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Migrations, public goods and taxes*

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Abstract

This paper examines how and why people migrate between two regions with asymmetric size. The agglomeration force comes from the scale economies in the provision of local public goods, whereas the dispersion force comes from congestion in consumption of public goods. Public goods considered resemble club goods (or public goods with congestion) and people are heterogeneous in their migration costs.

We find that the large countries can be destination of migrants for sufficiently high provision of public goods, even when the large country taxes too much. The high provision of public good offsets the congestion effect. While, the small country can be the destination of migrants for two reasons. Firstly, when public good supply is intermediate, people move to avoid congestion in the large country and to benefit from low taxation in the small one. Finally, when the provision of public goods is low, people move towards the small countries just to avoid congestion.

Keywords: Migration, public goods, congestion.
Jel classification: H0; F3.

1 Introduction

In this contribution, we study the effects of individuals' migration decisions when these individuals have to arbitrate between the public facilities offered in their home country, and the psychological costs of moving from their home country to abroad. The problems raised by migration flows constitute a growing concern for today's governments. Migrations can create conflicting aspirations between the incumbent residents of a country and the new migrants: migrants are often viewed by the residents as populations enjoying national prerogatives while they are simultaneously perceived as not contributing enough to national richness, or steeling jobs to the local population. For example, migrants may

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create significant externalities on the incumbent residents through capital and labor markets and as well public finance channels (see Borjas, 1994, 1995, 1999, Razin and Sadka, 2004, Friedberg and Hunt, 1995, Leiner 1997, among others). Such feelings reinforce the natural tendency to hypernationalism, amplifying thereby the credit given to extremist political views. Furthermore, they make more difficult the cultural assimilation of migrants in their new environment. It is thus important to better understand the mechanisms underlying migration movements. In particular, one must examine whether migration flows have automatic stabilizers preventing the development of such conflicting aspirations between migrants and residents, and dampening the migration movements generated by the asymmetric richness among countries. Each region (or country) provides its residents with several advantages, like its network of roads, its social protection, its military organization for public defense, its education facilities. The production of these public goods is financed by levying taxes on the residents. When evaluating whether they should change their location, citizens in a given country balance the benefits expected from the public goods provided in this country with those derived from the existence of public goods in rival countries. Of course, this evaluation must also take into account the costs resulting from moving from one country to another, as well as the differentials in taxes to be paid, and in wages to be obtained, in each of them. The cost of moving abroad is not the same across the population of residents. Some of them are strongly attached to their relatives living in their residential area, while others are considerably more mobile, simply because they have weaker links with people living around them (for instance, see Beine *et al* (2009) for the role of diasporas on migration). National traditions, patriotism, and historical origins constitute significant values for some individuals, while they let others, -who feel like *citizens of the World*-, almost indifferent. Meteorological conditions can also influence individuals' moving costs: for some of them, the passage from a sunny to a humid country is like a cataclysm while, for others, it does not entail any disagreement at all! It follows that individuals, placed otherwise in similar situations, appear as heterogeneous in their willingness to move abroad to find better conditions in their economic environment.

Endowed with its power to levy taxes and provide public goods, the government plays a crucial role in the individual citizens' decisions about where to locate. Regions with a rich panoply of public goods generally exert a powerful attraction on the natives of countries (or regions) not so well endowed with public facilities. Individuals are attracted by richness, not only for private goods but as well for public facilities. For instance, a large fraction of migrations from the developing countries to those belonging to the European Community can probably be explained by the existence in the latter of a social protection and other social benefits which do not exist to such an extent in the former. Similarly, educational advantages of a country over another can drain a significant fraction of the young citizens' population from the latter to the former. As stated by Tiebout (1956), individuals respond to regional discrepancies in the economic environment by "voting with their feet".

Of course, the importance of the endowment in public facilities depends on the size of fiscal pressure, since expenses entailed by the provision of public goods have to be covered by the National Budget. The latter in turn is related to the size of the population: the larger the population in the country, the larger the tax base and the resulting product of taxes and, thus, the ability of the government to make many public facilities available to the population. Notice however that, beyond some level of use, public goods can also generate negative externalities for their users. As club goods, the consumption of public goods may suffer from congestion. When migration flows are not too excessive, they are attractive for the residents who enjoy the positive externality resulting from the increase in size in total population, and the resulting increase in the provision of public facilities. But, beyond a certain level of population, including migrants, the congestion cost resulting from migration may well overweigh the positive externality created initially by the population size effect.

Interestingly enough, the above complex relationships generate subtle interactions between the fiscal policies of different regions. Increasing the fiscal pressure imposed to the residents in a given country increases its National Budget and, thus, its potential endowment in public facilities. This in turn makes the country more attractive to the citizens of the other country and, as long as congestion effects do not take place, generate thereby migration movements from the latter toward the former. As a result, the relative size of the populations in the two countries is modified. Thus, the fiscal policy adopted in a region can serve to regulate migration toward that region, or *vice versa*. But the same reasoning applies *mutatis mutandis* to the other region: the fiscal policy adopted in the latter can create migration movements in the opposite direction. Clearly, this opens the door to strategic interaction between the countries since both of them share the power of influencing the size of migration flows between them. In this paper we are precisely interested in studying the migration movements resulting from these combined forces, in the framework of a stylized model embodying two countries.

The basic ingredients of the model are as follows. There are two countries, home H and foreign F, with country F being smaller in terms of its population. Citizens can freely move from one country to the other. Residents are heterogeneous because they incur different migration costs. Each resident compares the level of utility obtained at home with the utility obtained abroad. The first term of this comparison writes as the sum of the utility derived from the consumption of home public goods and disutility resulting from home taxes levied on home wages. The second term consists of the same magnitudes, but applied now to foreign conditions: utility derived from the consumption of foreign public goods and disutility resulting from foreign taxes levied on foreign wages. However, this second term must also include the migration cost since the resident has now to migrate abroad in order to benefit from the corresponding foreign conditions. Individuals' utility is assumed to be initially increasing in the amount of public good. However, as club goods, the consumption of public goods suffers from congestion. This means that, up to a certain extent of migration flows, public goods are viewed as attractive. But, beyond a certain size of population, including migrants, congestion costs start to overweigh the positive effects of migration, which affects negatively the utility of the citizens. Countries set simultaneously the amount of public goods, which is assumed to be proportional to population size, or to the product of taxes, to the extent that public goods are assumed to be produced with a linear technology (constant returns to scale). With this respect, the size of the constant returns parameter, called hereafter the *public good multiplier*, is an important ingredient of the model since it determines the rate at which the product of taxes is transformed into an amount of public good. Countries are assumed to play a two stage game. In the first stage, each government is assumed to set its tax rate. It is supposed to maximize tax revenue, taking into consideration the possible migration flow initiated as a consequence of its fiscal pressure. In the second stage, residents in each country decide whether to stay in their own country or to migrate.

The analysis of the subgame perfect equilibrium of the above two stage game leads to the following conclusions. First, a country can experience an inflow of migrants if and only if the size of its population is strictly smaller than the size of the population in the rival country. Furthermore, the larger its initial size, the larger the inflow. Furthermore, this country sets a higher or lower tax rate than its rival whenever the public good multiplier exceeds or not some threshold value.

Another outcome of this analysis follows from the comparison of the countries' sizes resulting from migration in the above game, with the size which would be optimal from the viewpoint of the citizens in their country, given the size existing before the migration had taken place. When the initial size of the country is very small, migration entails that its population suffers from congestion. Accordingly one must expect that the optimal desired size of the country would be smaller than at equilibrium. This is indeed the case because the Government ignores this fact when maximizing tax revenue against the rival country. For intermediate sizes, the welfare optimal size can be larger or smaller than the one achieved at the subgame perfect equilibrium, according to the relative magnitude of the public good multiplier.

Our paper creates a bridge between two strands of literature. On the one hand, our paper shares with previous papers (see for instance, Brueckner, 2000, Burbidge and Myers, 1994, or Wilson, 1997), the idea that migrating people can create congestion in the use of local public goods. However, our focus is not only on the effect of migration on the population of the destination country. In fact, due to the strategic interaction between the two countries in their attempt to attract mobile factors of production, the present paper also shares some features with the large body of literature devoted to tax competition among jurisdictions of asymmetric size, like in Bucovetsky, 1991, Wilson, 1991, Kanbur and Keen, 1993, and Trandel, 1994. We take a tax competition perspective to analyze migration, and we introduce the level of public good as a strategic choice for attractiveness of the country as done for capital attractiveness in Thisse and Ypersele, 2002, or Zissimos and Wooders, 2008. Thus, differently from Brueckner, 2000, Burbidge and Myers, 1994, or Wilson, 1997, we highlight the role played by the size of the countries on defining their strategies of tax and public goods levels and, thus, ultimately on defining migration flows.

The paper is organized as follows. Section 2 describes a general setup of the model with two countries. Section 3 represents the same model with specific ingredients of utility function including private-public consumption. Section 4 analyzes migration when it takes place from home to foreign country, while Section 5 analyzes migration when it proceeds in the opposite direction. The analysis in Sections 4 and 5 are performed assuming the tax competition of Leviathan government which reshapes population. In Section 6, we examine what is the desirable size of the population in the receiving country from a welfare perspective. We end up with a short conclusion.

2 The general setup

Consider two countries, H and F, of uneven population size, whose governments impose taxes on residents and supply each a public good. The population in each country is distributed over *types* and the set of types in each country is represented by the $[0, 1]$ interval with different densities: λ_h in H and λ_f in F, $\lambda_h \neq \lambda_f$. The heterogeneity of population is due to the individual level of home attachment. As a consequence, a citizen of country i located at position x, faces a migration cost x moving to country j. Clearly, similarly to the population, this cost is uniformly distributed over the support interval $[0, 1]$. Types are ranked by increasing order of disutility incurred from migration.

Consider a timeline of two periods. In the first period, people belong to a origin country. In the second period, people may decide to migrate[1].

Let us normalize the world population to 1. Then, country's H population λ_h in the first period is given by $[0, \lambda_0]$ where the subscript 0 indicates the time period. Then the population λ_F of country F is simply $(\lambda_0, 1]$. Given tax rates t_h, t_f, the individual of type x from country H obtains a utility level u_h if he/she decides to stay equal to $u_h = u(t_h, \lambda_h)$,where this utility function is twice differentiable and combines private and public consumption. Moreover $\frac{\partial u(t_h,\lambda_h)}{\partial t_h} < 0$ while $\frac{\partial u(t_h,\lambda_h)}{\partial \lambda_h}$ can assume any sign depending on the level of λ_h. If $\frac{\partial u(t_h,\lambda_h)}{\partial \lambda_h} > 0$, then the congestion effect of migrants in the consumption of public goods is dominated by the positive effect of higher tax revenue brought by migrants. When instead $\frac{\partial u(t_h,\lambda_h)}{\partial \lambda_h} < 0$, the reverse holds true. If he/she decides to migrate, this utility level (denote it by U_h) obtains as $U_h = u(t_f, \lambda_f) - x$, with $\frac{\partial u(t_f,\lambda_f)}{\partial t_f} < 0, \frac{\partial u(t_f,\lambda_f)}{\partial \lambda_f} \lesseqgtr 0$. It follows that a strictly positive migration flow takes place from country H to country F if, and only if, the set of types $\{y/y \leq x\}$ where x solves the equation $u_h = U_h$, i.e.

$$x = u(t_f, \lambda_f) - u(t_h, \lambda_h) \quad (1)$$

[1]This assumption marks a difference of our paper with that of Tiebout (1956).While we assume an initial exogenous allocation of people in countries, and study how this allocation changes via migration flows, Tiebout (1956) does not assume the existence of an initial allocation of people among the regions, but studies how people, which are initially not identified to belong to a region, decide to allocate among them.

has strictly positive measure. Population of countries in the second period, denoted by λ_h^1 and λ_f^1, after a migration from country H to country F has taken place, are given by

$$\lambda_f^1 = \lambda_f + x\lambda_h \quad (2)$$
$$\lambda_h^1 = (1-x)\lambda_h \quad (3)$$

When the governments take into account the future redistribution in the population resulting from the use of their tax strategies, it gives rise to a tax game between the two countries, with tax rates strategies t_h and t_f, and payoffs corresponding to the income tax revenues of the governments resulting from the reshuffling in the population which follows from the use of these tax rates. Assuming tax revenue maximizing governments, governments decide non/cooperatively the taxes (t_h, t_f). Their choice is obtained by solving the following problem

$$\max_{t_h} \Pi_h(t_h, t_f) = (1 - (u(t_f, \lambda_f) - u(t_h, \lambda_h)))\, \lambda_h t_h \quad (4)$$
$$\max_{t_f} \Pi_f(t_h, t_f) = (\lambda_f + (u(t_f, \lambda_f) - u(t_h, \lambda_h))\, \lambda_h\,)t_f,$$
$$s.t. x, t_h, t_f > 0. \quad (5)$$

The first order conditions write as:

$$\frac{\partial \Pi_h(t_h, t_f)}{\partial t_h} = \lambda_h - \lambda_h \left[u(t_f, \lambda_f) - u(t_{h,}\lambda_h) - t_h \frac{\partial u(t_{h,}\lambda_h)}{\partial t_h} \right] = 0 \quad (6)$$
$$\frac{\partial \Pi_f(t_h, t_f)}{\partial t_f} = \lambda_f + \lambda_h \left[t_f \frac{\partial u(t_{f,}\lambda_f)}{\partial t_f} + u(t_f; \lambda_f) - u(t_{h,}\lambda_h) \right] = 0$$

To ensure uniqueness of the equilibrium and an interior solution of equilibrium tax rates, we shall impose the following second order conditions

$$\frac{\partial^2 \Pi_h(t_h, t_f)}{\partial t_h} < 0 \text{ and } \frac{\partial^2 \Pi_f(t_h, t_f)}{\partial t_f} < 0$$

which boil down to

$$2\frac{\partial u(t_{i,}\lambda_i)}{\partial t_i} + t_i \frac{\partial^2 u(t_{i,}\lambda_i)}{\partial t_i} < 0, \; i = h, f.$$

Note that when the utility function is linear with respect to the amount of taxes, $\frac{\partial^2 u(t_{i,}\lambda_i)}{\partial t_i} = 0$, the condition for an interior solution is simply

$$\frac{\partial u(t_{i,}\lambda_i)}{\partial t_i} < 0.$$

The solution of the system of (6) reveals the equilibrium taxes :

$$t_h^*(\lambda_h, \lambda_f) = \frac{(u_h(\lambda_h) - u_f(\lambda_f))\,\lambda_h - \lambda_f}{\lambda_h \frac{\partial u(t_f, \lambda_f)}{\partial t_f}}$$

$$t_f^*(\lambda_h, \lambda_f) = \frac{u_f(\lambda_f) - u_h(\lambda_h) - 1}{\lambda_h \frac{\partial u(t_h, \lambda_h)}{\partial t_h}}$$

and the corresponding equilibrium migration flow as :

$$x^*(t_h^*, t_f^*) = u(t_f^*, \lambda_f) - u(t_h^*, \lambda_h)$$

whenever $0 < x^*(t_h^*, t_f^*) < 1$ and equilibrium taxes are nonnegative and interior to the admissible interval $[0, 1]$.

3 The model

In this section, we proceed by describing the more specific model used in order to get a closed form, and to understand better the mechanics underlying the process of migration. There are two goods in the economy, produced with labor as unique input. Each citizen is endowed with one unit of labor serving as input for the production of both the public and the private good. The government owns a linear technology, used for the production of the public good, $\Phi(z) = kz$, $k < 1$, with z denoting the amount of input which we assume to be equal to the amount of labor made available to the government. As for the production of the private good, citizens have access to a technology Ψ which transforms one unit of labor into one unit of the private good, i.e $\Psi(z) = z$. The labor used by the government in the production of the public good is obtained by diverting some fraction $t_i, i = H, F$, from each individual's labor endowment, which we called hereafter the tax imposed in country i. The government pays each worker in units of the private good, at its marginal product, namely, kt.The private good is also produced by each individual using the fraction $1 - t_i$ of labor remaining available after taxation. Accordingly, at the end of the production process, each individual in country i is endowed with a bundle of goods equal to $(kt_i + (1 - t_i), k\lambda_i t_i)$ where the first component corresponds to the amount of the private good, received from the government as a salary and the one individually produced, and the second to the amount of the public good produced and made available by the government.

Moreover, we assume that, corresponding to this bundle, each citizen receives a utility level equal to

$$kt_i + (1 - t_i) + \left[k(1 - \lambda_i)t_i - (1 - \lambda_i)^2\right] : \qquad (7)$$

the two first terms of this expression refer to the utility obtained from consuming the private good while the term $k(1 - \lambda_i)t_i - (1 - \lambda_i)^2$ measures his/her utility when consuming the public good produced in country i, $(k\lambda_i t_i)$. Thus, we assume that utility is linearly decreasing in the amount of taxes and additive between the utility of the private good and the utility of the public good. Furthermore, while the utility is always increasing with the size of the population λ_i when kt_i is larger than 2, while it may well decrease in the opposite case. This is due to the term λ_i^2 which represents a negative externality arising from congestion when the public good is rivalrous. In the case of two countries, residents may initially benefit from additional migrants because of their additional contribution to the amount of public good produced, but migration can also bring congestion costs when the public good is rivalrous.

Finally, in this specification of the general model, our analysis is restricted to a level of size asymmetry between the two countries guaranteeing the positivity of taxes at equilibrium, namely, we assume that $\lambda_0 < \frac{1}{4}$. The reason of this restriction is that, when, countries are too much alike (so that λ_0 is close to $\frac{1}{2}$), the congestion in the use of the public good due to population movements is very similar in the two countries. Therefore, the main rationale for migration is based on tax purposes, making tax competition so harsh that taxes are driven down to zero. Assuming a sufficient asymmetry makes that congestion matters in the migration process, thus allowing us to study the variety of policy mixes of governments when countries are really different in terms of populations' size[2].

In order to identify the Nash equilibria of this tax game according to the level of the public good multiplier k, we need to assume *ex-ante* their existence according to three possible cases: when there is a strictly positive migration from the large country to the small one, a strictly positive migration from the small country to the large one, and no migration. We start with the first case.

4 Migration from H to F

Given tax rates t_h, t_f, and total income, the individual of type x from country H obtains a utility level U_h if he/she decides to stay equal to (7) while, if he/she decides to migrate, her utility is $U_h^{'} = kt_f + (1 - t_f) + \left[k\lambda_0 t_f - \lambda_0^2\right] - x$. It follows that a strictly positive migration flow takes place from country H to country F if, and only if, the set of types $\{y/y \leq x\}$ where x solves the equation $U_h' = U_h$, i.e.

$$x = \max\left\{\left(t_h - t_f + k\lambda_0\left(t_f + t_h\right) + k\left(t_f - 2t_h\right) + 1 - 2\lambda_0\right); 0\right\} \tag{8}$$

has positive measure. To examine when this condition is satisfied, let us evaluate the sign of x at a Nash equilibrium, assuming initially its existence. The size of the migration flow depends on the difference between taxes and on the size asymmetry of populations, which affects the utility from consuming the public good. Population in country F next period, when a migration from country H to country F has taken place, is given by

$$\lambda_1(\lambda_0) = \lambda_0 + x(1 - \lambda_0). \tag{9}$$

[2] The model is easily solved in absence of such an assumption.

Substituting x, as in (8), in (9), we get

$$\lambda_1(\lambda_0) = \lambda_0 + (1-\lambda_0)\left(t_h - t_f + k\lambda_0\left(t_f + t_h\right) + k\left(t_f - 2t_h\right) + 1 - 2\lambda_0\right). \quad (10)$$

Now consider the tax game between countries H and F, with strategies t_h and t_f, and payoffs $\Pi_h(t_h, t_f) = (1-\lambda_1(\lambda_0))t_h$ and $\Pi_f(t_h, t_f) = \lambda_1(\lambda_0)t_f$ for countries H and F, respectively. We assume that each government maximizes its payoff given the tax strategy of its opponent, taking into account the future redistribution in the population resulting from the use of the tax strategies. To identify a Nash equilibrium of this game (if it exists), we must solve the maximization problem :

$$\begin{aligned} \max_{t_h} \Pi_h(t_h, t_f) &= (1-\lambda_1(\lambda_0))t_h && (11) \\ \max_{t_f} \Pi_f(t_h, t_f) &= \lambda_1(\lambda_0)t_f, \\ s.t. m_h, m_f, x, t_h, t_f &> 0. && (12) \end{aligned}$$

In the coming propositions and in Appendix 2, we show that there exist two types of equilibria in which people migrate from H to F. Such equilibria are unique and correspond to two separated intervals of values for the public good multiplier k. In the first type equilibrium, the level of k is *relatively low*, namely, $0 < k < \frac{2\lambda_0^2 - 5\lambda_0 + 2}{5\lambda_0^2 - 11\lambda_0 + 5}$, and the net income to citizens after taxation is positive. While in the other type of equilibrium, the level of k lies in the interval $\frac{2\lambda_0^2 - 5\lambda_0 + 2}{5\lambda_0^2 - 11\lambda_0 + 5} < k < \frac{1}{2-\lambda_0}$ and in the home country the net of income to citizens after taxation is equal to zero, while in the foreign country it remains strictly positive. We proceed in detail here for the analysis of the first type of equilibrium, and send the analysis of the other type in Appendix 2.

Substituting (10) into (11), we get the equilibrium of the tax game, whenever it exists. Second order conditions for an interior maximum require that $k < \frac{1}{1+\lambda_0}$ and $k < \frac{1}{2-\lambda_0}$. Notice that, due to the assumption $\lambda_0 < \frac{1}{4}$, $k < \frac{1}{2-\lambda_0}$ hence $k < \frac{1}{1+\lambda_0}$. In this range of values of k, solving (11), we easily obtain the tax rates t_h^* and t_f^*, respectively, which are given by:

$$\begin{aligned} t_h^* &= \frac{\left(2\lambda_0^2 - 2\lambda_0 - 1\right)}{3\left(k\lambda_0 - 2k + 1\right)\left(\lambda_0 - 1\right)}, && (13) \\ t_f^* &= \frac{2\left(\lambda_0^2 - \lambda_0 + 1\right)}{3\left(k + k\lambda_0 - 1\right)\left(\lambda_0 - 1\right)}. \end{aligned}$$

The resulting equilibrium flow of migrants from H to F obtains as:

$$0 < x^* = \frac{\left(\lambda_0 - 2\right)\left(2\lambda_0 - 1\right)}{3\left(1 - \lambda_0\right)} < 1$$

It is easy to check that $0 < x^* < 1$ for any admissible λ_0.[3] In Appendix 1, we show the conditions under which the income levels at equilibrium are positive.

[3] $1 > x^* \Leftrightarrow 1 - \frac{(\lambda_0-2)(2\lambda_0-1)}{3(1-\lambda_0)} = \frac{1}{3}\frac{-2\lambda_0+2\lambda_0^2-1}{\lambda_0-1} > 0$ Q.E.D.

The resulting population λ_1^* at equilibrium in the foreign country is thus equal to $\lambda_1^* = \lambda_0 + x^*(1-\lambda_0)$.

Hence,

Proposition 1 *There exists a unique Nash equilibrium of the tax game where the smaller country experiences a strictly positive inflow of migrants if and only if the public good multiplier is relatively low.*

Proof. In this range of k both maximization problems given by (11) are concave. ■

The above proposition identifies a threshold on k under which the equilibrium migration flow takes place from the large country to the small one because the level of the public good multiplier is low and, consequently, the congestion in using the public good in the large country is higher than the congestion in the small one. Does it imply that the small country is able to tax more than the large country because people escape congestion? To answer this question we compare the level of equilibrium taxes (13) :

Proposition 2 *The smaller country sets a strictly higher (resp. lower) tax than its rival whenever the public good multiplier k satisfies the inequality* $0 < k < \frac{1-2\lambda_0}{3(1-\lambda_0)}$ *(resp.* $\frac{1-2\lambda_0}{3(1-\lambda_0)} < k < \frac{2\lambda_0^2-5\lambda_0+2}{5\lambda_0^2-11\lambda_0+5}$*).*

Proof. Comparing t_h^* and t_f^*,we observe that $t_h^* > t_f^*$ iff $\frac{1-2\lambda_0}{3(1-\lambda_0)} < k$. Note that $\frac{1-2\lambda_0}{3(1-\lambda_0)} < \frac{1-\lambda_0-2\lambda_0^2}{2\lambda_0-5\lambda_0^2+1}$.Consequently, in the constellation of parameters where a unique Nash equilibrium of the tax game exists, namely $0 < k < \frac{2\lambda_0^2-5\lambda_0+2}{5\lambda_0^2-11\lambda_0+5}$, the tax levied by the smaller country can be smaller but also higher than the tax levied by the larger country. Q.E.D. ■

Thus, the answer to the above question is positive: in some range of k values the small country imposes higher taxes than the large country taking advantage from the fact that people escape from congestion in H.

The above proposition describes two scenarios, where a migration flow takes place from H to F, but the driving forces of the flow are different. When the public good multiplier is quite small, the likeliness of high congestion is high. Indeed, for this range of k, citizens quit the large country H, because they avoid the congestion in using the public good. This is true no matter the tax advantage supplied in the small country. But when the public good multiplier increases, the congestion and the public good effect start to offset each other. Thus, the small country can be attractive for tax reasons. Consequently, citizens quit H to take advantage of low taxes in F. This result marks a difference of our paper with the tax competition literature as Bucovetsky (1991), Wilson (1991), Kanbur and Keen (1993) and Trandel (1994), who find that small countries are systematically undercutting taxes in order to attract capital or mobile labor. In these papers, congestion is absent and, therefore, the capital flight or labor movements are only due to tax differences.

In Appendix 2, it is shown that outside the range mentioned above, namely $\frac{2\lambda_0^2-5\lambda_0+2}{5\lambda_0^2-11\lambda_0+5} < k < \frac{1}{2-\lambda_0}$, the smaller country experiences a strictly positive inflow of migrants and sets a strictly lower tax at equilibrium (interior solution) than its rival (corner solution). Intuition of that statement is a continuation of proposition 1, as equilibrium taxes given above are increasing functions of k: higher levels of the public good multiplier give the possibility for the countries to tax more. Especially the bigger country benefits from a higher provision of the public good, so that its equilibrium tax reaches the upper bound faster, at which net income starts to become not negative.

5 Migration from F to H

In this section, we start from the assumption that there exists a positive flow from F to H with the equilibrium taxes, when $k \geq 1/(2-\lambda_0)$. After proving the uniqueness of the equilibrium in the unique range $k < 1/(2-\lambda_0)$, naturally we can state that the analysis, when applied to the range $k \geq 1/(2-\lambda_0)$, will identify the equilibria when a migration takes place from F to H or when no migration takes place. Similarly to the above section, given taxes t_h, t_f, the individual of type x from country F obtains a utility level $U_f = t_f(k-1) + 1 + \lambda_0(kt_f - \lambda_0)$ if he/she decides to stay while, if he/she decides to migrate, this utility level (denote it by U_f') obtains as $U_f' = t_h(k-1) + 1 - x + (kt_h - 1 + \lambda_0)(1-\lambda_0)$

It follows that a strictly positive migration flow takes place from country F to country H if, and only if, the set of types $\{y/y \leq x\}$ where x solves the equation $U_f = U_f'$, i.e.

$$x = \max\left\{(t_f - t_h + 2\lambda_0 - kt_f + 2kt_h - kt_f\lambda_0 - kt_h\lambda_0 - 1)\,;0\right\}.$$

The size of the migration flow depends again on the difference in taxes and on the size asymmetry of the populations. Similarly to the above section, governments' maximization problems are

$$\begin{aligned} \max_{t_h} \Pi_h(t_h, t_f) &= (1-\lambda_1(\lambda_0))t_h && (14) \\ \max_{t_f} \Pi_f(t_h, t_f) &= \lambda_1(\lambda_0)t_f && (15) \\ s.t. m_h, m_f, x, t_h, t_f &\geq 0. \end{aligned}$$

Notice that the problem faced by the government in country F is strictly concave in the set $k < \frac{1}{1+\lambda_0}$. Therefore, in this set, we can use the FOC to identify its best reply. The problem faced by the government of country H is convex when $k > \frac{1}{2-\lambda_0}$. Therefore, the tax in this country reaches its maximum at the level of tax at which net income is still non negative. Similarly to the above section we develop the full analysis of the types of equilibria in Appendix 3. As above, there exist two types of equilibrium taxes corresponding to different levels of k.

Here, we develop the scenario where at least one of the equilibrium taxes is an interior solution. The constellation of parameters where this equilibrium arises is $\frac{1-2\lambda_0}{2-5\lambda_0} < k < \frac{1}{1+\lambda_0}$. We call the levels of k belonging to this interval the *relatively high* public good multiplier's values. In this scenario, solving (14), we easily obtain the candidates equilibrium taxes t_h^* and t_f^*, given by

$$\begin{aligned} t_h^* &= \frac{1}{1-k} \\ t_f^* &= \frac{3k\lambda_0 + 3 - 4k - 2\lambda_0}{2\left(k + k\lambda_0 - 1\right)\left(k-1\right)}. \end{aligned} \tag{16}$$

The resulting equilibrium flow of migrants from H to F obtains as:

$$x^* = \frac{3k\lambda_0 - 2\lambda_0 - 2k + 1}{2k - 2} > 0.$$

The resulting population λ_1^* at equilibrium in the foreign country is thus equal to $\lambda_1^* = \lambda_0 \left(1 - x^*\right)$.

In Appendix 3, we show that the level of incomes are non negative at equilibrium. Hence,

Proposition 3 *When the public good multiplier is relatively high, the bigger country experiences a strictly positive inflow of migrants and sets a strictly higher tax (corner solution) than its rival (interior solution).*

Proof. Provided the initial assumption of this section i.e. $k \geq \frac{1}{2-\lambda_0}$, second order condition for the home country's maximization problems violates and for the foreign one it requires $k < \frac{1}{1+\lambda_0}$ to be satisfied. ■

The driving force for the migration flow from F to H is the public good effect: When k is large, the positive effect of the consumption of the public good dominates both the tax and the congestion effects. Therefore, consumers quit F and leave towards the large country H to consume the public good.

When the public good multiplier is relatively high and therefore positive externalities from tax collection is high, both countries try not only to keep their residents, but also to attract immigrants. In this case, the small country is disadvantaged, but it tries however to keep its residents by setting lower taxes.

From the above analyses, it follows that migration flows are driven by three major forces: the level of public good, the level of taxes and the congestion level. For a high provision of public facilities, the benefit obtained from the public good consumption overweighs the disutility from congestion and high taxes. Due to its larger amount of public goods, the larger country has the power to attract migrants even though it taxes more than the smaller country: the benefit from public goods' overweighs the disutility from congestion and higher tax. When the provision of public good consumption is moderate, the small country has advantage attracting migrants by levying lower taxes. Finally, when the level of public good consumption is sufficiently low, the congestion effect dominates the benefits obtained from public good consumption and even lower taxes, so that the small country now becomes the destination place for migrants coming from the larger country.

To conclude, it is interesting to notice that in a different framework, Hindriks et al. (2008) find that the small country can win the competition for mobile capital by supplying a more appealing environment for capital through public investments rather than only be competitive in terms of capital taxation.

6 Optimal size of countries

When the size of populations is driven by competition between Leviathan governments, the resulting countries' size can be far from being optimal from the perspectives of the hosting population's welfare.

When considering the equilibrium size of countries, we observe the trade-off between benefits and costs from a larger size. The cost *per capita* of many public goods is lower in larger countries, as more taxpayers pay for them, and the benefit of the larger country increases up to the point of congestion. Alesina and Spolaore (1997) have calculated the optimal number and size of countries with various states of the governments. Here we do not have such an ambition. We simply intend to find the optimal relative distribution of populations. To do so, define λ_1^d the population level which would maximize the welfare of the foreign country, given the equilibrium taxes we have identified above. We are led to compare λ_1^d with the population λ_1^* resulting from the equilibrium taxes. We want to see whether the population level corresponding to the choice of the governments coincides or departs from the population level in the country that maximizes the utility of the representative resident.

The utility obtained by the representative individual living in country F obtains as :

$$U_F = 1 - t_f + (kt_f - \lambda_1)\lambda_1$$

which has to be maximized with respect to the size λ_1.Solving the first order condition, we get the optimal welfare size λ_1^d for the foreign country, namely

$$\lambda_1^d = \frac{1}{2}kt_f^*(k).$$

For tractability, we demonstrate how the equilibrium size is different from the optimal welfare size of the foreign country within two intervals of the public good multiplier's values when taxes are kept at their corresponding equilibrium values.

Proposition 4 *When the migration flow takes place from H to F, then $\lambda_1^* \geq \lambda_1^d$. By contrast, when the migration flow takes place from F to H, then $\lambda_1^* < \lambda_0^d$ if $k < \frac{3-2\lambda_0}{4-3\lambda_0}$, otherwise $\lambda_1^* > \lambda_0^d$.*
Proof. *See Appendix 4.* ■

When migration flow takes place from H to F, residents of the small country suffer immigration flows due to congestion. But the government ignores this fact when it maximizes tax revenue rather than the welfare of its natives.

When migration flow takes place from F to H, two scenarios are possible:

(i) When $k < \frac{3-2\lambda_0}{4-3\lambda_0}$, the population of the small country prefers a smaller outflow than the government does in order to avoid the decrease in its population which would entail a reduction in the amount of public good available. But, due to tax competition, the government in country F is challenged by the larger rival country.

(ii) When $k > \frac{3-2\lambda_0}{4-3\lambda_0}$, the population in country F would desire a smaller size than the one reached under tax competition because the higher k the higher tax rates in the tax competition setup. While, according to the desired level of population by people, the increase of k leads to a decrease of tax rates. Due to this lower level of tax rates people prefer a lower level of population than the government does.

7 Concluding remarks

This paper investigates the relationship between local public good consumption and migratory flows. Assuming that the local public good level is attractive for foreign citizens and not only for the native population in a country, we show how the differences of the public good level provided in different countries determine the direction and size of migration flows among them. We use a two-country model, where the decision of residents of the large or the small country about where to locate, is the result of the trade off between public good and private good consumption net of taxes and the cost of moving abroad. Countries provide a public good with congestion and set income taxes in order to maximize tax revenue.

The main results of the paper sort out three major scenarios: *(i)* the large country experiences immigration by supplying a high level of public good even though taxes are higher than the small country. This is the advantage of being large; *(ii)* the small country experiences immigration because it offers low income taxes supplying a moderate level of public good. And, finally, *(iii)* the small country experiences immigration supplying a low level of public good and taxing more than the large country. In this case, migrants escape congestion in public good consumption in the large country. This is the benefit of being small.

High levels of public good provision may guarantee strong positive networks externalities and have cumulative effects. Moreover, the large country taking advantage of its size can afford high taxes in order to increase tax revenue needed for the provision of public goods. This is the process of *agglomeration,* as defined in the economic geography literature (see Fujita and Thisse, 2002). While, small countries can attract migrants (or keep home their residents) by fixing small taxes. Empirically, it is proved that migrants do react on tax differences when choosing their location (Liebig and Sousa-Poza, 2005).

The scenario we want to emphasize is the case where the congestion effect defines the direction of migrants (scenario *(iii)*). In this case, the small country is neither attractive for public facilities nor for small taxes, but it takes advantage from the level of congestion in public goods into the large country. These dynamics correspond to a small value of the public good multiplier. For instance, the public good multiplier can be small due to an inefficient management of public money, which translate into low level of public services and high congestion in the large country.

Interestingly enough, a parallelism can be made between our paper and the analysis of network goods with congestion. For example, consider the case of a mobile operator company. While there are clear benefits from having a large network of consumers, it may damage the quality of the service due to congestion. Some consumers might end up preferring a smaller network mobile operator even paying more, because they read the signal of high price and small network as a signal of higher quality service, image and luxury.

Finally, we want to point out that our theoretical study does not account for alternative reasons of migration, cultural differences, diaspora effects or family ties [4]. It would be a natural further step to this paper to analyze the consequences of such alternative causalities on migration flows. Another future path of research would consist in discriminating among public goods with the aim of identifying which type of public good attracts which type of migrants. For example, it is believed that high skilled (educated) individuals have lower mobility costs and tend to locate in countries where there is a good education system and a significant cultural environment. This study would allow a more refined analysis of migrants' motivations and the impact of governments' policies on migration flows.

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Appendices

Appendix 1

This appendix refers to the Section 4, where we show the nonnegativiness of equilibrium income levels when k is satisfies the following range
$0 < k < \frac{-\left(5\lambda_0 - 2\lambda_0^2 - 2\right)}{\left(5\lambda_0^2 - 11\lambda_0 + 5\right)}$ and equilibrium taxes are give by (13), equilibrium m_h^* writes as:

$$m_h^* = \frac{1}{3}\frac{\left(5\lambda_0^2 - 11\lambda_0 + 5\right)k + \left(5\lambda_0 - 2\lambda_0^2 - 2\right)}{\left(\lambda_0 - 1\right)\left(-2k + k\lambda_0 + 1\right)}$$

From the SOC recall that, $k\lambda_0 - 2k + 1 > 0$, so $m_h^* > 0$ if
$\left(5\lambda_0^2 - 11\lambda_0 + 5\right)k + \left(5\lambda_0 - 2\lambda_0^2 - 2\right) < 0$. The latter condition writes as:

$$k < \frac{2\lambda_0^2 - 5\lambda_0 + 2}{5\lambda_0^2 - 11\lambda_0 + 5} \tag{17}$$

Equilibrium m_f^* becomes:

$$m_f^* = \frac{\left(5\lambda_0^2 - 2\lambda_0 - 1\right)k + \left(1 - \lambda_0 - 2\lambda_0^2\right)}{3\left(k + k\lambda_0 - 1\right)\left(\lambda_0 - 1\right)}$$

From the SOC recall that, $k + k\lambda_0 - 1 < 0$, so
$m_h^* > 0$ if $\left(5\lambda_0^2 - 2\lambda_0 - 1\right)k + \left(1 - \lambda_0 - 2\lambda_0^2\right) > 0$. The latter condition writes as:

$$k < \frac{1 - \lambda_0 - 2\lambda_0^2}{2\lambda_0 - 5\lambda_0^2 + 1} \tag{18}$$

Notice that $\frac{2\lambda_0^2 - 5\lambda_0 + 2}{5\lambda_0^2 - 11\lambda_0 + 5} \leq \frac{1 - \lambda_0 - 2\lambda_0^2}{2\lambda_0 - 5\lambda_0^2 + 1}$, which means that the condition (7) guarantees non negativeness of net taxed incomes in the both country. Q.E.D

Appendix 2

This appendix proves the statement in Section 4 on existence of a strictly positive inflow of migrants in the smaller country in the following range of the public good multiplier $\frac{2\lambda_0^2-5\lambda_0+2}{5\lambda_0^2-11\lambda_0+5} < k < \frac{1}{2-\lambda_0}$.

In this range of k, both maximization problems given by (11) are concave, as we have seen from the proof of Proposition 1, when $k < \frac{2\lambda_0^2-5\lambda_0+2}{5\lambda_0^2-11\lambda_0+5}$ violates, m_h becomes negative, in this case t_h solves the condition for non negativeness of the net income (maximum possible tax at which the income is not negative).

$$t_h^* = \frac{1}{1-k}$$

Provided that $k < 1$, then t_h^* is strictly positive.

The reaction function for the equilibrium tax rate of th foreign country is:

$$t_f = \frac{2\lambda_0 - t_h - 2\lambda_0^2 + 2kt_h + \lambda_0 t_h - 3k\lambda_0 t_h + k\lambda_0^2 t_h - 1}{2(1-\lambda_0)(k+k\lambda_0-1)}$$

The candidate equilibrium tax is:

$$t_f^* = \frac{1}{2}\frac{\left(3\lambda_0^2 - 5\lambda_0 + 3\right)k + \left(3\lambda_0 - 2\lambda_0^2 - 2\right)}{(\lambda_0-1)(k+k\lambda_0-1)(k-1)}$$

Recall from the SOC that, $k + k\lambda_0 - 1 < 0$, so $t_f^* > 0$ iff $\left(3\lambda_0^2 - 5\lambda_0 + 3\right)k + \left(3\lambda_0 - 2\lambda_0^2 - 2\right)$. Here we use the constraint condition $k < \frac{1}{2-\lambda_0}$, which gives us:

$\left(3\lambda_0^2 - 5\lambda_0 + 3\right)k + \left(3\lambda_0 - 2\lambda_0^2 - 2\right) < \left(3\lambda_0^2 - 5\lambda_0 + 3\right)\frac{1}{2-\lambda_0} + \left(3\lambda_0 - 2\lambda_0^2 - 2\right) =$

$= (\lambda_0 - 1)\frac{2\lambda_0 - 2\lambda_0^2 - 1}{\lambda_0 - 2} < 0$.Q.E.D

Nonnegativity of income: We plug the tax rate calculated above in the income function of the foreign country, and we get:

$$m_f^* = \frac{1}{2}\frac{\left(5\lambda_0^2 - 5\lambda_0 + 1\right)k + \left(\lambda_0 - 2\lambda_0^2\right)}{(\lambda_0-1)(k+k\lambda_0-1)} \tag{19}$$

where $m_f > 0$ iff $\left(5\lambda_0^2 - 5\lambda_0 + 1\right)k + \left(\lambda_0 - 2\lambda_0^2\right) > 0$. This is always true as $\lambda_0 < \frac{1}{4}$.Q.E.D

After checking the positiveness of the equilibrium net incomes, we check the validity of an initial assumption on positivity of the equilibrium flow:

$$x^* = \frac{1}{2}\frac{\left(3\lambda_0^2 - 7\lambda_0 + 3\right)k + \left(5\lambda_0 - 2\lambda_0^2 - 2\right)}{(1-\lambda_0)(k-1)} \tag{20}$$

where $x^* > 0$ iff $\left(3\lambda_0^2 - 7\lambda_0 + 3\right)k + \left(5\lambda_0 - 2\lambda_0^2 - 2\right) < 0$. Recall that $k < \frac{1}{2-\lambda_0}$ and $\lambda_0 < \frac{1}{4}$, so we write:

$\left(3\lambda_0^2 - 7\lambda_0 + 3\right)k + \left(5\lambda_0 - 2\lambda_0^2 - 2\right) < (\lambda_0 - 1)\frac{4\lambda_0 - 2\lambda_0^2 - 1}{\lambda_0 - 2} < 0$. Q.E.D

Note that migration is not full, which means that $x^*$is less than 1 for any feasible value of k :
$1-x^*=1-\frac{1}{2}\frac{\left(3\lambda_0^2-7\lambda_0+3\right)k+\left(5\lambda_0-2\lambda_0^2-2\right)}{(1-\lambda_0)(k-1)}=\frac{1}{2}\frac{\left(3\lambda_0^2-5\lambda_0+1\right)k+\left(3\lambda_0-2\lambda_0^2\right)}{(\lambda_0-1)(k-1)}>0$.Q.E.D

Appendix 3

This appendix refers to Section 5, where we show the nonnegativiness of equilibrium net income levels when k is satisfies the following range $\frac{1-2\lambda_0}{2-5\lambda_0}<k<\frac{1}{1+\lambda_0}$ and equilibrium taxes are give by 16, equilibrium m_f^* writes as:

$$m_f^*=\frac{1}{2}\frac{\left(5\lambda_0-2\right)k+\left(1-2\lambda_0\right)}{k+k\lambda_0-1}$$

Recall that $k+k\lambda_0-1<0$, so m_f^* is positive iff $\left(5\lambda_0-2\right)k+\left(1-2\lambda_0\right)<0$ which writes as:

$$\frac{1-2\lambda_0}{2-5\lambda_0}<k$$

If $\frac{1-2\lambda_0}{2-5\lambda_0}<k<\frac{1}{1+\lambda_0}$ violates, we have a corner solution for the pair of taxes: (i) If $\frac{(1-2\lambda_0)}{-(5\lambda_0-2)}<k$ violates, then candidate m_f^* given above becomes negative. Hence, taxes reach their maximum value, which solves nonnegativeness of the net income; (ii) If $k<\frac{1}{\lambda_0+1}$ violates, then maximization problem for the foreign country becomes convex and the equilibrium tax rates are upper bounded with the same constraint as in the previous case:

$$t_h^*=t_f^*=\frac{1}{1-k} \tag{21}$$

The resulting equilibrium flow of migrants from F to H obtains as:

$$x^*=\left(2\lambda_0-1\right)\frac{2k-1}{k-1}>0$$

Recall that: $\frac{1}{2}<\frac{1}{2-\lambda_0}<k<1$, which guarantees x to be positive. Note that $1>x^*$ if $1-x^*=\frac{k(4\lambda_0-3)-2\lambda_0+2}{1-k}>0$. The latter inequality satisfies iff $k\left(4\lambda_0-3\right)+2-2\lambda_0>0$. So, $k<\frac{2-2\lambda_0}{3-4\lambda_0}$, guarantees the survival of the small country, otherwise when k reaches its highest range, H country gains attractiveness being big and augmenting public good by higher k, so full migration is expected.

Appendix 4

We consider the desirable size of the F country from the perspectives of resident population, when taxes are given by (13):

$$\frac{\partial}{\partial\lambda_0}U_f=\frac{\partial}{\partial\lambda_0}\left(t_f(k-1)+1+(kt_f-\lambda_0)\lambda_0\right)=kt_f-2\lambda_0=0$$

The solution is given by:

$$\lambda_0^d = \frac{kt_f}{2}$$

We selected two intervals for k to demonstrate the difference between equilibrium and desired size of the foreign country.

1. First, we consider the scenario, where the migration takes from place from H to F, this scenario belongs to the range $k < \frac{2\lambda_0^2-5\lambda_0+2}{5\lambda_0^2-11\lambda_0+5}$. Hence, the optimal λ_0^d when taxes are given by (13) is:

$$\lambda_0^d = \frac{\left(\lambda_0^2 - \lambda_0 + 1\right) k}{3\left(\lambda_0 - 1\right)\left(k + k\lambda_0 - 1\right)}$$

The equilibrium size reached after the free movement of migrants with the same equilibrium pairs of taxes writes as:

$$\lambda_1^* = \frac{2\left(\lambda_0^2 - \lambda_0 + 1\right)}{3}$$

We compare equilibrium and optimal sizes of the country F: $\lambda_1^* > \lambda_0^d$ if $\lambda_1^* - \lambda_0^d = \frac{1}{3}\left(\lambda_0^2 - \lambda_0 + 1\right)\frac{\left(2\lambda_0^2-3\right)k+(2-2\lambda_0)}{(\lambda_0-1)(k+k\lambda_0-1)} > 0$ and this is true if $\left(2\lambda_0^2 - 3\right)k + (2 - 2\lambda_0)$ is positive. Recall that $k < \frac{2\lambda_0^2-5\lambda_0+2}{5\lambda_0^2-11\lambda_0+5}$, which gives us $\left(2\lambda_0^2 - 3\right)k + (2 - 2\lambda_0) > \frac{2\lambda_0^2-5\lambda_0+2}{5\lambda_0^2-11\lambda_0+5}\left(2\lambda_0^2 - 3\right) + (2 - 2\lambda_0) > 0$, Q.E.D

2. Secondly, we consider the scenario, where the migration takes place from F to H, which belongs to the range $\frac{1-2\lambda_0}{2-5\lambda_0} < k < \frac{1}{1+\lambda_0}$ and in this range the optimal λ_0^d when taxes are given by (16) is:

$$\lambda_0^d = \frac{1}{4}k\frac{-4k - 2\lambda_0 + 3k\lambda_0 + 3}{(k-1)\left(k + k\lambda_0 - 1\right)}$$

The equilibrium size reached after the free movement of migrants writes as:

$$\lambda_1^* = \frac{\left(3k\lambda_0 + 3 - 4k - 2\lambda_0\right)\lambda_0}{2\left(1 - k\right)}$$

We compare equilibrium and desired sizes of the country F:
$\lambda_1^* > \lambda_0^d$ if $\lambda_1^* - \lambda_0^d = \frac{1}{4}\left(4k + 2\lambda_0 - 3k\lambda_0 - 3\right)\frac{k-2\lambda_0+2k\lambda_0+2k\lambda_0^2}{(k-1)(k+k\lambda_0-1)} > 0$, this is true if $4k + 2\lambda_0 - 3k\lambda_0 - 3 = (4 - 3\lambda_0)k + (2\lambda_0 - 3) > 0$ which writes as $k > \frac{3-2\lambda_0}{4-3\lambda_0}$. Consequently: $\lambda_1^* < \lambda_0^d$ if $k < \frac{3-2\lambda_0}{4-3\lambda_0}$. Q.E.D.