Mathematics Seminar
of the
University of Luxembourg
in cooperation with the
Luxembourg Mathematical Society

December 2006

5 December 2006 at 2 pm **Room 3.04 bs**

Mathematics Colloquium
(lecture in the frame of the Seminar for doctorands)
Roland Friedrich
Max-Planck-Institut Bonn

Stochastic Loewner Evolutions and some of its Ramifications

Abstract

In this talk we shall give an overview of various aspects of Stochastic Loewner Evolutions (SLE). SLE developed spectacularly in the last couple of years, starting with an ingenious idea of how to describe possible scaling limits of various discrete 2D models. So, in particular we shall talk about the underlying and unifying framework that permitted to generalize it to non trivial topologies as e.g. arbitrary domains or surfaces, or to singular geometries like polygons. Further we shall also briefly mention its connections with representation theory of the Virasoro algebra and Conformal Field Theory. The talk is of interest to a broader mathematical or physical audience.

5 December 2006 at 5 pm **Room 3.04 bs**

Johannes Huebschmann
Université des Sciences et Technologies de Lille

Singular Poisson-Kähler geometry of stratified Kähler spaces

Abstract

A *stratified Kähler space* is a stratified symplectic space together with a complex analytic structure which is compatible with the stratified symplectic structure; in particular

each stratum is a Kähler manifold in an obvious fashion. The notion of stratified Kähler space establishes an intimate relationship between nilpotent orbits, singular reduction, invariant theory, reductive dual pairs, Jordan triple systems, symmetric domains, and prehomogeneous spaces. The purpose of the talk is to illustrate the significance of stratified Kähler spaces.

Examples of stratified Kähler spaces abound. The closure of a holomorphic nilpotent orbit carries a normal Kähler structure. Symplectic reduction carries a Kähler manifold to a normal stratified Kähler space in such a way that the sheaf of germs of polarized functions coincides with the ordinary sheaf of germs of holomorphic functions. Projectivization of holomorphic nilpotent orbits yields exotic stratified Kähler structures on complex projective spaces and on certain complex projective varieties including complex projective quadrics. Other examples come from certain moduli spaces of holomorphic vector bundles on a Riemann surface and variants thereof; in physics language, these are spaces of conformal blocks. Still other physical examples are reduced spaces arising from angular momentum.

In the world of singular Poisson-Kähler geometry, reduction after quantization coincides with quantization after reduction: For a stratified symplectic space, the concept of stratified polarization, which is defined in terms of an appropriate Lie-Rinehart algebra, encapsulates polarizations on the strata and, moreover, the behaviour of the polarizations across the strata. Exploiting the notion of stratified Kähler space, one can prove that, given a Kähler manifold, reduction after quantization coincides with quantization after reduction in the sense that not only the reduced and unreduced quantum phase spaces correspond but the invariant unreduced and reduced quantum observables as well.

12 December 2006 at 5 pm **Room 3.04 bs**

Mathematics Colloquium
(lecture in the frame of the Mathematics Seminar)

Karl-Theodor Sturm
University of Bonn

Optimal Transportation, Ricci Curvature and Diffusions on the L2-Wasserstein Space

Abstract

We introduce and analyze generalized Ricci curvature bounds for metric measure spaces (M, d, m), based on convexity properties of the relative entropy $\mathrm{Ent}(\cdot|m)$. For Riemannian manifolds, $\mathrm{Curv}(M, d, m) \geq K$ if and only if $\mathrm{Ric}_M \geq K$ on M; for the Wiener space, $\mathrm{Curv}(M, d, m) = 1$.

One of the main results is that these lower curvature bounds are stable under (e.g. measured Gromov-Hausdorff) convergence. This solves one of the basic problems in this field, open for many years.

Furthermore, we introduce a (more restrictive) curvature-dimension condition $\mathrm{CD}(K, N)$ which implies sharp versions of the Brunn-Minkowski inequality, of the Bishop-Gromov volume comparison theorem and of the Bonnet-Myers theorem. Moreover, it allows to construct a canonical Dirichlet form with Gaussian bounds for the corresponding heat kernel.

Finally, we indicate how to construct a canonical reversible process on the L^2-Wasserstein space of probability measures $\mathcal{P}(\mathbb{R})$, regarded as an infinite dimensional Riemannian manifold. This process has an invariant measure $\mathbb{P}^\beta$ which may be characterized as the 'uniform distribution' on $\mathcal{P}(\mathbb{R})$ with weight function $\frac{1}{Z}\exp(-\beta \cdot \mathrm{Ent}(\cdot|m))$ where m denotes a given finite measure on $\mathbb{R}$. One of the key results is the quasi-invariance of this measure $\mathbb{P}^\beta$ under push forwards $\mu \mapsto h_*\mu$ by means of smooth diffeomorphisms h of $\mathbb{R}$.

13 (!) December 2006 at 11.15 am **Room 3.04 bs**

Mathematics Colloquium
(lecture in the frame of the Probability Seminar)
Max von Renesse
TU Berlin

Entropic Measure and Wasserstein Diffusion of Probability Measures on the Unit Interval

Abstract

This is a report on recent results obtained jointly with K-T Sturm (Bonn). We construct a diffusion process on the space of probability measures on the unit interval by Dirichlet form methods. For this we construct a Gibbs type invariant measure with the Boltzmann entropy function as Hamiltonian. We show an integration by parts formula for this measure and demonstrate, that choosing the appropriate notion of a gradient, the intrinsic metric of the process is the quadratic Wasserstein distance.

18 (!) December 2006 at 5 pm **Room 3.04 bs**

Robert Wolak
Jagiellonian University of Krakow

Geometry of singular spaces and foliations

Abstract

We will show how singular stratified spaces arise naturally as leaf closure spaces of singular Riemannian foliations. Then we discuss the relation between foliated structures and geometric structures on the leaf closure space giving conditions which assure that given structures project onto the leaf closure space. The foliated space can be considered as a desingularization of its leaf space. This class includes the orbit spaces of compact group actions and orbifolds in particular. We shall discuss Riemannian, Kähler, symplectic, Poisson and Finsler structures as well as linear connections and (transversally) harmonic maps.