FACULTY OF LAW, ECONOMICS AND FINANCE

CREA Discussion Paper 2011-18

Center for Research in Economic Analysis
University of Luxembourg

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December, 2011

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Property Rights, Optimal Public Enforcement, and Growth

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Abstract: We study the link between public enforcement of property rights, innovation investments, and economic growth in an endogenous growth framework with an expanding set of product varieties. We find that a government may assure positive equilibrium growth through public employment in the enforcement of property rights, if the economic environment is sufficiently favorable to growth and/or public enforcement is sufficiently effective. However, in terms of welfare an equilibrium path without property rights protection and growth might be preferable. In this case the enforcement of property rights involves too much reallocation of labor from production and research towards the public sector.

Keywords: Technological Change, Economic Development, Property Rights, Public Employment

JEL-Classification: O10, H11, D23

Revised Version: November 27, 2011.

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1 Introduction

The legal framework of an economy is often thought of as defining the rules of the game that economic agents play. However, what really matters for the incentives of these agents is the strength of the "rule of law". While precise definitions of this term are hard to come by, they usually involve the notion of an economy's degree of property rights protection, the enforceability of contracts, the likelihood of crime and violence, and the effectiveness of an economy's judiciary (see, e. g., Kaufmann et al., 2007; Weil, 2009, p. 346). The rule of law cannot be taken for granted in most parts of the world. Nobel laureate Douglas North even concluded that the inability of countries to develop effective, low-cost enforcement of contracts is the most important cause of the historical stagnation and the contemporary underdevelopment of today's low-income countries (North, 1990, p. 54).

The focus of the present paper is on the link between property rights protection, the incentives to engage in innovation investments, and endogenous economic growth. On the positive side, we want to know under what conditions a growth policy of public property rights protection may be effective. On the normative side, we ask whether better public enforcement of property rights is desirable and discuss the circumstances under which the government should intervene and hire public employees to protect property rights. To fix ideas, one may think of public employment as policemen, tax inspectors, lawyers and judges, prison guards or administrators that all help to enforce property rights.

We address these questions in an endogenous growth framework where growth is the result of an expanding set of product varieties in the sense of Grossman and Helpman (1991). The degree of property rights protection is parameterized by the fraction of profits in the intermediate-good sector that is appropriated by its owners. It endogenously depends on the share of public employment in the total workforce. Weaker property rights deter innovation investments and reduce economic growth. Thus, our framework is consistent with the empirical literature that establishes a positive relationship between the strength of property rights and economic growth (see, e. g., Knack and Keefer, 1995; Barro, 1996; Clague et al., 1999; Aron, 2000, provides a comprehensive survey of the empirical literature). It is also in line with recent empirical support for a positive link between property rights protection and entrepreneurship (Estrin et al., 2009).

Our main results can be summarized as follows. We first show that a unique dynamic general equilibrium exists. Whether an economy in its steady state experiences strictly positive growth rates or not depends on its economic environment and the effectiveness of public employment in the enforcement of property rights. However, even if public protection of property rights triggers positive growth rates, this may not be optimal from a welfare point of view. Indeed, we characterize environments where no growth can be better than some growth. In these cases the level of public employment that assures positive growth requires too much reallocation of labor from manufacturing and research to the public sector. The social costs of public enforcement of property rights then outweigh the social benefits from positive growth. This result illustrates that public employment may be too ineffective to solve the problem of property rights protection. It suggests that public enforcement of property rights may itself be a "white elephant" in the sense of Robinson and Torvik (2005), i. e., a public project with negative social surplus.

Our paper relates and contributes to at least two different strands of the literature linking property rights enforcement to economic growth.

First, it makes a contribution to the literature on predation and economic growth starting with Grossman and Kim (1996) and Tornell (1997). Papers that, similarly to ours, focus on *public* enforcement of property rights include Economides et al. (2007), Zak (2002), and Dincer and Ellis (2005). These studies explicitly model individuals' decision how to allocate their resources to productive and expropriative activities. Individuals have access to an expropriation technology that, among others, depends on governmental activity. Moreover, its specific design determines how many and what type of equilibria exist. In contrast to these papers, we do not model this decision of individuals. Rather, we assume that under imperfect property rights resources are diverted from the production to the household sector. Moreover, the strength of property rights is directly a function of public employment in the enforcement of property rights. Hence, the focus of our analysis is on the solution of the problem of property rights protection rather than on the conditions that may cause weak property rights. An advantage of this approach is that the equilibrium we identify is unique and allows for clear-cut predictions. Moreover, coordination failure is not an issue.

Our result that positive growth can lead to lower social welfare than stagnation if public enforcement is sufficiently ineffective complements the findings of Gonzalez (2007). This author shows that increased *private* enforcement of property rights and faster growth can be Pareto-dominated by an allocation with lower growth and less private enforcement. Similarly, Gonzalez and Neary (2008) show that, in the absence of public enforcement, (second-best) optimal fiscal policy may call for a reduction in growth in order to mitigate the diversion of resources associated with private enforcement. Intuitively, one may have thought that public enforcement can, at least in the absence of corruption, solve the problem associated with private enforcement. However, our result suggests that this does not have to be the case.

Second, we contribute to the literature that studies the relationship between intellectual property rights (IPR), i. e., the danger of imitation and the erosion of monopoly power, growth, and welfare in the framework of the variety expansion growth model. Related studies include Kwan and Lai (2003) and Furukawa (2007). They analyze the social benefits and costs of IPR protection assuming that the government can choose the degree of IPR protection at no cost.[1] By contrast, this paper focuses on the role of property rights over profits. In our model the strength of property rights determines the share of monopoly profits that is appropriated by its owners. Moreover, we argue that the enforcement of property rights through governments is endogenous and costly. Accordingly, the optimal degree of law enforcement equilibrates the advantage of better incentives and faster growth to the disadvantage of foregone consumption due to the reallocation of labor into public employment.

The remainder of this paper is organized as follows. Section 2 presents the details of the model. In Section 3, we derive the dynamic general equilibrium and establish the Pareto-optimal allocation as a benchmark. Our main results appear in Section 4. First, we determine the conditions under which the possibility of public growth-enhancing policies exists. Second, we characterize the second-best optimal share of public employment in the enforcement of property rights. Section 5 concludes. Proofs are relegated to the Appendix. The latter also contains the details on the calibrations underlying Figures 1 to 3.

[1]See, e. g., Eicher and García-Peñalosa (2008) for an analysis of the role of private investments in the endogenous degree of IPR protection and their impact on economic growth.

2 The Model

We consider a closed economy with four sectors and a government. *Households* work, consume, and save. The *final-good sector* produces a consumption good out of a variety of existing intermediate goods. The *intermediate-good sector* consists of monopolistically competitive firms that manufacture one intermediate good using labor as the only input. The blueprint for the production of each intermediate good is invented in a *research sector*. Property rights in the intermediate-good sector are imperfectly protected in the sense that only a certain fraction of profits can be appropriated by their owners. This may be due to, e. g., illegal conduct of competitors or expropriation by a criminal organization. In any case, these actions generate "laundered" income that increases consumption and savings of the household sector. The degree of property rights protection endogenously depends on government activity. Specifically, the *government* can use tax resources to hire a fraction of the workforce as public enforcers of property rights.

The Household Sector

There is a continuum of identical households of mass 1. We study their behavior through the lens of a single representative household that supplies the time-invariant aggregate labor endowment L inelastically to the intermediate-good, the research and the public sector. Her consumption-savings decision maximizes intertemporal utility

$$U = \int_0^\infty \ln c(t)\, e^{-\rho t} dt, \tag{1}$$

where $c(t)$ is consumption at date t and $\rho > 0$ the subjective discount rate. Henceforth, we suppress time arguments whenever this does not cause confusion. At each t, household net income comprises labor income, wL, returns on assets, $r\Omega$, and laundered income, M, less taxes, T. The household's flow budget constraint is then given by

$$\dot{\Omega} = wL + r\Omega + M - T - p_c c, \quad \text{with} \quad \Omega(0) > 0. \tag{2}$$

Here, w denotes the wage rate at t, r the rate of return on assets, and p_c the price of the consumption good. The budget constraint (2) captures the fact that in a closed economy total laundered income is used for consumption or saving of the household sector.

The household's maximization of (1) is subject to (2) and a No-Ponzi condition. Following Grossman and Helpman (1991), we choose consumption expenditure as the numéraire, i. e., $p_c c = 1$ at all t. Then, the Euler condition implies $r = \rho$, and the transversality condition is $\lim_{t\to\infty} e^{-\rho t}\Omega(t) = 0$.

The Final-Good Sector

The final-good firms produce the consumption good c out of a variety of exiting intermediates according to the production function

$$c = \left[A^{(\sigma-1)(1-\alpha)} \int_0^A x(j)^\alpha dj\right]^{1/\alpha}, \tag{3}$$

where $A \in \mathbb{R}_{++}$ is the "number" of available intermediate goods at t and $x(j)$, $j \in [0, A]$ denotes the quantity of intermediate-good input j used at t. The parameter $\alpha \in (0, 1)$ determines the elasticity of substitution between any pair of intermediates, $\epsilon \equiv 1/(1 - \alpha)$. Following Ethier (1982), the term in front of the integral introduces $\sigma > 0$ as a measure of the gains from specialization. As σ increases, these gains become more pronounced, for $\sigma \to 0$ they vanish.

The representative producer of c is competitive and chooses $\{x\,(j)\}_{j=0}^{A}$ to maximize $p_c c - \int_0^A p(j)x(j)dj$ at all t, where $p(j)$ is the price of input j.

The Intermediate-Good Sector

Each intermediate-good firm $j \in [0, A]$ produces a single intermediate good in a monopolistically competitive environment with demand $x(j) = cp(j)^{-\epsilon}/P$, where $P \equiv [A^{(\sigma-1)(1-\alpha)} \int_0^A p(j')^{1-\epsilon}dj']^{\epsilon/(\epsilon-1)}$. The production function for all varieties is $x(j) = l(j)$, where $l(j)$ is the amount of labor hired by firm j. The price $p(j)$ charged by intermediate-good firm j maximizes his profits $\pi(j) = q\,[p(j) - w]\,cp(j)^{-\epsilon}/P$. Here, $q \in [0, 1]$ denotes the strength of property rights protection. The weaker the degree of property rights protection, i. e., the lower q, the lower are net profits of intermediate-good producers. Intermediate-good producers regard q as a given constant. The resulting monopoly price satisfies $p(j) = p = w/\alpha$ such that $x(j) = x = cA^{\frac{\sigma}{1-\epsilon}-1}$ and $\pi(j) = \pi = q(1 - \alpha)px$.

The Research Sector

Previous to the marketing of an intermediate good it is invented by competitive research firms. The production function of the research sector for new intermediates is

$$\dot{A} = AL_A/a, \tag{4}$$

where L_A is the aggregate amount of labor used for research and a is a productivity parameter. Once a new variety is invented, it is sold by auction to the highest bidder who also receives a perpetual patent. Accordingly, the price for such a patent at t is $v(t) = \int_t^\infty \pi(s)e^{-\rho(s-t)}ds$. The profit-maximization problem of the representative research firm is then to choose L_A that maximizes $vAL_A/a - wL_A$. For L_A to be finite the first-order condition is

$$v \leq \frac{wa}{A} \quad \text{with} \quad \text{``} = \text{''}, \quad \text{if} \quad \dot{A} > 0. \tag{5}$$

Government Activity and Property Rights

The government levies a lump-sum tax T and uses these receipts to hire a fraction $\delta \in [0, 1]$ of the total workforce as public workers, L_P. Their task is to help enforce property rights.

Under a balanced government budget we have for all t

$$T = wL_P = w\delta L. \tag{6}$$

We stipulate that the degree of property rights protection, q, depends on the share of public employment in the total workforce, $\delta = L_P/L$, according to

$$q = F(\delta) \quad \text{with} \quad F : [0, 1] \to [0, 1]. \tag{7}$$

Here, F is $\mathcal{C}^2$ with $F(0) = 0$, $F(1) = 1$, $F' > 0 > F''$, and $\lim_{\delta \to 0} F' = \infty$. This reduced form relationship captures the idea that the government via increased spending relative to the size of the economy can strengthen property rights, though at a declining rate. Naturally, public employment is bounded by the aggregate labor force. Without public employment in the enforcement of property rights firms loose all profits. If the government hired the total labor force as public enforcers, property rights in the intermediate-good sector would be fully secured. Moreover, the function F fulfills an Inada-type condition, reflecting the idea that the productivity of public employees in generating higher degree of property rights protection is very high for low levels of δ. Note also that q is a flow variable, i. e., the enforcement level of property rights has to be maintained constantly.

3 Equilibrium and Welfare

In this section we derive the dynamic general equilibrium (DGE) for a given δ and compare it to the Pareto-optimal allocation. The findings of this section provide the groundwork for the policy analysis that follows.

3.1 Equilibrium

Given δ, the DGE consists of an allocation $\{c(t), \Omega(t), M(t), x(j,t), l(j,t), L_x(t), L_A(t), L_P(t), A(t)\}_{t=0}^{t=\infty}$ and a price system $\{r(t), p_c(t), w(t), p(j,t), v(j,t)\}_{t=0}^{t=\infty}$ such that households, final-good, intermediate-good and research firms behave optimally at all t, the government has a balanced budget, the degree of property rights enforcement is given by (7), there is full employment of labor, i. e., at all t, $L_x(t) + L_A(t) + L_P(t) = L$, and the capital market values firms according to fundamentals and $\Omega(t) = A(t)v(t)$.

The following proposition establishes the existence of a unique steady-state equilibrium with and without growth.

Proposition 1 *The steady-state growth rate of consumption is given by*

$$g_c^* \quad = \quad \frac{\sigma}{\epsilon - 1} max\left\{0, \frac{(1-\alpha)(1-\delta)F(\delta)L - a\alpha\rho}{a\left[F(\delta)(1-\alpha)+\alpha\right]}\right\}. \tag{8}$$

The economy immediately jumps to the unique steady state for any admissible set of initial conditions.

According to Proposition 1, public employment in the enforcement of property rights affects the consumption growth rate g_c^* in three ways. On the one hand, a higher δ reduces the labor force available for research and intermediate-good production as reflected by the term $(1-\delta)L$. Through this channel, a higher δ reduces g_c^*. The remaining two channels affect the equilibrium value of firms, $\Omega = F(\delta)(1-\alpha)/(g_A(\delta)+\rho)$, where g_A is the growth rate of A. The equilibrium value of firms increases in δ because of better property rights. This effect is mitigated since better property rights also increase the growth rate of A.

Finally, the steady-state growth rate is determined by the parameters σ, a, ρ, α, and L that characterize the economic environment. Quite intuitively, for $g_c^* > 0$ we have

$$\frac{\partial g_c^*}{\partial \sigma} > 0, \quad \frac{\partial g_c^*}{\partial a} < 0, \quad \frac{\partial g_c^*}{\partial \rho} < 0, \quad \frac{\partial g_c^*}{\partial \alpha} < 0, \quad \frac{\partial g_c^*}{\partial L} > 0. \tag{9}$$

Hence, the economic environment is more prone to research and growth the smaller a, ρ, and α and the greater σ and L. Intuitively, the smaller a, the greater is the productivity in the research sector and the greater is the research output. The smaller the discount rate, ρ, the greater is the incentive to save and to acquire equity shares issued by research firms. The lower the degree of substitutability of intermediate goods, α, the greater are the monopoly profits in the intermediate-good sector. With stronger gains from specialization, i. e., with a higher σ, consumption growth is faster. Finally, the higher the aggregate labor endowment, L, the more labor can be allocated to research, manufacturing and public employment.

Observe that g_c^* of (8) collapses to

$$g_c = \frac{\sigma}{\epsilon - 1} max \left\{ 0, \frac{(1-\alpha)L}{a} - \alpha\rho \right\} \tag{10}$$

if $\delta = 0$ and $q = F(\delta) = 1$ could hold simultaneously. This is the result established for an economy where property rights are fully secured without government intervention as envisaged by Grossman and Helpman (1991). Intuitively, a meaningful trade-off between better property rights protection and its costs can only arise if $g_c > 0$ in the world of Grossman and Helpman. Therefore, for the remainder of our analysis we assume this to be the case:

Assumption 1 It holds that $a\alpha\rho/[(1-\alpha)L] < 1 \Rightarrow g_c > 0$.

With this assumption it is straightforward to show that $g_c > g_c^*$ for all admissible parameter values. Thus, in a world where property rights need protection the equilibrium growth rate is always smaller.

3.2 The Pareto-Optimal Allocation (First-best)

To derive the Pareto-optimal growth rate, we consider a social planner who allocates the factors of production and outputs to households and firms. Naturally, this allocation is independent of the degree of property rights protection.

Due to the decreasing marginal product of the intermediate goods in the production of the final good, the social planner chooses a symmetric configuration $c = A^{\sigma/(\epsilon-1)}L_x$ at all t. The inter-temporal optimization determines the allocation of labor between manufacturing and research. Formally, the planner maximizes U of (1), invoking full employment and the production function of the research sector. This problem has previously been solved by Bénassy (1998) and de Groot and Nahuis (1998). In our notation their result appears in the following proposition.

Proposition 2 *The Pareto-efficient growth rate of consumption is*

$$g_c^P = \max\left\{0, \frac{\sigma L}{(\epsilon-1)a} - \rho\right\}. \tag{11}$$

Comparing the Pareto-optimal rate (11) to the equilibrium growth rate under secure property rights (10) it holds that

$$g_c^P \gtreqless g_c \Leftrightarrow \sigma \gtreqless \frac{a\rho}{(1-\alpha)(L+a\rho)} \in (0,1). \tag{12}$$

Proposition 2 shows that the Pareto-efficient growth rate may be smaller than the equilibrium growth rate if the gains from specialization are sufficiently small. Since the problem in less developed and transition economies seems to be that there is inefficiently low growth, we shall from now on focus on the case where $g_c^P > g_c$.[2]

[2]Recall that $g_c > g_c^*$ under Assumption 1. Therefore, we also have $g_c^P > g_c^*$ for any admissible δ. Thus, we neglect the case where $g_c^P < g_c^*$ may hold for some δ.

4 Optimal Government Policy (Second-Best)

This section establishes the optimal level of public employment that the government should hire to protect property rights. The government maximizes the welfare of the representative household attainable in the equilibrium of Proposition 1. It is in this sense that the optimal policy is second-best. Moreover, we establish that the second-best growth rate of the economy is strictly smaller than the growth-maximizing one. To address these issues, it is necessary to enquire first into the possibility of growth-enhancing policies.

4.1 Government Policy and Steady-State Growth

The first statement of the following proposition characterizes the conditions under which government investment in the enforcement of property rights may trigger positive growth rates. For this case, the second statement establishes the growth-maximizing government policy.

Proposition 3 *(Policy and Growth)*

1. *If* $\hat{\delta} = \arg\max_{\delta \in [0,1]} (1-\delta)F(\delta)$ *is such that* $(1-\hat{\delta})F(\hat{\delta}) \leq a\alpha\rho/[(1-\alpha)L]$, *then* $g_c^* = 0$ *for all* $\delta \in [0,1]$.

2. *If* $(1-\hat{\delta})F(\hat{\delta}) > a\alpha\rho/[(1-\alpha)L]$, *then there is* $(\delta_{min}, \delta_{max})$ *with* $0 < \delta_{min} < \delta_{max} < 1$ *such that* $g_c^* > 0$ *for all* $\delta \in (\delta_{min}, \delta_{max})$. *In this case, there is a unique* $\delta^* \in (\delta_{min}, \delta_{max})$ *that maximizes* g_c^*.

Roughly speaking, Proposition 3 states that public employment may increase the equilibrium growth rate if its effectiveness in the production of property rights protection as represented by the function F is sufficiently strong for the economic environment. Intuitively, the product $(1-\delta)F(\delta)$ represents the two opposing effects of δ on g_c^* that appear in the numerator of (8). Since $F(0) = 0$, $F'(\delta)$ is (very) large for small values of δ and $F(1) = 1$, this product has some global maximum on $(0,1)$. The value of this maximum will crucially depend on F. For instance, if $F(\delta) = \delta^\nu$, $0 < \nu < 1$, then $F_\nu(\delta) < 0$. Therefore, $1/\nu$ may serve as a measure of the effectiveness of the public sector. Accordingly, $(1-\hat{\delta})F(\hat{\delta})$ increases with this measure.

According to Statement 1 of Proposition 3, the steady-state growth rate is zero if the economic environment is not sufficiently favorable to growth and/or public employment is not sufficiently effective. However, government intervention may trigger positive growth. Then, according to Statement 2 of Proposition 3 a growth-maximizing share of government activity, δ^*, exists. It balances at the margin the two opposing effects of government activity mentioned above.

4.2 Optimal Property Rights Protection

To derive the welfare-maximizing policy, we first solve the integral of (1) using $c(t) = c_0\, e^{g_c^* t}$ to obtain household welfare in equilibrium as

$$U = \frac{1}{\rho}\left(\ln c_0 + \frac{g_c^*}{\rho}\right). \tag{13}$$

Here c_0 denotes the initial level of consumption at $t = 0$. Using the equilibrium conditions $c = A^{\sigma/(\epsilon-1)}Ax$ and $L_x = Ax$ we obtain c_0, for a given initial quantity of intermediates A_0 and $0 < \delta_{min} < \delta_{max} < 1$ as[3]

$$c_0 = A_0^{\frac{\sigma}{\epsilon-1}} L_x = \begin{cases} A_0^{\frac{\sigma}{\epsilon-1}}(1-\delta)L & \text{if } \delta \in [0, \delta_{min}] \cup [\delta_{max}, 1] \\ \\ \frac{\alpha((1-\delta)L + a\rho)A_0^{\frac{\sigma}{\epsilon-1}}}{[F(\delta)(1-\alpha)+\alpha]} & \text{if } \delta \in [\delta_{min}, \delta_{max}]. \end{cases} \tag{14}$$

Observe that the expressions of the latter equation are valid in the circumstances described

[3]To derive c_0 for $\delta \in [\delta_{min}, \delta_{max}]$ consider that in this case $L_x = \alpha/w$, where the steady-state wage rate is determined by condition (5), which has to hold with equality in a steady state with positive R&D activity, i. e., $w = vA/a$. Moreover, the aggregate value of equities, $\Omega = vA$, is constant in the steady state. From $A\,(s) = A_0\, e^{g_A^* s}$, where $g_A^* = [(\epsilon - 1)/\sigma]g_c^*$ (see proof of Proposition 3), and $v(t) = \int_t^\infty \frac{1-\alpha}{A(s)} e^{-\rho(s-t)} ds$ one finds that $vA = F(\delta)(1-\alpha)/(g_A^* + \rho)$ and thus $w = [F(\delta)(1-\alpha) + \alpha] / ((1-\delta)L + a\rho)$. Note, that the former also implies that the initial value $A_0 > 0$ determines $v\,(0)$ such that $\Omega_0 = F(\tau)\,(1-\alpha)\,/\,(g_A^* + \rho)$.

in Statement 2 of Proposition 3.[4] In this case, the welfare function is given by

$$U(\delta) = \begin{cases} \frac{1}{\rho} \ln \left[A_0^{\frac{\sigma}{\epsilon-1}} (1-\delta) L \right] & \text{if } \delta \in [0, \delta_{min}] \cup [\delta_{max}, 1] \\ \\ \frac{1}{\rho} \ln \left[\frac{\alpha((1-\delta)L + a\rho) A_0^{\frac{\sigma}{\epsilon-1}}}{[F(\delta)(1-\alpha)+\alpha]} \right] + \frac{g_c^*}{\rho^2} & \text{if } \delta \in [\delta_{min}, \delta_{max}]. \end{cases} \tag{15}$$

As c_0, also U is piecewise-defined in regimes with and without growth. However, one readily verifies that U is continuous for all $\delta \in [0, 1]$.

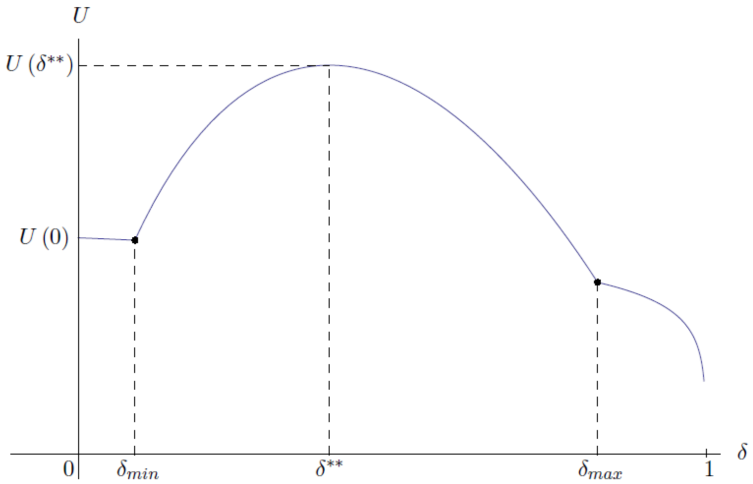

Figure 1: Welfare function with a global maximum at $\delta^{**} \in (0, 1)$.

To develop an understanding of the value of δ that maximizes U on $[0, 1]$, the following remarks prove useful.

First, if the government chooses $\delta \in [0, \delta_{min}] \cup [\delta_{max}, 1]$, then there is no research. Hence, $L_x = (1-\delta)L$, and $g_c^* = 0$. Accordingly, in these intervals a rise in δ reduces consumption in all periods by reducing the labor force available for production and welfare declines monotonically in δ (see, e. g., Figure 1).

Second, for levels of $\delta \in [\delta_{min}, \delta_{max}]$, there is research and growth. Therefore, a rise in δ has a level effect on current consumption and a growth effect. The level effect is due to the reallocation of labor from manufacturing to research and public employment. The sign of the growth effect depends on the level of δ with respect to its growth-maximizing value δ^*.

[4]If Statement 1 of Proposition 3 applies, then $c_0 = A_0^{\sigma/(\epsilon-1)}(1-\delta)L$ and $g_c^* = 0$ for all δ. As will become clear below, in this case welfare would decline monotonically in δ such that the welfare maximum is attained at $\delta = 0$. In what follows we shall neglect this case.

Close to δ_{min} the growth effect increases g_c^*. If the positive growth effect of a higher share of public employment outweighs the negative effects on the level of initial consumption near δ_{min}, then the welfare function is inversely U-shaped on $[\delta_{min}, \delta_{max}]$. Otherwise, U continues to decline in δ. Examples for the first case are given in Figures 1 and 3. The second case is depicted in Figure 2.

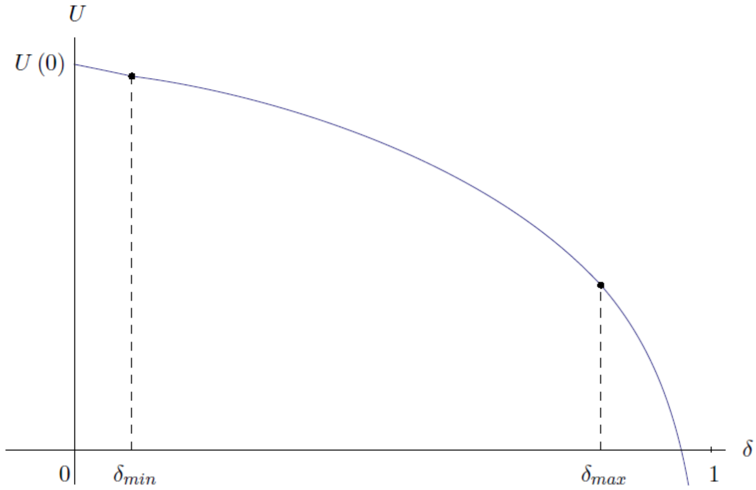

Figure 2: Monotonically declining welfare function.

The following proposition sharpens this intuition and fully characterizes the optimal share of public employment.

Proposition 4 *(Policy and Welfare) Suppose Statement 2 of Proposition 3 holds.*

1. *If* $dU/d\delta|_{\delta=\delta_{min}} > 0$*, then there exists* $\underset{\delta\in[\delta_{min},\delta_{max}]}{\arg\max}\ U = \delta^{**} \in (\delta_{min}, \delta_{max})$*. If additionally* $U(\delta^{**}) > U(0)$*, then* δ^{**} *maximizes* U *on* $[0,1]$*. If* $U(\delta^{**}) < U(0)$*, then* $\delta = 0$ *maximizes* U *on* $[0,1]$*. If* $dU/d\delta|_{\delta=\delta_{min}} \leq 0$*, then* $\delta = 0$ *maximizes* U *on* $[0,1]$*.*[5]

2. *It holds that* $\delta^{**} < \delta^*$*.*

Statement 1 of Proposition 4 shows that in terms of welfare no growth can be better than some growth. The two possible scenarios are depicted in Figures 2 and 3. In these economic environments the level of public employment that guarantees positive growth rates requires a substantial reallocation of the labor force away from manufacturing and

[5]In the non-generic case where $U(0) = U(\delta^{**})$, the solution of $\max_{\delta\in[0,1]} U$ is not unique.

research towards the public sector. Therefore, the negative static welfare effect of public employment outweighs the welfare benefits from growth. As shown in Figure 2, this may occur at the margin δ_{min} such that $dU/d\delta|_{\delta=\delta_{min}} \leq 0$. Figure 3 depicts the case where $dU/d\delta|_{\delta=\delta_{min}} > 0$ with a local maximum at δ^{**} and $U(\delta^{**}) < U(0)$.

This result suggests that public employment may be insufficiently effective to solve the problem of property rights protection.[6] Our result that positive growth can lead to lower social welfare than stagnation confirms the findings of Gonzalez (2007) and Gonzalez and Neary (2008) in an environment where the enforcement of property rights is not a private matter.

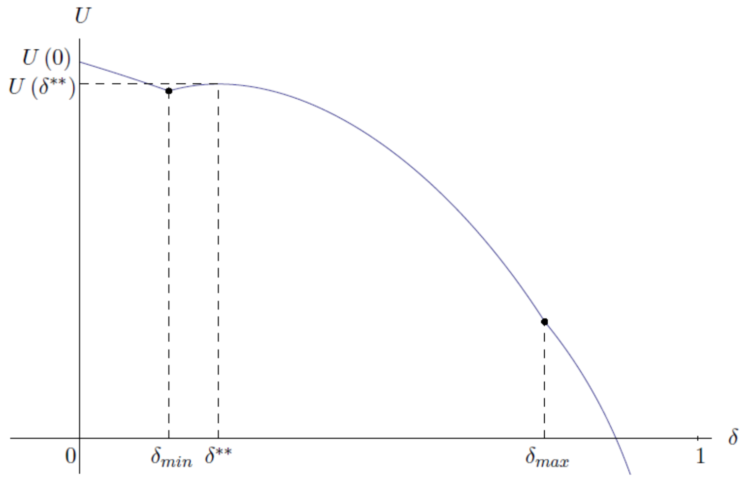

Figure 3: Welfare function with a local maximum at $\delta^{**} \in (0, 1)$.

Statement 2 of Proposition 4 implies that even if the welfare-maximizing public employment share is positive, it always falls short of the growth-maximizing one, i. e., $\delta^{**} < \delta^{*}$. To grasp the intuition for this, recall that $U = \frac{1}{\rho}\left(\ln c_0 + \frac{g_c^*}{\rho}\right)$. The second term, i. e., the consumption growth rate is maximized at δ^*. By contrast, the first term (which corresponds to the static welfare effect) negatively depends on δ because a rise in δ reduces the resources available for final-good production. Thus, the public employment share that maximizes U has to be smaller than the one that maximizes g_c^*.

[6] It's worth noting that Proposition 4 holds independent of whether $g_c^P \geq g_c$ or not. In the examples depicted in Figures 1, 2 and 3 it holds that $g_c^P > g_c > g_c^*$, i.e., there is inefficiently low growth in equilibrium.

5 Concluding Remarks

We have studied the interdependence between property rights, optimal public enforcement, innovation, and endogenous economic growth in an economy where growth results from an expanding set of product varieties. The strength of property rights enforcement determines the profit that firms expect from an innovation investment. It is determined by governments hiring a fraction of the labor force. Our results may be summarized as follows.

On the positive side, we identity the conditions under which a government is able to assure strictly positive equilibrium growth through public enforcement of property rights. This is the case if the economic environment is sufficiently prone to growth and/or public employment is sufficiently effective.

On the normative side, we determine the optimal enforcement policy of a government able to protect property rights and to generate growth through public employment. We find that in terms of welfare, implementing an equilibrium path with no property rights protection and no growth may be preferable to one with some positive degree of property rights protection and strictly positive growth. The former equilibrium path is optimal if the welfare costs of the reallocation of labor away from manufacturing and research towards public employment are too high. The latter solution arises only if the economic environment is sufficiently favorable to growth and public employment is sufficiently effective. The government may then choose an optimal level of public enforcement that allows for strictly positive growth. However, the optimal policy always involves imperfect enforcement of property rights and the implemented growth rate is strictly smaller than the highest possible growth rate.

Appendix

Proof of Proposition 1

We start the derivation of the steady-state growth rate by looking at the labor market.[7] The linear production function of intermediates implies for a symmetric configuration that the aggregate labor demand of this sector is $L_x = Ax$. Moreover, constant returns to scale in the production of the final consumption good and our normalization imply $1 = Apx$. Thus, $L_x = \alpha/w$. Aggregate labor demand in the research sector obtains from the production function of research as $L_A = ag_A$, where $g_A \equiv \dot{A}/A$. Hence, the labor market equilibrium condition $(1-\delta)L = L_x + L_A$ holds if and only if $g_A = (1-\delta)L/a - \alpha/(wa)$. When employment in research is positive, we need $v = wa/A$ (see equation 5). Hence, a necessary condition for positive steady-state growth of A is $v \geq \alpha a/(AL)$. Defining $V \equiv \Omega^{-1} = 1/(Av)$ as the inverse of the economy's equity value, we obtain

$$g_A = \max\left\{0, \frac{(1-\delta)L}{a} - \alpha V\right\}. \tag{16}$$

For the capital market to be in equilibrium, the return that a shareholder can expect must be equal to the return of a riskless loan. As the former is the sum of dividends and capital gains and the latter is equal to ρ, we obtain as a no-arbitrage condition $\rho = (\pi + \dot{v})/v$, where instantaneous net profits with $1 = Apx$ are $\pi = q(1-\alpha)/A$. Then, observing that $\dot{V}/V = -g_A - \dot{v}/v$, we have

$$\frac{\dot{V}}{V} \;=\; -g_A - \rho + q(1-\alpha)V. \tag{17}$$

Equations (16) and (17) jointly describe the equilibrium paths of V and g_A. Setting $\dot{V} = 0$ in (17) and substituting $V = \frac{g_A+\rho}{q(1-\alpha)}$ and $q = F(\delta)$ in (16) delivers the unique equilibrium growth rate of intermediates

$$g_A^* \;=\; \max\left\{0, \frac{(1-\alpha)(1-\delta)F(\delta)L - a\alpha\rho}{a\left[F(\delta)(1-\alpha)+\alpha\right]}\right\}. \tag{18}$$

[7] This proof extends the one of Grossman and Helpman (1991, p. 57-62) to an environment with $\sigma \neq 1$ and imperfect property rights.

For any symmetric configuration it holds that $c = A^{\sigma/(\epsilon-1)} L_x$. Hence, $\dot{c}/c = \sigma g_A/(\epsilon - 1) + \dot{L}_x/L_x$. As L_A and L_x have to be constant in the steady state, g_c^* is also unique and given by (8).

To demonstrate that there are no transitional dynamics we show that starting the economy outside of the steady state leads either to $V \to \infty$, $g_A = 0$ or $V \to 0$, $g_A \to L/a > 0$. Both cases violate rational expectations.

Consider the first case. As $V \equiv 1/(Av) \to \infty$ it must be that $v \to 0$ since A cannot decline. However, with $g_A = 0$ and $\pi = q(1-\alpha)/A$ the value of a patent v obtains as

$$v(t) = \int_t^\infty \frac{q(1-\alpha)}{A(t)} e^{-\rho(s-t)} ds = \frac{q(1-\alpha)}{\rho A(t)} > 0,$$

i. e., without innovations the monopoly profits and their present value remain positive. We arrive at a contradiction to $v \to 0$.

The second case has $g_A > 0$ which implies that $A(s) > A(t)$ for all $s > t$, so that

$$v(t) = \int_t^\infty \frac{q(1-\alpha)}{A(s)} e^{-\rho(s-t)} ds < \int_t^\infty \frac{q(1-\alpha)}{A(t)} e^{-\rho(s-t)} ds = \frac{q(1-\alpha)}{\rho A(t)},$$

or $V > \rho/[q(1-\alpha)]$ which contradicts $V \to 0$. ■

Proof of Proposition 2

See Bénassy (1998) or de Groot and Nahuis (1998).

Proof of Proposition 3

The equilibrium growth rate (8) is equal to zero if

$$F(\delta)(1-\delta) \leq \frac{\alpha \rho a}{(1-\alpha)L}. \tag{19}$$

Define the left-hand side of (19) as $LHS(\delta)$. $LHS(\delta)$ is greater or equal than zero for all $\delta \in [0,1]$ with $LHS(0) = LHS(1) = 0$. Moreover, $\partial LHS/\partial\delta = -F + (1-\delta)F'$ is positive for values of δ close to zero and negative for values of δ close to one. Finally,

$\partial^2 LHS/\partial\delta^2 < 0$. Thus, there exists a unique $\hat{\delta} \in (0,1)$ such that $\partial LHS/\partial\delta = 0$. If $\hat{\delta} = \arg\max[(1-\delta)F(\delta)]$ is such that $(1-\hat{\delta})F(\hat{\delta}) \leq \alpha\rho a/[(1-\alpha)L]$, then $g_c^* = 0$ for any δ. This proves Statement 1.

For g_c^* to become positive in (18) the government has to set δ such that

$$F(\delta)(1-\delta) > \frac{\alpha\rho a}{(1-\alpha)L}. \tag{20}$$

If $\hat{\delta} = \arg\max[(1-\delta)F(\delta)]$ is such that $(1-\hat{\delta})F(\hat{\delta}) > \alpha\rho a/[(1-\alpha)L]$, then there exist δ_{min} and δ_{max} with $0 < \delta_{min} < \delta_{max} < 1$ such that $(1-\delta_{min})F(\delta_{min}) = \alpha\rho a/[(1-\alpha)L] = (1-\delta_{max})F(\delta_{max})$. Then, for all $\delta \in (\delta_{min}, \delta_{max})$ it holds that $g_c^* > 0$. This proves the first part of Statement 2. Moreover, for all $\delta \in (\delta_{min}, \delta_{max})$, g_c^* takes strictly positive values. Tedious, but straightforward manipulations reveal that $\partial^2 g_c^*/\partial\delta^2 < 0$ such that g_c^* has a unique maximum. Denote $\delta^* = \underset{\delta\in(\delta_{min},\delta_{max})}{\arg\max}\ g_c^*$. This proves the second part of Statement 2. ■

Proof of Proposition 4

We proof each statement of the Proposition separately, starting with Statement 1.

1. On the intervals $[0, \delta_{min}]$ and $[\delta_{max}, 1]$, U is a monotonically declining and strictly concave function in δ. Moreover, $U(0) > U(\delta_{max})$. Thus, on the interval $[0, \delta_{min}] \cup [\delta_{max}, 1]$ U has its global maximum at $\delta = 0$.

   On the interval $[\delta_{min}, \delta_{max}]$, it holds for all $\delta > \delta^*$ that $\partial g_c^*/\partial\delta < 0$ such that U unambiguously declines in δ. However at δ_{min}, increasing δ has two opposing effects on U. A higher δ negatively impinges on welfare by lowering initial consumption c_0 and positively affects welfare by allowing for a higher consumption growth rate g_c^*. If the latter effect dominates the former, then $(dU/d\delta)|_{\delta=\delta_{min}} > 0$. Moreover, U is a continuous function and $U(\delta_{max}) < U(\delta_{min})$. Thus, if $(dU/d\delta)|_{\delta=\delta_{min}} > 0$ holds, then there exists an interior maximum of U in $[\delta_{min}, \delta_{max}]$. Denote $\delta^{**} = \underset{\delta\in[\delta_{min},\delta_{max}]}{\arg\max}\ U$.

From (15) the necessary and sufficient condition for $\left(dU/d\delta\right)|_{\delta=\delta_{min}} > 0$ is

$$\frac{1}{\rho}\frac{\partial g_c^*}{\partial \delta}\bigg|_{\delta=\delta_{min}} > \left|\frac{\partial \ln c_0}{\partial \delta}\right|\bigg|_{\delta=\delta_{min}} \quad \Leftrightarrow$$

$$\frac{(1-a)^2\sigma}{\alpha a\rho}\frac{F'(\delta_{min})\,\alpha[(1-\delta_{min})\,L+a\rho]-LF(\delta_{min})\,[F(\delta_{min})(1-\alpha)+\alpha]}{[F(\delta_{min})(1-\alpha)+\alpha]^2}$$

$$> \frac{L}{(1-\delta_{min})\,L+a\rho}+\frac{F'(\delta)\,(1-\alpha)}{[F(\delta_{min})(1-\alpha)+\alpha]}. \tag{21}$$

Using $F(\delta_{\min}) = (a\alpha\rho)/\left[(1-\alpha)(1-\delta_{\min})\,L\right]$ in (21) and rearranging yields

$$\sigma\frac{(1-\alpha)}{\alpha}\Bigg[\underbrace{(1-\delta_{\min})\,F'(\delta_{\min})-F(\delta_{\min})}_{>0}\Bigg]-\frac{a\rho F'(\delta_{\min})}{L} > \frac{a\alpha\rho}{(1-\alpha)(1-\delta_{\min})\,L}. \tag{22}$$

The term in square brackets on the left-hand side of (22) is positive because Statement 2 of Proposition 3 holds. Moreover, the right-hand side of (22) is equal to $F(\delta_{min})$, and therefore strictly smaller than 1. Thus, (22) is easily met, e. g., for sufficiently large values of σ.

For δ^{**} to maximize U on the whole interval $[0,1]$, we need in addition that $U(\delta^{**}) > U(0)$, i. e.,

$$\frac{(1-\alpha)(1-\delta^{**})\,F(\delta^{**})\,L-a\rho\alpha}{a\rho\,[F(\delta^{**})(1-\alpha)+\alpha]}-\ln\left[F(\delta^{**})(1-\alpha)+\alpha\right] > \ln\left[\frac{L}{\alpha\left((1-\delta^{**})\,L+a\rho\right)}\right].$$

Figure 1 depicts this case.

If $\left(dU/d\delta\right)|_{\delta=\delta_{min}} > 0$ and $U(\delta^{**}) < U(0)$, then $\delta = 0$ is the global maximizer of U. This case is illustrated in Figure 3.

If condition (22) does not hold, then U is a monotonically declining function in $[0,1]$ and is maximized at $\delta = 0$ (see Figure 2 for an example).

2. Recall that welfare is given by $U = \frac{1}{\rho}\left(\ln c_0 + \frac{g_c^*}{\rho}\right)$. The second term is maximized at $\delta^* > 0$. Moreover, $\partial \ln c_0/\partial\delta < 0$ for any δ. Thus, the welfare-maximizing share has to be strictly smaller than the growth-maximizing one, i. e., $\delta^{**} < \delta^*$.

■

Calibrations underlying Figures 1 - 3

All figures were produced with *Mathematica*. The notebooks are available upon request. The parameter values were not chosen to represent a particular economy but rather to construct cases that underline the main points of the paper. Clearly, all calibrations satisfy Assumption 1. Moreover, it holds that $g_c^P > g_c$, i. e., in equilibrium growth is inefficiently low.

All figures specify $F(\delta) = \delta^\nu$. The parameters of Figure 1 are $\nu = 3/4$, $A_0 = 2$, $L = 20$, $a = 3$, $\rho = 1/5$; $\sigma = 40$, $\alpha = 5/6$. The parameters of Figure 2 are $\nu = 3/4$, $A_0 = 2$, $L = 20$, $a = 3$, $\rho = 1/5$; $\sigma = .5$, $\alpha = 5/6$. The parameters of Figure 3 are $\nu = 3/4$, $A_0 = 2$, $L = 5$, $a = 1$, $\rho = 1/5$; $\sigma = 3$, $\alpha = 5/6$.

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