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Firms Location under Demand Heterogeneity

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Firms' Location under Demand Heterogeneity

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Abstract

In this paper we build an economic geography model where firms sell product varieties with heterogenous demands. We show that firms selling the products with higher demands select to set up their plants in larger countries. Larger countries do not only get better access to more varieties but also to the most demanded and valuable ones. The impact of such a spatial selection on firms' location choice depends on the skewness of the distribution of demand intensity across varieties. In a model where only capital moves across regions, demand heterogeneity generally diminishes the amount of capital invested in the larger country. In a model where the work force moves across regions, demand heterogeneity is shown to eliminate dramatic changes in the location patterns and to result in the asymmetric dispersion of workers, rather their symmetric dispersion or complete agglomeration in a specific region.

Keywords: heterogeneous taste and quality, spatial selection, economic geography, agglomeration, home market effect.

JEL classifications: F12, F15, R11, R12.

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1 Introduction

The present paper studies the impact of demand heterogeneity on trade and firms location. Firms sell product varieties with characteristics and uses that consumers value differently. Some firms end up selling product varieties that are highly demanded whereas others produce varieties with low demands. Firms are therefore heterogenous with respect to the intensity of demand for their products and therefore to the taste and preference that consumers express for their products. The role of demand heterogeneity on trade has recently been presented by Baldwin and Harrigan (2011) and Foster et al. (2008) who show that exporting firms quote higher prices than non-exporters. The impact of taste heterogeneity on trade has also been presented by Crozet et al. (2009) who show that Champagne and Burgundy wines are exported in larger quantities, to more numerous regions and at higher prices if they receive better quality ratings by reputed wine tasters (e.g. Robert Parker). However, heterogeneity in taste and demand is also likely to have an impact on firms' location decisions and therefore on the regional composition of industries. The present paper discusses this issue in more detail.

The causal relationship between firms' location and their product characteristics is well described in the business literature. Porter (1990) discusses it as the link between industrial clustering and product sophistication. This author offers many examples of sets of firms that sell higher added value products and choose to cluster in regions with larger markets. For instance, in 1818, Koenig and Bauer are known to have returned from London (U.K.) to Bavaria (Germany) to set up the production plant of their novel "rotary press" because Bavaria was one amongst the world's largest market for printing press. Other German producers of high quality press followed this path and moved their plants into Bavaria, making this region the world leader in the press industry in terms of sales and sophistication. Similarly, after World War II, the patient monitoring equipment industry has clustered in the U.S. because U.S. wealthy private hospitals had higher demands for sophisticated monitoring than many European countries with socialized medicine. More recently, in the seventies, the robotic industry has clustered in Japan because Japanese management teams had stronger engineering background and consequently higher demand for robotics (Porter, 1990, pp. 188-204). Those examples show that large markets are not only attractive to more firms but also to the most successful ones. To our knowledge, this causal relationship between firms' location decisions and the demand for their product characteristics has not been full explored in the economic literature.

The international trade literature has highlighted the role of firms' mobility on spatial economic discrepancies and the role of trade costs and country sizes on the spatial distribution of firms (Krugman

1991). However, this literature has not studied whether and how firms' mobility fosters discrepancies in the value of goods supplied by regions. Do larger regions (or cities) attract the firms producing the higher value, higher demand goods, giving an endogenous return premium to the industries located in those regions? Can peripheral regions (or cities) be left with the firms producing low value, low demand goods? In this context, we ask the two additional questions: Does such demand heterogeneity exacerbate or reduce the home market effect, according to which larger regions host a more than proportional share of industrial activity? What kind of demand heterogeneity has a stronger impact on the spatial distribution of firms?

In this paper, we extend Ottaviano *et al.* 's (2002) model in a way similar to Foster *et al.* (2008) by assuming that consumers share the same heterogenous preferences over a set of product varieties. More specifically, we assume that the intensity of preference for each variety is distributed according to a taste distribution which, for a given set of prices, maps into a demand distribution. As a result, product values and product demands differ across varieties but not across consumers. Each firm produces a distinct variety, competes under monopolistic competition and chooses its plant location in one of two regions. We sequentially envisage the situations where capital and workers are mobile. In a first footloose capital model, all consumers are immobile and capitalists allocate their capital to regions offering the highest return to capital (Martin and Rogers, 1995). In a second core periphery model, firms are run by mobile skilled workers who choose the residence and work location that offers the best outcome in terms of earnings and consumer surplus. Because skilled workers move with their firms, the demand for manufacturing varieties follows the firms' moves and creates a demand linkage. In each set up, we derive the price equilibrium conditions and the firms' location equilibrium conditions. We then discuss the impact of the demand distributions on the location equilibrium.

We obtain the following results. In both models, we firstly show that firms selling the goods with higher demand and higher value select the larger region. As a result, *larger regions do not only get better access to more varieties* as usually emphasized in the new economic geography literature but they also get *better access to the products on which consumers put a higher value.* We secondly discuss the impact of the shape of the taste and demand distribution functions on trade and firms' location choice. To our knowledge, this issue has been neglected in past works where Pareto distribution functions are used for their convenient analytical properties. We show that the skewness of the demand distribution has an important effect on firms location and on the home market effect. *The introduction of demand heterogeneity reduces the home market effect only if the demand distribution is unskewed or* *skewed towards high demand varieties.* That is, if high demand varieties are not too abundant. This conclusion applies for uniform and Pareto demand distribution functions. We further show how the impact of changes in demand distributions can be broken down between shifts in average demand and changes in the spread of those distributions. The demand distributions are non negligible determinants of firms' location pattern.[1]

We thirdly discuss the core periphery model and show how the number of equilibria, the possibilities of catastrophic changes and the effect of trade costs are related to demand distributions. Accordingly, we establish the condition under which those distributions yield a unique location equilibrium. We also show that *the introduction of demand heterogeneity eliminates the possibility of catastrophic changes* that exist in the same model with homogenous demands. Hence, demand heterogeneity is likely to help explain the empirical difficulty to verify catastrophic changes (e.g. Davis and Weinstein 2002). Finally, we show that demand heterogeneity should be considered *neither as a dispersion force nor as an agglomeration force.* Indeed, compared to the homogenous demand model, the introduction of heterogeneity has ambiguous effects on the dispersion and agglomeration of firms. In particular, *the introduction of demand heterogeneity makes an initially dispersed economy less dispersed and an initially agglomerated economy less agglomerated.* It is a force that entices skilled workers to agglomerate *partially.*

The remaining part of this introduction presents a deeper review of the literature.

Related literature

This paper relates to several strands of literature. Beyond the business literature mentioned above, the paper relates to the literature on trade and cost heterogeneity (Melitz, 2003; Helpman, Melitz and Yeaple, 2004; Falvey *et al.,* 2004; Melitz and Ottaviano, 2008; Baldwin and Robert-Nicoud, 2008). This literature responds to the empirical research on trade at firm level and explains the impact of trade liberalization on the export decisions of firms that are immobile and endowed with heterogenous productivity (Tybout and Westbrook, 1995; Bernard, Eaton, Jensen and Kortum, 2003; Bernard, Jensen, and Schott 2004). However, this literature has recently been qualified by a research agenda concerned with the empirical finding of a positive correlation between firms' export prices and export status. Foster *et al.* (2008), Nanova and Zhang (2009) and Baldwin and Harrigan (2011) indeed suggest that trade is explained better by demand heterogeneity than by cost heterogeneity because exporting

[1] Cabral and Mata (2001) study the distribution of firm size in more depth.

firms can be shown to export higher value or more demanded goods. In the same spirit, it has been shown that firms react to trade integration by restructuring their activities along their product quality ladder, which indicates the importance of quality in firms' strategy (e.g. Fontagné *et al.*, 1998; Kluger and Verhoogen, 2009).

This new literature on quality and trade generally builds on the theoretical extension of monopolistic competition model to quality, where quality is represented by a demand shifter. As in Schott (2004), this literature and this paper takes the view that a product has higher quality than another, if, for the same price, the former product has a higher demand. The theoretical foundation of this literature is generally based on the assumption of a set of horizontally differentiated varieties augmented by different quality levels, which stands apart from the traditional literature on trade and vertical product differentiation (an exception is Fajgelbaum *et al.*, 2009). This literature rather assumes that countries produce a same variety that has a different quality, which entices the richer country to specialize in the high quality variety (Linden 1961; Falvey 1981; Falvey and Kierzkowski 1987; Flam and Helpman 1987; Stockey 1991). This link between quality and income is nevertheless supported by the data (Schott, 2004; Hummels and Klenow, 2005; Hallak 2006).

To sum up, the trade literature has very recently revived the issue of firm heterogeneity and product quality. However, to our knowledge, this literature has not taken on board the implications of heterogeneity and quality on the firms' incentives to locate across regions and workers' to choose their residence place. As many papers in the literature, our paper also departs from models related to Flam and Helpman (1987) that focus on a free trade context and on income effects to show that high income regions produce higher value added products. By contrast, our paper focuses on heterogenous but horizontally differentiated varieties. The paper shows that region size asymmetries rather than income effects are sufficient factors to produce geographical discrepancies in the prices and added values of local productions. This paper also discusses the impact of trade costs in more detail.

The contribution of the paper is to study the spatial sorting of an industry endowed with a continuous distribution of varieties for which consumers have different tastes and demands. This contribution is a natural continuation of the research agenda of new economic geography about firms and workers' mobility and aims to integrate some recent stylized facts about quality in the trade literature. Our footloose capital model is meant to explain the mobility and selection of firms across regions while our core periphery model discusses the mobility and sorting of individuals across the cities or provinces of a country where individuals are free to move. The idea that heterogenous firms select location of production according to their production performance is not new. The idea that more efficient firms move and sort out in larger markets goes back at least to Syverson (2004), Nocke (2006), Okubo and Baldwin (2006), and Okubo et al. (2010) who assume that firms differ in cost. As presented in the above empirical studies, cost heterogeneity is not the only critical characteristic that explains trade patterns. So, a deeper investigation of the impact of demand heterogeneity is welcome. In this sense, the present paper presents an economic geography framework where firms' and workers' are mobile to a model where firms sell varieties with heterogenous demands. We discuss the possibility of firms' and workers' spatial selection, in which successful firms or skilled workers agglomerate in the large market and unsuccessful ones locate in the small market. This idea is consistent with Scott (2004) who gives evidence of international specialization *within* product ranges. In our model, capital holders who specialize in the high value varieties of a specific product range allocate their investments in the biggest market. However, in contrast to the main literature, such an outcome is related to the issue of market access rather than wage differentials.

Finally, it must be noted that sorting is a natural result in our linear quadratic model but it is not in CES models where sorting conditions turn out to be independent of firms' types (as already noted by Nocke, 2006). Our model allows us to perform a dedicated analysis on the impact of the distribution of heterogeneity across varieties. In contrast to the literature, our paper studies a large class of distributions of firms' characteristics which encompasses the discrete distribution studied in Okubo *et al.* (2010) and the Pareto distributions combined with CES preferences for their convenient analytical properties (e.g. Okubo and Baldwin, 2006).

The paper is structured as it follows. Section 2 presents the model and the short run equilibrium in product markets. Section 3 discusses the spatial distribution and selection of firms and capital in a footloose capital model with demand heterogeneity. Section 4 discusses the spatial distribution and sorting of workers in a core periphery model. Section 5 extends the model to a simple quality model where higher quality products are more expensive to produce. Section 6 concludes.

2 The model

In this section, we present the basic model and characterize the product market outcome for any given organizational structure and spatial distribution of firms. We start by describing consumers' preferences and their choices.

2.1 Preferences

Consider a world with two regions, labeled H and F. Variables associated with each region will be subscripted accordingly. We assume that there is a mass L of consumers, with a share $1/2 \leq \theta_H < 1$ located in region H. In what follows, we refer to H as the large and to F as the small region.

All consumers in region $i = H, F$ have identical quasi-linear preferences over a homogenous good and a continuum of horizontally differentiated varieties, indexed by $v \in \mathcal{V} \equiv [0,1]$. The utility of a representative agent in region i is given by the following quadratic function:

$$U_i = \int_{\mathcal{V}} \alpha(v) q_i(v)\mathrm{d}v - \frac{\beta - \gamma}{2} \int_{\mathcal{V}} [q_i(v)]^2 \mathrm{d}v - \frac{\gamma}{2} \left[\int_{\mathcal{V}} q_i(v)\mathrm{d}v \right]^2 + q_i^o, \tag{1}$$

where $q_i(v)$ denotes the consumption of variety v in region i and q_i^o stands for the consumption of the homogenous good in that same region. As in Ottaviano *et al.* (2002), γ is a measure of the degree of substitution between varieties whereas $\beta - \gamma$ (> 0) measures the consumer bias toward a more dispersed consumption of varieties.

As in Foster et al. (2008), the new element in this model is the function $\alpha(v) : \mathcal{V} \rightarrow [\alpha_l, \alpha_h]$, $0 < \alpha_l \leq \alpha_h$, that measures the *willingness to pay* for variety v.[2] Willingness to pay is heterogenous and reflects the intensity of consumer's preferences for each differentiated product v with respect to the homogenous good. Without loss of generality we assume that varieties are ranked according to the consumers' willingness to pay, i.e. such that $v > v' \iff \alpha(v) > \alpha(v')$. Because there is a unit mass of varieties, the inversion of the function α yields the taste cumulative distribution : $F_\alpha : [\alpha_l, \alpha_h] \rightarrow [0,1]$, $F_\alpha(x) =$Proba$[v : \alpha(v) \leq x] = \alpha^{-1}(x)$. The taste distribution density is then the function $f_\alpha : [\alpha_l, \alpha_h] \rightarrow \mathcal{R}$, $f_\alpha(x) = F'_\alpha(x) = 1/[\alpha'(\alpha^{-1}(x))]$. Because of this close relationship, we will refer to α as the *taste* (function) and to F_α and f_α as the *taste cumulative distribution* and the *taste distribution density* across varieties. Using this vocabulary, we denote the *average taste* $\int x \mathrm{d}F_\alpha(x)$ by the symbol $\overline{\alpha}$ (with an upper bar). Using the identities $x = \alpha(v)$ and $v = F_\alpha(\alpha(v))$, we successively get $\int x dF_\alpha(x) = \int \alpha(v) \mathrm{d}F_\alpha(\alpha(v)) = \int \alpha(v) dv$. So, the *average taste* can also be defined as

$$\overline{\alpha} \equiv \int_{\mathcal{V}} \alpha(v) dv$$

which will prove convenient. Note finally that the consumers have identical preferences: there is no prior 'regional preferences' (e.g. Tabuchi and Thisse, 2001) or 'local preferences' (e.g. Mossay, 2006).

[2] In the spirit of Baldwin and Harrigan (2008) $\widehat{\alpha}(v)$ can be called as the quality of variety v. We will have a closer look at this interpretation in Section 5.

Each agent maximizes his/her utility (1) subject to his/her budget constraint:

$$\int_{\mathcal{V}} p_i(v) q_i(v) \mathrm{d}v + p_i^o q_i^o \leq w_i + \overline{q}^o, \tag{2}$$

where $p_i(v)$ denotes the consumer price of variety v; w_i stands for the wage in region $i = H, F$. As is standard, we assume that consumers own a sufficiently large endowment $\overline{q}^o > 0$ of the numéraire so that they are not constrained in their consumption of differentiated varieties and spend the rest of their income on the homogenous numéraire good. As a by-product, this shifts all income effects on the homogenous good. As will become clear in the sequel, free trade in the homogenous good market leads to price equalization across regions, thus making this good a natural choice for the numéraire so that we can set $p_i^o = 1$, $i = H, F$.

We assume that all varieties are consumed. Maximizing the utility (1) subject to the budget constraint (2) yields the following first order condition:

$$\alpha(v) - (\beta - \gamma) q_i(v) - \gamma \int_{\mathcal{V}} q_i(\xi) d\xi - p_i(v) = 0 \tag{3}$$

Integrating the left hand side of this equality yields the average taste

$$\overline{\alpha} = \beta \int_{\mathcal{V}} q_i(v) \mathrm{d}v + \int_{\mathcal{V}} p_i(v) \mathrm{d}v \tag{4}$$

The last two expressions allows us to derive the individual demand for variety v as the following linear formula:

$$q_i(v) = a(v) - (b + c) p_i(v) + c\mathbb{P}_i \tag{5}$$

where

$$\mathbb{P}_i = \int_{\mathcal{V}} p_i(v) \mathrm{d}v \tag{6}$$

is the manufacturing price index in region i and where b and c are the following positive coefficients

$$b = \frac{1}{\beta} \quad \text{and} \quad c = \frac{\gamma}{\beta (\beta - \gamma)} \tag{7}$$

The parameter b measures the price sensitivity of demand and the parameter c the degree of product differentiation. In particular, when $c \to 0$ varieties are perfectly differentiated, whereas they become perfect substitutes when $c \to \infty$.

In expression (5), the function $a(v)$ is equal to

$$a(v) = \alpha(v) (b + c) - c\overline{\alpha} \tag{8}$$

and measures the *demand size* for variety v. This function is indeed equal to the consumer's demand when all prices are nil. At given prices and average taste, a change in $a(v)$ fully reflects a change in the taste $\alpha(v)$ of the variety v. Varieties for which consumers express a high taste have high demands. This is consistent with empirical works that attribute a higher quality to the goods that, for a same price, have higher demands (e.g. Hallak and Schott, 2011). For the sake of convenience, the minimum and the maximum demand size are defined as $a_l \equiv \min_v a(v) = \alpha_l(b+c)+\overline{\alpha}c$ and $a_h \equiv \max_v a(v) = \alpha_h(b+c)+\overline{\alpha}c$ while the *average demand size* can be defined as $\overline{a} \equiv \int x dF_a(x)$ or, equivalently,

$$\overline{a} \equiv \int_{\mathcal{V}} a(v)dv = \overline{\alpha}b$$

The cumulative distribution of demand size across varieties is equal to the function $F_a : [a_l, a_h] \to [0,1]$, $F_a(y)$ =Proba$[v : a(v) \leq y]$. This distribution is related the taste cumulative distribution by the relationship

$$F_\alpha(x) = F_a(x(b+c) - \overline{\alpha}c) \iff F_a(y) = F_\alpha\left(\frac{by+\overline{a}c}{b(b+c)}\right)$$

It is worth noting that the cumulative distribution of demand size F_a has an empirical content as this is the distribution that the econometrician is likely to measure. The taste cumulative distribution F_α cannot be measured.

To guarantee positive demand size $a(v) > 0$, we posit that $\alpha(v)/\overline{\alpha} > c/(b+c)$ for all v. So, the preference for the lowest quality should not be too low.

The indirect utility is computed in Appendix as it follows: $V_i = S_i + w_i + \overline{q}_i^o$ where

$$S_i = \frac{\overline{a}^2}{2b} - \overline{a}\int_{\mathcal{V}} p_i(v)\mathrm{d}v + \frac{b+c}{2}\int_{\mathcal{V}} [p_i(v)]^2\,\mathrm{d}v - \frac{c}{2}\left[\int_{\mathcal{V}} p_i(v)\mathrm{d}v\right]^2 + \frac{\mathrm{var}\,[a]}{2(b+c)} - \int_{\mathcal{V}}[a(v)-\overline{a}]p_i(v)\mathrm{d}v \quad (9)$$

If varieties have the same demand size ($a(v) = \overline{a}$), we get back to the Ottaviano *et al.'s* (2002) consumer surplus.

2.2 Price equilibrium and trade costs

Production takes place in two sectors. In the first sector, the homogenous good is produced under perfect competition using one unit of labor per unit of output. We assume that this good can be costlessly traded between regions, which implies that its price is internationally equalized and equal to wages. Normalizing wages to one we get $p_i^o = w_i = 1$ for $i = H, F$, which justifies our previous choice of this good as the numéraire.

In the second sector, called the manufacturing sector, each firm produces and sells a single differentiated manufacturing variety. Let $\mathcal{V}_i$ and n_i be the set and the mass of manufacturing firms located in region i. Naturally, we have that $n_i = \mu(\mathcal{V}_i) \equiv \int_0^1 d\mu_i(v)$ where $\mu(\mathcal{V}_i)$ is the measure of $\mathcal{V}_i$ and $\mu_i(v)$ is the measure of variety v if it is produced in region i ($\mu_H(v) + \mu_F(v) = 1$). In this section we derive the price equilibrium for a *given* location structure $(\mathcal{V}_H, \mathcal{V}_F)$ and for a given distribution of demand sizes across firms $(a(\cdot))$.

The demand for each variety in each market depends on the set of varieties produced domestically and on the set produced abroad. In accord with empirical evidences (e.g., Head and Mayer, 2000; Haskel and Wolf, 2001), we assume that product markets are segmented. Firms are hence free to set specific prices in each national market. The profit of a manufacturing firm established in region i is given by

$$\Pi_i(v) = L\theta_i p_{ii}(v) q_{ii}(v) + L\theta_j (p_{ij}(v) - \tau) q_{ij}(v) - r_i(v), \quad v \in \mathcal{V}_i \tag{10}$$

where L is the total population, θ_i is the share of population in region i, $r_i(v)$ is the remuneration of firm v's fixed factors and $q_{ij}(v)$ and $p_{ij}(v)$ is the price and demand of variety v when it is produced in region i and consumed in region j. By (5), the individual demand writes as

$$q_{ij}(v) = a(v) - (b + c) p_{ij}(v) + c\mathbb{P}_j$$

for all $i, j \in \{H, F\}$. Under monopolistic competition, firms are too small to affect the aggregate variables. So they set their prices $p_{ii}(v)$ and $p_{ij}(v)$ taking $\mathbb{P}_i, \mathbb{P}_j$ and $a(\cdot)$ as given. The optimal prices are computed as it follows:

$$p_{ii}(v) = \frac{a(v) + c\mathbb{P}_i}{2(b + c)} \quad \text{and} \quad p_{ij}(v) = p_{jj}(v) + \frac{\tau}{2} \tag{11}$$

which increases with the demand size of the variety offered and with the prices of the other varieties sold in the same market.

At the equilibrium in the product market, the firm's prices $(p_{ii}(v), p_{ij}(v))$ are consistent with aggregate prices or price indices $(\mathbb{P}_i, \mathbb{P}_j)$. The latter are successively equal to

$$\begin{aligned}
\mathbb{P}_i &= \int_0^1 p_{ii}(v) d\mu_i(v) + \int_0^1 p_{ji}(v) d\mu_j(v) \\
&= \int_0^1 p_{ii}(v) d\mu_i(v) + \int_0^1 \left(p_{ii}(v) + \frac{\tau}{2} \right) d\mu_j(v) \\
&= \int_0^1 p_{ii}(v) dv + \int_0^1 \frac{\tau}{2} d\mu_j(v) \\
&= \frac{\overline{a} + c\mathbb{P}_i}{2(b + c)} + \frac{\tau}{2} n_j
\end{aligned}$$

Solving for the fixed point yields

$$\mathbb{P}_i = \frac{\overline{a} + (b+c)\tau n_j}{2b+c}$$

so that equilibrium prices are equal to

$$p^*_{ii}(v) = \frac{1}{2}\frac{2\overline{a} + \tau n_j c}{2b+c} + \frac{a(v) - \overline{a}}{2\,(b+c)} \quad \text{and} \quad p^*_{ij}(v) = p^*_{jj}(v) + \frac{\tau}{2} \tag{12}$$

If varieties have the same specific demand sizes ($a(v) = \overline{a}$), we get back to the Ottaviano *et al.* (2002). Otherwise, the price is larger for any variety that offers a higher value to the consumer. The reader will observe that although the price of a variety depends specifically on the idiosyncratic taste for its own variety, it does not depend on where each other specific variety is produced. The price of a variety v depends only on the mass of varieties produced in each region. More formally, we mean that $p^*_{ii}(v)$ depends on (n_i, n_j) but not on the sets $(\mathcal{V}_i, \mathcal{V}_j)$. Indeed, in this linear demand model, the import price of a variety is adjusted to local price up to a constant equal to $\tau/2$, which does not depend on its demand size. The contribution of imported varieties in the local price index is therefore equal to $\tau/2$ times the mass of the importers, which is independent of any specific demand size. So, prices and price indices do not depend on the characteristics of each individual firm, but only on the mass of firms in each region. This independence of prices to the precise composition of local production turns out to be a useful property in the subsequent analysis of spatial selection.

Given the above prices, it is easy to show that production is equal to $q^*_{ii}(v) = (b+c)p^*_{ii}(v)$ and $q^*_{ij}(v) = (b+c)\left(p^*_j(v) - t/2\right)$ so that the profit of firm v located in region i can be written as

$$\Pi_i(v) = L(b+c)\left[\theta_i(p^*_{ii}(v))^2 + \theta_j\left(p^*_{jj}(v) - \frac{\tau}{2}\right)^2\right] - r_i(v) \tag{13}$$

Note that the operational profit (i.e. the squared bracket) increases in v since $p^*_{ii}(v)$ and $p^*_{jj}(v)$ increases with $\alpha(v)$. Finally to assure that trade remains feasible, we must impose that the consumption for any variety never falls to zero: $\min_{\{i,j,v\}} q^*_{ij}(v) > 0$. This is equivalent to the condition:

$$\tau < \tau^{\text{trade}} \equiv \frac{2\overline{a}}{2b+c} - \frac{\overline{a} - a_l}{b+c}$$

This is satisfied for a low enough trade cost τ and a high enough lower bounds for taste parameter α_l and/or demand size a_l.

In the next section, we analyze the equilibrium location of capital in a footloose capital model.

3 Spatial distribution of capital

In this section, we consider that capital is perfectly mobile whereas populations are not allowed to move across regions. This setup typically describes a context of international trade where capital allocates across countries. It may also refer to the allocation of investment within a country where individuals are not allowed or not used to move. For instance, the mobility of workers is constrained by internal migration restrictions in China, and by language and cultural barriers within the E.U.

As in Ottaviano and Thisse (2004), we consider a footloose capital model where consumers' location is exogenously given by the population shares (θ_i, θ_j) and where the home region has the largest population size: $\theta_H \geq \theta_F = 1 - \theta_H$. In this footloose capital model, the fixed factor is capital. To produce, each firms needs a unit of capital that is supplied by some capitalists holding a unit mass of capital. More specifically, the timing includes four stages. In the first stage, each immobile capitalist is endowed with a unit of capital that he/she invests in a variety before knowing the consumer's taste for this particular variety. Varieties are thus alike and capital is randomly spread across varieties. In the second stage, nature chooses the taste parameter of each variety according to the taste distribution F_α. Third, the capitalist sets up a firm and locates it to the region where his/her capital rent $r_i(v)$, $i \in \{H, F\}$, is the highest. Finally, labor and product markets clear. In accordance with the literature we define the location equilibrium in the manufacturing market as the distribution of firms such that product markets clear and no capital unit can earn a higher return in another location. Note that, in this model, the absence of income effect implies that capitalists consume the same set of manufacturing goods as other agents, irrespective of their actual idiosyncratic rents. The population shares of capitalists can thus simply be included in the consumers' population shares (θ_H, θ_F).

3.1 Spatial selection and location equilibrium

In equilibrium, the capital rent r_i exhausts profits so that $\Pi_i(v) = 0$. The capital rent differential between the two regions then writes as

$$\Delta r(v) \equiv r_H(v) - r_F(v) = L(b + c)\tau \left[\theta_H(p^*_{HH}(v) - \tau/4) - \theta_F(p^*_{FF}(v) - \tau/4)\right]$$

A *location equilibrium* is therefore a partition of the firms $(\mathcal{V}_H, \mathcal{V}_F)$ ($\mathcal{V}_H \cup \mathcal{V}_F = [0, 1]$ and $\mathcal{V}_H \cap \mathcal{V}_F = \varnothing$) such that capital owners do not wish to reallocate their capital. That is, it requires that $\Delta r(v) \geq 0$ if $v \in \mathcal{V}_H$ and $\Delta r(v) \leq 0$ if $v \in \mathcal{V}_F$.

Plugging equilibrium prices in the rent differential we get

$$\Delta r(v,n_H)=\frac{L\tau\left(b+c\right)}{2\left(2b+c\right)}\begin{bmatrix}\left(2b+c\right)\left(2\theta_H-1\right)\left(a(v)-\overline{a}\right)\\+\left(2\overline{a}-\tau b\right)\left(b+c\right)\left(2\theta_H-1\right)\\-c\tau\left(b+c\right)\left(n_H-1/2\right)\end{bmatrix} \tag{14}$$

This capital rent differential increases with the trade cost τ. An increase in the latter rises the incentives to locate in the region with larger population. The rent differential also increases with the demand size $a(v)$ and therefore the address v of the corresponding variety. Indeed, if $\theta > 1/2$, the *single crossing property*

$$\frac{\partial\Delta r}{\partial v}=L\tau\left(b+c\right)\left(\theta_H-1/2\right)a'(v)>0$$

holds for any variety v and any spatial distribution of firms $(\mathcal{V}_H,\mathcal{V}_F)$. In words, higher demand firms have higher profit levels and increase their profit by a larger amount when they self-select into the region with the larger population. This property guaranties that firms separate according to their demand size. It must be noted the separation of firms is obtained from the equilibrium condition because of the pro-competitive effects encompassed in the present linear demand model. By contrast, separation cannot be obtained from equilibrium condition in iso-elastic demand models because of the absence of pro-competitive effects. In the latter models, separation must be obtained using other ad-hoc, dynamic criteria (e.g. Baldwin and Okubo 2006).

At the equilibrium we can have three configurations: the capital fully flows in region H or region F, or it (unevenly) disperses across regions. More formally, we have that (i) $\Delta r(v,n_H)>0$ for all v, (ii) $\Delta r(v,n_H)<0$ for all v, or (iii) there exists a variety $\widetilde{v}\in\mathcal{V}$ such that $\Delta r(v,n_H)\geq 0$ iff $v\geq\widetilde{v}$. The variety $\widetilde{v}$ is the solution of $\Delta r(\widetilde{v},n_H)=0$ and divides the set of firms between the firms that are willing to locate either in region H or F. From (14), we therefore get the following lemma:

Lemma 1 (Spatial selection) *If $\theta_H>1/2$, $\mathcal{V}_H=\{v\mid a(v)>a(1-n_H)\}$ and $\mathcal{V}_F=\mathcal{V}\backslash\mathcal{V}_H$.*

Let us define the function

$$G(n_H)=a\left(1-n_H\right)-\overline{a}=\left(b+c\right)\left[\alpha\left(1-n_H\right)-\overline{\alpha}\right]$$

which is decreasing in n_H under our monotonicity assumption on $\alpha(v)$ and which crosses the zero axis for some $n_H\in[0,1]$. Plugging this in the capital rent differential we get

$$\Delta r^*(n_H)\equiv\frac{L\tau\left(b+c\right)}{2\left(2b+c\right)}\begin{bmatrix}\left(2b+c\right)\left(2\theta_H-1\right)G\left(n_H\right)\\+\left(2\overline{a}-\tau b\right)\left(b+c\right)\left(2\theta_H-1\right)\\-c\tau\left(b+c\right)\left(n_H-1/2\right)\end{bmatrix} \tag{15}$$

which is a decreasing function of n_H.

Given the above lemma, we can deduce that a *location equilibrium* is represented by the mass of firms $n_H^* \in [0,1]$ such that (i) $n_H^* = 1$ and $\Delta r^*(1) > 0$, (ii) $n_H^* = 0$ and $\Delta r^*(0) < 0$ or, (iii) $n_H^* \in (0,1)$ and $\Delta r^*(n_H^*) = 0$. Because Δr^* decreases with larger n_H, the location equilibrium exists and is unique. In the latter equilibrium, the return to capital is higher in the larger region. This is because lowest capital rent in the larger region ($v = 1 - n_H^*$) is just equal the highest capital rent in the other region.

Proposition 2 *In the footloose capital model ($\theta_H > 1/2$), a unique equilibrium exists where high value varieties are produced in the larger region. The return to capital is higher in the larger region.*

Note that capital has the same return before the realization of the demand for each variety. However, after the realization of the demand of each variety, the capital return varies according the nature of each variety and becomes larger in the bigger region once the more lucky capitalists relocates the production of his/her firm in the most attractive, larger market.

Finally, the spatial distribution of firms under homogenous taste and demand size is obtained by setting $G(n_H) = 0$ in the zero rent differential (15). The interior solution is equal to

$$n_H^o \equiv \frac{1}{2} + \frac{2\overline{a} - \tau b}{c\tau}\left(2\theta_H - 1\right) > \frac{1}{2} \qquad (16)$$

In this case, the larger region attracts more capital (see Ottaviano and Thisse, 2004). This however tells nothing about the idiosyncratic value of goods produces in that region. We now analyze how the taste distribution F_α impact on the spatial distribution of firms.

3.2 Taste distribution and capital allocation

We now study the impact of taste heterogeneity on capital allocation. More specifically, we want to know how capital allocates across regions when varieties become more or less heterogeneous. This investigation allows us to put into perspective the usual assumption of homogenous varieties in footloose capital models and to compare the impact of the possible assumptions on taste distributions on the economic geography (e.g. uniform, Pareto, normal...). This analysis can also be used do discuss technological progress. Indeed, if the latter enhances the value of varieties through time, it should also improve the quality and the willingness to pay for each variety and have an impact on spatial allocation of capital.

We begin by offering an intuitive explanation about the impact of the shape of the taste distribution on the spatial allocation of capital. Note at the outset that the capital rent diffential (15) increases with the function $G(n_H) = (b+c)(\alpha(1-n_H)-\overline{\alpha})$, which depends on the shape of the taste distribution, and in particular, on its skewness. To make this idea clear, Figure 1 displays two cases of taste distributions, F_α, one being skewed toward high taste varieties (left hand panel) and the other towards low taste varieties (right hand panel). Intuitively, a few firms produce the most popular varieties in the first case, whereas many firms produce those varieties in the other case. From Lemma 1, we know that the selection of firms takes place around the firm endowed with the cut-off taste parameter $\alpha(1-n_H)$. The figure shows that this cut-off taste parameter lies below the average taste parameter $\overline{\alpha}$ in the first case. As a result, the taste distribution decreases the capital rent differential and entices more capital dispersion. By contrast, in the second case, it increases the capital rent differential and entices more agglomeration. So, capital disperses (resp. agglomerates) more if the taste distribution is skewed towards high (resp. low) taste varieties. The following text formalizes this argument.

INSERT FIGURE 1 HERE

First, to determine the impact of the shape of the taste distribution on capital allocation, it is convenient to focus on Stuart and Ord's (1994) skewness coefficient, $sk \equiv \overline{\alpha} - \alpha(1/2)$, that measures the difference between the average and median taste parameters. If $sk \geq 0$, the taste distribution will be skewed toward high taste varieties; in other words, a few firms will produce the most popular varieties. Loosely speaking, a larger sk implies a fewer number of even more popular varieties. This coefficient can interestingly be used to express the function $G(n_H)$ as $[\alpha(1-n_H) - \alpha(1/2)] - sk$. Using the zero rent condition (15), it readily comes that, ceteris paribus, *an increase in the skewness towards high taste varieties, sk, decreases the number of firms and the capital invested in the larger region.* Also, setting $n_H = 1/2$ in (15), one readily checks that

$$n_H^* \geq 1/2 \iff sk \leq sk^* \equiv \frac{b+c}{2b+c}(\overline{a} - \tau b)$$

Therefore, the larger region attracts the more valuable firms and also more capital if the taste distribution is not too much skewed towards high taste varieties ($sk \leq sk^*$); that is, if the few popular varieties are not too popular. Otherwise, the smaller region attracts more capital, but the latter is invested in lower demand varieties. In this case, the ratio of capital to population in the larger country is clearly below one and there is a reversion of the home market effect. Note that $sk^* > 0$.

By contrast, if the taste distribution is too much skewed towards high taste varieties ($sk > sk^*$), there exists a small set of very popular varieties. Then, the larger region still attracts the most valuable firms but it attracts a smaller amount of them. Intuitively, a fewer number of even more popular varieties increases the average taste and demand size parameters so that less popular varieties suffer from a competitive disadvantage and are obliged to drastically lower their local prices in the larger region to remain competitive (see (12)). Less popular varieties are nevertheless able to reduce competition by relocating into the smaller market. *If the taste distribution is too much skewed towards high taste varieties, the small region does not only end up to produce the lower value goods but it also attracts a higher proportion of them.* By the same token, it thus attracts a higher proportion of capital units but the less productive ones.

Finally, note that the capitalists reallocate their capital away from the larger region if one introduces some demand heterogeneity even with an unskewed distribution. That is, $n_H^* < n_H^o$ if $sk = 0$. Indeed, since $n_H^o > 1/2$, the demand heterogeneity diminishes the capital rent differential (15) by the amount $G(n_H^o) = a(1 - n_H^o) - \overline{a} = a(1 - n_H^o) - a(1/2) < 0$. As $\Delta r^*(n_H^o) < 0$, some low demand firms prefer to relocate in the smaller region.

We summarize the above results in the following proposition:

Proposition 3 *The larger region attracts the more valuable firms and also more capital if the taste distribution is not too much skewed towards high taste varieties ($sk \leq sk^*$). Otherwise, it attracts a smaller amount of less capital that is nevertheless invested in the more valuable goods.*

The above discussion suggests that changes in the distribution of taste heterogeneity F_α may reduce or increase the capital invested in the larger region according to the skewness of the initial taste distribution. Note that changes in the distribution F_α alter both the average demand size $\overline{a}$ and the function $G(n_H)$. Therefore, the effect of taste heterogeneity can be broken down according to the changes that affect respectively $\overline{a}$ and $G(n_H)$. On the one hand, we observe that the value of $G(n_H)$ is invariant to any parallel shift in the taste distribution F_α because such a shift has the same impact on the average taste $\overline{\alpha}$ and the idiosyncratic taste parameter $\alpha(v)$. A parallel shift of F_α to the right therefore increases only the average taste $\overline{\alpha}$ and raises only the second term of the rent differential (15) which fosters the allocation of capital in the larger region.

On the other hand, let us consider an increase of the taste distribution spread around its mean. By definition, this change has no impact on the average taste parameter $\overline{\alpha}$. It however implies that a variety with a taste parameter below the average has a positive probability to worsen and a variety with a taste parameter above the average has a positive probability to improve. More simply, it implies that the taste parameter diminishes when it lies below the average taste and increases when it lies above it. In the Appendix, we show that an increase in the spread of the taste distribution reduces the number of firms in the large market if and only if $G(n_H^*) \leq 0$; that is, if $\alpha(1 - n_H^*) \leq \overline{\alpha}$. This means that the firm endowed with the average demand size is located in the larger region. Using the zero rent condition (15), the latter condition is satisfied if and only if $n_H^* \leq n_H^o$ where n_H^o is the spatial distribution of firms under homogenous taste (see (16)). This condition has the following consequence: an increase in the spread of the taste distribution reduces the number of firms in the larger region only if the spatial distribution of firms is initially more dispersed than that under homogenous taste. Otherwise, the increase in the spread induces more agglomeration in the larger region.

How does the impact of an increase in the spread of the taste distribution relate to the skewness of the taste distribution? We know from above that an increase in the skewness towards high taste varieties, sk, decreases the number of firms locating in the larger region. As a consequence, there exists a degree of skewness, sk^o (solution of $n_H^o = n_H^*$), above which firms are more (resp. less) dispersed than under no taste heterogeneity and, therefore, above which an increase in the spread of the taste distribution reduces (resp. augments) the number of firms in the large region. We also know from the argument above Proposition 3 that the capitalists reallocate their capital away from the larger region if one introduces some demand heterogeneity with an unskewed distribution ($n_H^* < n_H^o$ if $sk = 0$). This means that an increase in the spread of the taste distribution reduces the number of firms in the larger market for any unskewed distribution. As a result, it must be that $sk^o < 0$. We summarize our arguments in the following proposition.

Proposition 4 *(i) Any parallel shift of the taste distribution increases the number of firms and the capital invested in the larger region if and only if it raises the average taste parameter* $\overline{\alpha}$.
(ii) There exists a skewness of the taste distribution towards high taste varieties, $sk^o < 0$, *such that an increase in the spread of the taste distribution around its mean reduces the number of firms in the larger region if and only if* $sk \geq sk^o$.

Proof. See Appendix. ■

The proposition readily applies to a set of usual distribution functions. On the one hand, the capital invested in the larger market falls after an increase in the spread of uniform or normal distributions because those distributions are unskewed ($sk = 0$). This is also true for any symmetric distribution in the sense that $f_\alpha(\overline{\alpha} + x) = f_\alpha(\overline{\alpha} - x)$ because those are also unskewed. On the other hand, the capital invested in the large market also falls after an increase in the spread of Pareto and log normal distributions because the latter distributions are skewed toward high varieties ($sk > 0$). For those taste distributions, the introduction of heterogeneity implies a spatial selection process where high demand firms are enticed to enter the larger region and push away lower demand firms.

Finally, our analysis can be used to explain the impact of technology improvements on capital allocation. Of course, all depends on how technology affects the value of each variety. On the one hand, one may conceive that some technology improvements affect all varieties in the same way so that the taste distribution incurs a parallel shift towards higher taste parameters. For instance, in the last decades, improvements in micro-electronics have affected the quality of *all* products in *all* firms within many industries. Similarly, improvements in education are also likely to affect the quality of the workforce and thus the products across all firms. In this case, the model predicts an increase in capital investments in larger markets. So, technologies leading to common quality improvements reinforce the advantage of larger regions. On the other hand, one may conceive technological progress as a leapfrogging process where a few firms are able to implement drastic innovations and get very strong quality advantages. The recent success of Apple's products (Ipod, Iphone, ...) may be an example of such drastic quality improvement in telecommunication appliances. In such a case, technology improvements are likely to give rise to taste distributions that are skewed towards high taste varieties. Then, the model predicts that high technology firms locate in the larger market but that the latter market is more competitive and hosts a fewer number of them.

3.3 Uniform taste distribution

To clarify and extend our discussion, we study the case of a uniform taste distribution with density $f_\alpha = 1/\sigma$ where $\sigma > 0$. Under such distribution we get $\alpha(v) = \overline{\alpha} + \sigma\,(v - 1/2)$ where σ measures the 'spread' of the taste distribution ($\sigma < 2\overline{\alpha}$). This parameter takes larger values when the taste distribution spreads over wider supports $[\overline{\alpha} - \sigma/2,\, \overline{\alpha} + \sigma/2]$. The trade feasibility condition becomes $\tau < \tau^{\text{trade}} \equiv 2\overline{\alpha}b/\,(2b + c) - \sigma/2$. When $\sigma \to 0$ we return to a homogenous taste distribution.

We readily compute that $G(n_H) = \sigma(b + c)\,(1/2 - n_H)$. The spatial distribution of firms has then the following explicit expression:

$$n_H^* = \begin{cases} \widetilde{n}_H & \text{if} \quad \theta_H < \widetilde{\theta}_H^* \equiv \frac{1}{2} + \frac{1}{2}\frac{\tau c}{2b(2\overline{\alpha}-\tau)-\sigma(2b+c)} \\ 1 & \text{otherwise} \end{cases}$$

where $\widetilde{n}_H$ is the interior equilibrium spatial distribution of firms that solves $\Delta r^*(\widetilde{n}_H) = 0$ as

$$\widetilde{n}_H - 1/2 = \frac{n_H^o - 1/2}{1 + \frac{\sigma}{c\tau}\,(2\theta_H - 1)\,(2b + c)} \tag{17}$$

and where $\widetilde{\theta}_H^*$ is the threshold above which firms fully agglomerate in the larger region. On the one hand, this analysis under uniform taste distribution confirms our previous results. One indeed readily checks that the denominator in expression (17) increases with σ so that $d\widetilde{n}_H/d\sigma < 0$. *The amount of capital attracted by the larger region decreases for larger spreads of the taste distribution.* On the other hand, note that $\widetilde{\theta}_H^* = \widetilde{\theta}_H^o$ if $\sigma = 0$, that $d\widetilde{\theta}_H^*/d\sigma > 0$ so that $\widetilde{\theta}_H^* \geq \widetilde{\theta}_H^o$. So, larger spreads also reduce the set of parameters supporting full agglomeration of capital in the larger region.

The assumption of uniform taste distribution allows us to present additional results. First, we show in the Appendix that $d^2\widetilde{n}_H/d\theta_H d\sigma < 0$. So, *increases in the spread reduces the impact of region size asymmetries on the allocation of capital.* An increase in demand heterogeneity raises the value of those goods in the large region and makes competition tougher for firms selling lower value goods; low demand firms are deterred to locate there and capital moves away from the larger smaller region. Second, it can be shown that $dn_H^o/d\tau < d\widetilde{n}_H/d\tau < 0$. So, a fall in trade costs always increases the amount of capital allocated in the larger region but it increases this amount less when the spread of the taste distribution is larger. So, *stronger demand heterogeneity reduces the importance of trade costs on the unequal spatial distribution of capital across regions.*

Proposition 5 *Suppose a uniform taste distribution with spread σ. Then, an increase in the spread reduces the share of capital in the larger region. Larger spread reduces the impact of trade cost on the amount of capital allocated in the larger region.*

Up to now we have assumed that the work force was immobile. This has allowed us to study how capital investments would distribute across regions and to study how those could be altered by changes in trade costs and taste distributions. We now turn to the analysis of an economy where the work force can move across regions.

4 Spatial distribution of workers

In many contexts, workers are able move between locations. In the short run, workers have the freedom to choose their region or city of residence within the country borders. In the long run, migration policies usually allow workers and their descent to move across borders and to choose residence in the most attractive countries. The basic difference between the mobility of capital and workers is that workers bring their purchasing power with them when they choose relocate to a region, which creates the additional effect of a *demand linkage*. The new economic geography literature offers an extensive discussion of this effect in various models. We here choose to discuss the Ottaviano *et al.*'s (2002) model in which we integrate the above preferences for varieties with heterogeneous taste and demand.

In this model, the fixed factor of the firm is owned by a share of the working population that we call "skilled workers". Therefore, the unit mass of firms is owned by a unit mass of mobile skilled workers. The timing has three stages. In the first stage, each skilled worker is endowed with a variety. In the second stage taste parameter is drawn from the taste distribution F_α and each skilled worker chooses to set up and locate his/her firm to the region where his/her utility V_i, $i \in \{H, F\}$ is the highest. Finally, labor and product markets clear. In this model, each skilled worker does not only consider the rent $r_i(v)$ that he/she collects from his/her firm but also the consumer surplus he/she obtains in his/her region. In addition, we assume a mass A of immobile workers-consumers in each region. Mobile skilled workers and immobile workers consume in the region they reside so the mass of consumers in region H (resp. F) is equal to $\theta_H L = A + n_H$ (resp. $\theta_F L = A + n_F$) where the total number of individuals is equal to $L = 2A + n_H + n_F = 2A + 1$. The key feature of the core periphery model lies in the fact the skilled workers relocate their purchasing power when they move with their firms to another region.

4.1 Spatial sorting and location equilibrium

When skilled workers consider their location, they contemplate the utility differential between the two regions. Let us denote the utility differential as $\Delta V(v) = V_H(v) - V_F(v)$. A *location equilibrium* is then a partition of the skilled workers $(\mathcal{V}_H, \mathcal{V}_F)$ $(\mathcal{V}_H \cup \mathcal{V}_F = [0, 1]$, $\mathcal{V}_H \cap \mathcal{V}_F = \varnothing)$ such that skilled workers do not wish to relocate: $\Delta V(v) \geq 0$ if $v \in \mathcal{V}_H$ and $\Delta V(v) \leq 0$ if $v \in \mathcal{V}_F$.

An skilled worker's indirect utility is equal to $V_i(v) = S_i(v) + r_i(v)$ where the consumer surplus $S_i(v)$ and his/her earning $r_i(v)$ are given by expressions (9) and (13). In a specific region i, all skilled workers faces the same prices and therefore get the same consumer surplus $S_i(v) \equiv S_i$, irrespective of the variety they produce (see Appendix). The utility differential between skilled workers located within a same region i stems only from their earnings $r_i(v)$. For the same reason as in the footloose capital model, the earnings from the sales of the variety v depend on the idiosyncratic demand for this variety and on the masses of firms in each region. So, we can re-express the earnings as the function $r_i(v, n_i)$.

As a result we can apply the same spatial selection argument as in the footloose capital model. The only difference is that θ_H is now endogenous and given by $\theta_H L = A + (n_H - 1/2)$. Therefore a region that hosts more firms also hosts a larger population, which in turn entices skilled workers to sell varieties with high value products to locate there. So, we encounter three cases: the full agglomeration of skilled workers in region H or F and the (possibly uneven) dispersion of them. More formally, this implies the three following sets of conditions: first, $\Delta V(v, n_H) > 0$ for all v; second, $\Delta V(v, n_H) < 0$ for all v; and finally, if $n_H > 1/2$, there exists a skilled worker $\widetilde{v} \in [0,1]$ such that $\Delta V(v, n_H) \geq 0$ if $v \geq \widetilde{v}$. The skilled worker $\widetilde{v}$ divides the set of skilled workers between those that are willing to locate either in region H or in F. As a result the spatial sorting takes place according to the idiosyncratic demand of the variety produced by each skilled worker.

Lemma 6 (Spatial sorting) *If $\theta_H > 1/2$, $\mathcal{V}_H = \{v \mid a(v) > a(1-n_H)\}$ and $\mathcal{V}_H = \mathcal{V} \backslash \mathcal{V}_H$.*

Given this spatial sorting property, the skilled worker's utility differential can be written as a function of the mass of skilled workers located in each region $(n_H, 1-n_H)$. Whereas his/her earnings differential is equal to $\Delta r^*(v, n_H)$ as defined in expression (14), his/her consumer surplus is computed as (see Appendix)

$$\Delta S^*(n_H) \equiv S_H - S_F = \frac{\tau}{2}(2n_H - 1)(2\overline{a} - b\tau)\left(\frac{b+c}{2b+c}\right)^2 + \frac{\tau}{2}M(n_H) \tag{18}$$

where

$$M(n_H) \equiv \int_{1-n_H}^{1} (a(v) - \overline{a})\, dv = (b+c)\int_{1-n_H}^{1} (\alpha(v) - \overline{\alpha})\, dv \geq 0$$

Note that because $M'(n_H) = G(n_H)$ and because $G(n_H)$ is a decreasing function, $M(n_H)$ is concave function. More particularly, the function $M(n_H)$ firstly increases from zero at $n_H = 0$, then attains a maximum at n'_H where $G(n'_H) = 0$ and then decreases back to zero at $n_H = 1$.

The first term in expression (18) reflects the traditional demand linkage found in new economic geography. Domestic consumers indeed benefit from the agglomeration of firms in their region because they have access to more varieties at lower prices there. The second term in this expression reflects the roles of demand heterogeneity and sorting of skilled workers. *Consumers that belong to the larger region have access to the varieties they demand more because high demand firms sort out in their region.* As a result, taste heterogeneity rises the consumer surplus in that region and fosters further agglomeration.

Given the above argument, we can specify the skilled workers' utility differential by adding $\Delta S^*(n_H)$ to $\Delta r^*(n_H)$ and by using the relationship $(2\theta_H - 1)L = 2n_H - 1$. Let

$$\Phi(n_H) \equiv M(n_H) + G(n_H)(2n_H - 1)$$

which has the property $\Phi(1/2) = M(1/2) > 0$. If $n_H > 1/2$, the utility differential depends on the mass of skilled workers only, and can be written as

$$\Delta V^*(n_H) = \frac{\tau}{4}\frac{b+c}{(2b+c)^2}\{4\overline{a}(3b+2c) - \tau[2b(3b+2c) + Lc(2b+c)]\}(2n_H - 1) + \frac{\tau}{2}\Phi(n_H) \tag{19}$$

If $n_H < 1/2$, the spatial sorting takes place in the other direction and we naturally get $\Delta V^*(n_H) \equiv -\Delta V^*(1-n_H)$. Because $\lim_{\varepsilon\to 0+}\Delta V^*(1/2+\varepsilon) \propto M(1/2) > 0 > \lim_{\varepsilon\to 0+}\Delta V^*(1/2-\varepsilon) \propto -M(1/2)$, the function ΔV^* has a discontinuity at $n_H = 1/2$. Lemma 6 does not give any information about the distribution of skilled workers in the case where $\theta_H = 1/2$ and therefore $n_H = 1/2$. Thus, without loss of generality, we simply assume that the distribution of skilled workers and varieties across regions is random when $n_H = 1/2$. As a consequence, consumer surpluses are equal everywhere and utility differential is nil: $\Delta V^*(1/2) = 0$.

The above specification of utility differential allows us to become more precise about the definition of location equilibria. A *location equilibrium* is represented by the mass of skilled workers $n_H^* \in [0,1]$ such that (i) $n_H^* = 1$ and $\Delta V(1) > 0$, (ii) $n_H^* = 0$ and $\Delta V(0) < 0$, or (iii) $n_H^* \in (0,1)$ and $\Delta V(n_H^*) = 0$. The latter will be *asymptotically stable* if any small deviation from the equilibrium distribution leads back to the equilibrium distribution according to the following dynamics of skilled workers:

$$\frac{dn_H}{dt} = \begin{cases} \Delta V(n_H) & \text{if} \quad n_H \in (0,1/2)\cup(1/2,1) \\ 0 & \text{if} \quad n_H \in \{0,1/2,1\} \end{cases}$$

This proves to be true iff $d\Delta V/dn_H < 0$ at any interior equilibrium location $n_H^* \in (0,1)\backslash\{1/2\}$. Any corner location equilibrium $n_H^* \in \{0,1\}$ is also stable. Finally, the symmetric equilibrium, $n_H = 1/2$, is stable if $\lim_{\varepsilon\to 0}\Delta V(1/2+\varepsilon) - \Delta V(1/2-\varepsilon) < 0$, which is equivalent to $\lim_{\varepsilon\to 0}\Delta V(1/2+\varepsilon) < 0$.

Homogenous taste: It is instructive to begin by the description of the location equilibrium in the absence of heterogeneity. In that case, there is no sorting and we get $G(n_H) = M(n_H) = 0$ so that $\Delta V(n_H)$ is proportional to $2n_H - 1$. Therefore, firms will disperse in symmetric locations, $n_H^* = 1/2$, if $\tau > \tau^o$ where

$$\tau^o \equiv \frac{4(3b+2c)\overline{a}}{2b(3b+2c) + c(2b+c)L},$$

they fully agglomerate in one region, $n_H^* \in \{0,1\}$, if $\tau < \tau^o$, and they can allocate according to any spatial distribution if $\tau = \tau^o$. This is consistent with Ottaviano et al (2002).

This location equilibrium is illustrated in Figure 2 where each dashed (straight) line represents the skilled workers utility differential under homogenous taste. When $\tau > \tau^o$ (left hand panel), the utility gets larger in region H compared to region F when some skilled workers agglomerate in region H. This entices other skilled workers to take the same decision. When $\tau < \tau^o$ (right hand panel), the utility falls in region H compared to F when some skilled workers agglomerate in region H. As a result, those skilled workers have incentives to go back to a symmetric configuration. When $\tau = \tau^o$ (central panel), skilled workers are indifferent between locations.

INSERT FIGURE 2 HERE

Figure 2 also illustrates the impact of taste heterogeneity. Each solid curve plots the utility differential (19) under taste heterogeneity, which differs from homogenous taste utility differential only by the amount $\Phi(n_H)$. As shown in the figure, the new utility differential is larger for symmetric spatial distributions of skilled workers and smaller for spatial distributions close to full agglomeration. Intuitively, taste heterogeneity pushes the equilibrium towards a configuration between symmetry and full agglomeration. The following text formalizes this illustration. Let us first check the case of full agglomeration where $n_H = 1$.

Full agglomeration: In this case, we have that $\theta_H L = A + 1$, $G(1) = a_l - \overline{a} < 0$ and $M(1) = 0$. After some algebraic manipulations, one gets that $\Delta V(1) > 0$ if and only if

$$\tau < \tau^s \equiv \tau^o - \frac{2\left(\overline{a} - a_l\right)\left(2b + c\right)^2}{\left(b + c\right)\left[c\left(2b + c\right)L + 2b\left(3b + 2c\right)\right]} \tag{20}$$

The threshold τ^s defines the sustain point as the trade cost below which full agglomeration is sustainable. It is equal to τ^o when heterogeneity disappears ($a_l \to \overline{a}$) and decreases with any rise in heterogeneity as reflected by the term $\overline{a} - a_l$. So, for $\tau \in (\tau^s, \tau^o)$, skilled workers do not fully agglomerate under taste heterogeneity whereas they fully agglomerate under homogenous taste. Therefore *taste heterogeneity does not make full agglomeration more likely.*

Symmetric and interior equilibria: Let us now discuss the case of interior equilibria where $\Delta V(n_H^*) = 0$. In the presence of heterogeneity, the symmetric distribution of skilled workers is an equilibrium since $\Delta V(1/2) = 0$. It is however not stable because $\lim_{\varepsilon \to 0} \Delta V(1/2 + \varepsilon) \propto M(1/2) > 0$. If skilled workers locate symmetrically, the skilled workers producing highly demanded goods have a *common* incentive to sort out in one region and thereby to attract more consumers there. A small perturbation in the distribution of skilled workers triggers this effect and some skilled workers start agglomerating.

The number of interior equilibria depends on the properties of the function $\Phi(n_H)$. Notice that the latter function is continuous on $(1/2, 1]$, has a positive value at $n_H \to 1/2$ and is equal to the negative value $a_l - \overline{a}$ at $n_H = 1$. Its shape depends on the taste distribution. As a consequence, the number of roots of $\Delta V(n_H)$ and the number of equilibria also depend on the properties of this distribution. In the Appendix we show that the function $\Phi(n_H)$ accepts one and only one root on the interval $(1/2, 1]$ if

$$5a' > a'' \tag{21}$$

This sufficient condition implies that taste function α is not a too convex function or equivalently that the taste cumulative distribution F_α is not too concave. This condition applies for uniform and Pareto taste distributions (see Appendix). Let $\widetilde{n}$ be the solution of (19). Then, we derive the following Proposition.

Proposition 7 *Under Condition (21), there exist two sets of stable equilibria: full agglomeration (either $n_H^* = 1$ or $n_F^* = 1$) if $\tau < \tau^s$; and asymmetric dispersion (either $n_H^* = \widetilde{n}$ or $n_F^* = \widetilde{n} \in (1/2, 1)$) otherwise. In the latter equilibrium, one region hosts a larger group of skilled workers who produce the more demanded varieties.*

Proof. See Appendix. ■

The traditional view in new economic geography emphasizes the role of demand linkages which entices skilled workers to agglomerate in the larger region and makes the larger region host the production of a wider range of varieties. When consumer's taste is heterogenous, demand linkages combine with spatial sorting so that the skilled workers producing the most demanded varieties are enticed locate together in the larger, more attractive market and the skilled worker producing the least demanded varieties are enticed to avoid co-agglomeration with high-demand firms. In other words, demand heterogeneity is a factor working against a perfectly uneven spatial distribution of firms because low demand firms want to avoid the high demand firms that agglomerate in the larger market. Taste heterogeneity is also a factor working against a perfectly even spatial distribution of firms for the same reason: low demand firms prefer to avoid the high demand firms and concentrate in the less attractive, smaller market.

4.2 Trade costs

Given the above analysis, it is easy to discuss the impact of trade costs on the location of firms. We know that, when $\tau \in (\tau^s, \tau^o)$, skilled workers fully agglomerate under homogenous taste whereas they fully agglomerate under taste heterogeneity. Also, when $\tau \geq \tau^o$, firms evenly disperse under homogenous taste but never do so under taste heterogeneity. So, the equilibrium spatial distribution of firms implies less dispersion than under taste heterogeneity if τ is large enough and less agglomeration if τ is small enough.

In addition, it interesting to study the impact of a fall in trade costs on the spatial equilibrium pattern. In particular, it is known that the globalization process in the last century has triggered a fall in trade costs and trade barriers τ. Under homogenous taste for manufacturing varieties, a fall in trade costs around τ^o dramatically alters the location pattern of firms from even dispersion to full agglomeration. This is because expression (19) is multiplicative of $(2n_H - 1)$ when $\Phi = 0$. However, taste heterogeneity adds the term Φ in the expression (19) that is not multiplicative of $(2n_H - 1)$. As a result, there is no dramatic changes in the location of firms. This is shown in Figure 3 that depicts the graph of the spatial equilibrium with respect to trade costs. The equilibrium number of firms n_H under heterogeneity is represented by the solid lines and that under homogenous taste by the dashed lines.

INSERT FIGURE 3 HERE

We summarize these results in the following proposition:

Proposition 8 *Suppose that the location equilibrium is unique. Then,*
(i) as trade cost falls from $\tau = \tau^{trade}$ to 0, the equilibrium spatial distribution of skilled workers continuously moves from asymmetric dispersion to full agglomeration.
(ii) Compared to the case of homogenous taste, the equilibrium spatial distribution of skilled workers under taste heterogeneity involves less dispersion if $\tau > \tau^o$ and less agglomeration if $\tau \in (\tau^s, \tau^o)$.

We finally study the impact of changes in the distribution of taste across varieties.

4.3 Taste distribution and skilled workers' location

We first discuss the impact of a parallel shift of the taste distribution. Suppose that this shift increases all taste parameter $\alpha(v)$ and demand size $a(v)$ by a same amount. Since this shift does not alter the difference between idiosyncratic and average demand sizes, $a(v) - \overline{a}$, it has no impact on both the functions $G(n_H)$ and $\Phi(n_H)$. As a consequence, the parallel shift in the taste distribution raises only in the term $\overline{a}$ in the curly bracket of skilled worker's utility differential (19). One can readily check that this has two consequences. On the one hand, the parallel shift in the taste distribution raises the threshold τ^s so that full agglomeration is sustained for a larger set of trade costs. On the other hand, it increases the interior equilibrium number of skilled workers in the larger region ($n_H^* = \widetilde{n}$). Indeed, at this equilibrium, $dn_H^*/d\overline{a} = -\left[\partial\Delta V/\partial\overline{a}\right] / \left[\partial\Delta V/\partial n_H\right]$ is positive because the stability condition of this interior equilibrium implies $\partial\Delta V/\partial n_H < 0$ and because the parallel shift in the taste distribution implies $\partial\Delta V/\partial\overline{a} > 0$ (holding $\Phi(n_H)$ unchanged).

Proposition 9 *A parallel shift of taste distribution fosters the agglomeration of skilled workers if and only if the average demand size increases.*

Proof. See Appendix. ■

As in the previous model, we can study the impact of an increase in the spread of the taste distribution around its mean. The analysis unfortunately turns out to be more difficult because the term $\Phi(n_H)$ in the skilled worker's utility differential (19) is not a monotone function. Two cases are easy to analyze: full agglomeration and nearly symmetric dispersion. First, because the sustain point τ^s increases with larger difference $\overline{a} - a_l$ in expression (20), we can deduce that an increase in the spread diminishes τ^s and shrinks the set of parameters for which skilled workers fully agglomerate in a region. Therefore, *an increase in the spread of the taste distribution makes full agglomeration less likely.* This is because the skilled worker with lowest demand variety has a strong incentive to locate away from the skilled workers producing the best varieties. By doing so, she prefers the higher earnings she obtains in the less competitive peripheral region to the better access she could have to the more numerous and higher demand varieties in the core region. An increase in the spread raises her competitive disadvantage and makes her less likely to agglomerate with the other mobile workers.

The natural extension of the last property would suggest that an increase in the spread of the taste distribution around its mean should also make a nearly symmetric distribution of skilled workers more likely. As in the footloose capital model, heterogeneity would fosters firms' dispersion. However, this extension turns out to be false. In the Appendix, we show that such an increase in the spread reduces symmetry if the initial spatial equilibrium is sufficiently near to symmetry. The intuition is that, when skilled workers disperse, the intensity of competition is not much weaker in the smaller region and is not significantly altered by the increase in the spread of the taste distribution around its mean. However, such an increase raises the value of higher demanded varieties that are produced in the larger region and augments the consumption surplus there. So, in a nearly symmetry spatial distribution, a larger spread of the taste distribution results in a demand effect that entices mobile workers to locate near the more valuable varieties.

Proposition 10 An increase in the spread of the taste distribution around its mean makes both a full agglomeration of skilled workers and a near symmetric dispersion of skilled workers less likely.

Proof. See Appendix. ■

In the present model, the taste heterogeneity generates a dispersion or agglomeration force that depends the initial location of mobile workers. More precisely, a larger spread in this distribution implies a relocation of skilled workers towards the *partial agglomeration* pattern. This point is clarified in the case of a uniform taste distribution, which we explore below.

As in the previous model, this analysis can be used to explain the impact of technology improvements on the spatial distribution of workers. We may conceive that technology improvements affect all varieties in the same way (i.e. parallel shift in taste distribution). Then, this model predicts more agglomeration in some specific locales, a process that can be interpreted as the urbanization in some megapolises. We may also conceive technological progress as a leapfrogging process where a few firms are able to implement drastic innovations and get very strong quality advantages (i.e. an increase in the spread of the taste distribution). In this case, the models predicts the coexistence of dominating and dominated regions or cities, in which dominating ones are larger and produce higher value goods and dominated ones host the firms producing the lower value goods.

4.4 Uniform taste distribution

Let us consider again the case of a uniform taste distribution with density $f_\alpha = 1/\sigma$ so that $\alpha(v) = \overline{\alpha} + \sigma\,(v - 1/2)$ where σ measures the 'spread' of the taste distribution and σ^{-1} its density. Then, one computes that $M(n_H) = \frac{1}{2}\sigma(b+c)n_H\,(1-n_H)$ so that

$$\Phi(n_H) = \frac{1}{2}\sigma\,(b+c)\left(5n_H - 5n_H^2 - 1\right) \tag{22}$$

Note that this polynomial has a unique root $n_H = \frac{1}{2} + \sqrt{\frac{1}{20}} \simeq 0.72$ on the interval $(1/2, 1]$ and takes positive values on $(0.5, 0.72]$ and negative values on $(0.72, 1]$. The value of this polynomial is multiplied by the spread of the taste distribution σ. So, an increase in σ raises the effect of taste heterogeneity in expression (19). Hence, an increase in the spread of this distribution increases the number of skilled workers and varieties in the larger region (up to $n_H^* = 0.72$) if $\tau > \tau^o$ but reduces it (down to $n_H^* = 0.72$)

if $\tau^s < \tau < \tau^o$. In any case, *a larger spread in this distribution implies a relocation of skilled workers towards the partial agglomeration configuration where* $n_H^* = 0.72$. This corresponds to the spatial distribution of firms when the trade cost is just equal to τ^o.

To complete our analysis we compute the interior equilibrium $\widetilde{n}$ as the solution of $\Delta V(\widetilde{n}) = 0$. That is,

$$\widetilde{n} = \frac{1}{2} - x + \sqrt{\frac{1}{20} + x^2} \quad \text{where} \quad x = \frac{\tau - \tau^o}{\sigma} \frac{2b\,(3b + 2c) + c\,(2b + c)\,L}{5\,(2b + c)^2}$$

and where $\widetilde{n}$ decreases in x. Hence, as made clear above, the number of firms in the larger region rises with an increase of the spread σ if and only if $\tau > \tau^o$. Also, the number of firms in the larger region rises as trade costs τ fall, which confirms traditional results of economic geography.

Proposition 11 *Suppose a uniform taste distribution. Then, (i) a larger spread in this distribution implies a relocation of skilled workers towards the partial agglomeration configuration where* $n_H^* = 0.72$. *(ii) The number of firms in the larger region rises as trade costs* τ *fall.*

5 Costly quality

In the above model, consumers have varying willingness to pay for the varieties whereas firms incur the same variable cost to produce each of them. In reality, more valuable varieties often offer more characteristics and have stronger technological and/labor content than less valuable ones. So, more valuable varieties cost more to firms and can become less attractive to produce. One may wonder whether the combination of heterogenous taste and cost still yields the same qualitative results as in the previous section. Towards this aim, we focus on the particular case where the preference for and the cost of varieties are positively correlated.

For the sake of the argument, let the taste parameter $\alpha(v)$ represents the (exogenous) number of characteristics embedded in a specific variety v. As in Stockey (1991), consumers demand more the products with higher $\alpha(v)$ because the latter offer a larger spectrum of characteristics. For simplicity, let $m_o \in [0, 1)$ be the production cost of a single characteristic. So, the marginal cost of producing variety v is equal to $m(v) = m_o \alpha(v)$. Under this specification, one can compute that the profit maximizing price of a variety v (expression (11)) is augmented by the constant $m(v)/2$ and each price indices $\mathbb{P}_i$ by the constant $\alpha m_o(b + c)/(2b + c)$. In the footloose capital model, one computes the rent differential for the firm producing variety v as

$$\Delta r^*(v, n_H) = \frac{L\tau\,(b+c)}{2\,(2b+c)} \left[\begin{array}{c} (2b+c)\,(2\theta_H - 1)\,(a(v) - \overline{a})\,(1 - m_o) \\ +\,(2\theta_H - 1)\,(b+c)\,(2\overline{a}\,(1 - m_o) - b\tau) \\ -\tau c\,(b+c)\,(n_H - 1/2) \end{array} \right]$$

This expression very similar to (15) and simplifies to it when $m_o = 0$. It is obvious that firms' self-selection takes place in the same way as before even if higher value products are more costly to produce. Results are thus qualitatively the same. The cost of a characteristic m_o has nevertheless the following additional impact on location incentives. As shown in the above expression, each term related the demand sizes $\overline{a}$ and $a(v)$ are deflated by $1 - m_o$. So, a higher cost m_o reduces both the idiosyncratic advantage of each capitalist (first term in the square bracket) as well the aggregate advantage to locate in this market (second term). This is because a rise in the cost of a characteristic m_o implies a less than proportionate increase in firms' prices and makes competition tougher. Firms therefore have higher incentives to disperse. In this case, location equilibrium n_H^* is equal to the one we analyzed earlier where we now replace τ by $\tau/(1 - m_o)$.

In the core periphery model, location equilibrium is determined by the above rent differential plus the consumer surplus differential. As before, the skilled worker's consumer surplus does not depend on the idiosyncratic taste of his/her variety. So, the spatial sorting of skilled workers is again given by the above rent differential. The consumer surplus differential can then be computed as

$$S_H - S_F = \frac{\tau}{2}\,(2n_H - 1)\,[2\overline{a}\,(1 - m_o) - b\tau] \left(\frac{b+c}{2b+c}\right)^2 + \frac{\tau}{2}\,(1 - m_o)\,M(n_H)$$

This expression similar to (18) and reduces to it when $m_o = 0$. The cost m_o introduces two additional effects. On the one hand, it reduces the aggregate economic value of goods and therefore the consumer surplus differential (see the term $\overline{a}\,(1 - m_o)$). On the other hand it also reduces the consumer's benefit of having direct access to the more valuable varieties in their own region (see last term) because their surplus from those varieties is smaller. Note that the demand sizes in the consumer surplus differential are expressed as $\overline{a}\,(1 - m_o)$ and $(1 - m_o)\,M(n_H) = \int_{1-n_H}^{1} (1 - m_o)\,(a(v) - \overline{a})\,dv$. Therefore the equilibrium n_H^* of the core periphery model remains the same as the one we analyzed earlier where we also replace τ by $\tau/(1 - m_o)$. Skilled workers have higher incentives to disperse for larger m_o.

6 Conclusion

Product quality is an important issue in the business and trade literature. Business studies have emphasized the role of product quality in the process of firms' clustering. Past trade literature has studied the relationship between trade patterns and the quality of imports. A recent literature is reviving this research agenda in the context of the heterogeneity of firms' product quality or demand. The present paper integrates the two perspectives in two regional economic models where either capital or workers move across regions. More particularly, the paper investigates the firms' location in two new economic geography models where consumers value differently the product variety produced by each firm. In contrast with the trade literature on quality (and cost) heterogeneity, our paper stresses the role of spatial selection and firms' location rather than the role of export strategy and the resulting trade pattern. In our setting, the large markets attract the firms producing the varieties that consumers find more valuable. The resulting tougher competition in those markets forces the firms with lower quality or lower demand products to locate in smaller markets.

We obtain several interesting results. We show that firms selling the higher added value goods sort out into the region hosting the largest number of consumers. Larger regions thus get better access to the products that consumers value and demand more highly. We also show that the effect of spatial selection on firms' spatial distribution crucially depends on the properties of the taste distribution across varieties. For strong skewness in taste and demand distributions, only a smaller number of (high quality) firms survive in the larger region. This reverses the traditional home market effect argument, according to which larger markets get higher level of capital investment per capita. Finally, we show that taste heterogeneity smooths the agglomeration patterns but that it should be considered neither as a dispersion force nor as an agglomeration force. Indeed, the introduction of taste heterogeneity makes an initially dispersed economy less dispersed and an initially agglomerated economy less agglomerated.

The present model can be extended in several ways. For instance, cost heterogeneity and beachhead type of export cost could be added to our taste heterogeneity model. The relationship between export behavior and quality could be carefully explored. Also, empirical tests of our theoretical results could be performed with trade data or regional data on prices or unit values.

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Appendix

Consumer surplus

Consumer surplus: The consumer surplus in region i is equal to

$$S_i = \int_0^1 \alpha(v) q_i(v)\mathrm{d}v - \frac{\beta-\gamma}{2}\int_0^1 [q_i(v)]^2\mathrm{d}v - \frac{\gamma}{2}\left[\int_0^1 q_i(v)\mathrm{d}v\right]^2 - \int_0^1 p_i(v)q_i(v)\mathrm{d}v$$

This can be written as

$$S_i = \frac{1}{2}\int_0^1 q_i(v)\left\{2\alpha(v) - (\beta-\gamma)\, q_i(v) - \gamma\int_0^1 q_i(\xi)\mathrm{d}\xi - 2p_i(v)\right\}\mathrm{d}v$$

where, using the first order condition of the consumer decision (3) we successively get

$$\begin{aligned} S_i &= \frac{1}{2}\int_0^1 q_i(v)\left\{\alpha(v) - p_i(v)\right\} dv \\ &= \frac{1}{2(\beta-\gamma)}\int_0^1 \left\{\alpha(v) - \gamma\int_0^1 q_i(\xi)d\xi - p_i(v)\right\}\left\{\alpha(v) - p_i(v)\right\}\mathrm{d}v \end{aligned}$$

Substituting the integral $\int_0^1 q_i(\xi)d\xi$ by (4) in the last expression and expanding it, we obtain the following expression on consumer surplus as a function of prices only:

$$2(\beta-\gamma)\, S_i = \int_0^1 \alpha(v)^2\mathrm{d}v - 2\int_0^1 \alpha(v)p_i(v)\mathrm{d}v + \int_0^1 p_i(v)^2\mathrm{d}v - \frac{\gamma}{\beta}(\overline{\alpha} - \mathbb{P}_i)^2$$

We now substitute β by $1/b$ and γ by $c/\left[b\,(b+c)\right]$ and we use the inverse of equality $a(v) = \alpha(v)\,(b+c) - c\alpha$ and we use the definitions of the means, $\overline{a} = \overline{\alpha}b$, and the variance, $\mathrm{var}[a(v)] \equiv \int_0^1 \left(a(v)^2 - \overline{a}^2\right) dv = (b+c)^2 \int_0^1 (\alpha(v)^2 - \overline{\alpha}^2)\mathrm{d}v$ to get

$$S_i = \frac{\mathrm{var}\left[a(v)\right]}{2(b+c)} - \int_0^1 [a(v)p_i(v) - \overline{a}\mathbb{P}_i]\mathrm{d}v + \frac{\overline{a}^2}{2b} - \overline{a}\mathbb{P}_i + \frac{b+c}{2}\int_0^1 p_i(v)^2\mathrm{d}v - \frac{c}{2}\mathbb{P}_i^2$$

Because $\int_0^1 [a(v)p_i(v) - \overline{a}\mathbb{P}_i]\mathrm{d}v = \int_0^1 [a(v) - \overline{a}]p_i(v)\mathrm{d}v$, we can write

$$S_i = \frac{\overline{a}^2}{2b} - \overline{a}\mathbb{P}_i + \frac{b+c}{2}\int_0^1 p_i(v)^2\mathrm{d}v - \frac{c}{2}\mathbb{P}_i^2 - \int_0^1 [a(v) - \overline{a}]p_i(v)\mathrm{d}v + \frac{\mathrm{var}\,[a(v)]}{2\,(b+c)} \tag{23}$$

Consumer surplus differential: We now derive the surplus differential $S_H(v) - S_F(v)$. One can write the consumer surplus (23) for the case of two regions as

$$S_i(v) = \begin{cases} \frac{\overline{a}^2}{2b} + \frac{\mathrm{var}[a]}{2(b+c)} \\ -\overline{a}\left[\int_{\mathcal{V}_i} p_{ii}(\xi)\mathrm{d}\mu_i(\xi) + \int_{\mathcal{V}_j} p_{ji}(\xi)\mathrm{d}\mu_j(\xi)\right] \\ +\frac{b+c}{2}\left\{\int_{\mathcal{V}_i} [p_{ii}(\xi)]^2\,\mathrm{d}\mu_i(\xi) + \int_{\mathcal{V}_j} [p_{ji}(\xi)]^2\,\mathrm{d}\mu_j(\xi)\right\} \\ -\frac{c}{2}\left[\int_{\mathcal{V}_i} p_{ii}(\xi)\mathrm{d}\mu_i(\xi) + \int_{\mathcal{V}_j} p_{ji}(\xi)\mathrm{d}\mu_j(\xi)\right]^2 \\ -\left[\int_{\mathcal{V}_i} [a(\xi) - \overline{a}]p_{ii}(\xi)\mathrm{d}\mu_i(\xi) + \int_{\mathcal{V}_j} [a(\xi) - \overline{a}]p_{ji}(\xi)\mathrm{d}\mu_j(\xi)\right] \end{cases}$$

or equivalently

$$S_i(v) = \begin{cases} \frac{\overline{a}^2}{2b} + \frac{\mathrm{var}[a]}{2(b+c)} \\ -\overline{a}\mathbb{P}_i \\ +\frac{b+c}{2}\left\{\int_{\mathcal{V}_i} [p_{ii}(\xi)]^2\,\mathrm{d}\mu_i(\xi) + \int_{\mathcal{V}_j} [p_{ji}(\xi)]^2\,\mathrm{d}\mu_j(\xi)\right\} \\ -\frac{c}{2}\mathbb{P}_i^2 \\ -\left[\int_{\mathcal{V}_i} [a(\xi) - \overline{a}]p_{ii}(\xi)\mathrm{d}\mu_i(\xi) + \int_{\mathcal{V}_j} [a(\xi) - \overline{a}]p_{ji}(\xi)\mathrm{d}\mu_j(\xi)\right] \end{cases} \tag{24}$$

where $\mu_i(\xi)$ is the measure of variety ξ on the set $\mathcal{V}_i$ and where $\mathbb{P}_i = \int_{\mathcal{V}_i} p_{ii}(\xi)\mathrm{d}\mu_i(\xi) + \int_{\mathcal{V}_j} p_{ji}(\xi)\mathrm{d}\mu_j(\xi)$. This expression is independent of the variety produced by the skilled worker, ξ, so that his/her surplus and surplus differential are independent of ξ: $S_i(v) \equiv S_i$ and $S_H(v) - S_F(v) \equiv S_H - S_F$. We successively derive the surplus differential $S_H - S_F$ for each term of expression (24).

Since the first term in expression (24) is a constant it does not impact on the surplus differential. Using (6) and (12), the sum of the second term and fourth terms in expression (24) yields the following contribution of to the surplus differential $S_H - S_F$:

$$\begin{aligned} -\left(\overline{a}\mathbb{P}_H + \frac{c}{2}\mathbb{P}_H^2\right) + \left(\overline{a}\mathbb{P}_F + \frac{c}{2}\mathbb{P}_F^2\right) &= -\left(\mathbb{P}_H - \mathbb{P}_F\right)\left(\overline{a} + \frac{c}{2}\left(\mathbb{P}_H + \mathbb{P}_F\right)\right) \\ &= -\frac{(b+c)\tau\left(n_F - n_H\right)}{2b+c}\left(\overline{a} + \frac{c}{2}\frac{2\overline{a} + (b+c)\tau}{2b+c}\right) \\ &= \tau\left(n_H - n_F\right)\frac{\left(4\overline{a} + c\tau\right)\left(b+c\right)^2}{2\left(2b+c\right)^2} \end{aligned}$$

Third, the last term in the square bracket of expression (24) writes as

$$\int_{\mathcal{V}_i} [a(\xi) - \overline{a}]\left[p_{ii}^o + \frac{a(v) - \overline{a}}{2\,(b+c)}\right]\mathrm{d}\mu_i(\xi) + \int_{\mathcal{V}_j} [a(\xi) - \overline{a}]\left[p_{ii}^o + \frac{a(v) - \overline{a}}{2\,(b+c)} + \frac{\tau}{2}\right]\mathrm{d}\mu_j(\xi)$$

where we have defined

$$p_{ii}^o \equiv \frac{1}{2}\frac{2\overline{a}+\tau n_j c}{2b+c}$$

This can be re-written as

$$p_{ii}^o \int_{\mathcal{V}} \left(a(\xi)-\overline{a}\right) \mathrm{d}\mu(\xi) + \frac{1}{2\left(b+c\right)} \int_{\mathcal{V}} [a(\xi)-\overline{a}]^2 \mathrm{d}\mu(\xi) + \frac{\tau}{2} \int_{\mathcal{V}_j} \left(a(\xi)-\overline{a}\right) \mathrm{d}\mu_j(\xi)$$

where $\mu(\xi)$ is the measure of variety ξ on the set $\mathcal{V}$ which is independent of ξ. The first term of this expression is nil and the second term is a constant. So, the contribution of this term in the surplus differential $S_H - S_F$ (which includes the minus sign in front of this square bracket) is simply equal to

$$\tau M(n_H)$$

where

$$M(n_H) = \frac{1}{2}\int_{\mathcal{V}_H} \left(a(\xi)-\overline{a}\right) d\mu_H(\xi) - \frac{1}{2}\int_{\mathcal{V}_F} \left(a(\xi)-\overline{a}\right) d\mu_F(\xi)$$

Given that best highest demand varieties sort in region H, $\mathcal{V}_H = \{v : a(v) > a(1-n_H)\} = (1-n_H, 1]$, we can sequentially write

$$\begin{aligned} M(n_H) &= \frac{1}{2}\int_{1-n_H}^{1} \left(a(\xi)-\overline{a}\right) d\xi - \frac{1}{2}\int_{0}^{1-n_H} \left(a(\xi)-\overline{a}\right) d\xi \\ &= \int_{1-n_H}^{1} \left(a(\xi)-\overline{a}\right) d\xi - \underbrace{\frac{1}{2}\int_{0}^{1} \left(a(\xi)-\overline{a}\right) d\xi}_{0} \\ &= \int_{1-n_H}^{1} \left(a(\xi)-\overline{a}\right) d\xi \end{aligned}$$

Finally, the curly bracket in the third term of expression (24) can be written as

$$\begin{aligned} \int_{\mathcal{V}_i} [p_{ii}(\xi)]^2 \,\mathrm{d}\mu_i(\xi) + \int_{\mathcal{V}_j} [p_{ji}(\xi)]^2 \,\mathrm{d}\mu_j(\xi) &= \int_{\mathcal{V}_i} \left[p_{ii}^o + \frac{a(\xi)-\overline{a}}{2\left(b+c\right)}\right]^2 \mathrm{d}\mu_i(\xi) \\ &+ \int_{\mathcal{V}_j} \left[p_{ii}^o + \frac{a(\xi)-\overline{a}}{2\left(b+c\right)} + \frac{\tau}{2}\right]^2 \mathrm{d}\mu_j(\xi) \\ &= \int_{\mathcal{V}} \left[\left(p_{ii}^o\right)^2 + 2p_{ii}^o \frac{a(\xi)-\overline{a}}{2\left(b+c\right)} + \left(\frac{a(\xi)-\overline{a}}{2\left(b+c\right)}\right)^2\right] \mathrm{d}\mu(\xi) \\ &+ \int_{\mathcal{V}_j} \left[\tau p_{ii}^o + \tau\frac{a(\xi)-\overline{a}}{2\left(b+c\right)} + \frac{\tau^2}{4}\right] \mathrm{d}\mu_j(\xi) \end{aligned}$$

This expression simplifies to

$$\left(p_{ii}^o\right)^2 + 0 + \frac{\mathrm{var}[a]}{\left[2\left(b+c\right)\right]^2} + \tau p_{ii}^o n_j + \frac{\tau}{2\left(b+c\right)} \int_{\mathcal{V}_j} \left(\widehat{\overline{a}}(\xi) - a\right) \mathrm{d}\mu_j(\xi) + \frac{\tau^2}{4} n_j$$

So, we can write the difference of the curly bracket in the third term of (24) as

$$\left(p^o_{HH}\right)^2 - \left(p^o_{FF}\right)^2 + \tau\left(p^o_{HH}n_F - p^o_{FF}n_H\right) - \frac{\tau}{b+c}M(n_H) + \frac{\tau^2}{4}\left(n_F - n_H\right)$$

After plugging the values of (p^o_{HH}, p^o_{FF}) we get

$$\tau\left(n_F - n_H\right)(b+c)\frac{2\overline{a} + \tau(b+c)}{(2b+c)^2} - \frac{\tau}{b+c}M(n_H)$$

The contribution of this term in the surplus differential $S_H - S_F$ is equal to this last expression times $(b+c)/2$.

Adding up those terms we get

$$S_H - S_F = \frac{1}{2}\tau\left(n_H - n_F\right)(2\overline{a} - b\tau)\left(\frac{b+c}{2b+c}\right)^2 + \frac{1}{2}\tau M(n_H) \tag{25}$$

Proof of Proposition 5

(i) This part is proved in the text.

(ii) Suppose two taste cumulative distributions F^1_α and F^2_α so that $F^2_\alpha(x) \equiv F^1_\alpha(x - \delta)\ \forall x \in [\alpha_l, \alpha_h]$ and $\delta > 0$. It naturally comes that $\alpha_2(v) = \alpha_1(v) - \delta$ and $\overline{\alpha}_2 = \overline{\alpha}_1 - \delta$ as well as $\overline{\alpha}_2 = \overline{\alpha}_1 - \delta b$. This implies that $G_1(n^*_H) = (b+c)\left[\alpha_1(1-n^*_H) - \overline{\alpha}_1\right]$ is equal to $G_2(n^*_H) = (b+c)\left[\alpha_2(1-n^*_H) - \overline{\alpha}_2\right]$. So, this shift has only an impact on the second line of the rent differential (15).

(iii) Suppose two taste cumulative distributions F^1_α and F^2_α so that a variety with a taste parameter below the average has a positive probability to worsen (i.e. $F^1_\alpha(x) \le F^2_\alpha(x)$ if $x < \overline{\alpha} \equiv \overline{\alpha}_1 = \overline{\alpha}_2$) and a variety with a taste parameter above the average has a positive probability to improve (i.e. $1 - F^1_\alpha(x) < 1 - F^2_\alpha(x) \iff F^1_\alpha(x) > F^2_\alpha(x)$ if $x > \overline{a}$). This property implies that $\alpha_1(v) \ge \alpha_2(v) \iff \alpha_1(v) \le \overline{\alpha} \iff \alpha_1(v) \le \alpha_1(1/2) + sk_1$. Let n^1_H and n^2_H be the location equilibrium under each taste distribution function.

We first prove that $sk_1 \ge 0$ implies the condition $n^1_H - n^2_H \ge 0$ for any distributions F^1_α and F^2_α that are close enough. Suppose that F^1_α and F^2_α are close enough so that there exists a small enough $\varepsilon > 0$ such that $|n^1_H - n^2_H| < \varepsilon$. By (15), the equilibrium conditions $\Delta r^*(n^1_H) = 0$ and $\Delta r^*(n^2_H) = 0$ imply that $n^1_H - n^2_H = k\left[G_1\left(n^1_H\right) - G_2\left(n^2_H\right)\right]$ where $k = (2b+c)(2\theta_H - 1)/\left[c\tau(b+c)\right] > 0$. Because the function G_2 is continuous, there exists a positive scalar δ such that $G_2\left(n^1_H\right) - \delta < G_2\left(n^2_H\right) < G_2\left(n^1_H\right) + \delta$. Therefore, $k\left[G_1\left(n^1_H\right) - G_2\left(n^2_H\right) - \delta\right] < n^1_H - n^2_H < k\left[G_1\left(n^1_H\right) - G_2\left(n^2_H\right) + \delta\right]$. When $\varepsilon \to 0$, we get $\delta \to 0$ so that $\delta < k\left|G_1\left(n^1_H\right) - G_2\left(n^1_H\right)\right|$ and that $n^1_H - n^2_H \ge 0$ if and only if $G_1\left(n^1_H\right) - G_2\left(n^1_H\right) \ge 0$.

The latter condition is true if $\alpha_1(1-n_H^1) \geq \alpha_2(1-n_H^1)$, or equivalently, if $\alpha_1(1-n_H^1) \leq \overline{\alpha}$. Finally, the property then applies for two distant distributions F_α^1 and F_α^2 by constructing a series of distributions $\{F_\alpha^z\}$ where $z = 1 + l/N$ and $l \in \{1, ..., N\}$ such that $\overline{\alpha}_z = \overline{\alpha}$, $\alpha_z(1-n_H^z) \leq \overline{\alpha}$ and that two consecutive distributions F_α^z and $F_\alpha^{z'}$ are close enough. Applying the above argument, we get $n_H^1 - n_H^2 \geq 0$ if $\alpha_1(1-n_H^1) \leq \overline{\alpha}$ and $\alpha_2(1-n_H^2) \leq \overline{\alpha}$, and conversely, $n_H^1 - n_H^2 < 0$ if $\alpha_1(1-n_H^1) > \overline{\alpha}$ and $\alpha_2(1-n_H^2) > \overline{\alpha}$.

Proof of Proposition 5

First one can show that

$$\frac{d^2 \widetilde{n}_H}{d\sigma d\theta_H} < 0 \iff \frac{d^2 \ln(\widetilde{n}_H - 1/2)}{d\sigma d(2\theta_H - 1)} < 0 \iff -\frac{c\tau (2b+c)}{(c\tau + \sigma(2b+c)(2\theta_H - 1))^2} < 0$$

which is true. Second, we have

$$\begin{aligned} \frac{d \ln(\widetilde{n}_H - 1/2)}{d\tau} &= -\frac{2\overline{a}c + (2\theta_H - 1)\sigma b (2b+c)}{(2\overline{a} - b\tau)[c\tau + (2\theta_H - 1)\sigma(2b+c)]} < 0 \\ \frac{d^2 \ln(\widetilde{n}_H - 1/2)}{d\sigma d\tau} &= \frac{(2\theta_H - 1)(2b+c)c}{(c\tau + (2b+c)\sigma(2\theta_H - 1))^2} > 0 \end{aligned}$$

so that $(d/d\tau)\ln(n_H^o - 1/2) < (d/d\tau)\ln(\widetilde{n}_H - 1/2) < 0$. This yields $dn_H^o/d\tau < d\widetilde{n}_H/d\tau < 0$.

Proof of Proposition 7

We need to show that the function $\Delta V(n_H)$ has at most one root on the interval $(1/2, 1]$. Because $\lim_{\varepsilon \to 0} \Delta V(1/2 + \varepsilon) > 0$ a sufficient condition is that the function $\Delta V(n_H)$ is concave on $(1/2, 1]$. This will be true if the function $\Phi(n_H)$ is also concave on this interval. Using $M' = G$, we successively get that

$$\begin{aligned} \Phi' &= 3G + (2n_H - 1) G' \\ \Phi'' &= 5G' + (2n_H - 1) G'' \end{aligned}$$

For $\Phi'' < 0$ on the interval $(1/2, 1]$, we must have that $-5G' > (2n_H - 1) G''$ for all $n_H \in (1/2, 1]$. Since $G' < 0$, this is true if $G'' < 0$. If $G'' > 0$, a sufficient condition is simply that $-5G' > G''$ for all $n_H \in (1/2, 1]$. Since $G'(n_H) = -(b+c)\alpha'(1-n_H)$ and $G''(n_H) = (b+c)\alpha''(1-n_H)$. As a result, a sufficient condition for a concave $\Phi(n_H)$ is that

$$5\alpha'(1-n_H) > \alpha''(1-n_H) \tag{26}$$

for all $n_H \in (1/2, 1]$. That is α is not a too convex function and, by the same token, F_α is not a too concave function. The sufficient condition (26) applies for uniform taste distribution since this implies that $F_\alpha'' = \alpha'' = 0$. It indeed applies for taste Pareto distribution $F = 1 - (x/\alpha_l)^{-k}$ provided that it has a finite average; that is, if $k > 1$. In this case, $\alpha = \alpha_l (1 - v)^{-1/k}$ so that condition (26) becomes $5n_H > (1 + 1/k)$, which is true for all $n_H \in (1/2, 1]$ and $k > 1$.

Proof of Proposition 9

Suppose that two taste distributions F_α^1 and F_α^2 so that $F_\alpha^2(x) \equiv F_\alpha^1(x+\sigma)\ \forall x \in [\alpha_l, \alpha_h]$ and $\sigma > 0$. This means that $\alpha_2(v) = \alpha_1(v) + \sigma$ so that the willingness to pay for each variety increases by σ. We know from our previous analysis that $a_1(v) - \overline{a}_1 = a_2(v) - \overline{a}_2$, $G_1(n_H^*) = G_2(n_H^*)$. As a result, it readily comes that $M_1(n_H) = \int_{1-n_H}^{1} (a_1(v) - \overline{a}_1)\, dv = \int_{1-n_H}^{1} (a_2(v) - \overline{a}_2)\, dv = M_2(n_H)$ so that $\Phi_1(n_H^*) = \Phi_2(n_H^*)$.

Proof of Proposition 10

We here study the impact of increase in spread of the taste distribution around its mean. We use the same definition as in Proposition 4: a variety with a taste parameter below the average has a positive probability to worsen (i.e. $F_\alpha^1(x) \leq F_\alpha^2(x)$ if $x < \overline{a} \equiv \overline{a}_1 = \overline{a}_2$) and a variety with a taste parameter above the average has a positive probability to improve (i.e. $F_\alpha^1(x) > F_\alpha^2(x)$ if $x > \overline{a}$). For simplicity let us focus on taste distributions that are not skewed ($sk = 0$) or that are skewed towards high taste varieties ($sk > 0$). From the previous discussion we know that $G_1 < 0$ so that $G_2 < G_1$. This also implies that $a_2(v) > a_1(v) \iff a_1(v) > \overline{a} \iff v > 1/2$. In particular this gives $a_l^2 \equiv a_2(0) < a_1(0) \equiv a_l^1$. An increase in the spread fosters agglomeration if it increases the value of $\Phi(n_H)$ in the entrepreneurs' utility differential (19) and fosters dispersion otherwise. Two important situations are easy to characterize. First suppose that the initial taste distribution F_α^1 yields a full agglomeration equilibrium ($n_H^1 = 1$). It must be that $\Delta V_1^*(1) \geq 0$. Then, since $M_1(1) = M_2(1) = 0$ and since $\Phi_2(1) - \Phi_2(1) \propto G_2(1) - G_1(1) = (b + c)(a_l^2 - a_l^1) < 0$, we have that $\Delta V_2^* < \Delta V_1^*$. If the difference $G_2(1) - G_1(1)$, which is proportional to $a_l^2 - a_l^1$, is sufficiently negative, the utility differential can be reversed and full agglomeration is no longer a spatial equilibrium for the new taste distribution F_α^2. So, *an increase in spread of the taste distribution around its mean makes the full agglomeration equilibrium less likely.*

Second, suppose that the initial taste distribution yields a spatial equilibrium that is close to the symmetric spatial distribution so that $\Delta V_1^*(n_H^1) = 0$ and $n_H^1 = 1/2 + \varepsilon$, where $\varepsilon > 0$ is small. It must be that $\Delta V_2^*(n_H^1) > \Delta V_1^*(n_H^1) \iff \Phi_2(n_H^1) > \Phi_1(n_H^1) \iff \int_{1/2-\varepsilon}^1 (a_2(v) - a_1(v))\,dv + 2\varepsilon\,[a_2(1/2-\varepsilon) - a_1(1/2-\varepsilon)] > 0$, which is true if ε is sufficiently small because $a_2(v) - a_1(v) > 0$ for $v > 1/2$. So, *an increase in spread of the taste distribution around its mean decreases the likelihood to get an equilibrium that is close to symmetry.*

Proof of Proposition 11

To prove Proposition 11, suppose that $\tau = \tau^o$ so that the first term in expression (19) is equal to zero. This case is displayed in central panel of Figure 3. The location equilibrium is simply given by the zero of the above polynomial (22); that is, by $n_H^* = 0.72$. When the spread σ rises, the polynomial (22) increases for $n_H < 0.72$ and falls for $n_H > 0.72$. Suppose now that $\tau > \tau^o$ so that expression (19) smaller than (22) for any $n_H > 1/2$. As it can be seen in left hand side panel of Figure 3, the number of entrepreneurs n_H^* is smaller than 0.72 and increases with larger spread σ. The opposite argument applies when $\tau < \tau^o$. So a rise in σ implies a convergence of the equilibrium distribution of entrepreneurs to $n_H = 0.72$.

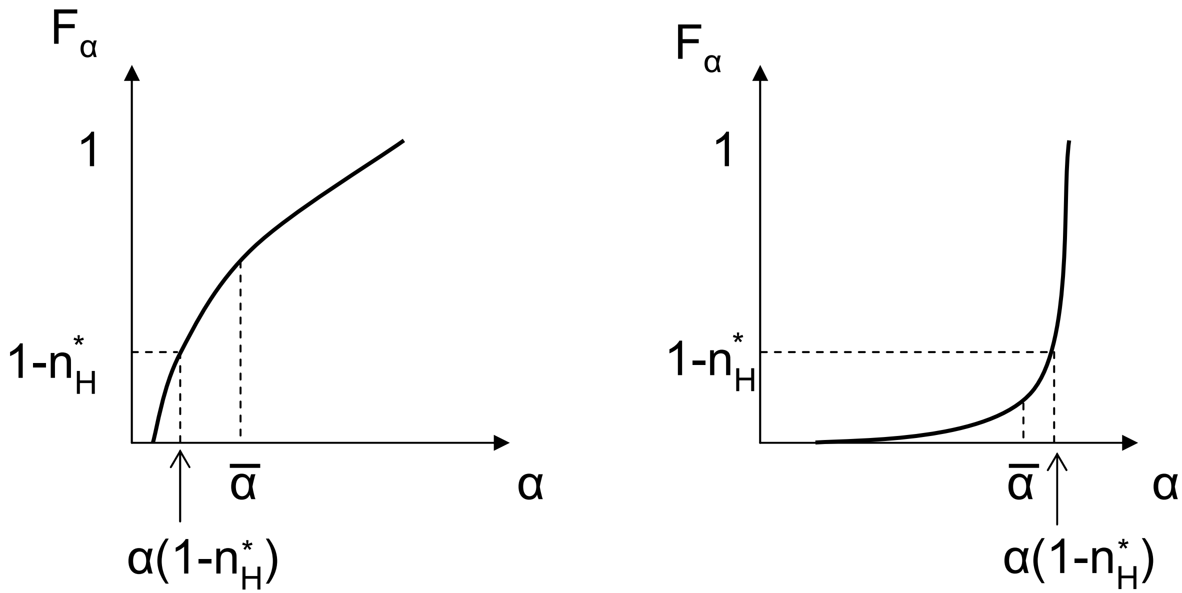

Figure 1: Skewness of taste distributions and spatial allocation of capital.

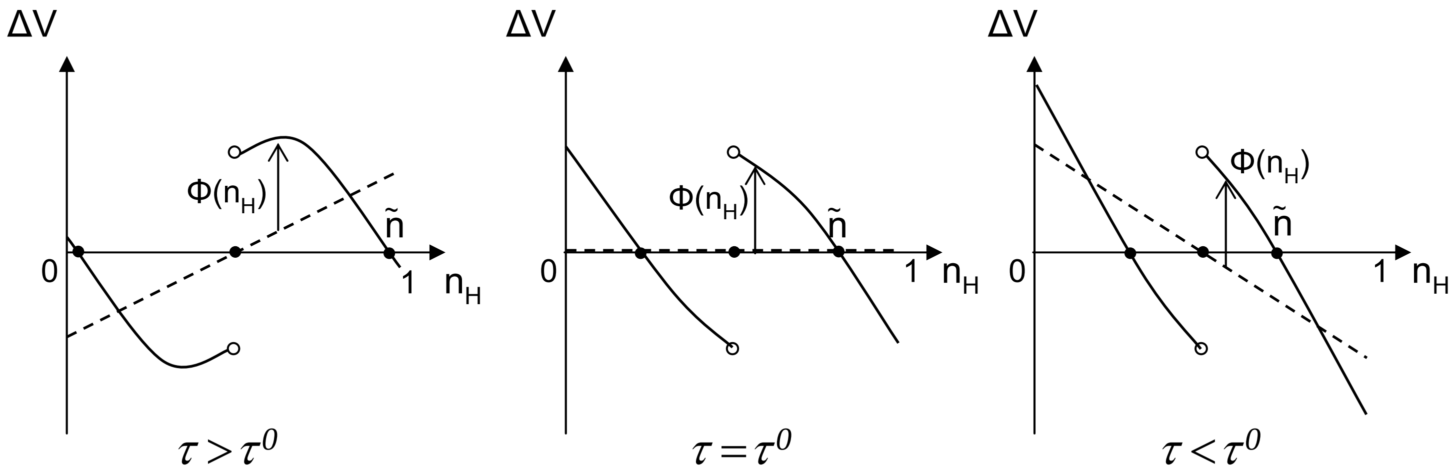

Figure 2: Utility differential of skilled workers.

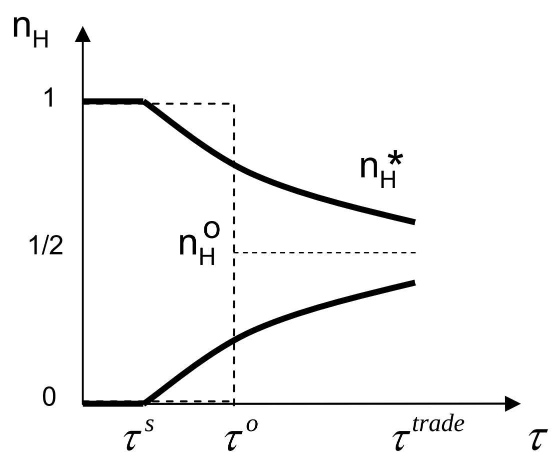

Figure 3: Location equilibrium of skilled workers.