

MATHEMATICS SEMINAR  
of the  
UNIVERSITY OF LUXEMBOURG  
in cooperation with the  
LUXEMBOURG MATHEMATICAL SOCIETY

April 2007

17 April 2007 at 5 pm

Room 3.04 bs

Friedrich Wagemann  
University of Nantes

**Gerbes and crossed modules of Lie groupoids**

Abstract

In categories  $\mathcal{C}$  like the category of groups or (Lie-)algebras, *crossed modules* are known to be linked to abelian 3-cohomology. A crossed module is, roughly speaking, a morphism  $\mu : X \rightarrow Y$  between objects  $X, Y$  of the category  $\mathcal{C}$ , together with a compatible action of  $Y$  on  $X$ . To such a crossed module, one associates a 4-term exact sequence obtained by passing to kernel and cokernel of  $\mu$ .

On the other hand, integral singular 3-cohomology of a manifold  $M$  can be described in terms of *gerbes with band*  $S^1$ . This comes from the interpretation of this kind of gerbes as  $PU(\mathcal{H})$ -principal bundles  $P$  on  $M$ , where  $\mathcal{H}$  is an infinite dimensional, separable Hilbert space. More precisely, lifting the structure group of  $P$  to  $U(\mathcal{H})$  defines locally (on  $M$ ) the category of principal  $U(\mathcal{H})$ -bundles  $\hat{P}$  whose quotient by the center is  $P$ . This sheaf of categories is a gerbe with band  $S^1$ .

The purpose of my talk is to show in general that the lifting of the structure group  $K$  of a  $K$ -principal bundle  $P$  on  $M$  to a central extension  $\hat{K}$  of  $K$  by an abelian group  $Z$  gives rise to a gerbe with trivial band  $Z$  on  $M$ . The key point for this is to interpret the data in terms of a crossed module of Lie groupoids, based on the *Atiyah groupoid* associated to  $P$ . Crossed module of Lie groupoids whose kernel is a trivial bundle  $Z \times M$  with abelian  $Z$  and whose cokernel is the pair groupoid  $M \times M$  are shown to be in one-to-one correspondence to gerbes with abelian trivial band. The corresponding 3-cohomology obstruction class of the crossed module gives the Dixmier-Douady class of the gerbe.

The origin of this joint work Camille Laurent-Gengoux goes back to open questions of Karl-Hermann Neeb who defined obstruction classes in the de Rham 3-cohomology of  $M$  using a crossed module of topological Lie algebras associated to  $P$  and

$$1 \rightarrow Z \rightarrow \hat{K} \rightarrow K \rightarrow 1.$$

The interpretation of the lifting problem in terms of crossed modules of Lie groupoids is due to Kirill Mackenzie and his school.

17 April 2007 at 6:30 pm

Bâtiment de Recherche B, ground floor, common room

**Annual meeting of the Luxembourg Mathematical Society**

24 April 2007 at 5 pm

Room 3.04 bs

Paulo Antunes  
École Polytechnique Palaiseau

**Big bracket and Lie algebroid structures**

Abstract

We will see how structures such as Lie (bi)algebroids and Courant algebroids appear as hamiltonians on a supermanifold with the 'big bracket' as natural Poisson bracket in the cotangent space. We will also show how the 'big bracket' in this supermanifold setting allow us to characterize other algebraic structures.