR
of the
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January 2010

12 January 2010, at 5 pm **Room B02**

Chris Rogers
University of California at Riverside

An invitation to higher symplectic geometry

Abstract

In this lecture I will describe two approaches towards generalizing symplectic geometry: "2-plectic geometry" and "Courant algebroids". Just as a symplectic manifold is equipped with a closed non-degenerate 2-form, a "2-plectic manifold" is equipped with a closed non-degenerate 3-form. The Poisson bracket makes the smooth functions on a symplectic manifold form a Lie algebra. Similarly, any 2-plectic manifold gives a "Lie 2-algebra" - the higher analogue of a Lie algebra, where the usual laws hold only up to isomorphism. Closed 3-forms and Lie 2-algebras also play a role in the theory of Courant algebroids. Roughly, Courant algebroids are vector bundles that generalize the structures found in tangent bundles and quadratic Lie algebras. Certain Courant algebroids which naturally arise in 2-dimensional variational problems are classified up to isomorphism by the third de Rham cohomology of the underlying base space. I will explain these ideas further and describe how these two different generalizations of symplectic geometry are intimately related both algebraically and geometrically.

26 January 2010, at 5 pm **Room B02**

Salem Ben Said
University Henri Poincaré, Nancy

Interpolation between two different minimal representations for two different groups

Abstract

The classical Fourier transform is one of the most basic objects in analysis; it may be understood as belonging to a one-parameter group of unitary operators on $L^2(\mathbb{R}^N)$, and this group may even be extended holomorphically to a semigroup (the *Hermite semigroup*) $I(z)$ generated by the self-adjoint operator $\Delta - \|x\|^2$. This is a holomorphic semigroup of bounded operators depending on a complex variable z in the complex right half-plane, viz. $I(z+w) = I(z)I(w)$. The structure of this semigroup and its properties may be appreciated without any reference to representation theory, whereas the link itself is rich as was revealed beautifully by R. Howe in connection with the Schrödinger model of the Weil representation.

The aim of this talk is to consider the Dunkl Laplacian Δ_k and to construct a deformation of the classical situation, namely, a generalization $\mathscr{F}_{k,a}$ of the Fourier transform, and the holomorphic semigroup $\mathscr{I}_{k,a}(z)$ with infinitesimal generator $\|x\|^{2-a}\Delta_k - \|x\|^a$, acting on a concrete Hilbert space deforming $L^2(\mathbb{R}^N)$. We analyze these operators $\mathscr{F}_{k,a}$ and $\mathscr{I}_{k,a}(z)$ in the context of integral operators as well as representation theory. Particular attention will be given to the cases $a = 1$ and $a = 2$. This is a joint work with T. Kobayashi and B. Ørsted.

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March 2009

3 March 2009, at 5 pm **Room 3.04 bs**

Robert Coquereaux
CNRS, Centre de Physique Théorique, Luminy

Quantum subgroups of Lie groups and modular invariance

Abstract

From quantum groups at roots of unity, or from affine Lie algebras at some level,ă one can construct a monoidal category of representations that admits, for special values of the chosen root (or of the level), module-categories, ie additive categories on which the previous one acts. In the case of quantum SU2, those "quantum subgroups" are classified by the usual ADE Dynkin diagrams.ă This classification is equivalent to another problem solved long ago in the case of SU2 by theoretical physicists, in the context of conformal field theories with boundaries, namely the classification of modular-invariant sesquilinear forms,ă for the Hurwitz - Verlinde representations of SL(2,Z). Each such quantum subgroup is associated with a weak Hopf algebra of a special kind (an Ocneanu quantum groupoid) that admits two, usually distinct, representations theories whose multiplicative structures can be encoded by graphs: the fusion graph and the graph of quantum symmetries. The purpose of the seminar is to provide a general introduction to the above ideas and to describe what happens when SU2 is replaced by more general Lie groups. This leads in particular to higher analogues of Coxeter-Dynkin diagrams (that will be presented for SU3 and SU4) and to higher graphs of quantum symmetries.

17 March 2009, at 5 pm **Room 3.04 bs**

Janusz Grabowski
Polish Academy of Sciences

Geometry of quantum systems: density states and entanglement

Abstract

Various problems concerning the geometry of the space of Hermitian operators on a Hilbert space H are addressed. In particular, we study the canonical Poisson and Riemann-Jordan tensors and the corresponding foliations into Kähler submanifolds. It is also shown that the space $D(H)$ of density states on an n-dimensional Hilbert space H is naturally a manifold stratified space with the stratification induced by the the rank of the state. This stratification is maximal in the sense that every smooth curve in $D(H)$, viewed as a subset of the dual $u^*(H)$ to the Lie algebra of the unitary group $U(H)$, at every point must be tangent to the strata it crosses. For a quantum composite system entangled states are defined in a geometrical way and an abstract criterion of entanglement is proved.

24 March 2009, at 5 pm **Room 3.04 bs**

Dmitri Alekseevsky
Edinburgh University and Maxwell Institute for Mathematical Sciences

Para-CR structures and related structures

Abstract

A para-CR structure is a para-complex analogue of a CR structure. It is defined as a distribution H on a manifold M together with a para-complex structure K on H, i.e. a field of endomorphisms K such that $K^2 = \mathrm{Id}$ and the eigendistributions $H^{\pm}$ of K are involutive. Many notions and results of CR geometry remain valid in para-CR case. We present a survey of basic facts of para-CR geometry. A description of maximally homogeneous para-CR manifolds of semisimple type will be given. We consider also some structures subordinated to para-CR structure, for example, quaternionic para-CR structure, which is a para-analogue of 3-Sasakian structure, and pseudo-conformal quaternionic para-CR structure and describe their relations with pseudo-hyperKähler structure and pseudo-quaternionic Kähler structure. An interesting special case of para-CR structures consists of non degenerate codimension one para-CR structures. Such structure can be defined as a decomposition $H = H^+ + H^-$ of a contact distribution H into direct sum of two integrable Lagrangian subdistributions. We discuss relations of such structures with second order ODE discovered by P. Nurowski and G.A.J. Sparling and to parabolic Monge-Ampere equations.

31 March 2009, at 5pm **room 3.04 bs**

Prof. Andreas Kollross (Universität Augsburg)

Low cohomogeneity and polar actions on symmetric spaces.

Abstract:

A Lie group action on a Riemannian manifold is called polar if there exists a section, i.e. a submanifold which meets all orbits orthogonally. A natural example is given by the action of a compact Lie group on itself by conjugation, where the maximal tori are sections.
Another class of examples is given by actions of cohomogeneity one. I will talk about classification results for polar actions on symmetric spaces.

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April 2010

13 April 2010, at 5 pm **Room B02**

Stéphane Guillermou
University Joseph Fourier, Grenoble

Microlocal theory of sheaves and symplectic geometry

Abstract

The microsupport of a sheaf on a manifold, introduced by Kashiwara and Schapira, is a subset of the cotangent bundle of the manifold. It is involutive with respect to the natural symplectic structure of the cotangent bundle.

We will recall some aspects of this theory and we will see that it gives a link between symplectic transformations on the cotangent bundle and operations on sheaves. Then we will explain how Tamarkin used it in a recent paper to recover some results ("non-displaceability") of symplectic geometry.

20 April 2010, at 5 pm **Room B02**

Charles-Michel Marle
University Pierre et Marie Curie, Paris

The hidden symmetries of the Kepler problem

Abstract

The *Kepler problem* is the mathematical description of all possible motions of a point mass in a central attractive Newtonian field. Solutions of that problem approximatively describe the motions of planets in the solar system.

We will first recall the equations of that problem, and how they can be solved. The total energy (kinetic plus potential) of the point mass remains constant during the motion. The orbit of the point mass is a conic section, with the attracting center as focus point: an ellipse if the total energy is negative, a parabola if it is zero and an hyperbola (or rather a connected component of an hyperbola) if it is positive (we have chosen for the zero level of the potential energy the potential energy of the point mass when it is at infinity).

We will then define the *space of motions* and, for each value e of the total energy, the *subspace of motions energy e*. After reduction (in the sense of Marsden and Weinstein) that reduced space is in one-one correspondence with the space of oriented circles in space, contained in a plane through the origin, and such that the power of the origin with respect to these circles is equal to a constant times e. For each energy level, that reduced space can be identified with an open, dense subspace of the space of oriented geodesics of an homogeneous Riemannian space with constant curvature.

That property will be used to *regularize* the problem, in order to account for singular orbits which end by a collision with (or begin by an ejection from) the attracting center. For that purpose, the problem is transformed by an inverse stereographic projection (generalized for positive energy) or by an inversion with respect to the attractive center (for energy zero).

After regularization, new *symmetries* of the Kepler problem appear. The natural symmetry group of that problem is the 3-dimensional group $\mathrm{SO}(3)$ of rotations of the 3-dimensional Euclidean space around the attractive center. After regularization, the symmetry group of the reduced space of motions with a fixed energy e is 6-dimensional: for $e < 0$, that group is the group $\mathrm{SO}(4)$ of rotations of a 4-dimensional Euclidean space around one of its points; for $e = 0$, the group of Euclidean displacements (generated by rotations and translations) of a 3-dimensional Euclidean space; and for $e > 0$, the Lorentz group $\mathrm{SO}(3,1)$. Additional symmetries so obtained explain the existence of the vector-valued first integral sometimes called the *Laplace-Runge-Lenz vector*, discovered by J. Hermann (1678–1733).

27 April 2010, at 5 pm **Room B02**

Paul Baum
Penn State University

What is *K*-theory and what is it good for?

Abstract

This talk will consist of four points:

1. The basic definition of K-theory
2. A brief history of K-theory
3. Algebraic versus topological K-theory
4. The unity of K-theory

The talk is intended for non-specialists, so the basic definitions will be carefully stated.

29 April 2010, at 2 pm **Room A02**

Paul Baum
Penn State University

What is K-homology?

Abstract

K-homology is the dual theory to K-theory. In algebraic geometry, if X is a projective algebraic variety (which may have singularities) then the K-homology of X is the Grothendieck group of coherent algebraic sheaves on X. In topology, there are three ways to define K-homology:

1. (Homotopy theory) K-homology is the homology theory determined by the Bott spectrum.
2. (Functional analysis) The K-homology of a topological space X is the group of abstract elliptic operators on X.
3. (Geometric cycles) The K-homology of a topological space X is the group of K-cycles on X.

This talk will give the definition (following Atiyah, Brown-Douglas-Fillmore, and Kasparov) of K-homology as abstract elliptic operators. The geometric cycle approach (due to Baum-Douglas) will also be indicated. This second definition of K-homology is closely connected to the D-branes of string theory. K-homology will then be used to state the BC (Baum-Connes) conjecture.

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May 2010

11 May 2010, at 5 pm **Room B.02**

Ivan Nourdin
University Paris 6

Universal Gaussian fluctuations of non-Hermitian matrix ensembles

Abstract

My goal is to explain how to prove multi-dimensional central limittheorems for the spectral moments (of arbitrary degrees) associated with random matrices with real-valued i.i.d. entries, satisfying some appropriate moment conditions. The used techniques rely on a universality principle for the Gaussian Wiener chaos, as well as on some combinatorial estimates. All necessary probabilistic notions will be introduced and discussed during the talk. This presentation is based on a joint work with Giovanni Peccati.

18 May 2010, at 5 pm **Room B.02**

Sergei Akbarov
Russian Institute of Scientific and Technical Information

Duality for Stein groups with algebraic connected component of identity

Abstract

We describe a new way to generalize Pontryagin duality from the category of commutative Stein groups to the category of (not necessarily commutative) Stein groups with algebraic connected component of identity (ArXiv.0806.3205). Our approach is based on the idea of replacing the category of Banach spaces with the category of stereotype spaces (i.e. topological vector spaces, which are reflexive with respect to the topology of uniform convergence on totally bounded sets). This yields to a generalization, where the enveloping category consists of Hopf algebras in the symmetrical monoidal category of stereotype spaces. As an application to quantum groups we show how this duality works in the case of the group “az+b”.

25 May 2010, at 5 pm **Room B.02**

Gregory Ginot
University Paris 6

Higher Hochschild (co)homology

Abstract

The Hochschild (co)homology is a useful tool which appears in algebra, deformation theory, algebraic geometry and algebraic topology. It comes with many algebraic structures, some of them we will recall. It is folklore philosophy to think as these structures as being related to some kind of circle action on Hochschild (co)chain complexes.ă We will explain a precise mathematical way which allowsă to see the Hochschild (co)chain complex as a cohomology theory functorially modelled on the circle. We will then explain how this theory can be generalized to include any topological spaces in place of the circle.

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June 2010

1 June 2010, at 5 pm **Room B.02**

Jean-Louis Tu
Paul Verlaine University, Metz

~~**TBA**~~ The Baum-Connes and the coarse Baum-Connes conjecture

Let G be a locally compact group. The Baum-Connes conjecture allows to compute the K-theory group of the reduced C*-algebra of G. It is related to many domains of mathematics such as differential geometry, index theory, harmonic analysis and geometry of groups. We will present a survey of some past and recent developments on this subject.

15 June 2010, at 5 pm **Room B.02**

Vincenzo Nesi
University of Rome 1, La Sapienza

Planar harmonic maps

Abstract

A planar harmonic mapping $U = (u^1, u^2)$ on the unit disk $B \subset \mathbb{R}^2$ is simply a pair of harmonic functions on B. The theme of the talk is establishing conditions under which U is a global homeomorphism.

Given a homeomorphism Φ of ∂B onto a simple closed Jordan curve γ, set D to be the simply connected bounded open set determined by γ. A classical result of H. Kneser (1926) establishes that, if D is convex, the harmonic extension of Φ is a homeomorphism of $\overline{B}$ onto $\overline{D} \equiv \gamma \cup D$.

I will then present the main result. If $\Phi \in C^{1,\alpha}(\partial B)$, then we give a necessary and sufficient condition for U to be a diffeomorphism of $\overline{B}$ onto $\overline{D}$ so providing a sharp version of H. Kneser's Theorem. Finally, if time permits, I will present versions of Kneser's theorem which are valid when considering L^∞ elliptic operators rather than the Laplace operator with applications to composite materials. The talk is based upon a joint work with Giovanni Alessandrini, Università degli Studi di Trieste.

General Mathematics Seminar
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October 2010

Tuesday, Oct 5, 2010 at 17:00 Campus Kirchberg, room A02

Jean-Louis Loday
(CNRS, Strasbourg)

Hidden structures in homological algebra

If a chain complex is equipped with some compatible algebraic structure, then its homology gets equipped with this algebraic structure. The surprise is that, in most cases, there is a hidden algebraic structure on this homology. We will give several elementary examples and show how the notions of spectral sequence, Connes boundary map B, A-infinity algebra and MacLane invariant of a crossed module come naturally out of this principle. I'll end up with a problem in biology.

General Mathematics Seminar
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October 2010

Tuesday, October 19, 2010, at 17:00 **Campus Kirchberg, room A02**

Prof. E. M. Semenov
(Voronezh University, Russia)

Invariant Banach limits

Abstract:

A linear functional $B \in l_\infty^*$ is said to be a Banach limit if $B(1,1,\ldots) = 1$, $B \geq 0$ and $B(Tx) = Bx$ for any $x \in l_\infty$ where T is the translation operator, that is $T(x_1, x_2, \ldots) = (x_2, x_3, \ldots)$. We present a set of easily verifiable sufficient conditions on an operator $H \in L(l_\infty)$, guaranteeing the existence of a Banach limit B s.t. $B = BH$. We apply our results to the classical Cesaro operator. We present another application to geometry of non-separable Banach spaces.

Joint work with F. A. Sukochev (Sydney University, Australia).

General Mathematics Seminar of the University of Luxembourg

In cooperation with the Luxembourg Mathematical Society

October 2010

Tuesday, Oct 26, 2010 at 17:00 Campus Kirchberg, room A02

Kalyan B. Sinha
(Jawaharlal Nehru Centre for Advanced Scientific Research
and Indian Institute of Science, Bangalore, India)

Introduction to Quantum Stochastic Calculus and Applications

Many avenues exist for extending probability theory to a non-Kolmogoroffian one – one of these is the one following from the Quantum theory. A system of filtrations can be constructed in the Fock space by an increasing family of noncommutative *-algebras, often containing commuting subfiltrations, most noteworthy amongst them are the classical ones coming from the Brownian motion and Poisson process. Non-commutative diffusions can be constructed in Fock space starting with a stochastic differential equation, driven by three fundamental non-commuting martingales.

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November 2010

Tuesday, Nov 9, 2010, 17:00 Campus Kirchberg, room A02

Ioannis Dokas
(Department of Mathematics and Statistics,University of Cyprus, Nicosia,Cyprus)

Structures in prime characteristic with additional operations

First we will give elements of the theory of restricted Lie algebras. Then moving to the pre-Lie context we introduce and study the notion of restricted pre-Lie algebra which is the analogue of the notion of restricted Lie algebra in theLie context.

General Mathematics Seminar
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November 2010

Tuesday, November 16, 2010, at 17:00 **Campus Kirchberg, room A02**

Prof. Sergiu I. Vacaru
(University Al. I. Cuza, Iasi, Romania)

Almost Kahler - Ricci Flows and Deformation Quantization of Gravity

Abstract:

We show that the Einstein and Horava-Lifshitz gravity theories can be defined canonically in terms of almost Kaehler-Finsler geometric objects. This allows us to perform a (Fedosov type) deformation quantization of such gravity models. The constructions can be generalized to Ricci flows of classical and quantum gravity theories.

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November 2010

Tuesday, November 23, 2010, at 17:00 **Campus Kirchberg, room A02**

Arthemy V. Kiselev
(Utrecht, The Netherlands)

Variational Lie algebroids and homological evolutionary vector fields

Abstract:

The construction of Lie algebroids over smooth manifolds is important in differential geometry (particularly, in Poisson geometry) and appears in various models of mathematical physics (e.g., in Poisson sigma-models). We extend the classical definition of Lie algebroids over smooth manifolds [e.g., Vaintrob'1997] to the construction of variational Lie algebroids over infinite jet spaces. We define these structures in a standard way via vector bundles and also through homological vector fields $Q^2 = 0$ on infinite jet super-bundles, then proving the equivalence. Our generalization of the classical construction manifestly respects the geometry which appears under mappings between smooth manifolds. For this reason, the variational picture, which we develop here, more fully grasps the geometry of strings in space-time [Alexandrov, Kontsevich, Zaboronsky, Schwarz'1997]. [This talk follows the paper joint with J.W. van de Leur: arXiv:math.DG/1006.4227v2 (October 31, 2010), 15p.]

General Mathematics Seminar
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November 2010

Tuesday, November 30, 2010, at 17:00 **Campus Kirchberg, room A02**

Christian Pauly
(University of Montpellier)

On the monodromy of the Hitchin connection

Abstract:

In this talk I will show that the monodromy representation of the projective Hitchin connection on the sheaf of generalized theta functions on the moduli space of vector bundles over a curve has an element of infinite order in its image. I will explain the link with conformal blocks and KZ equations and I will give some applications in the context of the Grothendieck conjectures on the p-curvatures of a local system.