

General Mathematics Seminar
Of the
University of Luxembourg
In cooperation with the
Luxembourg Mathematical Society

January 2011

Tuesday, January 4, 2011, at 17:00

Campus Kirchberg, room A02

Walter van Suijlekom
(Radboud University, Nijmegen)

Gauge theories and noncommutative manifolds

Abstract:

In this talk, we will discuss some aspects of the intrinsic gauge theoretical nature of noncommutative manifolds. Following Connes, we describe a noncommutative (Riemannian, spin) manifold by its fundamental class in K -homology. Among other functional analytical data, such a K -cycle consists of a (noncommutative) C^* -algebra. As a consequence of noncommutativity, there might exist non-trivial inner automorphisms; these will be referred to as gauge transformations.

The key example that motivates this terminology from physics is when one replaces the algebra of functions on a manifold by matrix-valued functions. The resulting Morita equivalence describes ordinary Yang-Mills theory as formulated in terms of vector bundles and connections thereon. If time permits, we will consider a second class of examples, so-called toric noncommutative manifolds.

General Mathematics Seminar
of the
University of Luxembourg
in cooperation with the
Luxembourg Mathematical Society

January 2011

Tuesday, January 25, 2011, at 17:00

Campus Kirchberg, room A02

Arkady Onishchik
(Yaroslavl University, Russia)

Homogeneous complex supermanifolds associated to compact Hermitian symmetric spaces

Abstract:

I would like to discuss the following classification problem: given a complex flag homogeneous space $M = G/P$ (G is a semi-simple complex Lie group, P its parabolic subgroup), to describe all homogeneous complex supermanifolds $(M; \mathcal{O})$ with reduction M . In the case when $M = Gr_{4;2}$ is the Grassmann manifold of 2-planes in \mathbb{C}^4 this problem was formulated by Yu.I. Manin; it was motivated by certain physical models. He also gave an example of a non-split homogeneous complex supermanifold with this reduction; this is the so-called Π - symmetric supergrassmannian $\Pi Gr_{4|4;2|2}$. My goal is to discuss certain general approaches to this classification problem and to formulate certain results in the case when M is an irreducible compact Hermitian symmetric space.

First, I consider the case when the supermanifold $(M; \mathcal{O})$ is split and \mathcal{O} is determined by the homogeneous vector bundle over M induced by a representation ϕ of P . If ϕ is completely reducible, then homogeneous supermanifolds of this sort may be described in terms of the highest weights of ϕ .

The most difficult is the case of a non-split supermanifold $(M; \mathcal{O})$. There is the following conjecture: if, in our situation, the retract of this supermanifold is determined by an irreducible representation of P , then this retract is isomorphic to $(M; \Omega)$, where Ω is the sheaf of holomorphic differential forms on M . It is also possible to describe non-split homogeneous supermanifolds with this retract. The conjecture is proved for the following symmetric spaces: $M = Gr_{n;k}, 3 \leq k \leq n - k; Gr_{2n+1;2}, n \geq 2; Gr_{4;2}, Gr_{6;2}; Sp_{2n}/U_n, n \geq 2; E_6/D_5 \times T; E_7/E_6 \times T$.