General Mathematics Seminar
of the
University of Luxembourg
in cooperation with the
Luxembourg Mathematical Society

March 2011

Tuesday, March 08, 2011, at 17:00 **Campus Kirchberg, room B02**

Elizaveta Vishnyakova
(Tver'-Bochum)

Non-split supermanifolds

Abstract:

It is well known that the category of real Lie supergroups is equivalent to the category of the so called Harish-Chandra pairs. That means that a Lie supergroup depends only on the underlying Lie group and its Lie superalgebra with certain compatibility conditions. More precisely, the structure sheaf of a Lie group and the group morphisms can be explicitly described in terms of the corresponding Lie superalgebra. Hence, some geometric properties of homogeneous supermanifolds can be characterized in terms of Lie superalgebras. We will discuss necessary and sufficient conditions for a complex homogeneous supermanifold to be split, i.e. its structure sheaf is not isomorphic to $\bigwedge \mathscr{E}$, where $\mathscr{E}$ is a locally free sheaf. More precisely, the following theorem is proved (2010).

Theorem *Let $(G, \mathscr{O}_G)$ be a complex Lie supergroup with the Lie superalgebra $\mathfrak{g} = \mathfrak{g}_{\bar{0}} \oplus \mathfrak{g}_{\bar{1}}$. If $[\mathfrak{g}_{\bar{1}}, \mathfrak{g}_{\bar{1}}] = 0$, then all $(G, \mathscr{O}_G)$-homogeneous supermanifolds $(M, \mathscr{O}_M)$ are split.*

Conversely, if a complex homogeneous supermanifold $(M, \mathscr{O}_M)$ is split, then there is a Lie supergroup $(G, \mathscr{O}_G)$ with $[\mathfrak{g}_{\bar{1}}, \mathfrak{g}_{\bar{1}}] = 0$, where $\mathfrak{g} = \mathfrak{g}_{\bar{0}} \oplus \mathfrak{g}_{\bar{1}} = \mathrm{Lie}(G, \mathscr{O}_G)$ such that $(G, \mathscr{O}_G)$ acts on $(M, \mathscr{O}_M)$ transitively.

The second question, which we are going to discuss, is

How to find out, whether a complex supermanifold is split or non-split?

A method, suggested by A.L.Onishchik, is to study grading operators on the structure sheaf of a supermanifold. We are going to use this method for the flag supermanifolds introduced by Yu.I. Manin. Here we need the description of Lie superalgebra of vector fields on flag supermanifolds, given by A.L. Onishchik, A.A. Serov, E. Vishnyakova, V. Bunegina and others.

Note that from the results of a paper written by I. Penkov and I. Skornyakov (1983) it follows, that certain flag supermanifolds are non-split.

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Tuesday, March 15, 2011, at 17:00 **Campus Kirchberg, Room- B02**

Pierre Bieliavsky
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Rankin-Cohen brackets and $\mathfrak{sl}_2$-equivariant quantizations

Abstract:

Rankin-Cohen brackets are $\mathfrak{sl}_2$-equivariant bilinear maps on holomorphic modular forms. Twenty years ago, Zagier gave a combinatorial formula for a bilinear formal product in terms of Rankin-Cohen brackets that turns the space of modular forms into a formal noncommutative associative $\mathfrak{sl}_2$-module algebra. We will present a geometric approach to Zagier's product and some of its extensions in the context of (non-formal) quantization of $\mathfrak{sl}_2$-surfaces.