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Explaining the Decline in the US Labor Share: Taxation and Automation

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Abstract

This study provides evidence for the US that the secular decline in the labor share is not only explained by technical change or globalization, but also by the dynamics of factor taxation, automation capital, and population growth. First, we empirically find indications of co-integration for the 1974-2008 period. Permanent effects on factor shares emanate from relative factor taxation. The latter also have a lasting effect on the use of robots. Variance decompositions reveal that taxing contributes to changes in the two income shares and in automation capital. Second, we analyse and calibrate a neoclassical growth model extended to include factor taxation, automation capital, and capital adjustment costs. The model is able to replicate the dynamics of the observed functional income distribution in the US during the 1965-2015 period. Counterfactual experiments suggest that the fall in the labor share would have been significantly smaller if labor and capital income tax rates had remained at their respective level of the 1960s.

1 Introduction

The functional income distribution in most OECD countries has changed significantly over recent decades. Figure 1.1 displays the labor share of the United States, Japan, and the Euro Area (EA-12) from 1960 to 2018. During this period these labor shares declined by roughly 5 to 15 percentage points.

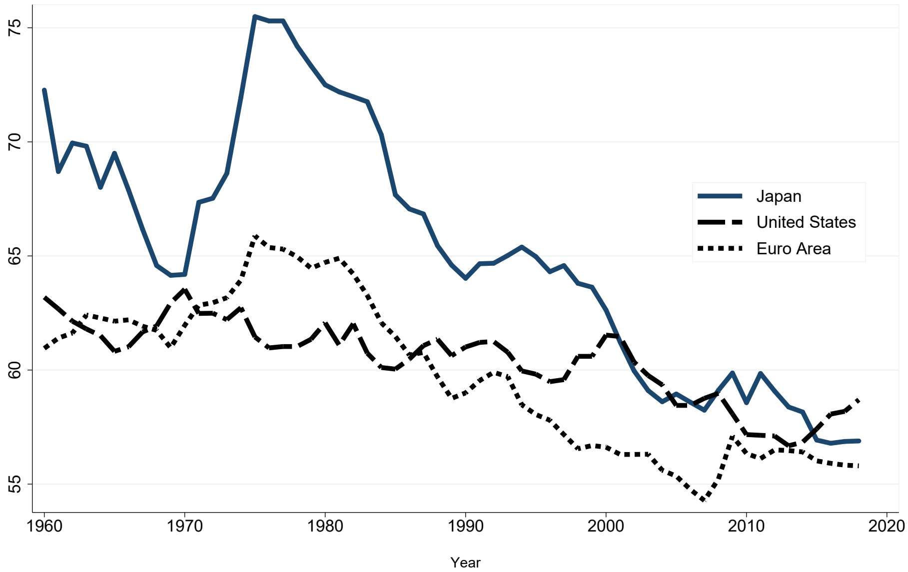
The existing literature emphasizes the role of several factors that contributed to this decline. They include skill-biased technological change (Goldin and Katz, 2008), declining investment good prices (Karabarbounis and Neiman, 2014), a sufficiently high elasticity of substitution between capital and labor (Piketty, 2014), globalization (Elsby et al. (2013), Helpman (2018)), "superstar firm" dynamics (Autor et al., 2019) or population aging (Irmen, 2020). In contrast to these studies, the present paper emphasizes the effect of taxation and population growth on the implementation of automation capital and the labor share.

The focus of our analysis is on the United States where two related evolutions accompany the decline in the labor share. First, as shown in the left panel of Figure 1.2, the difference in the effective tax rates on income from capital and labor shrinks from 1954 to 2010. Over this time span, capital income is taxed more heavily with an average effective tax rate of 41% compared to 23% for labor income. However, compared to the respective level in 1954 the effective tax rate on income from capital is much lower in 2010 whereas the one for labor income is much higher. Second, as shown in the right panel of Figure 1.2, in parallel with the declining labor share the population growth rate has fallen, in annualized terms, from 1.7% in 1950 to 0.7% in recent years.

Intuitively, the shrinking difference in the taxation of capital and labor and the decline in population growth induce firms to choose production processes that replace labor more and more with automation capital. On the one hand, the downward trend in the capital income tax rate increases the incentive to build up capital. As long as different types of capital are tied by a no-arbitrage condition, some of the additional capital will come in the form of automation capital.

On the other hand, the labor supply declines at the intensive margin if the labor income tax rate increases. In addition, a declining population growth rate reduces the labor supply at the extensive margin. As argued, e. g., by Heer and Irmen (2014), the relative decline in the labor force at both margins pushes wages up and, therefore, boosts the incentive to engage in labor-saving automation investments.

Figure 1.1: Labor share, 1960-2018: US, Japan, and Euro Area (EA-12).



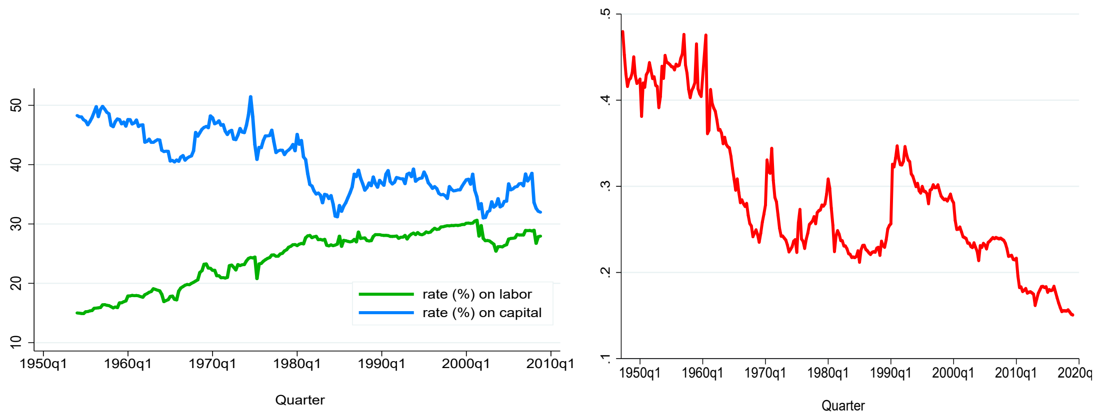
Note: Labor share is wage income in GDP at current prices; Source: AMECO database, OECD.

Hence, these tendencies strengthen the comparative advantage of automation capital in production. This leads to the prediction that the amount of automation capital per worker should increase. Figure 1.3 confirms this prediction: the time series of automation capital proxied by the (nowcasted) stock of robots per 1,000 (full-time) employees in the US from 1975 to 2010 is clearly increasing. Finally, as automation capital replaces labor, the labor share is expected to decline.

Our analysis derives two main sets of results. First, we provide empirical evidence for the US supporting the explanation for a declining labor share set out above. We find indications for cointegrating relationships over the period from the first quarter of 1974 to the fourth quarter of 2008 (henceforth, 1974:q1-2008:q4). Permanent effects on factor shares emanate from shocks in relative factor taxation. Changes in relative factor taxation also permanently and sizably affect the use of automation capital. The forecast error variance decomposition (FEVD) analysis of fitted vector error correction (VEC) models reveals that taxing policies account for up to roughly 22% of observed changes in the two income shares and for up to about 35% of the dynamics in automation capital.

The second set of results emanates from the quantitative analysis of a dynamic general equilibrium model that replicates the downward trend in the US labor share from 1965

Figure 1.2: Left panel: Quarterly US tax rates on income from labor and capital in percent; Right panel: US population growth rates in percent.

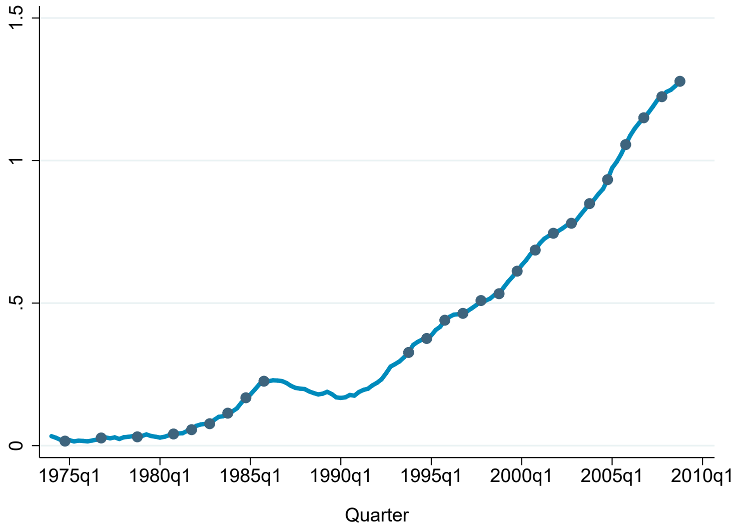


Sources: Left panel: Gomme et al. (2011); Right panel: BEA for population growth, seasonal adjustment: X12-ARIMA

to 2015. We derive these findings in a new variant of the neoclassical growth model where taxes on capital and labor income as well as population growth play a key role for the dynamics of the functional income distribution. We follow Steigum (2011) and distinguish two types of capital, traditional capital and automation capital. While traditional capital is complementary to labor and automation capital, the latter two factors of production are perfect substitutes. Hence, automation capital substitutes linearly for labor. We extend Steigum's framework in three directions. First, both types of capital investments are subject to adjustment costs. Second, we allow for an endogenous labor supply and, finally, include an active government that charges taxes to finance its consumption.

We use this model to compute the dynamics of the labor share, the stocks of traditional and automation capital, and the evolution of the (endogenous) labor supply over the period 1965-2029. We maintain that automation capital was not introduced into production processes until 1965. Our calibration incorporates the time series of effective capital and labor income tax rates as well as of the population growth rates that are displayed in Figure 1.2. The use of quadratic adjustment costs for both types of capital investments is in the tradition of Hayashi (1982). As a result, our model is able to closely replicate the downward trend in the labor share. In particular, in line with the data, we match the actual drop in the labor share from 62% to 57% between 1965 and 2015.

Figure 1.3: Robots per 1K full-time workers in the US: annual and nowcasted quarterly data.



Note: *Dotted* data points up to 1985:q4 are own calculations based on Tani (1989) and CPS, From 1994:q4 onwards they are obtained from Acemoglu and Restrepo (2019), original source: IFR; *Solid* blue line: nowcasted (pseudo-)quarterly data; detail on nowcasting is given in Appendix.

Sources: Tani (1989), Acemoglu and Restrepo (2019), IFR, CPS.

The present paper is related to several strands of the literature. First, our empirical analysis contributes to a young and growing body of empirical literature. It is concerned with the interplay of automation and institutional as well as macroeconomic variables in the context of economic growth. To some extent, this empirical literature has revived an old debate about the secular stagnation hypothesis (Hansen, 1939). Acemoglu and Restrepo (2017) find evidence in suppport of population ageing generating higher GDP per capita growth through giving a positive impulse to automation. In contrast, e.g., the conclusion of the study by Aksoy et al. (2019) goes in the opposite direction. So far, the focus has been nearly throughout on longitudinal data. The seminal study by Graetz and Michaels (2018) covers 14 years and 14 sectors in 17 countries. It is longitudinal with two cross-sectional dimensions consisting in economies and industries. Also the seminal empirical paper by Acemoglu and Restrepo (2019) is longitudinal in nature using data on robots exploiting the geographic variation within the United States. In the context of the rise of robots in China, Cheng et al. (2019) also rely on longitudinal survey-based manufacturing firm-level data. All of these seminal studies reveal insightful and, in parts, even causal statistical associations between automation, institutional policies, and macroeconomic variables. However, the empirical establishment of long run –in the sense of cointegrating– relationships between these variables is missing. We seek to fill this gap. Hitherto, attempts to contribute empirically in this direction are in their early infancy. We are only aware of such attempts by Bergholt et al. (2019). These authors estimate sign-restricted structural vector autoregressive (SVAR) models with permanent shocks to study "medium-run" trend relationships in this context.

Second, our theoretical analysis shows that an appropriately augmented variant of Steigum (2011)'s model provides a tractable framework of analysis for macroeconomic phenomena related to the substitution of labor with automation capital that may serve as an alternative to existing models used in the literature including, among others, Acemoglu and Restrepo (2018), Hémous and Olsen (2018), Eden and Gaggl (2018), or Irmen (2020). Our quantitative analysis suggests that the inclusion of adjustment costs in the spirit of Hayashi (1982) for both types of capital into Steigum's model proves particularly useful for a realistic description of the introduction and the buildup of robots.

Finally, and relatedly, our calibration analysis highlights an important role of factor taxation for the incentives to automate and the evolution of the US functional income distribution. This adds a new positive explanation for the observed decline in the US labor share to the literature mentioned above. In particular, our counterfactual experiments suggest that the labor share in 2015 could have been substantially higher had the tax rates on capital and labor income remained at their 1965 level. These findings support the normative assessment of Acemoglu et al. (2020) who argue that the US tax system generates excessive automation incentives and implies a suboptimally low labor share.

The paper is organized as follows. In Section 2, we present and interpret the time series evidence for the US economy relying on cointegration analysis and VEC models. An wide range of test details and robustness checks are relegated to the Appendix (see Appendix A.1-A.4). Section 3 studies a variant of the neoclassical growth model that distinguishes between traditional and automation capital and allows for factor taxation and capital adjustment costs. Sections 3.1-3.4 introduce the model and define the competitive equilibrium with dynamic taxation. Section 3.5 provides an analysis of the steady state without automation capital. Section 3.6 analyses the properties of the asymptotic balanced growth path with automation capital. Section 3.7 details how we calibrate the model. Section 3.8 features our results on the transition dynamics (the methodological description of the model's transition analysis is relegated to Appendix A.5). Here, we first devise a calibration that replicates the actual decline of the US labor share during the period 1965-2015. Second, we conduct counterfactual experiments suggesting that the effective tax rates on capital and labor income had a significant effect on the labor share over the considered period. Section 5 concludes.

2 Empirical Analysis

This section provides a test for and a quantification of long-run equilibrium relationships between the US income shares and factor taxation, automation capital, and population growth. We find permanent effects of factor taxation and population growth on factor shares and the use of robots, respectively. Pairwise error correcting or cointegrating relationships are established for the following pairs of variables: i) relative factor taxation and the capital share, ii) use of robots and the capital share, and iii) population growth and the labor share.

Establishing equilibrium restoring mechanisms in the short and medium-run requires historical series of adequate length and frequency. We document issues related to the compilation and the construction of such time series in the next section. Then, relevant details concerning our methodological framework and the identification strategy are provided. The section ends with an illustration of the responses of factor shares and automation capital to factor taxation and population growth. Detail on the contribution to variance of relative factor taxation to income shares and to the use of robots is provided.

2.1 Time Series

We work with time series in quarterly frequency for the US economy ranging from the first quarter of 1974 to the fourth quarter of 2008. The limiting factor with regard to the end of our sample period is the quarterly series for US effective tax rates on income from capital and labor provided by Gomme et al. (2011). The limiting factor for the start of our empirical analysis is Tani (1989). The latter provides the numerator of the first part of actual datapoints for the nowcast of the automation capital series as shown in Figure 1.3. Underlying is a series of biannual frequency for the 1970s and, from 1980-1985, of annual frequency. This series states the industrial robot population in the US, where an industrial robot is defined by the Industrial Organization for Standardization (ISO).[1] The denominator in the construction of datapoints prior to 1986 shown in Figure 1.3 is the corresponding annual average of the seasonally adjusted (SA) number of full-time employees in the US as collected in the Current Population Survey (CPS). The ISO normed definition of robots and the expression in units of "per thousand workers" allows us to combine it with the corresponding annual data from the International Federation of Robotics (IFR) as provided by Acemoglu and Restrepo (2019) and to nowcast a quarterly series of automation capital. Our nowcast is based on the procedure proposed by Shumway and Stoffer (2008). It relies on the Kalman filter in combination with the expectation maximization (EM) algorithm. A detailed outline of this technique is contained in the Appendix.

Fernald (2014) provides the capital share series in quarterly frequency. Quarterly series for the US population (in thousand) stem from the BEA and is provided in the FRED database. We seasonally adjust this series by means of an X12-ARIMA and consider its log first differences transform. Also from FRED we retrieve the BEA series of the US quarterly gross compensation of employees in the form of paid wages and salaries as well as the corresponding GDP series.

These procedures prepare the data set for a multivariate cointegration analysis. It comprises series of quarterly frequency for the US income shares, factor taxation, population growth, and automation capital. The sample period covers all quarters from 1974 to 2008.

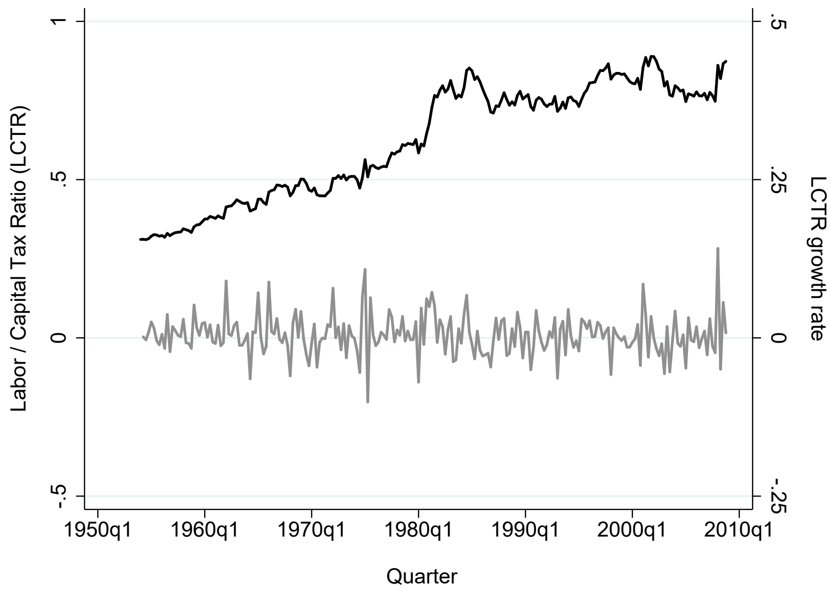
2.2 VEC Model Analysis

We rely on the maximum likelihood (ML) based framework for estimation and inference in cointegrating systems often referred to as the "Johansen approach" (Johansen et al., 1995). The variables of interest include the two effective tax rates as well as the two factor shares. Due to the natural log transformation, it is technically feasible to jointly consider the shares of labor and capital income. However, a (near perfect) collinearity prevents the joint integration of both income tax rates into a particular

[1] Accordingly, an "industrial robot is an automatic position-controlled reprogrammable multifunctional manipulator having several degrees of freedom capable of handling materials, parts, tools, or specialized devices through variable programmed motions for the performance of a variety of tasks" (Tani, 1989, p. 192).

VECM specification. An alternative to including both tax rates is the construction and use of the labor-tax-to-capital-tax ratio (LCTR, Figure 2.1) which serves as a workaround in our VECM specifications. It organically takes care of the historical tax policy mix.

Figure 2.1: Labor-tax-to-capital-tax ratio (LCTR), 1954:q1 to 2008:q4.



Note: Black – levels (left ordinate); grey – log first differences (right ordinate); Source: Gomme et al. (2011)

2.2.1 Johansen Procedure

Our reduced-form model space consists of three dimensions: a relatively exogenous variable X_t (population growth), a policy variable Y_t (the factor tax policy mix, i. e., the LCTR), and a multivariate group of response variables $\mathbf{W}_t$ (our automation capital proxy and the two factor shares), making it

$$\mathbf{Z}_t = [X_t, Y_t, \mathbf{W}_t] \Rightarrow \mathbf{Z}_t = \mathbf{A}_1\mathbf{Z}_{t-1} + \mathbf{A}_2\mathbf{Z}_{t-2} + ... + \mathbf{A}_p\mathbf{Z}_{t-p} + \mathbf{u}_t. \tag{2.1}$$

In standard VEC notation this becomes

$$\Delta\mathbf{Z}_t = \Gamma_1\Delta\mathbf{Z}_{t-1} + \Gamma_2\Delta\mathbf{Z}_{t-2} + ... + \Gamma_{p-1}\Delta\mathbf{Z}_{t-p-1} + \Pi\mathbf{Z}_{t-1} + \mathbf{u}_t, \tag{2.2}$$

where $\Gamma_i = (\mathbf{I} - \mathbf{A}_1 - \mathbf{A}_2 - ... - \mathbf{A}_p)$ for all $i = 1, ..., p$. Π can be thought of as consisting of an adjustment speed matrix $\mathbf{a}$, and a long-run coefficient matrix $\mathbf{b}$, such that $\Pi =$

$\mathbf{ab}'$, where $\mathbf{b}'\mathbf{Z}_{t-1}$ is the vectorial analogue of the error correction term in the Engel-Granger approach. For an exemplary unity lag order

$$\Delta \mathbf{Z}_t = \begin{pmatrix} \Delta Y_t \\ \Delta X_t \\ \Delta \mathbf{W}_t \end{pmatrix} = \Gamma_1 \begin{pmatrix} \Delta Y_{t-1} \\ \Delta X_{t-1} \\ \Delta \mathbf{W}_{t-1} \end{pmatrix} + \Pi \begin{pmatrix} Y_{t-1} \\ X_{t-1} \\ \mathbf{W}_{t-1} \end{pmatrix} + \mathbf{e}_t$$

$$= \Gamma_1 \begin{pmatrix} \Delta Y_{t-1} \\ \Delta X_{t-1} \\ \Delta W_{1\ t-1} \\ \Delta W_{2\ t-1} \\ \Delta W_{3\ t-1} \end{pmatrix} + \tag{2.3}$$

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{22} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \\ a_{51} & a_{52} & a_{53} & a_{54} \end{pmatrix} \begin{pmatrix} b_{11} & b_{21} & b_{31} & b_{41} & b_{51} \\ b_{12} & b_{22} & b_{32} & b_{42} & b_{52} \\ b_{13} & b_{23} & b_{33} & b_{43} & b_{53} \\ b_{14} & b_{24} & b_{34} & b_{44} & b_{54} \end{pmatrix} \begin{pmatrix} Y_{t-1} \\ X_{t-1} \\ W_{1\ t-1} \\ W_{2\ t-1} \\ W_{3\ t-1} \end{pmatrix} + \mathbf{e}_t.$$

The central requirement of cointegration (CI) is a reduced rank of $\Pi = \underset{n\times r}{\mathbf{a}} \left(\underset{n\times r}{\mathbf{b}} \right)'$, i. e., $\mathbf{Z}_t \sim I(1) \Rightarrow \Delta \mathbf{Z}_{t-1} \sim I(0) \Rightarrow \Pi \mathbf{Z}_{t-1} \overset{!}{\sim} I(0)$ for $\mathbf{u}_t \sim I(0)$. This allows for up to $n-1 = 5-1 = 4$ CI - relationships of the form $\mathbf{b}'\mathbf{Z}_{t-1} \sim I(0)$, $r \leq n-1$ cointegrating vectors $\in \Pi$, i. e., r columns of $\mathbf{b}$ form r linearly independent stationary combinations of variables $\in \mathbf{Z}_t$.

As a first step we perform augmented Dickey-Fuller (ADF) tests for all considered series. Throughout the unit root hypothesis cannot be rejected at a one percent level of significance according to the MacKinnon approximate p-values. For all log first differences transforms the null of a unit root is rejected at every conventional level of significance. The order of integration of variables is $I(1)$ (see the Appendix for details). In a second step the appropriate lag length choice is made resorting to likelihood ratio (LR) testing. Here, we follow Schwert (1989) and set $p_{\max} = \left[12 \cdot (T/100)^{\frac{1}{4}}\right]$ quarters where $T = 140$ and brackets denote the nearest integer part of the argument. Hence, $p_{\max} = 13$.

As $\Pi = -(\mathbf{I} - \mathbf{A}_1 - \mathbf{A}_2 - ... - \mathbf{A}_p)$, or equivalently $\Pi = \sum_{i=1}^{p} \mathbf{A}_i - \mathbf{I}$, the Johansen-procedure makes use of Granger's Representation Theorem which states that if matrix

Π has reduced rank $r < n$ with n denoting the number of non-stationary variables considered, then there exist $n \times r$ matrices $\mathbf{a}$ and $\mathbf{b}$ each with rank r such that $\Pi = \mathbf{ab}'$ and $\mathbf{b}'\mathbf{Z}_t \sim I(0)$; then, r is the number of cointegration relations (cointegrating rank) and each column of $\mathbf{b}$ is a cointegrating vector. However, before estimating Π, deterministic components of the general system

$$\begin{aligned} \Delta\mathbf{Z}_t &= \Gamma_1 \Delta\mathbf{Z}_{t-1} + ... + \Gamma_{p-1}\Delta\mathbf{Z}_{t-p-1} \\ &+\mathbf{a}\begin{pmatrix} \mathbf{b} \\ \mathbf{m}_1 \\ \mathbf{d}_1 \end{pmatrix}' \begin{pmatrix} \mathbf{Z}_{t-1} & 1 & t \end{pmatrix} + \mathbf{m}_2 + \mathbf{d}_2 t + \mathbf{u}_t \end{aligned} \quad (2.4)$$

must to be chosen. The above system can discriminate four central versions: (v1) no intercept or trend in the cointegrating equation (CE) or VAR part ($\mathbf{d}_1 = \mathbf{d}_2 = \mathbf{m}_1 = \mathbf{m}_2 = 0$); (v2) intercept and no trend in the CE part and neither intercept nor trend in the VAR part ($\mathbf{d}_1 = \mathbf{d}_2 = \mathbf{m}_2 = 0$), i. e., the no linear trend in data case (first differences have zero mean); (v3) intercept in the CE part and the VAR part, but no trends ($\mathbf{d}_1 = \mathbf{d}_2 = 0$), i. e., no linear trends in levels of data case; (v4) intercept in the CE part and the VAR part paralleled by a linear trend in the CE or in the VAR part, i. e., the linear trend in the CE case, sometimes referred to as the exogenous growth case. Following the Pantula Principle, we start with the most restrictive model, i. e., $r = 0$ in combination with v1, and move gradually to the least restrictive one where $r = n - 1$ in combination with v4. For each gradual step, the trace-test statistics is compared with the critical value and the iteration stops when for the first time the null of no cointegration is not rejected. This determines the order of Π, i. e., the number of cointegration vectors. Besides trace-based rank testing, we cross-check and validate our findings with maximum eigenvalue and information criteria-based cointegration rank tests (see the Appendix for details).

2.2.2 Identification Strategy

Our set-up implies an identification scheme for the VAR part of the VECM specification that resembles what has become known as "Slow-r-Fast" scheme in the literature (Stock and Watson, 2016, p. 455, pp. 477-478).[2] It delivers a block recursive scheme with

[2] The name stems from the order of partitioning of the dependent variables' vector by nature of its, partially sub-vectorial, elements. It has been used, in particular, to identify monetary policy shocks

an ordering of the respective elements of the partitioned $\mathbf{Z}_t$ vector that is not decisive for the system rotation matrix. In our application, population growth is supposed to be "the least" endogenous and does not respond to the policy variable, i. e., to either the labor or the capital tax rate or to the LCTR, and to any of the response variables. The latter are comprised of our automation capital measure and the two factor shares. As we are only interested in the responses to innovations in the tax policy variables, the ordering within $\mathbf{W}_t$ is uncritical. For the $\Delta\mathbf{Z}$ (VAR-) part of the respective VECM, the following block recursive scheme to identify responses to innovations ε, with orthogonalized analogues η, is implied

$$\begin{pmatrix} \eta_t^X \\ \eta_t^Y \\ \eta_t^{\mathbf{W}} \end{pmatrix} = \begin{pmatrix} H_{XX} & 0 & 0 \\ H_{YX} & H_{YY} & 0 \\ H_{\mathbf{W}X} & H_{\mathbf{W}Y} & H_{\mathbf{WW}} \end{pmatrix} \begin{pmatrix} \varepsilon_t^X \\ \varepsilon_t^Y \\ \varepsilon_t^{\mathbf{W}} \end{pmatrix} \text{ for } \Phi(L)\,\Delta\mathbf{Z}_t = \begin{pmatrix} \eta_t^X \\ \eta_t^Y \\ \eta_t^{\mathbf{W}} \end{pmatrix}, \quad (2.5)$$

where $\mathbf{Z}_t$ is partitioned $\mathbf{Z}_t = \begin{pmatrix} X_t & Y_t & \mathbf{W}_t \end{pmatrix}'$, and $H_{\mathbf{WW}}$ is squared.

2.3 Results

Our lag order selection of $p = 13$ for (2.1), (2.2), is supported by the adequate Likelihood ratio (LR) test. The Johansen testing procedure fails to reject the null of at most three cointegrating equations in (2.1), (2.2) for all versions, (v1) to (v4), of VECM representation (2.4). The second cointegration equation in the Johansen-normalization identification clearly indicates a statistically significant equilibrium relationship between the LCTR series and the two factor shares. Additionally, our specification is stable adhering to the implied eigenvalue stability condition (see the Appendix for details).

Impulse response (IR) functions of factor shares in response to a relative increase of taxing labor vis-à-vis capital, i. e., to a positve LCTR innovation, are given in Figure 2.2. A positive one percent LCTR shock implies a permanent increase in the capital share of production of 0.5%. The labor share response is negative (see the right schedule of Figure 2.2). Though also permanent in nature, it is quantitatively less pronounced.

(Bernanke et al., 2005). Under this scheme so-called slow-moving variables such as output and prices do not respond to monetary policy rate dynamics or to movements in fast-moving variables, such as expectational variables, within the period.

Figure 2.2: Orthogonalized IR functions of factor shares to LCTR shock.

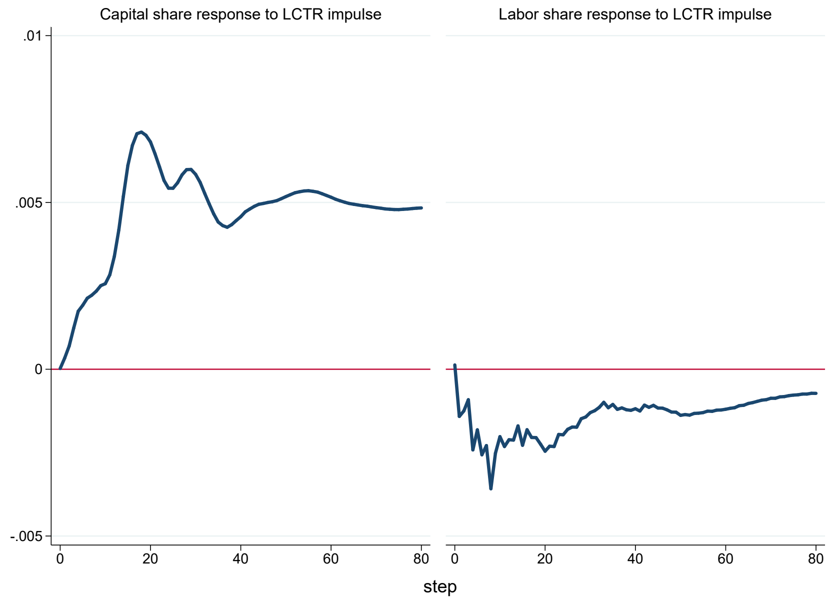


Table 2.1: FEVD values for an LCTR shock by response variable.

Step	Capital share	Labor share	Automation capital
10	0.010460	0.126208	0.296361
20	0.050732	0.118803	0.354056
30	0.080220	0.125973	0.347809
40	0.085780	0.127680	0.332902
50	0.086906	0.129162	0.325100
60	0.090602	0.128036	0.313584
70	0.091387	0.123696	0.299503
80	0.091690	0.117447	0.289272

Table 2.1 shows FEVD statistics for the three response variables, the capital share, the labor share, and automation capital, given a positive LCTR innovation. After about 30 to 40 quarters, relative factor taxation accounts for up to 9% (13%) of observed changes in the capital (labor) share and for up to 35% of the dynamics in the automation capital series. While the effect starts to peter out after 20 years (falling below 30%) for the use of robots, it only slightly falls for the two factor shares.

The VECM analysis, following the generic Johansen procedure and resting on a "Slow-

r-Fast"-style identification strategy (for its VAR part), has uncovered three significant long-run equilibrium correcting, i. e., cointegrating, relationships. These are given for relative factor taxation and the capital share of income, the use of robots and the capital share of income, and population growth, and the labor share. Furthermore, we have illustrated that shocks to relative factor taxation have permanent effects on factor shares. Corresponding variance decompositions reveal that innovations in the factor-tax policy mix contribute at some decade-long forecasting horizons to both changes in the two income shares and in automation capital.

3 A Neoclassical Growth Model with Dynamic Taxes, Automation Capital, and Adjustment Costs

We conduct our analysis in a framework that extends the neoclassical growth model in three ways. First, we follow Steigum (2011) and distinguish two types of capital in the aggregate production function, i. e., *traditional capital* (structures, machines) and *automation capital* (robots). Traditional capital and labor are imperfect substitutes whereas automation capital and labor are perfect substitutes. Second, we introduce adjustment costs associated with the installation of either type of capital. Finally and most importantly, we integrate dynamic taxes on capital and labor income to study their role for the installation of automation capital and the functional income distribution.

The economy comprises a household, a production, and a government sector in an infinite sequence of periods, $t = 0, 1, 2, ..., \infty$. At all t, there is a single manufactured good that serves as numéraire. This good may be directly consumed by households, invested, or collected by the government in the form of taxes. If invested, it serves in the next period either as traditional capital or as automation capital. If collected by the government, it is a means to provide contemporaneous government services to each member of the population.

3.1 Household Sector

There is a single representative household with N_t members. The household size grows at rate n_t, i. e.,

$$N_{t+1} = (1 + n_t)N_t, \quad N_0 > 0. \tag{3.1}$$

The household's inter-temporal utility is

$$U_0 = \sum_{t=0}^{\infty} \beta^t \left[u(c_t, 1 - l_t) + \nu(g_t) \right], \quad 0 < \beta < 1, \tag{3.2}$$

where c_t, l_t, and g_t denote, respectively, per-capita consumption of the manufactured good, the individual supply of working hours, and the per-capita consumption of the services provided by the government at t. In each period, the household's time budget is normalized to unity. Hence, $1 - l_t$ is leisure at t. We refer to g_t as individual government consumption at t.

The periodic utility of a household member is additively separable in the utility enjoyed from consumption and leisure, $u(.,.)$, and the utility derived from government consumption, $\nu(g_t)$. Hence, government consumption, g_t, does not affect the household's optimization with respect to consumption and labor.

The utility function $u : \mathbb{R}_+ \times [0, 1] \to \mathbb{R}$ is given by

$$u(c, 1 - l) = \frac{\left(c^\theta (1 - l)^{1-\theta}\right)^{1-\eta} - 1}{1 - \eta}, \quad 0 < \theta < 1, \quad \eta > 1, \tag{3.3}$$

where θ and $1 - \theta$ denote the weights attached to consumption and leisure.

Households own two kinds of assets, traditional capital, k_t, and automation capital, p_t (both in per capita terms). These stocks depreciate at the same rate, $\delta \in (0, 1)$, so that their respective accumulation is given by

$$(1 + n_t)k_{t+1} = (1 - \delta)k_t + i_t^k, \tag{3.4a}$$

$$(1 + n_t)p_{t+1} = (1 - \delta)p_t + i_t^p. \tag{3.4b}$$

Here, i_t^k and i_t^p denote per-capita investments in traditional and in automation capital, respectively. Following Hayashi (1982), we allow for symmetrical and quadratic adjustment costs for either type of capital. More precisely, for $x \in \{k, p\}$ an investment i_t^x requires

$$\phi(i_t^x, x_t) = i_t^x + \frac{a_1 \left(i_t^x\right)^2}{2(a_2 + x_t)}, \tag{3.5}$$

units of produced output at t where $a_1 > 0$ and $a_2 > 0$. The (small) constant a_2 allows for the transition from a regime with $p_t = 0$ into one with $p_t > 0$.[3]

[3] The usual specification of adjustment costs has $a_2 = 0$ (see, e. g., Heer and Scharrer (2018)).

Let w_t denote the real wage, r_t^k the rental rate of traditional capital, and r_t^p the rental rate of automation capital at t. Then, the household receives income from labor, $w_t l_t$, taxed at rate $\tau_t^w \in (0,1)$, and interest income on traditional capital, $r_t^k k_t$, and automation capital, $r_t^p p_t$, both taxed at rate $\tau_t^r \in (0,1)$. In addition, the household receives lump-sum government transfers, tr_t. Household income is spent on consumption, c_t, taxed at the constant rate $\tau^c > 0$, and on investment in both types of capital. Accordingly, the household's periodic budget constraint is

$$(1+\tau^c)c_t + \phi(i_t^k, k_t) + \phi(i_t^p, p_t) = (1-\tau_t^w)w_t l_t + (1-\tau_t^r)r_t^k k_t + (1-\tau_t^r)r_t^p p_t + tr_t. \quad (3.6)$$

For given values $k_0 > 0$ and $p_0 \geq 0$, the representative household's optimal plan is a sequence $\{c_t, l_t, i_t^k, i_t^p, k_{t+1}, p_{t+1}\}_{t=0}^{\infty}$ of per-capita variables that maximizes U_0 subject to (3.4a), (3.4b), (3.6), $l_t \in [0,1]$, and non-negativity constraints on c_t, i_t^k, i_t^p, k_{t+1}, p_{t+1}. The first-order conditions of the household problem are (a detailed derivation of these conditions can be found in Appendix A.5.1)

$$\lambda_t(1+\tau^c) = \theta c_t^{\theta(1-\eta)-1}(1-l_t)^{(1-\theta)(1-\eta)}, \quad (3.7a)$$

$$\lambda_t(1-\tau_t^w)w_t \leq (1-\theta)c_t^{\theta(1-\eta)}(1-l_t)^{(1-\theta)(1-\eta)-1} \text{with “<” only if } l_t = 0, \quad (3.7b)$$

$$q_t^k \leq \lambda_t \left(1 + \frac{a_1 i_t^k}{a_2 + k_t}\right) \text{ with “<” only if } i_t^k = 0, \quad (3.7c)$$

$$q_t^p \leq \lambda_t \left(1 + \frac{a_1 i_t^p}{a_2 + p_t}\right) \text{ with “<” only if } i_t^p = 0, \quad (3.7d)$$

$$q_t^k \geq \frac{\beta}{1+n_t}\left\{\lambda_{t+1}\left[(1-\tau_{t+1}^r)r_{t+1}^k + \frac{a_1\left(i_{t+1}^k\right)^2}{2(a_2+k_{t+1})^2}\right] + q_{t+1}^k(1-\delta)\right\}$$
$$\text{with “>” only if } k_{t+1} = 0, \quad (3.7e)$$

$$q_t^p \geq \frac{\beta}{1+n_t}\left\{\lambda_{t+1}\left[(1-\tau_{t+1}^r)r_{t+1}^p + \frac{a_1\left(i_{t+1}^p\right)^2}{2(a_2+p_{t+1})^2}\right] + q_{t+1}^p(1-\delta)\right\}$$
$$\text{with “>” only if } p_{t+1} = 0, \quad (3.7f)$$

$$0 = \lim_{t\to\infty} \beta^t \lambda_t k_{t+1}, \quad (3.7g)$$

$$0 = \lim_{t\to\infty} \beta^t \lambda_t p_{t+1}. \quad (3.7h)$$

Here, λ_t, q_t^k, and q_t^p are the respective Lagrange multipliers on the periodic budget constraint (3.6) and on the capital accumulation equations (3.4a) and (3.4b).

Conditions (3.7a) and (3.7b) characterize the contemporaneous trade-off between the consumption demand and the labor supply of a household member. Since $u(c, 1-l)$ is strictly concave on its domain and satisfies the Inada conditions

$$\lim_{c\to 0}\frac{\partial u(c,1-l)}{\partial c}=\infty \quad \text{and} \quad \lim_{l\to 1}\frac{\partial u(c,1-l)}{\partial l}=-\infty$$

the optimal plan involves $c_t > 0$ and $l_t < 1$. However, (3.7b) allows for a corner solution $l_t = 0$ that obtains if

$$c_t > \frac{\theta}{1-\theta}\frac{1-\tau_t^w}{1+\tau^c}w_t. \tag{3.8}$$

Hence, the individual labor supply vanishes if c_t, τ_t^w, or τ^c are sufficiently high relative to the wage.

Condition (3.7c) says that $i_t^k > 0$ if the marginal cost of producing one unit of traditional capital, $1 + a_1 i_t^k/(a_2 + k_t)$, expressed in period-$t$ utility as $\lambda_t \left(1 + a_1 i_t^k/(a_2 + k_t)\right)$ is equal to the marginal revenue from selling one unit of traditional capital expressed in period-t utility, $q_t^k \times 1$. Moreover, $i_t^k = 0$ if the costs exceed the benefit for the first marginal unit of investment in traditional capital. Mutatis mutandis, the interpretation of condition (3.7d) for automation capital is the same.

Condition (3.7f) says that $k_{t+1} > 0$ if the return associated with one additional unit of traditional capital per-capita in $t+1$ shared by a population that has grown by a factor $1 + n_t$ and expressed in period-t utility is equal to the marginal revenue from selling one unit of traditional capital expressed in period-t utility, $q_t^k \times 1$.[4] Mutatis mutandis, the interpretation of condition (3.7f) for automation capital is the same.

Finally, conditions (3.7g) and (3.7h) state the transversality conditions on both types of capital.

3.2 Production Sector

The production sector has a single competitive representative firm. Following Steigum (2011), this firm has access to the production function

$$Y_t = A\left[L_t + \kappa P_t\right]^{1-\alpha} K_t^{\alpha}, \quad A > 0, \tag{3.9}$$

[4]The return associated with one additional unit of traditional capital per-capita in $t+1$ has three components. First, there is the after-tax rate of return $(1-\tau_{t+1}^r)r_{t+1}^k$. Second, given i_{t+1}^k adjustment costs decline by $a_1\left(i_{t+1}^k\right)^2/\left(2(a_2+k_{t+1})^2\right)$. These returns can be consumed and add $\lambda_{t+1}\left[(1-\tau_{t+1}^r)r_{t+1}^k + a_1\left(i_{t+1}^k\right)^2/\left(2(a_2+k_{t+1})^2\right)\right]$ to period-$t+1$ utility. Third, the remaining $1-\delta$ units can be sold which generates a period-$t+1$ utility return equal to $q_{t+1}^k(1-\delta)$.

where L_t is employed hours worked, P_t the amount of hired automation capital, and K_t the amount of hired traditional capital. Hours worked and automation capital are perfect substitutes with a marginal rate of substitution equal to κ.[5]

The representative firm's optimal plan is a sequence $\{K_t, L_t, P_t, Y_t\}_{t=0}^{\infty}$ of factor demands and output supplies that maximizes the net present value of all current and future profits. This comes down to the maximization of periodic profits, Π_t, given by

$$\Pi_t = Y_t - w_t L_t - r_t^k K_t - r_t^p P_t.$$

The corresponding first-order conditions for all t are

$$(1-\alpha)A\left[\frac{K_t}{L_t+\kappa P_t}\right]^{\alpha} \leq w_t, \text{with “}<\text{” only if } L_t = 0, \tag{3.10a}$$

$$(1-\alpha)\kappa A\left[\frac{K_t}{L_t+\kappa P_t}\right]^{\alpha} \leq r_t^p, \text{with “}<\text{” only if } P_t = 0, \tag{3.10b}$$

$$\alpha A\left[\frac{L_t+\kappa P_t}{K_t}\right]^{1-\alpha} = r_t^k. \tag{3.10c}$$

These conditions reflect that K_t is essential but neither L_t nor P_t is.

Let LS_t denote the labor share at t. Using (3.10a) - (3.10c), the latter can be expressed as

$$LS_t = \begin{cases} 1-\alpha & \text{if} \quad L_t > 0,\ P_t = 0, \\ \frac{1-\alpha}{1+\kappa\frac{P_t}{L_t}} & \text{if} \quad L_t > 0,\ P_t > 0, \\ 0 & \text{if} \quad L_t = 0,\ P_t > 0. \end{cases} \tag{3.11}$$

Hence, in the presence of automation capital the labor share is smaller than $1-\alpha$. Moreover, it declines in the automation capital intensity, P_t/L_t. This is the result of two reinforcing effects that become visible if the labor share is expressed as the ratio of the marginal to the average product of hours worked, i. e., $LS_t = w_t/(Y_t/L_t)$. A higher automation capital intensity reduces the marginal product of hours worked and boosts its average product.

[5]Below, we choose κ to be smaller than unity to obtain reasonable results in our calibrations. In contrast to the actual evolution of the labor share, a specification involving $\kappa = 1$ (and a corresponding value $A = 0.83$ that generates an asymptotic growth rate of 2%) predicts that the labor share drops to 46% after the first year during which automation capital is used, i. e., when $P_t > 0$.

3.3 Government Sector

At all t, the government collects taxes and spends its receipts in the form of government consumption and lump-sum transfers while keeping its budget balanced. Let tax_t denote per-capita tax receipts. Then,

$$tax_t = \tau^c c_t + \tau_t^w w_t l_t + \tau_t^r r_t^p p_t + \tau_t^r r_t^k k_t. \tag{3.12}$$

Moreover, the government budget constraint reads

$$N_t g_t + N_t tr_t = N_t tax_t. \tag{3.13}$$

3.4 Dynamic Competitive Equilibrium with Dynamic Taxes

Given $K_0 > 0$, $P_0 \geq 0$, $N_0 > 0$, $N_t = (1+n)^t N_0$, and a sequence of tax rates $\left\{\tau^c, \tau_t^w, \tau_t^k\right\}_{t=0}^{\infty}$ the dynamic competitive equilibrium with dynamic taxes determines sequences of prices $\left\{w_t, r_t^k, r_t^p\right\}_{t=0}^{\infty}$, allocations $\left\{c_t, l_t, i_t^k, i_t^p, k_{t+1}, p_{t+1}, K_t, L_t, P_t, Y_t\right\}_{t=0}^{\infty}$, and government activities $\{g_t, tax_t, tr_t\}_{t=0}^{\infty}$. The equilibrium sequences are determined by the following conditions for all $t = 0, 1, 2, \cdots, \infty$:

(E1) The plan of the representative household satisfies conditions (3.7a) - (3.7h).

(E2) The plan of the representative firm satisfies (3.10a) - (3.10c).

(E3) The activities of the government satisfy (3.12) and (3.13).

(E4) The labor market clears, i. e.,

$$w_t \geq 0, \quad L_t \leq N_t l_t, \quad \text{and} \quad w_t(L_t - N_t l_t) = 0.$$

(E5) The market for both types of capital clear, i. e.,

$$r_t^k \geq 0, \quad K_t \leq N_t k_t, \quad \text{and} \quad r_t^k(K_t - N_t k_t) = 0,$$
$$r_t^p \geq 0, \quad P_t \leq N_t p_t, \quad \text{and} \quad r_t^p(P_t - N_t p_t) = 0.$$

(E6) The market of the final good clears, i. e.,

$$c_t + g_t + \phi(i_t^k, k_t) + \phi(i_t^p, p_t) = y_t. \tag{3.15}$$

Since the production function (3.9) exhibits constant returns to scale and factor markets are competitive, (E2) implies $\Pi_t = 0$. (E4) and (E5) describe the usual factor market clearing conditions. The price of labor cannot be negative, demand must not exceed supply, and an excess supply requires $w_t = 0$. Similarly, for the two stocks of capital.

Below it becomes clear that we have to deal with the cases where either $l_t = 0$, i. e., the supply of labor vanishes, or $p_t = 0$, i. e., the supply of automation capital vanishes. In the former case, the demand for hours worked must vanish, too. This is the case for all wages above $w_t = (1-\alpha)A(K_t/(\kappa P_t))^\alpha$ which is then the equilibrium wage consistent with $L_t = 0$. Combining the latter with equation (3.8) we obtain the equilibrium condition under which the individual labor supply vanishes as

$$c_t > \frac{\theta}{1-\theta}\frac{1-\tau^w}{1+\tau^c}(1-\alpha)A\left(\frac{K_t}{\kappa P_t}\right)^\alpha . \tag{3.16}$$

Similarly, if the supply of automation capital is zero, then its demand must vanish. This is the case for all rental rates above $r_t^p = (1-\alpha)\kappa A(K_t/L_t)^\alpha$ which is then the equilibrium rental rate for automation capital consistent with $P_t = 0$.

Finally, (E6) states the goods market equilibrium in per-capita terms, i. e., the supply of produced output is equal to private and government consumption demand plus investment outlays for both types of capital.

3.5 Initial Steady State without Automation Capital

Let period $t = 0$ correspond to the year 1965. Prior to this period, we assume that the economy is in steady state without automation capital. More precisely, the population growth rate n_0, and the tax rates τ_0^c, τ_0^w, and τ_0^r are constant and equal to the respective values prevailing in 1965. Moreover, the growth rate of the economy is set equal to zero.

Accordingly, the initial steady state is pinned down by the following 8 equations in the

8 endogenous variables k, l, i^k, c, w, r^k, q^k, and λ:

$$(1+\tau^c)\lambda = \theta c^{\theta(1-\eta)-1}(1-l)^{(1-\theta)(1-\eta)}, \tag{3.17a}$$

$$\frac{1-\tau^w}{1+\tau^c} w = \frac{1-\theta}{\theta}\frac{c}{1-l}, \tag{3.17b}$$

$$q^k = \lambda\left(1+\frac{a_1 i^k}{a_2+k}\right), \tag{3.17c}$$

$$\frac{q^k}{\lambda} = \frac{\beta}{1+n}(1-\tau^r)r^k + \frac{a_1(i^k)^2}{2(a_2+k)^2} + \frac{q^k}{\lambda}(1-\delta), \tag{3.17d}$$

$$i^k = (n+\delta)k, \tag{3.17e}$$

$$c+g+\phi(i^k,k) = Al^{1-\alpha}k^\alpha, \tag{3.17f}$$

$$w = (1-\alpha)A\left[\frac{k}{l}\right]^\alpha, \tag{3.17g}$$

$$r^k = \alpha A\left[\frac{l}{k}\right]^{1-\alpha}. \tag{3.17h}$$

Moreover, the labor share in the initial steady state is $LS = 1-\alpha$.

3.6 Automation Capital and the Asymptotic Balanced Growth Path

Define an asymptotic balanced growth path (ABGP) as an equilibrium path to which the economy tends as $t\to\infty$ that satisfies

$$\lim_{t\to\infty}\frac{y_{t+1}}{y_t} = \lim_{t\to\infty}\frac{k_{t+1}}{k_t} = \lim_{t\to\infty}\frac{p_{t+1}}{p_t} = \lim_{t\to\infty}\frac{i^k_{t+1}}{i^k_t} = \lim_{t\to\infty}\frac{i^p_{t+1}}{i^p_t} = \lim_{t\to\infty}\frac{c_{t+1}}{c_t} = 1+\gamma \tag{3.18}$$

for a constant population growth rate, $n_t = n$, constant tax rates, $\tau^r_t = \tau^r$ and $\tau^w_t = \tau^w$, as well as a constant ratio $g_t/y_t < 1$. Here, $\gamma > 0$ is the asymptotic growth rate of per-capita variables. This section derives key properties of the ABGP.

The accumulation equations (3.4a) and (3.4b) in conjunction with (3.18) deliver the asymptotic investment ratios

$$\frac{i^k}{k} = \frac{i^p}{p} = (1+n)(1+\gamma) - 1 + \delta. \tag{3.19}$$

Next consider the first-order conditions with respect to i^k_t and i^p_t given by (3.7c) and (3.7d). Equations (3.18) and (3.19) imply that the ratio $i^k_t/(a_2+k_t)$ converges to $i^k_t/k_t = i^k/k$ whereas $i^p_t/(a_2+p_t)$ converges to $i^p_t/p_t = i^p/p$. Hence, asymptotically we have

$$\frac{q^k}{\lambda} = 1 + a_1\frac{i^k}{k} \quad \text{and} \quad \frac{q^p}{\lambda} = 1 + a_1\frac{i^p}{p}, \tag{3.20}$$

so that $q^k = q^p = q$. Moreover, λ_t and $q_t^k = q_t^p = q_t$ grow at the same rate. Using the latter in (3.7e) and (3.7f) reveals that the ABGP requires

$$\frac{q}{\lambda} = \frac{\beta}{1+n}\left\{\frac{\lambda_{t+1}}{\lambda_t}\left[(1-\tau^r)r_{t+1}^k + \frac{a_1}{2}\left(\frac{i^k}{k}\right)^2\right] + \frac{q_{t+1}}{q_t}\left(\frac{q}{\lambda}\right)(1-\delta)\right\}, \tag{3.21a}$$

$$\frac{q}{\lambda} = \frac{\beta}{1+n}\left\{\frac{\lambda_{t+1}}{\lambda_t}\left[(1-\tau^r)r_{t+1}^p + \frac{a_1}{2}\left(\frac{i^p}{p}\right)^2\right] + \frac{q_{t+1}}{q_t}\left(\frac{q}{\lambda}\right)(1-\delta)\right\}. \tag{3.21b}$$

Hence, asymptotically we must have $r^k = r^p$. From (3.10b) and (3.10c) expressed in per-capita variables the latter requires that

$$(1-\alpha)A\lim_{t\to\infty}\left(\frac{k_t}{l_t+\kappa p_t}\right)^{\alpha} = \alpha A\lim_{t\to\infty}\left(\frac{l_t+\kappa p_t}{k_t}\right)^{1-\alpha}. \tag{3.22}$$

Since $\lim_{t\to\infty} l_t \in [0,1]$ (as we show below) the ABGP has

$$\lim_{t\to\infty}\left(\frac{k_t}{l_t+\kappa p_t}\right)^{\alpha} = \left(\frac{k}{\kappa p}\right)^{\alpha} \quad \text{and} \quad \lim_{t\to\infty}\left(\frac{l_t+\kappa p_t}{k_t}\right)^{1-\alpha} = \left(\frac{\kappa p}{k}\right)^{1-\alpha}. \tag{3.23}$$

With the latter in (3.10b) and (3.10c) we obtain

$$\frac{k}{p} = \frac{\alpha}{1-\alpha} \tag{3.24}$$

and

$$r^k = r^p = A\alpha^{\alpha}(1-\alpha)^{1-\alpha}\kappa^{1-\alpha}. \tag{3.25}$$

Equation (3.24) implies that the right-hand side of (3.16) converges to the finite constant

$$\frac{\theta}{1-\theta}\frac{1-\tau^w}{1+\tau^c}(1-\alpha)^{1-\alpha}A\left(\frac{\alpha}{\kappa}\right)^{\alpha}.$$

Since c_t grows asymptotically at a positive rate (3.16) will be satisfied in finite time. Hence, the ABGP has $l = 0$. From (3.7a) this implies that asymptotically

$$\frac{\lambda_{t+1}}{\lambda_t} = \frac{q_{t+1}}{q_t} = (1+\gamma)^{\theta(1-\eta)-1}.$$

Moreover, the asymptotic production function (3.9) is $Y_t = A\left(\kappa P_t\right)^{1-\alpha} K_t^{\alpha}$. As P_t and K_t grow at the same rate along the ABGP, the models asymptotic behavior mimics the one of the AK-model of, e. g., Frankel (1962) or Romer (1986), and $\gamma > 0$ will be endogenously determined.

3.7 Calibration

We assume that the economy is in a steady state without automation capital prior to the year 1965. Therefore, the income tax rates τ^w and τ^r as well as the population growth rate are set equal to their 1965 values. Then, we pick the values for A and κ.

In the initial steady state, both k_t and i_t^k are stationary. Hence, capital accumulation is such that $i^k = (n+\delta)k$ as stated in (3.17e). In conjunction with (3.17h), the first-order conditions for i_t^k and k_{t+1}, i. e., equations (3.17c) and (3.17d), evaluated at the steady state become

$$\frac{q^k}{\lambda} = 1 + a_1 \frac{n+\delta}{a_2+k} k, \tag{3.26a}$$

$$\frac{q^k}{\lambda} = \frac{\beta}{1+n} \left\{ (1-\tau^r)\alpha A k^{\alpha-1} l^{1-\alpha} + a_1 \frac{(n+\delta)^2 k^2}{2(a_2+k)^2} + \frac{q^k}{\lambda}(1-\delta) \right\}. \tag{3.26b}$$

The latter two equations can be solved for q^k/λ and k as functions of A for the following numerical calibrations. We set $l = 0.3$ and use a standard parameterization for the discount factor $\beta = 0.96$ and the depreciation rate $\delta = 7\%$ (see, e. g., Trabandt and Uhlig (2011)). The parameters of the adjustment cost function, $a_1 = 12$, is taken from Heer and Schubert (2012). The constant a_2 is set equal to 0.1. The initial labor share of 62% is matched with a choice of $\alpha = 38\%$. With values for k and l at hand, we compute y, i^k, w, and r^k for a given value of A.

Following Trabandt and Uhlig (2011), we fix the government consumption share in GDP at 18%. The resource constraint, $c = y - \phi(i^k, k) - g$ pins down steady-state consumption, c. Given the first-order conditions with respect to labor and consumption, we set $\theta = 0.3938$. Finally, we derive the steady-state values of λ and q using the value of 1/2 for the intertemporal elasticity of substitution, $1/\eta$.

To compute the asymptotic growth rate of per-capita variables, γ, we express (3.21a) along the ABGP as

$$\frac{q}{\lambda} = \frac{\beta(1+\gamma)^{\theta(1-\eta)-1}}{1+n} \left\{ (1-\tau^r) A \alpha^\alpha (1-\alpha)^{1-\alpha} \kappa^{1-\alpha} + \frac{a_1}{2}\left(\frac{i^k}{k}\right)^2 + \frac{q}{\lambda}(1-\delta) \right\}.$$

and solve the latter in conjunction with (3.19), (3.20), and (3.25) for r^k, q/λ, γ and i^k/k. We iterate this procedure trying different values of A and κ until γ is equal to 2.0% and the decline in the labor share is approximately equal to that observed during 1965-2010. This leads to a choice of $A = 9.6$ and $\kappa = 0.019$.

In the computations, we assume that, after the final period of transition, k, p, i^k, i^p, and c all grow at the common rate γ. Moreover, we keep the assumption that the government-GDP share remains constant during the transition and in ABGP.

3.8 Transition Analysis

This section has two parts. First, we study the transition from 1965-2015 of our benchmark economy with automation capital, a strictly positive labor supply, and the dynamic tax rates.[6] The main result is that the model explains the actual drop in the labor share from 62% to 57% that took place between 1965 and 2015. Second, we run counterfactual experiments to highlight the role of the tax rates on capital and labor income for the decline in the labor share. We demonstrate that the labor share would have been several percentage points higher if either the tax rate on labor income alone or both tax rates had remained at their 1965 level.

3.8.1 Benchmark

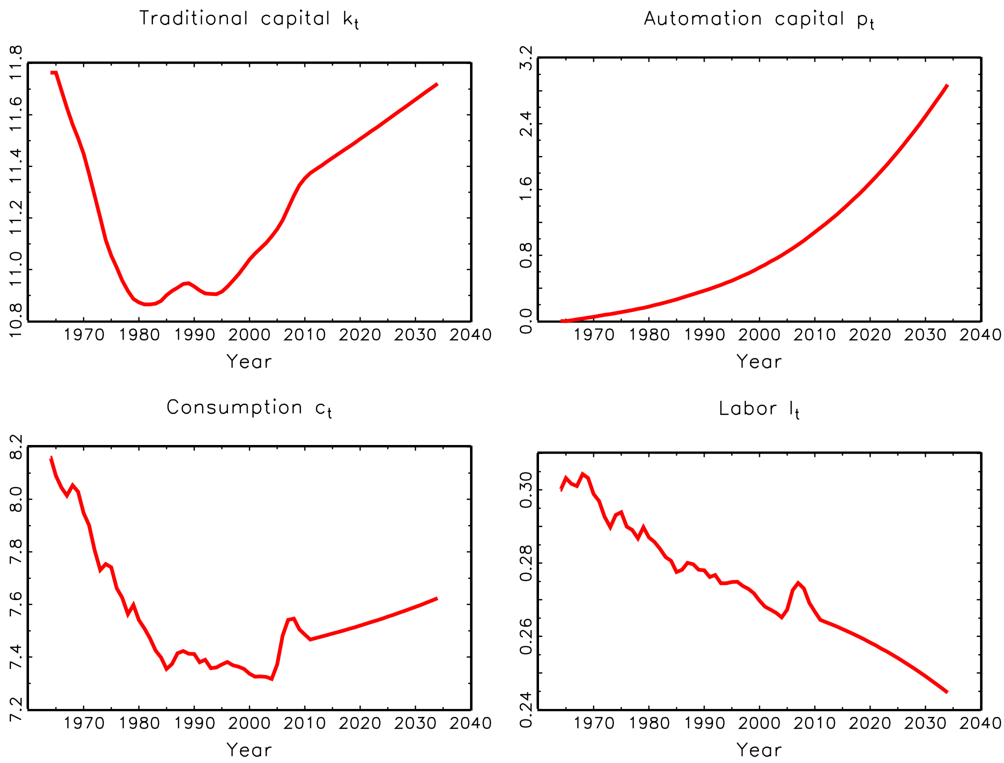
Figures 3.1 and 3.2 illustrate the transition dynamics of our model economy for the time span 1965-2030. The transition is computed under the following assumptions:

1. The economy is in steady state prior to 1965 with a labor share equal to 62%. The tax rates on labor and capital income and the population growth rate are set equal to their value in 1965, i. e., 17.9%, 47.3%, and 1.7%, respectively.
2. The transition starts in 1965. The initial capital stock is given by the steady-state stock of traditional capital. Households build up savings and supply labor according to their Euler equations for both types of capital and their first-order conditions with respect to consumption and labor.
3. For the period 1965-2010 the tax rates and the population growth rates are equal to their empirical counterparts.
4. In 1965, the household also starts to invest in automation capital.

[6]In our numerical analysis, we compute the transition dynamics over the period 1965-2175. By 2175, the deviation of the final values from their asymptotic steady state values is less than 0.001%. For reasons of exposition, we only display the first part of the transition. Some additional details on the computation of the transitional dynamics can be found in Appendix A.5.2.

5. After 2010, the tax rates on labor and capital income as well as the population growth rate remain constant at 28.3%, 37.1%, and 0.9%, respectively.

Figure 3.1: Transition Dynamics in the Model with Automation Capital and Adjustment Costs, Part I.

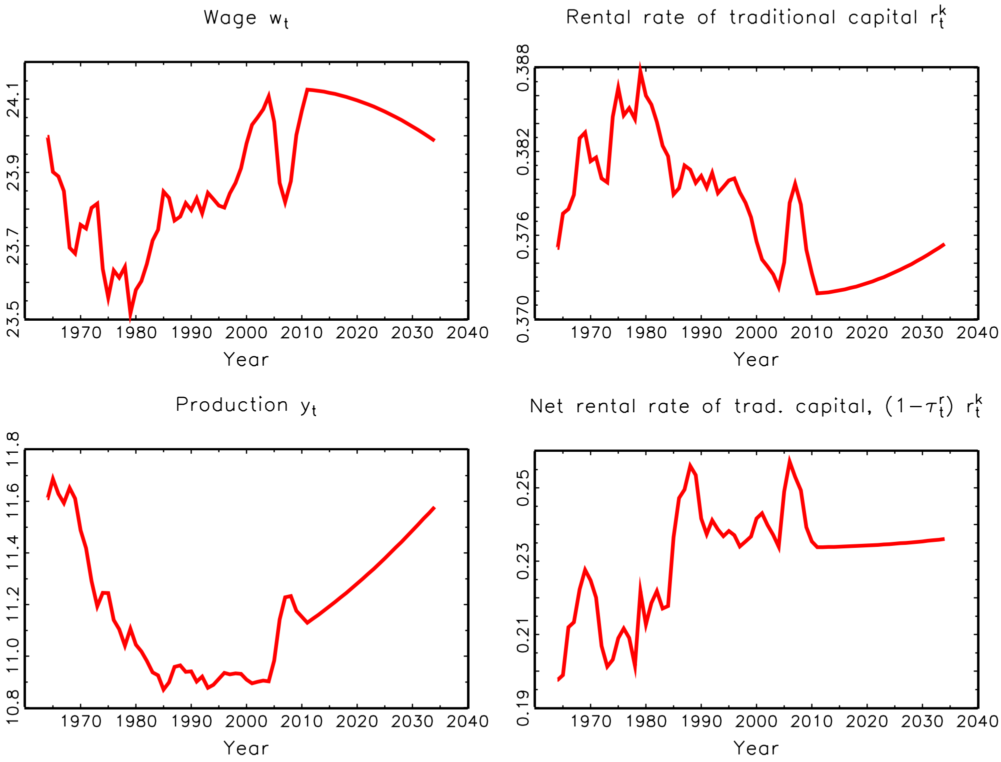


The first row of Figure 3.1 reveals that the household initially adjusts its asset portfolio and shifts wealth from traditional capital (upper left panel), k_t, to automation capital (upper right panel), p_t. After approximately 30 years, both types of capital start to grow over time. As we argue above, the asymptotic behavior of our model is similar to an AK-model, i. e., the economy converges to a balanced growth path with a growth rate of per-capita variables equal to 2.0%.

In the first year of the transition, the labor supply increases slightly from 0.3 in 1964 to 0.303 in 1965. As initially labor income taxes are low the household substitutes labor intertemporally so that l_t remains higher than 0.3 until 1973. Between 1965 and 2015, the labor supply drops from 0.303 to 0.262 (lower right panel).[7] This is the effect of

[7]Eventually, the labor supply vanishes completely. In our benchmark, $l_t = 0$ is reached in the year 2132.

Figure 3.2: Transition Dynamics in the Model with Automation Capital and Adjustment Costs, Part II.



two channels which operate simultaneously. First, automation capital replaces labor in production. Second, the incentive to supply labor falls with the increase in the tax rate on labor income. Experiment 1 below suggests that the increase in τ_t^l from 17.9% to 28.3% during 1965-2010 is the driving force behind the decline in l_t.

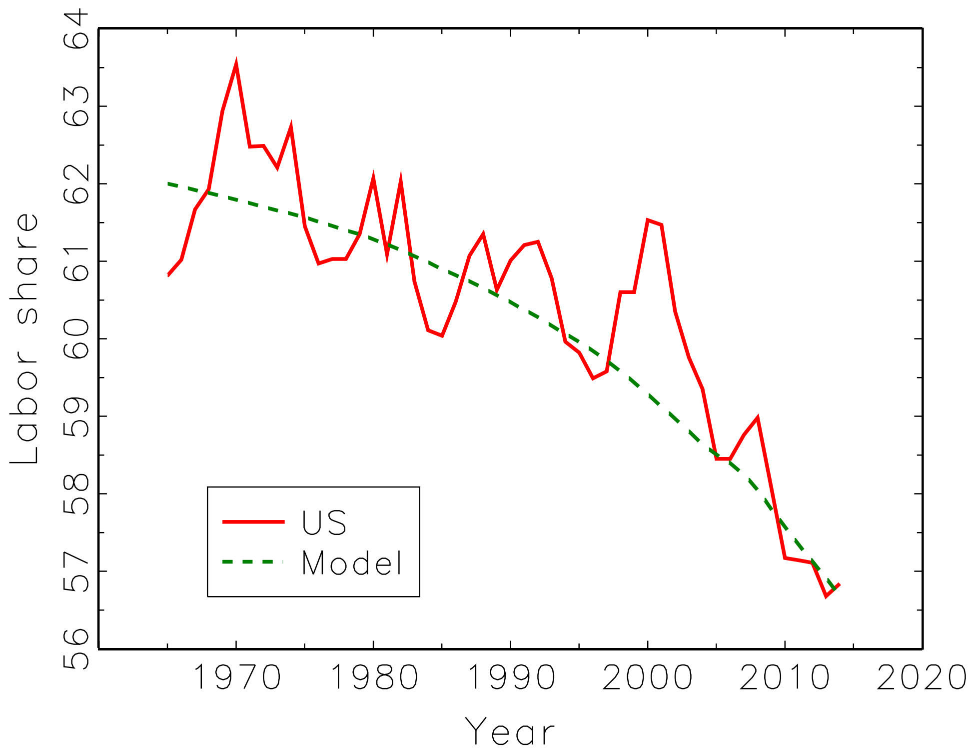
Per-capita consumption (lower left panel in Figure 3.1) falls between 1970 and 1985 for two reasons. First, the net wage income declines with a falling labor supply and increasing tax rates on labor income. During 2004-2010, the labor supply displays a hump-shaped increase which reflects the temporary fall in the labor income tax rate, τ_t^w, during these years (see Figure 1.2). Second, the household increases savings as, on average, the tax rates on capital income fall during 1965-2010 (see again Figure 1.2). Therefore, consumption declines during the first 30 years of the transition. Eventually, the growth effect sets in and consumption starts to increase after 2004. In the long run, consumption grows at the endogenous asymptotic growth rate. Notice the hump-shaped dynamics of consumption during the years 2004-2011 that mirrors the evolution of the labor supply and, hence, of (net) wage income.

The evolution of the factor prices w_t and r_t^k over 1965-2030 are displayed in the upper row of Figure 3.2. Their evolution reflects the first-order conditions (3.10a) - (3.10c) that define a factor-price frontier linking r_t^k, r_t^p, and w_t according to

$$r_t^k = \alpha(1-\alpha)^{\frac{1-\alpha}{\alpha}} \kappa^{\frac{1-\alpha}{\alpha}} A^{\frac{1}{\alpha}} (r_t^p)^{-\frac{1-\alpha}{\alpha}} \quad \text{and} \quad w_t = \frac{r_t^p}{\kappa}.$$

Since traditional capital, k_t, decreases and automation capital increases during 1965-1980, the wage, w_t, falls at the beginning of the transition. Consequently, the rental rate on traditional capital, r_t^k, rises during this period.[8]

Figure 3.3: Labor Share Dynamics.



The evolution of the labor share, LS_t, is illustrated in Figure 3.3. The model (broken green line) is able to replicate the downward trend of the labor share (solid red line) during 1965-2015. In fact, as observed empirically the model generates a drop in the labor share by five percentage points between 1965 and 2015. This effect is explained

[8]Asymptotically, the rental rates of traditional and automation capital approach the value given in (3.25).

by the substitution of labor with automation capital. The income share accruing to these two production factors is constant and equal to $1 - \alpha = 62\%$. However, the relative income share of automation capital increases over time at the expense of the residual share for labor.

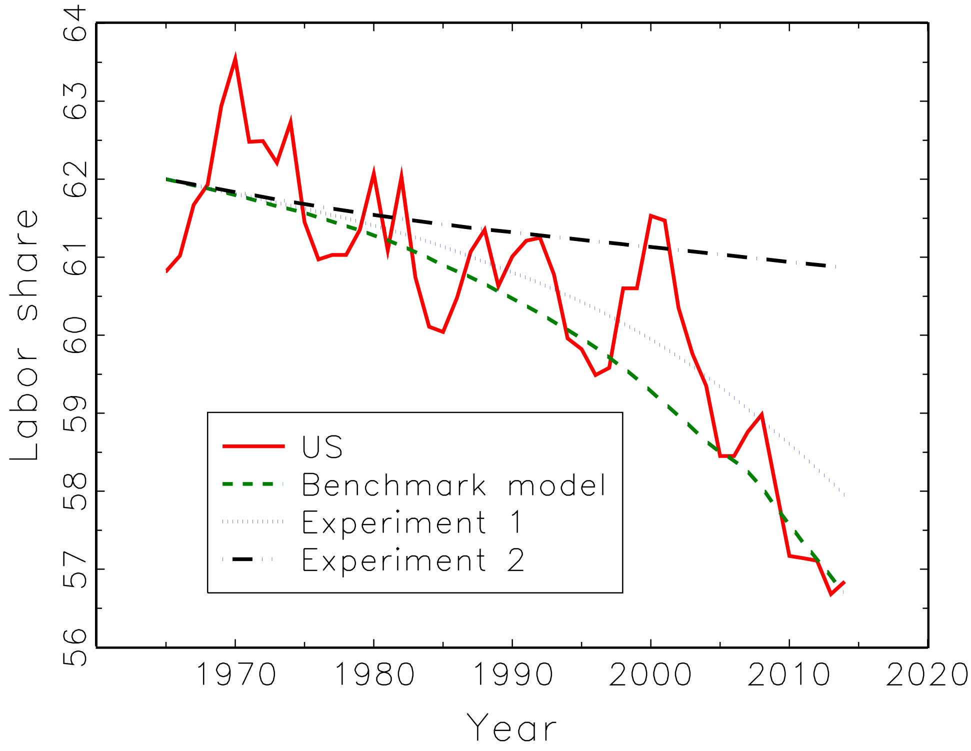
3.8.2 Experiments

We conduct two counterfactual experiments. In Experiment 1 the tax rate on labor income remains constant at its (low) 1965 level of $\tau_t^w = \tau^w = 17.9\%$ whereas the tax on capital income varies in line with the empirical evidence shown in Figure 1.2. Experiment 2 leaves both tax rates constant at their 1965 level, i. e., in addition the tax rate on capital income remains constant at its (high) 1965 level of $\tau_t^r = \tau^r = 37.1\%$. The counterfactual transition of the labor share is shown in Figure 3.4. The actual US labor share and the labor share of our benchmark model appear again as the solid red and the broken green line. In Experiment 1 the decline of the labor share (broken dotted blue line) is less pronounced than in reality and the benchmark. In particular, in 2015 it is 1.3 percentage points higher at 57.9%. Experiment 2 generates an even shallower decline in the labor share to 60.9% (dashed black line). We conclude that the observed increase in the tax on labor income and the decline in the tax on capital income from 1965 to 2015 played an important role for the evolution of the labor share over this period.

To understand the dynamics of the labor share in these experiments relative to the benchmark it is instructive to compare the co-evolutions of the remaining endogenous variables. For this purpose, Figure 3.5 illustrates the dynamics of traditional capital, k_t, automation capital, p_t, consumption, c_t, the labor supply, l_t, the wage, w_t, and the rental rate of traditional capital, r_t^k, (from the upper left to the lower right panel) in these experiments and the benchmark.

In Experiment 1 the lower labor income tax rate increases the labor supply in the year 2015 from 0.2605 in the benchmark to 0.292. As a consequence, net labor income increases which results in a strong rise of consumption and a moderate rise of savings (the sum of investments in traditional and automation capital, not presented). As is evident from the bottom-left panel of Figure 3.5, the increase in the labor supply also implies in a rise of the rental rate of capital, r_t^k. Therefore, individual household members adjust their portfolio composition and shift wealth from automation to traditional

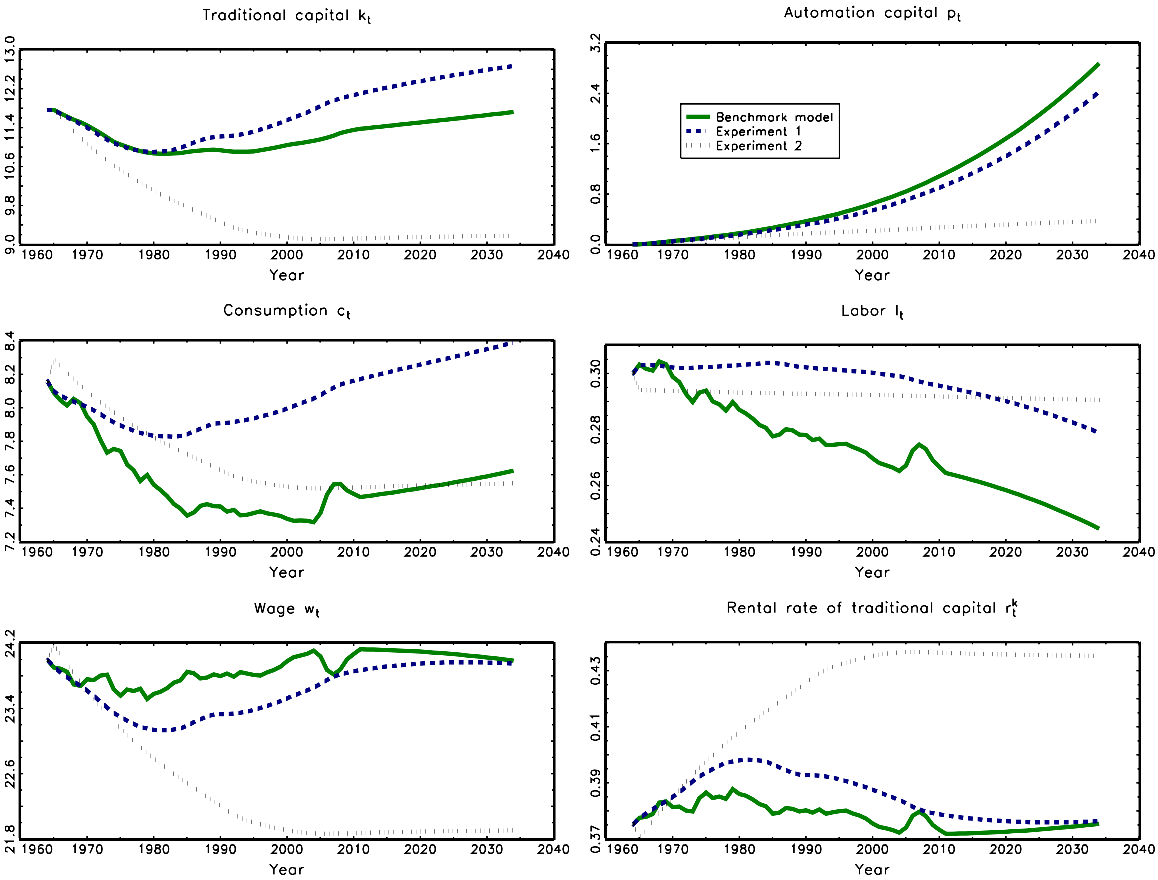
Figure 3.4: Labor Share and Income Taxes τ^w and τ^r.



capital over time. The top row of Figure 3.5 reveals that k_t is larger and p_t lower in Experiment 1 than in the benchmark. Accordingly, with a higher labor supply, l_t, and less automation capital, p_t, the labor share increases.

In Experiment 2 the capital income tax rate is permanently higher than in Experiment 1. Therefore, the after-tax rate of return from traditional capital, k_t, falls by approximately 25% in 2015 from 12.2% in Experiment 1 to 9.1% in Experiment 2. Since the growth rate of per-capita variables is endogenous and depends on the capital income tax rate, τ^k, growth slows down and approaches asymptotically 0.85%. For this reason, output, consumption and investment are significantly smaller in Experiment 2 than in Experiment 1. In particular, in 2015, automation capital, p_t, in Experiment 2 (1) only amounts to 0.29 (1.08) units of contemporaneous output which — after noticing that the labor supply does not react strongly to the higher capital income tax rate, τ_t^r — explains the much higher labor share of 60.9%.

Figure 3.5: Transition Dynamics in Experiments 1 and 2.



4 Conclusion

In the empirical part, we find three significant long-run equilibrium correcting, i. e. cointegrating, relationships for the 1974-2008 period. They are given for our factor-tax policy mix variable and the capital share, our use of robots variable and the capital share, and population growth and the labor share. Permanent effects on factor shares emanate from shocks in relative factor taxation. The latter also permanently affect the use of robots. Variance decompositions reveal that taxing factors contributes long-lastingly to the variation both in the two income shares and in automation capital. Overall, our findings give grounds for setting up and simulating a neoclassical growth model augmented by automation capital, capital adjustment costs, and factor taxation.

In our simulations, we find that tax rates on both labor and capital income have a significant effect on the functional income distribution and, in particular, on the labor share of income. For the US economy, the motivating empirical effect amounts to approximately 4 percentage points over the period 1965-2015. We demonstrate that this stylized fact can be reproduced by a neoclassical growth model with automation capital. Our growth model also predicts a continuing fall in the labor share over the coming decades. However, we would like to interpret this latter finding in a cautious way because we neglect other aggravating factors like artificial intelligence (AI). In our model, the productivity of the automation capital does not increase over time. In future research, we plan to endogenize the investment in AI and its effects on the functional income distribution.

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5 Appendix

In the Appendix, we describe some methodological and mathematical details along with summary statistics of time series and time series tests.

First, accompanying the empirical part of the paper, we provide detail on the used nowcasting technique based on the procedure proposed Shumway and Stoffer (2008) relying on the Kalman filter in combination with the expectation maximization (EM) algorithm. It is used to generate a quarterly nowcast of the (bi-)annual, shortfall plagued automation capital series based on robots per 1,000 workers figures by Tani (1989), Acemoglu and Restrepo (2019), the IFR, and the CPS. Proceedingly, detailed results of the Johansen testing procedure (adhering to the Pantula Principle) as a necessary prerequisite for our VECM estimation and analysis is given.

Secondly, accompanying the theoretical part of the paper, we derive some properties of the model with automation capital. In addition, we present some details on the computation of the transition dynamics. In essence, we describe the solution of a large-scale system of difference equations in four endogenous variables over a time horizon of 210 periods.

A.1 Nowcasting (Bi-)annual Automation Capital to Obtain a Quarterly Series

A.1.1 Time Series

To-be-nowcasted time series. The series we seek to nowcast – or to generate (pseudo-)quarterly data for – is the use of industrial robots per 1,000 workers in the US as provided in Acemoglu and Restrepo (2019). The original source is the IFR. This series is of annual frequency and starts in 1993. It ends in 2014. However, due to data limitations with regard to other series relevant for our analysis we end it in 2008. We merge these data with observations that we construct in the following way. Tani (1989) in his Tab. 1 (col. 3, p. 193) provides data for the industrial robot population in the US for the years 1974, 1976, 1978, and in annual frequency from 1980 to 1985. As Tani (1989) in his Tab. 3 standardizes these data to workers in the manufacturing sector only, we refrain from using his standardized series, but divide the non-standardized robots figures through the respective annual averages of the US full time employees, SA (in 1K), data that we obtain from the CPS. The result is a series of mixed bi-annual, annual frequency with missing values for 1986 to 1992. Hence, covered years are 1974, 1976, 1978, 1980-1985, 1993-2008. Graphically this series is made of the dots shown in Figure 1.3. As this series represents a stock variable (robots per 1,000 employees) for particular years, we might interpret it as end-of-year or q4-values.

General strategy and information set series. Given the (quasi-)q4-data of robots per 1,000 workers, the procedure runs in two main steps. In the first step, the missing (quasi-)q4-, or annual, values are generated using information from other use of automation capital related variables, for which we have data over the entire period and, at best, at a quarterly observation frequency. In a second step, for the obtained complete annual frequency series, running from 1974 to 2008, an analogue nowcasting approach is followed to generate a (pseudo-)quarterly series. Our baseline information set essentially uses variables from the Fernald (2014) database in contemporaneous and first lag expression that recently have been shown by Graetz and Michaels (2018) to be profoundly and significantly associated with robots input: hours worked, labor productivity, different estimates of labor quality (i.e. labor composition), total factor productivity (TFP), and utilitization-adjusted TFP. Additionally, we also consider the US tax rate on labor income provided by Gomme et al. (2011) as firms adopt robots mainly for saving on labor costs (besides ensuring uniform quality). Generally, labor costs depend on labor productivity and taxation.

A.1.2 Method

Starting point of the procedure is the notion of a general state space model for an n-dimensional time series $\boldsymbol{y}_t$ consisting of a measurement equation that relates the observed data to an m-dimensional state vector $\boldsymbol{\alpha}_t$. The generation of the state vector $\boldsymbol{\alpha}_t$ from the past state $\boldsymbol{\alpha}_{t-1}$, for $t = 1, ..., T$, is determined by the state equation. The measurement equation has the form

$$\boldsymbol{y}_t = \boldsymbol{Z}_t\boldsymbol{\alpha}_t + \boldsymbol{d}_t + \boldsymbol{u}_t, \quad t = 1, ..., T. \tag{A.1.1}$$

In (A.1.1), $\boldsymbol{Z}_t$ is an $n \times m$ matrix called measurement or observation matrix, $\boldsymbol{d}_t$ is an $n \times 1$ vector and $\boldsymbol{u}_t \sim$ iid $N(\boldsymbol{0}, \boldsymbol{H}_t)$ is an error vector. The state equation is given by

$$\boldsymbol{\alpha}_t = \boldsymbol{T}_t\boldsymbol{\alpha}_{t-1} + \boldsymbol{c}_t + \boldsymbol{R}_t\boldsymbol{\nu}_t, \quad t = 1, ..., T. \tag{A.1.2}$$

In (A.1.2), $\boldsymbol{T}_t$ is an $m \times m$ matrix called transition matrix, $\boldsymbol{c}_t$ is an $m \times 1$ vector, $\boldsymbol{R}_t$ is an $m \times g$ matrix and $\boldsymbol{\nu}_t \sim$ iid $N(\boldsymbol{0}, \boldsymbol{Q}_t)$ is a $g \times 1$ error vector. The matrices $\boldsymbol{Z}_t$, $\boldsymbol{d}_t$, $\boldsymbol{H}_t$, $\boldsymbol{T}_t$, $\boldsymbol{c}_t$, $\boldsymbol{R}_t$ and $\boldsymbol{Q}_t$ are referred to as system matrices. Usually, it is assumed that the errors of the measurement and the transition equation are uncorrelated, i.e.

$$E[\boldsymbol{u}_t \boldsymbol{\nu}_t'] = \boldsymbol{0} \quad \forall s, t = 1, ..., T.$$

Furthermore, it is assumed that the initial state is given by a normal vector

$$\boldsymbol{\alpha}_0 \sim N(\boldsymbol{a}_0, \boldsymbol{P}_0); \quad E[\boldsymbol{u}_t \boldsymbol{a}_0'] = \boldsymbol{0}, \; E[\boldsymbol{\nu}_t \boldsymbol{a}_0'] = 0, \quad t = 1, ..., T.$$

In our application of a state-space model, as defined by (A.1.1) and (A.1.2), we seek to generate estimators for the underlying unobserved signal $\boldsymbol{\alpha}_t$ given the data $\boldsymbol{y}_s$, for $s = 1, ..., S$. Whenever $s = t$ this problem is called filtering, while we speak of smoothing if $s > t$ and forecasting in case $s < t$. The problem of finding such estimators is solved by the Kalman Filter (KF), Kalman Smoother (KS) and forecasting recursions, respectively. The KF is a set of recursion equations (prediction equations and updating equations) that determine the optimal estimates for the state vector $\boldsymbol{\alpha}_t$ given the information available at t (henceforth, I_t). The following definitions are used

$$\boldsymbol{a}_t := E[\boldsymbol{\alpha}_t | I_t] \tag{A.1.3}$$

and

$$\boldsymbol{P}_t := E[(\boldsymbol{\alpha}_t - \boldsymbol{a}_t)(\boldsymbol{\alpha}_t - \boldsymbol{a}_t)' | I_t]. \tag{A.1.4}$$

That is, $\boldsymbol{a}_t$ is the optimal estimator of $\boldsymbol{\alpha}_t$ based on I_t and $\boldsymbol{P}_t$ is the mean square error (MSE) matrix of $\boldsymbol{a}_t$.

Prediction equations. Given $\boldsymbol{a}_{t-1}$ and $\boldsymbol{P}_{t-1}$,

$$\begin{aligned} \boldsymbol{a}_{t|t-1} &= E[\boldsymbol{\alpha}_t | I_{t-1}] \\ &= \boldsymbol{T}_t \boldsymbol{a}_{t-1} + \boldsymbol{c}_t & \text{(A.1.5)} \\ \boldsymbol{P}_{t|t-1} &= E[(\boldsymbol{\alpha}_t - \boldsymbol{a}_{t-1})(\boldsymbol{\alpha}_t - \boldsymbol{a}_{t-1})' | I_{t-1}] \\ &= \boldsymbol{T}_t \boldsymbol{P}_{t-1} \boldsymbol{T}_t' + \boldsymbol{R}_t \boldsymbol{Q}_t \boldsymbol{R}_t. & \text{(A.1.6)} \end{aligned}$$

And the optimal predictor of $\boldsymbol{y}_t$ is obtained from

$$\begin{aligned} \boldsymbol{y}_{t|t-1} &= \boldsymbol{Z}_t \boldsymbol{a}_{t|t-1} + \boldsymbol{d}_t \\ &= \boldsymbol{Z}_t \boldsymbol{T}_t \boldsymbol{a}_{t-1} + \boldsymbol{Z}_t \boldsymbol{c}_t + \boldsymbol{d}_t \\ &= \boldsymbol{Z}_t (\boldsymbol{T}_t \boldsymbol{a}_{t-1} + \boldsymbol{c}_t) + \boldsymbol{d}_t. & \text{(A.1.7)} \end{aligned}$$

The corresponding prediction error and its MSE matrix are

$$\begin{aligned}\boldsymbol{e}_t &= \boldsymbol{y}_t - \boldsymbol{y}_{t|t-1} \\ &= \boldsymbol{y}_t - \boldsymbol{Z}_t\boldsymbol{a}_{t|t-1} - \boldsymbol{d}_t \\ &= \boldsymbol{Z}_t\boldsymbol{\alpha}_t + \boldsymbol{d}_t + \boldsymbol{u}_t - \boldsymbol{Z}_t\boldsymbol{a}_{t|t-1} - \boldsymbol{d}_t \\ &= \boldsymbol{Z}_t(\boldsymbol{\alpha}_t - \boldsymbol{a}_{t|t-1}) + \boldsymbol{u}_t \end{aligned} \tag{A.1.8}$$

and

$$E[\boldsymbol{e}_t\boldsymbol{e}_t'] \coloneqq \boldsymbol{F}_t = \boldsymbol{Z}_t\boldsymbol{P}_{t|t-1}\boldsymbol{Z}_t' + \boldsymbol{H}_t. \tag{A.1.9}$$

Updating equations. The moment $\boldsymbol{y}_t$ is the optimal predictor observed and its MSE matrix are updated according to

$$\begin{aligned}\boldsymbol{a}_t &= \boldsymbol{a}_{t|t-1} + \boldsymbol{P}_{t|t-1}\boldsymbol{Z}_t'\boldsymbol{F}_t^{-1}(\boldsymbol{y}_t - \boldsymbol{y}_{t|t-1}) \\ &= \boldsymbol{a}_{t|t-1} + \boldsymbol{P}_{t|t-1}\boldsymbol{Z}_t'\boldsymbol{F}_t^{-1}(\boldsymbol{y}_t - \boldsymbol{Z}_t\boldsymbol{\alpha}_{t|t-1} - \boldsymbol{d}_t) \\ &= \boldsymbol{a}_{t|t-1} + \boldsymbol{P}_{t|t-1}\boldsymbol{Z}_t'\boldsymbol{F}_t^{-1}\boldsymbol{e}_t \end{aligned} \tag{A.1.10}$$

$$\boldsymbol{P}_t = \boldsymbol{P}_{t|t-1} - \underbrace{\boldsymbol{P}_{t|t-1}\boldsymbol{Z}_t\boldsymbol{F}_t^{-1}}_{\text{Kalman Gain}}\boldsymbol{Z}_t\boldsymbol{P}_{t|t-1}. \tag{A.1.11}$$

Filter derivation. The KF-derivation makes use of the following properties of a bivariate normal distribution. Given y, the distribution of x is normal with

$$E[x|y] = \boldsymbol{\mu}_{x|y} = \boldsymbol{\mu}_x + \boldsymbol{\Sigma}_{xy}\boldsymbol{\Sigma}_{yy}^{-1}(\boldsymbol{y} - \boldsymbol{\mu}_y) \tag{A.1.12}$$

$$Var(x|y) = \boldsymbol{\Sigma}_{xx} - \boldsymbol{\Sigma}_{xy}\boldsymbol{\Sigma}_{yy}^{-1}\boldsymbol{\Sigma}_{yx}. \tag{A.1.13}$$

For the state vector at $t = 1$,

$$\boldsymbol{\alpha}_1 = \boldsymbol{T}_1\boldsymbol{\alpha}_0 + \boldsymbol{c}_1 + \boldsymbol{R}_1\boldsymbol{\nu}_1,$$

with $\boldsymbol{\alpha}_0 \sim N(\boldsymbol{a}_0, \boldsymbol{P}_0)$, $\boldsymbol{\nu}_1 \sim N(\boldsymbol{0}, \boldsymbol{Q}_1)$ and $E[\boldsymbol{\alpha}_0\boldsymbol{\nu}_1'] = \boldsymbol{0}$. In a linear Gaussian state-space model the initial state vector is normally distributed with

$$\boldsymbol{a}_{1|0} \coloneqq E[\boldsymbol{\alpha}_1] = \boldsymbol{T}_1\boldsymbol{a}_0 + \boldsymbol{c}_1 \tag{A.1.14}$$

$$\boldsymbol{P}_{1|0} \coloneqq Var(\boldsymbol{\alpha}_1) = \boldsymbol{T}_1\boldsymbol{P}_{1|0}\boldsymbol{T}_1' + \boldsymbol{R}_1\boldsymbol{Q}_1\boldsymbol{R}_1'.$$

From the measurement equation, it follows that

$$\boldsymbol{y}_1 = \boldsymbol{Z}_1\boldsymbol{\alpha}_1 + \boldsymbol{d}_1 + \boldsymbol{u}_1,$$

with $\boldsymbol{u}_1 \sim N(\boldsymbol{0}, \boldsymbol{H}_1)$ s.t.

$$\begin{aligned} \boldsymbol{y}_{1|0} &:= E[\boldsymbol{y}_1] = \boldsymbol{Z}_1 \boldsymbol{a}_{1|0} + \boldsymbol{d}_1 \qquad \text{(A.1.15)} \\ Var(\boldsymbol{y}_1) &= E[(\boldsymbol{y}_1 - \boldsymbol{y}_{1|0})(\boldsymbol{y}_1 - \boldsymbol{y}_{1|0})'] \\ &= E[(\boldsymbol{Z}_1\{\boldsymbol{\alpha}_1 - \boldsymbol{a}_{1|0}\} + \boldsymbol{u}_1)(\boldsymbol{Z}_1\{\boldsymbol{\alpha}_1 - \boldsymbol{a}_{1|0}\} + \boldsymbol{u}_1)'] \\ &= \boldsymbol{Z}_1 \boldsymbol{P}_{1|0} \boldsymbol{Z}_1' + \boldsymbol{H}_1. \end{aligned}$$

Equations (A.1.14) and (A.1.15) are the prediction equations for $\boldsymbol{\alpha}_1$ and $\boldsymbol{y}_1$ at $t = 0$.

In a next crucial step one has to find the distribution of $\boldsymbol{\alpha}_1$ conditional on $\boldsymbol{y}_1$ being observed (updating). For this purpose the joint normal distribution of $(\boldsymbol{\alpha}_1', \boldsymbol{y}_1')$ must be determined. In finding the joint normal distribution we use

$$\begin{aligned} \boldsymbol{\alpha}_1 &= \boldsymbol{a}_{1|0} + (\boldsymbol{\alpha}_1 - \boldsymbol{a}_{1|0}) \\ \boldsymbol{y}_1 &= \boldsymbol{y}_{1|0} + \boldsymbol{y}_1 - \boldsymbol{y}_{1|0} \\ &= \boldsymbol{Z}_1 \boldsymbol{a}_{1|0} + \boldsymbol{d}_1 + \boldsymbol{Z}_1(\boldsymbol{\alpha}_1 - \boldsymbol{a}_{1|0}) + \boldsymbol{u}_1. \end{aligned}$$

Note that since

$$\begin{aligned} Cov(\boldsymbol{\alpha}_1, \boldsymbol{y}_1) &= E[(\boldsymbol{\alpha}_1 - \boldsymbol{a}_{1|0})(\boldsymbol{y}_1 - \boldsymbol{y}_{1|0})'] \\ &= E[(\boldsymbol{\alpha}_1 - \boldsymbol{a}_{1|0})(\boldsymbol{Z}_1\{\boldsymbol{\alpha}_1 - \boldsymbol{a}_{1|0}\} + \boldsymbol{u}_1)'] \\ &= E[(\boldsymbol{\alpha}_1 - \boldsymbol{a}_{1|0})(\{\boldsymbol{\alpha}_1 - \boldsymbol{a}_{1|0}\}\boldsymbol{Z}_1' + \boldsymbol{u}_1')] \\ &= E[(\boldsymbol{\alpha}_1 - \boldsymbol{a}_{1|0})(\boldsymbol{\alpha}_1 - \boldsymbol{a}_{1|0})\boldsymbol{Z}_1'] + E[(\boldsymbol{\alpha}_1 - \boldsymbol{a}_{1|0})\boldsymbol{u}_1'] \\ &= \boldsymbol{P}_{1|0}\boldsymbol{Z}_1', \end{aligned}$$

$$\begin{pmatrix} \boldsymbol{\alpha}_1 \\ \boldsymbol{y}_1 \end{pmatrix} \sim N\left(\begin{pmatrix} \boldsymbol{a}_{1|0} \\ \boldsymbol{Z}_1 \boldsymbol{a}_{1|0} + \boldsymbol{d}_1 \end{pmatrix}, \begin{pmatrix} \boldsymbol{P}_{1|0} & \boldsymbol{P}_{1|0}\boldsymbol{Z}_1' \\ \boldsymbol{Z}_1\boldsymbol{P}_{1|0} & \boldsymbol{Z}_1\boldsymbol{P}_{1|0}\boldsymbol{Z}_1' + \boldsymbol{H}_1 \end{pmatrix} \right).$$

In combination with (A.1.12) and (A.1.13), $(\boldsymbol{\alpha}_1|\boldsymbol{y}_1) \sim N(\boldsymbol{a}_1, \boldsymbol{P}_1)$ follows with

$$\begin{aligned} \boldsymbol{a}_1 &= \boldsymbol{a}_{1|0} + \boldsymbol{P}_{1|0}\boldsymbol{Z}_1'(\boldsymbol{Z}_1\boldsymbol{P}_{1|0}\boldsymbol{Z}_1' + \boldsymbol{H}_1)^{-1}(\boldsymbol{y}_1 - \boldsymbol{Z}_1\boldsymbol{a}_{1|0} - \boldsymbol{d}_1) \\ &= \boldsymbol{a}_{1|0} + \boldsymbol{P}_{1|0}\boldsymbol{Z}_1'\boldsymbol{F}_1^{-1}\boldsymbol{e}_1 \qquad \text{(A.1.16)} \\ \boldsymbol{P}_1 &= \boldsymbol{P}_{1|0} - \boldsymbol{P}_{1|0}\boldsymbol{Z}_1'(\boldsymbol{Z}_1\boldsymbol{P}_{1|0}\boldsymbol{Z}_1' + \boldsymbol{H}_1)^{-1}\boldsymbol{Z}_1\boldsymbol{P}_{1|0} \\ &= \boldsymbol{P}_{1|0} - \boldsymbol{P}_{1|0}\boldsymbol{Z}_1'\boldsymbol{F}_1^{-1}\boldsymbol{Z}_1\boldsymbol{P}_{1|0}. \qquad \text{(A.1.17)} \end{aligned}$$

Note that (A.1.16) and (A.1.17) are the Kalman Filter updating equations for $t = 1$.

ML-estimation and EM algorithm. Let $\boldsymbol{\theta}$ denote the parameters of the state-space model. These parameters are embodied in the system matrices. The likelihood of the state-space model is calculated based on the prediction errors $\boldsymbol{e}_t$ with $t = 1, ..., T$. The prediction error decomposition of the (negative) log-likelihood is

$$-2 \ln L(\boldsymbol{\theta}|\boldsymbol{y}) = nT \ln(2\pi) + \sum_{t=1}^{T} \ln |\boldsymbol{F}_t(\boldsymbol{\theta})| + \sum_{t=1}^{T} \boldsymbol{e}_t'(\boldsymbol{\theta})\boldsymbol{F}_t^{-1}(\boldsymbol{\theta})\boldsymbol{e}_t(\boldsymbol{\theta}). \quad \text{(A.1.18)}$$

Shumway and Stoffer (2008) proposed a procedure based on the EM algorithm that is conceptually simpler and more efficient than alternative procedures such as the Newton-Raphson algorithm. The basic idea is that if all states $\boldsymbol{\alpha}_T = \{\boldsymbol{\alpha}_t\}_{t=0}^T$ together with the observations $\boldsymbol{y}_T = \{\boldsymbol{y}_t\}_{t=1}^T$ were observed, one could consider the entire data space $\{\boldsymbol{\alpha}_T, \boldsymbol{y}_T\}$. The complete data likelihood might, thus, be written as

$$\begin{aligned}-2 \ln L(\boldsymbol{\theta}|\boldsymbol{\alpha}, \boldsymbol{y}) = {} & \ln |\boldsymbol{F}_0| + (\boldsymbol{\alpha}_0 - \boldsymbol{a}_0)'\boldsymbol{F}_0^{-1}(\boldsymbol{\alpha}_0 - \boldsymbol{a}_0) \\ & + n \ln |\boldsymbol{Q}_t| + \sum_{t=1}^{T}(\boldsymbol{\alpha}_t - \boldsymbol{T}_t\boldsymbol{\alpha}_{t-1})'\boldsymbol{Q}_t^{-1}(\boldsymbol{\alpha}_t - \boldsymbol{T}_t\boldsymbol{\alpha}_{t-1}) \\ & + n \ln |\boldsymbol{H}_t| + \sum_{t=1}^{T}(\boldsymbol{y}_t - \boldsymbol{Z}_t\boldsymbol{\alpha}_t)'\boldsymbol{H}_t^{-1}(\boldsymbol{y}_t - \boldsymbol{Z}_t\boldsymbol{\alpha}_t).\end{aligned} \quad \text{(A.1.19)}$$

Given the *complete* data without any missing values and mostly in the desired (quarterly) frequency, one could easily obtain the ML-estimates of $\boldsymbol{\theta}$. However, as this is not the case, we may find the ML-estimates based on the *incomplete* data with short-fall by successively maximizing the conditional expectation of the complete data likelihood. This is done in the following steps:

1. Find some initial values for parameters $\boldsymbol{\theta}^{(0)}$,
2. Calculate the incomplete data likelihood $-\ln L(\boldsymbol{\theta}^{(j-1)}|\boldsymbol{y})$; see equation (A.1.18),
3. At iteration $j = 1, 2, ...$ use the KF and KS to obtain smoothed values for $\boldsymbol{\alpha}_t^{(S)}$, $\boldsymbol{P}_t^{(S)}$ and $\boldsymbol{P}_{t|t-1}^{(S)}$ for $t = 1, ..., T$ based on the parameters $\boldsymbol{\theta}^{(j-1)}$. Use the smoothed

values to calculate the conditional expectation of the complete data likelihood

$$
\begin{aligned}
Q(\boldsymbol{\theta}|\boldsymbol{\theta}^{(j-1)}) =& E\left\{-2\ \ln\ L(\boldsymbol{\theta}|\boldsymbol{\alpha},\boldsymbol{y})|\boldsymbol{y}_n,\boldsymbol{\theta}^{(j-1)}\right\} \\
&= \ln\ |\boldsymbol{F}_0| +\ \mathrm{tr}\ \left\{\boldsymbol{F}_0^{-1}\left[\boldsymbol{P}_0^{(S)} + \left(\boldsymbol{\alpha}_0^{(S)} - \boldsymbol{a}_0\right)\left(\boldsymbol{\alpha}_0^{(S)} - \boldsymbol{a}_0\right)'\right]\right\} \\
&\quad + n\ \ln\ |\boldsymbol{Q}_t| +\ \mathrm{tr}\ \left\{\boldsymbol{Q}^{-1}\left[S_{11} - S_{10}\boldsymbol{Z}_t' - \boldsymbol{Z}_t S_{10} + \boldsymbol{Z}_t S_{00}\boldsymbol{Z}_t'\right]\right\} \\
&\quad + n\ \ln\ \boldsymbol{H} \\
&\quad +\ \mathrm{tr}\ \left\{\boldsymbol{H}^{-1}\sum_{t=1}^{T}\left[\left(\boldsymbol{y}_t - \boldsymbol{Z}_t\boldsymbol{\alpha}_t^{(S)}\right)\left(\boldsymbol{y}_t - \boldsymbol{Z}_t\boldsymbol{\alpha}_t^{(S)}\right)' + \boldsymbol{Z}_t\boldsymbol{P}_t^{(S)}\boldsymbol{Z}_t'\right]\right\},
\end{aligned}
$$

where

$$
\begin{aligned}
S_{11} &= \sum_{t=1}^{T}\left(\boldsymbol{\alpha}_t^{(S)}\boldsymbol{\alpha}_t^{(S)\prime} + \boldsymbol{P}_t^{(S)}\right), \\
S_{10} &= \sum_{t=1}^{T}\left(\boldsymbol{\alpha}_t^{(S)}\boldsymbol{\alpha}_{t|t-1}^{(S)\prime} + \boldsymbol{P}_{t|t-1}^{(S)}\right) \text{ and} \\
S_{00} &= \sum_{t=1}^{T}\left(\boldsymbol{\alpha}_{t|t-1}^{(S)}\boldsymbol{\alpha}_{t|t-1}^{(S)\prime} + \boldsymbol{P}_{t|t-1}^{(S)}\right).
\end{aligned}
$$

4. Update $\boldsymbol{\theta}_0$ according to

$$
\begin{aligned}
\boldsymbol{T}_t^{(j)} &= S_{10}S_{00}^{-1}, \\
\boldsymbol{Q}_t^{(j)} &= n^{-1}\left(S_{11} - S_{10}S_{00}^{-1}S_{10}'\right) \text{ and} \\
\boldsymbol{H}_t^{(j)} &= n^{-1}\sum_{t=1}^{T}\left[\left(\boldsymbol{y}_t - \boldsymbol{Z}_t\boldsymbol{\alpha}_t^{(S)}\right)\left(\boldsymbol{y}_t - \boldsymbol{Z}_t\boldsymbol{\alpha}_t^{(S)}\right)' + \boldsymbol{Z}_t\boldsymbol{P}_t^{(S)}\boldsymbol{Z}_t'\right]
\end{aligned}
$$

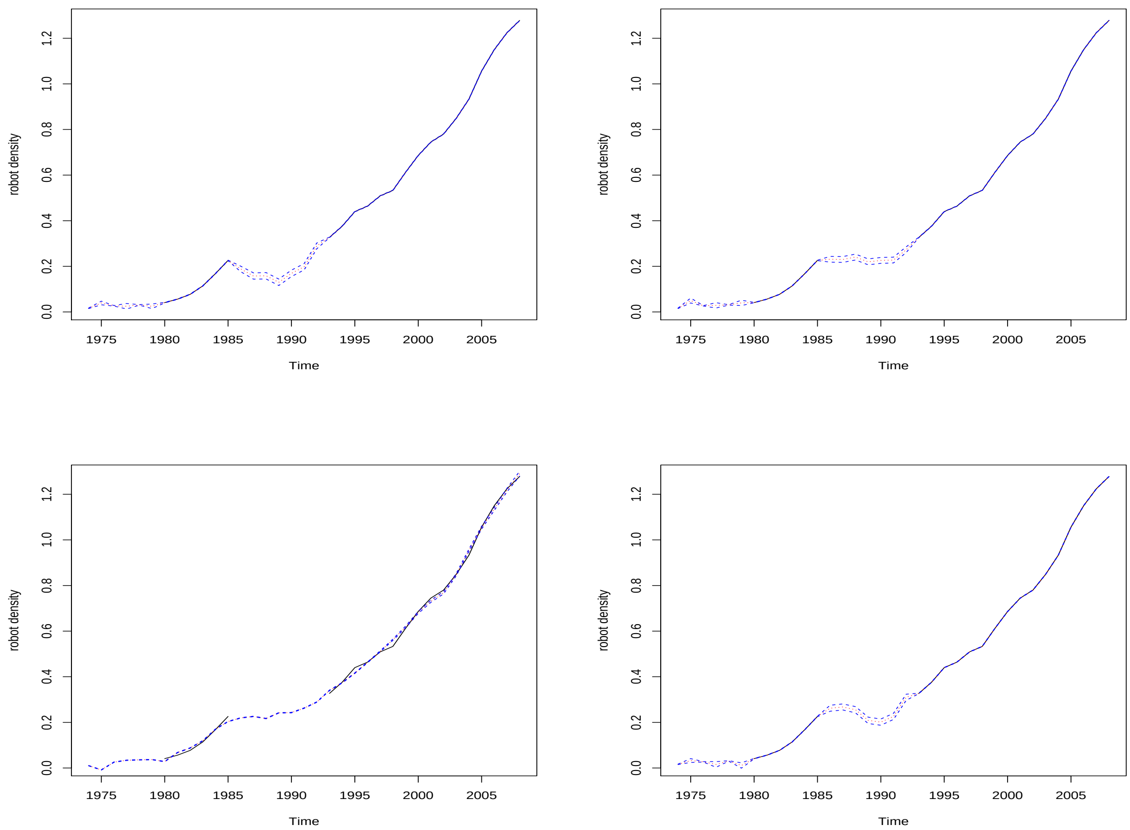
to obtain $\boldsymbol{\theta}^{(j)}$.

5. Repeat steps 2 to 4 until convergence is achieved (i.e.until parameters or likelihood values stabilize in the sense of differing from their predecessor values by some predetermined small amount κ only).

A.1.3 Application

Annual series nowcast. Having sketched the nowcasting procedure and indicators set, referred to as information set I_t above, we run the annual series (or q4-value) nowcast for four differently sized set of indicators: I_1 (index t dropped for notational ease) considers 16 series, i.e. in contemporaneous and first lag expression: hours worked, labor productivity, actually used labor composition/quality, labor composition/quality as reported by the Bureau of Labor Statistics (BLS), TFP, utilitization-adjusted TFP, and the tax rate on labor income. The next two considered sets, I_2 and I_3, are similar in size and nowcasting performance: Set I_3 comprises ten series by dropping the two labor composition/quality indicator series and the utilization-adjusted TFP series. Using either of the two TFP series does not alter the nowcasted values. It merely changes fourth or higher decimal places. The same applies to the two different labor quality indicators. I_2 is of the same size as I_3 but into account labor composition/quality indicator and leaves out the labor tax series. Information set I_4 compared to I_3 includes the labor tax indicator and drops labor productivity.

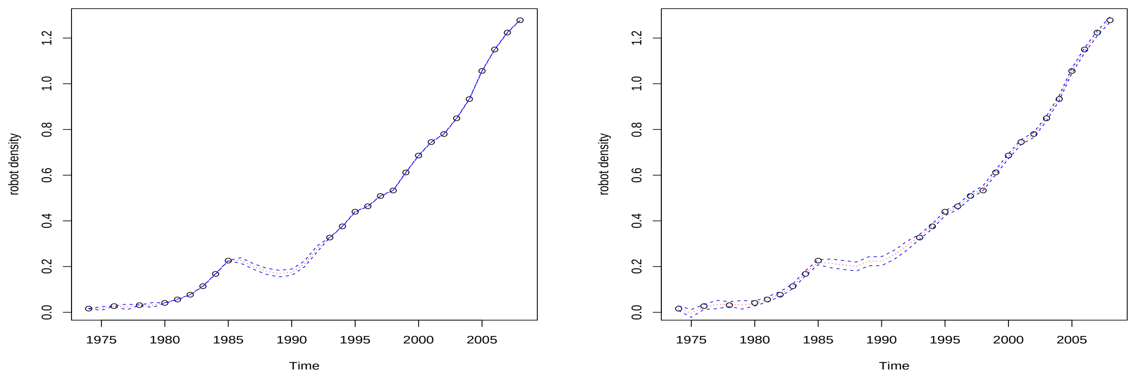
Figure A.1: Annual nowcast of robots per 1K workers for different information sets



Note: Underlying indicator sets, from left to right, first row: I_1, I_2, second row: I_3, I_4 (from left to right); black line and dots: empirical values, red dots: smoother values, blue dashed lines: 95 % C.I. of prediction errors

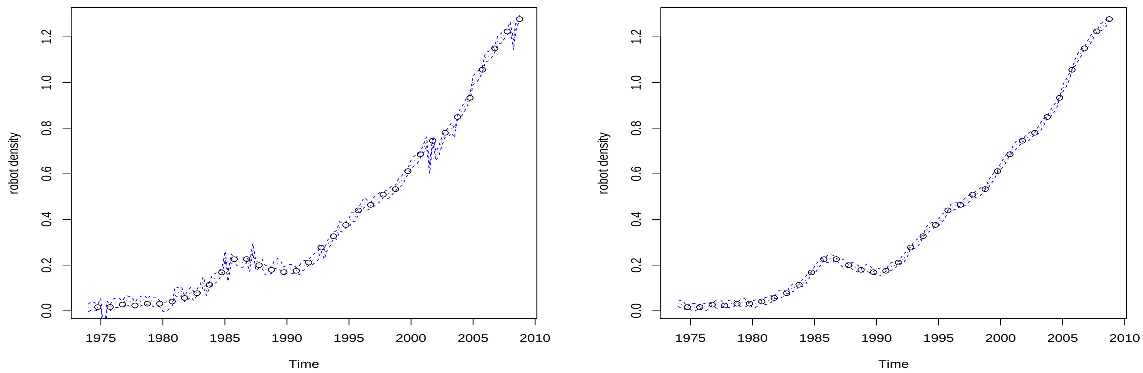
In line with intuition that – both and primarily – productivity and costs matter with regard to automization, information set I_3 (lower left schedule in Figure A.1) generates the most accurate and reasonable annual series nowast.

Figure A.2: Observation-equation-variance sensitivity of I_3-based nowcast



Note: Variances in observation equation matrix ($\boldsymbol{H}_t$) doubled from 1 (left) to 2 (right schedule); black circles: empirical values, red dots: smoother values, blue dashed lines: 95 % C.I. of prediction errors

Figure A.3: Observation/state-equation-variance sensitivity of quarterly nowcast



Note: Quarterly I_3-based (filling up I_3-based annual nowcast); variances in observation (state) equation matrix $\boldsymbol{H}_t$ ($\boldsymbol{Q}_t$) are set to 0.01 (1) for left schedule and 0.01 (0.01) for right schedule, respectively; black circles: empirical and nowcasted (q4-/fourth quarter) annual values; remaining legend as for Figure A.2

As can be seen from Figure A.2 our nowcasts are slightly sensitive with regard to observation equation variances and produce more accurate predictions of the empirical observations for lower values. Thus, we proceed with the I_3-based annual nowcast with the lower observation equation variance values.

Quarterly series nowcast. For our quarterly series nowcast, we fill up the I_3-based annual projections and empirical values but now rely on an I_3 analogue using all quarterly instead of just q4 information. The result is shown for different observation and state equation variance values, $\boldsymbol{H}_t$ and $\boldsymbol{Q}_t$, in Figure A.3. The right schedule circle and red dot values correspond to our nowcasted quarterly series of choice and corresponds to the time series (solid blue line) displayed in Figure 1.3.

A.2 VEC Model Analysis

A.2.1 Johansen Procedure

Table A.1: Unit root (UR) and stationarity test statistics

	ADF I	ADF II	PP	KPSS
Log levels:				
Population growth	–1.732	–1.679	–1.840	0.399***
Factor tax policy mix (LCTR)	–1.717	–2.003	–5.379	0.988***
Robots per 1K workers	–2.841*	–2.792	–0.481	0.504***
Wage share	–1.253	–2.861	–2.338	0.290***
Capital share	–1.377	–1.937	–1.876	0.361***
Log first differences:				
Population growth	–11.36***	–11.39***	–11.36***	0.0631
Factor tax policy mix (LCTR)	–7.887***	–17.13***	–14.08***	0.0359
Robots per 1K workers	–5.086***	–5.468***	–10.61***	0.1030
Wage share	–15.81***	–9.181***	–12.52***	0.0415
Capital share	–6.547***	–6.618***	–4.111***	0.0381

Note: ADF – Augmented Dickey-Fuller (UR under null; I/II = without/with linear trend component); PP – Phillips-Perron (UR under null); KPSS – Kwiatkowski-Phillips-Schmidt-Shin (stationarity under null; with automatic bandwidth selection and autocovariances weighted by quadratic spectral kernel); * $p < .10$, ** $p < .05$, *** $p < .01$.

Table A.2: Johansen test procedure and test statistics for cointegrating equations

		Trace		Max EV		Info Criterion
Model	Max Rank	Stats	5% c.v.	Stats	5% c.v.	HQIC
(v1)	0	101.04	59.46	43.72	30.04	–35.52362
	1	57.31	39.89	30.31	23.8	–35.64426
	2	27.00	24.31	17.77	17.89	–35.70900
	3	9.23*	12.53	9.21	11.44	–35.72472*
	4	0.01	3.84	0.01	3.84	–35.72275
(v2)	0	145.46	76.07	68.08	34.4	–35.52362
	1	77.38	53.12	30.36	28.14	–35.81121
	2	47.02	34.91	23.44	22.00	–35.85150
	3	23.58	19.96	17.56	15.67	–35.88699
	4	6.01*	9.42	6.01	9.24	–35.92591*
(v3)	0	133.92	68.52	68.07	33.46	–35.49021
	1	65.85	47.21	30.23	27.07	–35.80259
	2	35.62	29.68	21.39	20.97	–35.86672
	3	14.22*	15.41	12.56	14.07	–35.91094
	4	1.66	3.76	1.66	3.76	–35.93535*
(v4)	0	207.72	87.31	116.61	37.52	–35.49021
	1	91.10	62.99	36.70	31.46	–36.15999
	2	54.39	42.44	25.86	25.54	–36.25025
	3	28.53	25.32	17.42	18.96	–36.30482
	4	11.11*	12.25	11.11	12.52	–36.3426*
(v5)	0	188.16	77.74	111.36	36.41	–35.51998
	1	76.79	54.64	36.56	30.33	–36.17327
	2	40.23	34.55	25.80	23.78	–36.28723
	3	14.43*	18.17	13.46	16.87	–36.36616
	4	0.97	3.74	0.97	3.74	–36.39762*

Note: Trace – trace test; Max EV – maximum eigenvalue test; Info Criterion – information criterion (IC) with IC of choice: HQIC – Hannan-Quinn information criterion; procedure starts with test for zero cointegrating equations (CE), i.e. a maximum rank of zero, and then accepts the first null that is not rejected (indicated by '$*$'); the Pantula Principle sequence is: (v1) no intercept or trend in CE or in VAR part; (v2) intercept and no trend in CE part and neither intercept nor trend in VAR part; (v3) intercept in CE part and VAR part, but no trends; (v4) intercept in CE part and in VAR part paralleled by linear trend in CE part or in VAR part.

As can be seen from Tables A.1 and A.2, all series in log levels used in the fitted VECM, as described in the empirical part of the paper, are $I(1)$ and the result of the Johansen test procedure is that there are, at least, three cointegrating relationships. The latter concern population growth and the wage share, the factor tax policy mix (LCTR) and the capital share, and robots density and capital share, respectively. However, the last of these CE relationships is statistically significant at a 68% level of significance only according to the z-statistics of the Johansen normalized restriction test.

A post-estimation stability check confirms three (two) imposed unit moduli of eigenvalues of the companion matrix of our fitted VECM with two excact unit eigenvalues and one very close to one, i.e. with a value of .97. All remaining moduli of eigenvalues of the companion matrix are strictly less than one indicating stability. Serial correlation of residuals is clearly rejected by appropriate Lagrange Multiplier tests (LM tests).

Against the backdrop of the performed tests (with further detail available on request) we assess our VECM specification as being, all in all, acceptable.

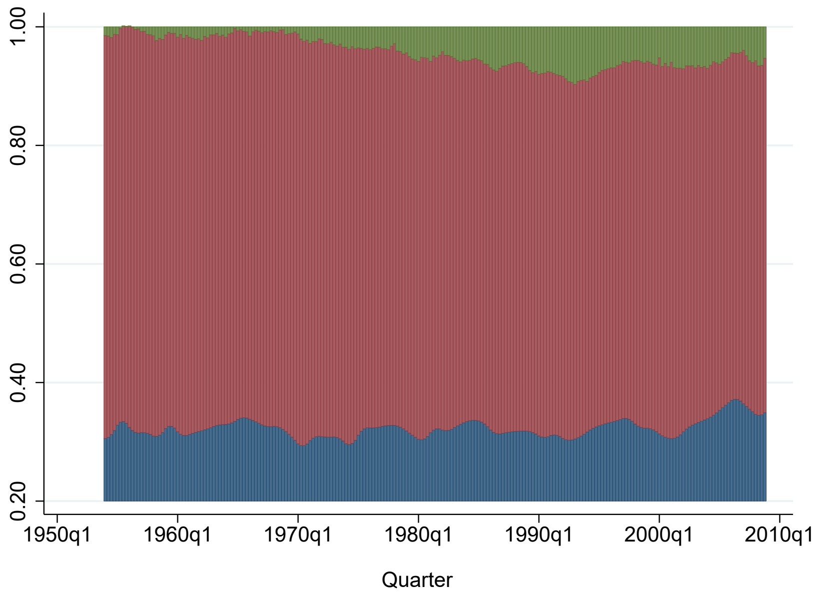
A.3 Construction of the Adjusted Labor Share

Our adjusted labor income share is based on BEA time series for GDP and GDI, that is, compensation of employees, paid wages and salaries. The share has been adjusted assuming that a one third add-on to GDI is attributable to the self-employed. Just like any adjustment of the empirical labor share, it represents a crude approximation. Contrary to alternative approaches, it precludes double counting.

A mean difference test for the two factor shares, capital income share and labor income share, summing to one (with mean rounded based on the digit at fourth decimal place) fails to reject the null of unity at any conventional level of significance.

An implied aggregate elasticity of substitution between capital and labor σ that is close to but slightly less than unity, $\sigma \leq 1$, is in line with recent evidence based on longitudinal data and consistent estimates by Glover and Short (2020). A $\sigma \lesssim 1$ (see the sum of the red and blue area in the following figure), as used in the empirical part of our paper, excludes the simple capital deepening explanation of the global decline in the labor share. For $\sigma \lesssim 1$, the fall in the labor's share cannot be rationalized by "rising effective capital ratios through physical investment in response to the fall in investment prices" (Glover and Short, 2020, p. 35).

Figure A.4: Capital income share, labor income share, and the elasticity of substitution



Note: Blue area: capital income share, red area: labor income share; green area: rest to unity

A.4 Summary Statistics of Series Used in VECM

Table A.3: Summary statistics of series used (as log levels) in VECM

Series	Mean	Std. dev.	Min	Max	Skewness	Kurtosis
Population growth	0.0026	0.0004	0.0021	0.0035	0.7574	2.0393
Factor tax policy mix (LCTR)	0.7417	0.0985	0.4733	0.8891	–1.0011	3.0182
Robots per 1K workers	0.3936	0.3695	0.0149	1.0278	0.9128	2.0670
Wage share	0.6341	0.0299	0.5827	0.6914	0.0355	2.0890
Capital share	0.3247	0.0164	0.2953	0.3717	0.9221	3.0684

Note: Summarized series were transformed to log levels before using them in the VECM as outlined in the empirical part of the paper; robots per 1K workers is the pseudo-quarterly series nowcasted as described above; throughout the observation period ranges from 1974:q1 to 2008:q4 (N obs = 140). All sources of data are given in the text (empirical part of the paper).

A.5 A Neoclassical Growth Model with Dynamic Taxes, Automation Capital, and Adjustment Costs - Analytical Details

A.5.1 Household Optimization Problem

The Lagrangean of the household optimization problem is

$$\begin{aligned}
\mathcal{L} =& \sum_{t=0}^{\infty} \beta^t \Bigg\{ \frac{\left(c_t^{\theta}(1-l_t)^{1-\theta}\right)^{1-\eta} - 1}{1-\eta} \\
& + \lambda_t \Big[(1-\tau_t^w) w_t l_t + (1-\tau_t^r) r_t^k k_t + (1-\tau_t^r) r_t^p p_t + tr_t - (1+\tau^c) c_t \\
& - \phi(i_t^k, k_t) - \phi(i_t^p, p_t) \Big] \\
& + q_t^k \left[i_t^k + (1-\delta) k_t - (1+n_t) k_{t+1} \right] \\
& + q_t^p \left[i_t^p + (1-\delta) p_t - (1+n_t) p_{t+1} \right] \\
& + \mu_t^l l_t + \mu_t^{i^k} i_t^k + \mu_t^{i^p} i_t^p + \mu_{t+1}^k k_{t+1} + \mu_{t+1}^p p_{t+1} \Bigg\},
\end{aligned}$$

where μ_t^x, $x \in \{l, i^k, i^p, k, p\}$, denotes the Lagrange multiplier on the respective constraint $l_t \geq 0$, $i_t^k \geq 0$, $i_t^p \geq 0$, $k_t \geq 0$, and $p_t \geq 0$.

Since the utility function $u(c, 1-l)$ of (3.3) is strictly concave on its domain and satisfies the Inada conditions mentioned in the main text the first-order conditions for the household's optimization problem are

$$\lambda_t(1+\tau^c) = \theta c_t^{\theta(1-\eta)-1}(1-l_t)^{(1-\theta)(1-\eta)}, \tag{A.5.1a}$$

$$\lambda_t(1-\tau_t^w) w_t + \mu_t^l = (1-\theta) c_t^{\theta(1-\eta)} (1-l_t)^{(1-\theta)(1-\eta)-1}, \tag{A.5.1b}$$

$$q_t^k + \mu_t^{i^k} = \lambda_t \phi_{i^k}(i_t^k, k_t), \tag{A.5.1c}$$

$$q_t^p + \mu_t^{i^p} = \lambda_t \phi_{i^p}(i_t^p, p_t), \tag{A.5.1d}$$

$$q_t^k - \frac{\mu_{t+1}^k}{1+n_t} = \frac{\beta}{1+n_t} \left\{ \lambda_{t+1} \left[(1-\tau_{t+1}^r) r_{t+1}^k - \phi_k(i_{t+1}^k, k_{t+1}) \right] + q_{t+1}^k (1-\delta) \right\}, \tag{A.5.1e}$$

$$q_t^p - \frac{\mu_{t+1}^p}{1+n_t} = \frac{\beta}{1+n_t} \left\{ \lambda_{t+1} \left[(1-\tau_{t+1}^r) r_{t+1}^p - \phi_p(i_{t+1}^p, p_{t+1}) \right] + q_{t+1}^p (1-\delta) \right\}, \tag{A.5.1f}$$

$$\mu_t^l l_t = 0, \quad \mu_t^{i^k} i_t^k = 0, \quad \mu_t^{i^p} i_t^p = 0, \quad \mu_{t+1}^k k_{t+1} = 0, \quad \mu_{t+1}^p p_{t+1} = 0,$$

$$0 = \lim_{t\to\infty} \beta^t \lambda_t k_{t+1}, \tag{A.5.1g}$$

$$0 = \lim_{t\to\infty} \beta^t \lambda_t p_{t+1}. \tag{A.5.1h}$$

Here, $\phi_{i^x}(i^x, x)$, $x \in \{k, p\}$, denotes the first derivative of the adjustment cost function $\phi(i^x, x)$ with respect to investment i^x. Similarly, ϕ_x denotes the first derivative of this function with respect to the second argument $x \in \{k, p\}$. For the specification of the adjustment cost function ϕ in (3.5), the above first-order conditions boil down to those of (3.7). Standard arguments show that the relevant second-order conditions are satisfied.

A.5.2 Details on the Computation of the Transition Dynamics

To compute the transition dynamics, we need to solve a difference equation system in the state variables $\{k_t, p_t\}_{t=1965}^{2175}$. We choose a time horizon of 210 years (= periods) so that the growth rates of the variables stabilize and are numerically close to their exact asymptotic counterparts.

As endogenous variables of our difference equation system, we use consumption, c_t, investment in both capital stocks, i_t^k and i_t^p, and labor l_t. The difference equations system include the household's first-order conditions (3.7c), (3.7e), (3.7f), and the resource constraint (3.15). With the endogenous variables, it is straightforward to compute the dynamics of the two capital stocks, k_t and p_t, from (3.4) for given initial values $k_{1965} > 0$ and $p_{1965} = 0$ taken from the steady state without automation capital.

Using k_t, p_t, and l_t, we compute the factor prices w_t, r_t^k, and p_t. From the first-order conditions (3.7a), (3.7c) and (3.7d) we calculate λ_t, q_t^k, and q_t^p. Hence, the values of all variables that show up in the equilibrium conditions of our model are pinned down. For the endogenous variables in the year 2176 that are also needed to compute the transition dynamics, we assume that c_t, i_t^k, and i_t^p grow at the asymptotic growth rate, while l_t is equal to zero (which occurs in the year 2132 in our simulation of the benchmark economy).

In essence, we have to solve a non-linear equations problem in $4 \times 210 = 840$ variables. This is a non-trivial task. The problem is to find a good initial value for the endogenous variables during 1965-2175. We, therefore, proceed as follows:[9]

Algorithm: Computation of the Transition Dynamics in the Benchmark Model with Automation Capital

Step 1: Compute the initial steady state in the year 1964 without automation capital.

[9]The Gauss computer code is available from the authors upon request.

Step 2: Compute the final steady state of the model with $\kappa = 0$ and, hence, without automation capital, for the tax rates and the population growth rate prevailing in 2015.

Step 3: Project a transition path for the model without automation capital (with $\kappa = 0$) for $\{i_t^k, c_t, l_t\}_{t=1965}^{2015}$ in the form of a linear adjustment.

Step 4: Solve the simple model without automation capital ($\kappa = 0$).

Step 5: Use the transition path from Step 4 with $p_t \equiv 0$ for $t = 1965, \ldots, 2015$ as initial guess for the computation of the transition in the model with automation capital. Assume that the variables $\{i_t^k, i_t^p, c_t\}$ grow at the rate γ after the final period, while l_t falls at the rate γ.

Step 6: Iterate over the time horizon of the transition T by incremental steps of one year. Use the transition path of the previous iteration as an initial guess assuming that in the period $T + 1$, the variables $\{i_t^k, i_t^p, c_t\}$ grow at the rate γ, while l_t falls at rate γ if it is larger than zero or remains equal to zero otherwise.

Step 7: Stop when the dynamics of the model during the period 1965-2015 do not change any more and the endogenous variables have reached their asymptotic values in the period T.