WORKING GROUP: TOPICS ON p-ADIC HODGE THEORY AND (φ, Γ)-MODULES

Fontaine's program in p-adic Hodge theory describes and classifies p-adic representations of the absolute Galois group of a finite extension of $\mathbb{Q}_p$. To those local Galois representations one can attach certain semilinear invariants called (φ, Γ)-modules which have the advantage to be very concrete and explicit. Their cohomology groups can be used, for instance, to compute the Iwasawa cohomology of local Galois representations. They also play an crucial role in Colmez's construction of a p-adic local Langlands correspondence for $\mathrm{GL}_2(\mathbb{Q}_p)$.

The aim of this workshop is to give an overview of the basic concepts in p-adic Hodge theory and, in the last two talks, discuss some applications to Iwasawa theory.

Time: Thursdays 11-12:30.

Talk 1: Overview of the workshop - 17 March. We will explain the central role played by p-adic Hodge theory in the formulation of certain conjectures in number theory (Fontaine-Mazur conjecture, Tamagawa number conjecture, special elements in Iwasawa theory, etc. [BK90, Fon92, Col00]).

Talk 2: (Relative) Lubin-Tate group laws - 24 March. § 1 and § 2 in [Sch21]. Define formal group laws and explain why they give rise to actual abelian groups on $\mathfrak{m}_K$. Define relative Lubin-Tate group laws and explain their dependence to the choice of Frobenius series and of the uniformizer.

Talk 3: Lubin-Tate extensions and local class field theory - 31 March. § 3 and parts of § 5 in [Sch21]. Introduce $\mathcal{F}_n$, $\chi_{E/L,n}$ and $\chi_{E/L}$. Describe the higher ramification theory of the Lubin-Tate extensions E_n/E. State and briefly explain the results in § 5.2, especially 5.8, 5.9, 5.11, 5.12.

Talk 4: Witt vectors - 7 April. Recall the basic theory of Witt vectors [Ber10, Section 6]. Discuss the topology of the ring of Witt vectors of a valued field, as in [Ber10, Section 16]. Introduce the rings E and $\tilde{E}$ following [Ber10, Section 15].

Talk 5: (φ, Γ)-modules - 21 April. Introduce the rings $\tilde{A}, \tilde{B}, A, B$, and E_K, A_K, B_K for a p-adic field K, following [Ber10, Section 17]. Use them to define (φ, Γ)-modules as in [Ber10, Section 18] and to prove [Ber10, Theorem 18.8].

Talk 6: Overconvergent (φ, Γ)-modules - 28 April. Introduce overconvergent elements of $\tilde{A}, \tilde{B}$ and the Colmez-Tate-Sen conditions; this material is contained in [Ber10, Sections 21-24]. Define overconvergent (φ, Γ)-modules as in [Ber10, Section 25] and prove Corollary 25.3.

Talk 7: p-adic Hodge theory - 5 May. Define the rings B_{dR}, B_{cris}, B_{st} and give their basic properties following [Ber10, Sections 26-28].

Talk 8: Crystalline and semistable representations - 12 May. Extract from [Ber10, Section 29] a proof of Corollary 29.15. Define crystalline and semistable representations and filtered (φ, N)-modules as in [Ber10, Section 30], and prove the equivalences of categories of [Ber10, Proposition 31.1].

Talk 9: Cohomology of (φ, Γ)-modules - 19 May. Chap. 5 in [Col04]. Definition of the cohomology of (φ, Γ)-modules and its identification with Galois cohomology (Thm. 5.2.2). Application to Tate's Euler-Poincaré formula (Thm. 5.3.17). Description of Tate's local pairing in terms of residues (§ 5.4).

Talk 10: Iwasawa cohomology - 26 May. Chap. 6 § 1-2 in [Col04]. Definition of Iwasawa cohomology $H^i_{\mathrm{Iw}}(V)$ and its description in terms of the Dieudonné module $D(V)$. Introduce the big exponential map Exp^*. Describe $D(\mathbb{Z}_p(1))^{\psi=1}$ and $H^1_{\mathrm{Iw}}(\mathbb{Z}_p(1))$ via Kummer theory as in Chap. 7 § 1-2 in [Col04].

Talk 11 (if time permits): Coleman's power series and an explicit reciprocity law. Chap. 7 § 3-5 in [Col04]. State and prove Thm 7.3.1. State Theorem 7.4.1 and explain the point (iii) of the following Remark (the measure μ_a there is the same as the measure λ_a in § 1.5). State and prove Lem. 1.5.1 and Prop. 1.5.2). Explain the strategy of the proof of the explicit reciprocity law outlined in § 7.5.1.

References

[Ber10] Laurent Berger. Galois representations and (φ, Γ)-modules. http://perso.ens-lyon.fr/laurent.berger/autrestextes/CoursIHP2010.pdf, 2010.

[BK90] Spencer Bloch and Kazuya Kato. L-functions and Tamagawa numbers of motives. In *The Grothendieck Festschrift, Vol. I*, volume 86 of *Progr. Math.*, pages 333–400. Birkhäuser Boston, Boston, MA, 1990.

[Col00] Pierre Colmez. Fonctions L p-adiques. Number 266, pages Exp. No. 851, 3, 21–58. 2000. Séminaire Bourbaki, Vol. 1998/99.

[Col04] Pierre Colmez. Fontaine's rings and p-adic L-functions. https://webusers.imj-prg.fr/~pierre.colmez/tsinghua.pdf, 2004.

[Fon92] Jean-Marc Fontaine. Valeurs spéciales des fonctions L des motifs. Number 206, pages Exp. No. 751, 4, 205–249. 1992. Séminaire Bourbaki, Vol. 1991/92.

[Sch21] Peter Schneider. Lubin-tate theory. https://ivv5hpp.uni-muenster.de/u/pschnei/publ/lectnotes/Lubin-Tate.pdf, 2021.