FACULTY OF LAW, ECONOMICS AND FINANCE

CREA
Discussion
Paper
2011-15

Center for Research in Economic Analysis
University of Luxembourg

Does the seller of a house facing a large number of buyers always decrease its price when its first offer is rejected?

available online : http://wwwfr.uni.lu/recherche/fdef/crea/publications2/discussion_papers/2011

Jean J. Gabszewicz, CORE Université catholique de Louvain.
Tanguy van Ypersele, GREQAM, Marseille.
Skerdilajda Zanaj, CREA, University of Luxembourg.

November, 2011

For editorial correspondence, please contact: crea@uni.lu
University of Luxembourg
Faculty of Law, Economics and Finance
162A, avenue de la Faïencerie
L-1511 Luxembourg

Does the seller of a house facing a large number of buyers always decrease its price when its first offer is rejected ?

Jean J. Gabszewicz*, Tanguy van Ypersele†, Skerdilajda Zanaj‡

September 2011

Abstract

This paper investigates the optimal price sequence of a two period tentative to sell an indivisible good, with take-it-or-leave-it offers, in which the seller faces ambiguity about the buyers' willingness to pay. If the first round fails, the seller updates its beliefs on the state of the market in accordance with Bayes rule and quotes a second and final price. We show that the optimal sequence of prices can be increasing. Furthermore, we describe the optimal sequence of prices with a myopic seller who does not update his beliefs in the second period. In this case, the optimal price sequence is always decreasing.

Keywords: ambiguity, sequential bilateral trade, bayesian vs myopic behaviour.

Jel classifications: D8, D82, D89.

1 Introduction

Consider the owner of a house, or of any indivisibe object, who is willing to sell it while facing a large number of potential buyers. This is a situation often encountered in the real estate market when the seller does not use an auction mechanism or any other available device, but simple sets and advertises a price, then waiting for a buyer whose reservation price exceeds the price. In this situation, the seller is like a monopolist since, not only he/she is the only owner of the house, but the buyers' side he/she faces is atomistic. Thus, the potential buyers are like price taking agents in a monopoly market. All of those who are potential candidates to buy the house perfectly know that, if the price set by the seller exceeds their reservation price, the seller can always wait enough time to be sure to find another potential buyer with a reservation price larger or equal

*CORE, University of Louvain.
†GREQAM, Marseille. Corresponding author: tanguy.vanypersele@univmed.fr
‡CREA, University of Luxembourg.

to their own reservation price. It is a good reason to accept the price as given. Of course, there is a penalty incurred by the seller while waiting for another potential buyer: This penalty takes the form of the discount rate to be applied to the amount of the sale, due to the time delay in obtaining it. This delay can be *a priori* very long, especially when the seller does not know with certainty the explicit value of the reservation prices existing in the large population of potential buyers, and when these potential buyers arrive at random to possibly make the deal. In several circumstances, the seller cannot wait for ever before finding a buyer whose reservation price exceeds the price. For instance, when the seller of the house needs the amount of the sale in order to buy another one, its impatience constrains him to sell the good in a rather short delay.

In this note, we propose a formal model capturing the essential ingredients of the above description, and we analyse the optimal strategies for selling the house, when the seller is uncertain about the distribution of the buyers' willingness to pay[1]. First we assume that the seller, with one indivisible object to sell, has only two periods to succeed to sell the house (for instance for the reason given above: the seller of the house needs the amount of money resulting from the sale in order to buy another one)[2]. Buyers are assumed to arrive randomly, one at a time. The seller sets a price for the object, while the buyer can either accept or reject the offer (take-it-or-leave-it offer). If the offer is accepted, then the object is transferred at the announced price; otherwise, the process continues for a next period, and stops afterwards. The seller can set another price in the second period. We are interested in the optimal price sequence in this repeated retrial of selling an object, which appears as typical in the housing market[3]. Then, we examine how a myopic seller, who does not update his/her *a priori* beliefs, would select the optimal price sequence. One could think that a first rejection should induce the seller of the house to increase the chances to sell it by reducing the price in the second trial. Surprisingly, we show that, even if this is always the case when the seller is myopic, the price of the house can well increase when the seller updates his beliefs in a Bayesian way. To do so, we first consider a selling process in which a sophisticated seller learns from a first rejection to buy, and updates *à la Bayes* his/her beliefs concerning the distribution of the willingness to pay over the buyers' population. Our main motivation is to answer the following question: *does a rational seller always decrease its price in the second period?* Surprisingly we show that, in several cases, a Bayesian seller is willing to increase its price at the second period.

Most of the literature related to the above problem is formulated assuming the context of bilateral trade with one seller and one buyer bargaining about the exchange price. This context is very different from the one adopted here and

[1] Uncertainty about the distribution of an event is often referred to as *ambiguity,* distinguishing between known (the probability and payoff are precise) and unknown (ambiguity in the probability and/or payoff) uncertainty.

[2] This is assumed to be private information available to the seller but not to the potential buyers.

[3] Because our attention is focused in the housing market we do not assume an auction mechanism as in Myerson (1981) who describe a mechanism typical of art objects auctions.

resorts to game theory as the natural analytical instrument to develop models capturing this bilateral situation (see Rubinstein (1982), Binmore (1980), Chatterjee and Samuelson (1980), Crawford (1981), Sobel and Takahashi (1981) and Fudenberg and Tirole (1983)). In particluar, Fudenberg and Tirole (1983) build a bilateral sequential bargaining game and use the concept of perfect Bayesian equilibrium. These authors focus on issues of commitment and information transfer in a game where both the seller and the buyer update their beliefs about their opponents in accordance with Bayes rule. In this paper, Fudenberg and Tirole (1983) do also obtain the surprising result of prices increasing over time but such a result is due to the combination of information transfer and the lack of precommitment embodied in the perfectness of the definition of the equilibria. Even if our context is very different, we have identified a similar property, in a much simpler setup.

2 The model

Consider a unit of an indivisible good, such as a house. The owner of the good who wants to sell it receives each period the visit of a potential buyer, whose reservation price is equal to λ, a random variable.

Assume that the seller faces ambiguity concerning the state of the market. More specifically, the seller does not know whether the willingness to pay λ of buyers is distributed uniformly over the distribution $[0, \underline{\lambda}]$, the *narrow range distribution,* or over the distribution $[0, \bar{\lambda}]$, the *wide range distribution*, with $\bar{\lambda} > \underline{\lambda}$. Thus, the range of reservation prices either includes only rather low values for the object or, on the contrary, extends to include also higher values. Define $\bar{F}(\lambda)$ the cumulative function of the first distribution and $\underline{F}(\lambda)$ the cumulative function of the second:

$$\bar{F}(\lambda) = \begin{cases} \frac{\lambda}{\bar{\lambda}} & \text{for } \lambda < \bar{\lambda} \\ 1 & \text{for } \lambda > \bar{\lambda} \end{cases} \text{ and } \underline{F}(\lambda) = \begin{cases} \frac{\lambda}{\underline{\lambda}} & \text{for } \lambda < \underline{\lambda} \\ 1 & \text{for } \lambda > \underline{\lambda}. \end{cases}$$

Before the transaction takes place, the seller believes that, with probability π_0 the wide range distribution is the good one. Then, the probability that a visitor has a willingness to pay smaller than λ is given by

$$H_0(\lambda) = \begin{cases} \pi_0 \frac{\lambda}{\bar{\lambda}} + (1 - \pi_0)\frac{\lambda}{\underline{\lambda}} & \text{for } 0 < \lambda < \underline{\lambda} \\ \pi_0 \frac{\lambda}{\bar{\lambda}} + (1 - \pi_0) & \text{for } \underline{\lambda} < \lambda < \bar{\lambda} \\ 1 & \text{for } \bar{\lambda} < \lambda. \end{cases}$$

In the following we shall restrict the analysis to the case when $\pi_0 \in [0, \frac{1}{4}] \cup]\frac{1}{2}, 1]$[4].

Assume that the seller sets in period zero a price p_0 to his/her first visitor, and that the transaction fails. Then, clearly, the willingness to buy of the visitor is smaller than p_0. Hence, if the price announced is p_0, the transaction fails with probability $H_0(p_0)$. In case of failure to sell in the first round, the seller learns

[4] This condition simplifies considerably the analysis. However, the optimal choice could easily be obtained without it.

from this event and can update his/her prior on probability π_0 according to the following rule:

$$\pi_1(p_0) = P(\bar{\lambda}|\lambda < p_0) = \frac{P(\lambda < p_0|\bar{\lambda})\pi_0}{P(\lambda < p_0|\bar{\lambda})\pi_0 + P(\lambda < p_0|\underline{\lambda})(1-\pi_0)} = \bar{F}(p_0)\frac{\pi_0}{H_0(p_0)}.$$

It is therefore necessary to distinguish two cases: (a) the price announced p_0 is smaller than the highest value of willingness to pay in narrow range distribution i.e. $\underline{\lambda}$; (b) the price announced p_0 is higher than $\underline{\lambda}$[5].

Case a $p_0 < \underline{\lambda}$. It follows that the updated probability $\pi_1^a(p_0)$ is given by

$$\pi_1^a(p_0) = \frac{\underline{\lambda}\pi_0}{\underline{\lambda}\pi_0 + (1-\pi_0)\bar{\lambda}}. \tag{1}$$

Case b $p_0 \geq \underline{\lambda}$. Then, the updated probability $\pi_1^b(p_0)$ is given by

$$\pi_1^b(p_0) = \frac{p_0\pi_0}{p_0\pi_0 + (1-\pi_0)\bar{\lambda}}. \tag{2}$$

The cumulative distribution of the first period $H_1(\lambda)$ is as $H_0(\lambda)$ but now the probability that the wide range distribution is the good one is π_1,with $\pi_1 = \pi_1^a$ if $p_0 < \underline{\lambda}$ and $\pi_1 = \pi_1^b$ if $p_0 \geq \underline{\lambda}$.

Our main concern consists in comparing the price quoted by the seller before the possible rejection of his/her first proposal and the price quoted *ex-post,* when rejection has taken place and the resulting information is taken into account *via* the Bayesian revision of the probability related to the willingness to pay distribution. In particular, we want to examine whether Bayesian revision always implies a decrease in the optimal price between period 0 and period 1, as one would expect a priori; or, on the contrary, whether an increasing optimal sequence could occur. To this end, we fully develop the payoff maximization solution for all admissible values of the parameters and we use this analysis to explore the properties of the optimal price sequence. Afterwards, we compare these properties with those arising in the case of a myopic seller.

The optimization problem is solved backwards assuming that the price p_0 has been selected in period zero.

2.1 Payoff maximisation in period one

Let the price p_0 be announced in period one. Then, the revision of the guess on the distribution of buyers yields π_1 as given by (1) or (2), which defines $H_1(p_1)$.In the second period, the seller maximizes its payoff $\Pi_1(p_1) = p_1(1-H_1(p_1))$. Under the condition $\frac{\underline{\lambda}}{\bar{\lambda}} > 1 - \frac{1}{2\pi_1}$, $p_1^* < \underline{\lambda}$, the candidate optimal price p_1^* is

$$p_1^*(\pi_1) = \frac{1}{2}\frac{\bar{\lambda}\underline{\lambda}}{\underline{\lambda}\pi_1 + \bar{\lambda}(1-\pi_1)}. \tag{3}$$

[5]Notice that a rejection to buy when $p_0 > \underline{\lambda}$, does not provide full information on the distribution of buyer because the two distributions overlap.

The corresponding payoff is

$$\Pi_1(\pi_1) = \frac{1}{4}\frac{\bar{\lambda}\underline{\lambda}}{\underline{\lambda}\pi_1 + \bar{\lambda}(1-\pi_1)}.$$

When $\frac{\underline{\lambda}}{\bar{\lambda}} < \frac{1}{2}$, then, $p_1^{**} > \underline{\lambda}$. Hence, the candidate optimal price p_1^{**} is

$$p_1^{**}(\pi_1) = \frac{\bar{\lambda}}{2}, \tag{4}$$

with corresponding payoff

$$\Pi_1(\pi_1) = \frac{\bar{\lambda}}{4}\pi_1.$$

It is possible to check that $\Pi_1(p_1^{**}(\pi_1)) > \Pi_1(p_1^{*}(\pi_1))$ iff $\frac{\underline{\lambda}}{\bar{\lambda}} < \frac{\pi_1}{1+\pi_1}$. Thus, provided that the inequality $\frac{\underline{\lambda}}{\bar{\lambda}} < \frac{\pi_1}{1+\pi_1}$ holds[6], the seller announces in the second trial to sell the house, a price equal to $p_1^{**}(\pi_1) = \frac{\bar{\lambda}}{2}$. Otherwise the optimal price in the second trial is given by $p_1^{*}(\pi_1)$.

Depending on the price level p_0 set in the first period, the revision of the prior differs. Therefore, we have two cases.

Case a $p_0 < \underline{\lambda}$, Substituting the expression of $\pi_1^a(p_0)$ in the ratio $\frac{\pi_1}{1+\pi_1}$, we obtain

$$\pi_1 = \frac{\underline{\lambda}\pi_0}{\underline{\lambda}\pi_0 + (1-\pi_0)\bar{\lambda}}.$$

Note that in this case the probability π_1 is always smaller than π_0. Hence, after a rejection of a price that is smaller than $\underline{\lambda}$, the seller decreases the probability with which he believes that the good distribution has as support the interval $[0, \bar{\lambda}]$.

Case b $p_0 > \underline{\lambda}$, Substituting π_1 in (4) we get

$$\pi_1 = \frac{p_0\pi_0}{p_0\pi_0 + (1-\pi_0)\bar{\lambda}}.$$

where similarly to above π_1 is always smaller than π_0. Hence, regardless how small is the price quoted in period zero, a rejection decreases the probability that the true distribution is the wide range one. It is not so surprising that rejection in period 0, puts a downward pressure in the belief of the seller about the domain of buyers' preferences for the object on sale.

Summarizing the two cases

Lemma *If* $p_0 < \underline{\lambda}$,*the optimal price sequence in the second period is given by* $p_{1a}^{**} = \frac{\bar{\lambda}}{2}$ *for all* $\frac{\underline{\lambda}}{\bar{\lambda}} \in [0, \frac{\underline{\lambda}\pi_0}{2\underline{\lambda}\pi_0 + (1-\pi_0)\bar{\lambda}}]$ *and* $p_{1a}^{*} = \frac{\bar{\lambda}\underline{\lambda}}{2}\frac{\underline{\lambda}\pi_0 + \bar{\lambda}(1-\pi_0)}{\underline{\lambda}^2\pi_0 + \bar{\lambda}^2(1-\pi_0)}$ *otherwise.*

[6]Notice that $\frac{\pi_1}{1+\pi_1} \in]1 - \frac{1}{2\pi_1}, \frac{1}{2}[$.

If $p_0 > \underline{\lambda}$*, the equilibrium price* $p_1^{**} = \frac{\bar{\lambda}}{2}$ *for all* $\frac{\underline{\lambda}}{\bar{\lambda}} \in [0, \frac{p_0\pi_0}{2p_0\pi_0+(1-\pi_0)\bar{\lambda}}]$ *and* $p_{1b}^* = \frac{\bar{\lambda}\underline{\lambda}}{2}\frac{p_0\pi_0+\bar{\lambda}(1-\pi_0)}{p_0\underline{\lambda}\pi_0+\bar{\lambda}^2(1-\pi_0)}$ *otherwise.*

2.2 Payoff maximization in period zero

The payoff in the first period writes as

$$\Pi_0 = (1 - H_0(p_0))p_0 + H_0(p_0)\Pi_1(p_0)$$

Solving the first order conditions, the candidate payoff maximizing prices are given by[7]

$$p_0^* = \frac{1}{2}\frac{\bar{\lambda}\underline{\lambda}(1+\frac{\pi_0}{4})}{\underline{\lambda}\pi_0 + (1-\pi_0)\bar{\lambda}}$$

if $\pi_0 < 1/4$ for any $\frac{\bar{\lambda}}{\underline{\lambda}}$;if $\pi_0 > 1/2$ for $\frac{\bar{\lambda}}{\underline{\lambda}} < \frac{8\pi_0}{9\pi_0-4}$. The candidate price is

$$p_0^{**} = \frac{5}{8}\bar{\lambda}$$

if $\frac{\bar{\lambda}}{\underline{\lambda}} > \frac{8}{5}$.

Hence, provided that $\pi_0 < 1/4$, we have to compare the payoffs when $\frac{\bar{\lambda}}{\underline{\lambda}} > \frac{8}{5}$. Instead, if $\pi_0 > 1/2$, we have to compare the payoffs when $\frac{8}{5} < \frac{\bar{\lambda}}{\underline{\lambda}} < \frac{8\pi_0}{9\pi_0-4}$. The payoff $\Pi_0(p_0^*)$ and $\Pi_0(p_0^{**})$ writes as

$$\begin{cases} \Pi_0(p_0^*) = (\frac{1}{2}+\frac{\pi_0}{8})^2\frac{\bar{\lambda}\underline{\lambda}}{\underline{\lambda}\pi_0+(1-\pi_0)\bar{\lambda}} \\ \Pi_0(p_0^{**}) = (\frac{5}{8})^2\bar{\lambda}\pi_0 \end{cases}$$

It is possible to show that $\Pi_0(p_0^*) > \Pi_0(p_0^{**})$ iff $\frac{\bar{\lambda}}{\underline{\lambda}} > \frac{8}{25}\frac{2+3\pi_0}{\pi_0}$. We can also easily show that $\frac{8}{25}\frac{2+3\pi_0}{\pi_0} < \frac{8\pi_0}{9\pi_0-4}$ when $\pi_0 > 1/2$ and $\frac{8}{25}\frac{2+3\pi_0}{\pi_0} > \frac{8}{5}$ for all π_0. Hence, we can summarize the optimal sequence of prices as follows:

Lemma *If* $\frac{\bar{\lambda}}{\underline{\lambda}} \in \left[\frac{8}{5}, \frac{8}{25}\frac{2+3\pi_0}{\pi_0}\right]$*, the optimal price sequence is* $\left\{p_0^* = \frac{5}{8}\bar{\lambda}; p_1^* = \frac{\bar{\lambda}}{2}\right\}$*. The same optimal sequence applies in the set* $[\frac{8\pi_0}{9\pi_0-4}, \infty[$ *if* $\pi_0 > \frac{1}{2}$*. In all other cases, the optimal price sequence is* $\left\{p_0^* = \frac{1}{2}\frac{\bar{\lambda}\underline{\lambda}(1+\frac{\pi_0}{4})}{\underline{\lambda}\pi_0+(1-\pi_0)\bar{\lambda}}; p_1^* = \frac{\bar{\lambda}}{2}\right\}$.

Now we are equipped with the needed information to study the behavior of prices in the optimal sequence, in particular whether this optimal sequence could be increasing. Using the above Lemma 2 to compare prices of the first period with prices of the second period we observe that the optimal sequence of pricing $\left\{p_0^* = \frac{5}{8}\bar{\lambda}; p_1^* = \frac{\bar{\lambda}}{2}\right\}$ is always decreasing. Nevertheless, comparison of prices in the sequence $\left\{p_0^* = \frac{1}{2}\frac{\bar{\lambda}\underline{\lambda}(1+\frac{\pi_0}{4})}{\underline{\lambda}\pi_0+(1-\pi_0)\bar{\lambda}}; p_1^* = \frac{\bar{\lambda}}{2}\right\}$ reveals the following property :

[7] Notice that $\frac{8}{5} < \frac{8\pi_0}{9\pi_0-4}$ when $\pi_0 > 1/2$.

Proposition 1 *Assume a Bayesian seller. When* $\pi_0 < \frac{1}{4}$*, the price set by the seller increases from period zero to period one if* $\frac{\bar{\lambda}}{\underline{\lambda}} \in [\frac{(4-3\pi_0)}{4(1-\pi_0)}, \frac{8\pi_0}{9\pi_0-4}]$*. When* $\pi_0 > \frac{1}{2}$ *then the optimal price sequence is increasing if* $\frac{\bar{\lambda}}{\underline{\lambda}} > \frac{(4-3\pi_0)}{4(1-\pi_0)}$.

Proof. We compare $p_0^* = \frac{1}{2}\frac{\bar{\lambda}\underline{\lambda}}{\underline{\lambda}\pi_0+(1-\pi_0)\bar{\lambda}}(1+\frac{\pi_0}{4})$ and $p_1^* = \frac{\bar{\lambda}}{2}$, which can be an optimal sequence of prices in the set $\frac{\bar{\lambda}}{\underline{\lambda}} \in \left[\frac{8}{25}\frac{2+3\pi_0}{\pi_0}, \frac{8\pi_0}{9\pi_0-4}\right]$ when $\pi_0 > \frac{1}{2}$ and they are defined in the set $\frac{\bar{\lambda}}{\underline{\lambda}} \in \left(\frac{8}{25}\frac{2+3\pi_0}{\pi_0}, +\infty\right[$ when $\pi_0 < \frac{1}{4}$. Such prices satisfy $p_0^* < p_1^*$ if $\frac{\bar{\lambda}}{\underline{\lambda}} > \frac{(4-3\pi_0)}{4(1-\pi_0)}$. This inequality can be satisfied for $\pi_0 > \frac{1}{2}$ or $\pi_0 < \frac{1}{4}$ because $\left[\frac{(4-3\pi_0)}{4(1-\pi_0)}, \frac{8\pi_0}{9\pi_0-4}\right] \cap \left[\frac{8}{25}\frac{2+3\pi_0}{\pi_0}, \frac{8\pi_0}{9\pi_0-4}\right] \neq 0$ and $\frac{(4-3\pi_0)}{4(1-\pi_0)} < \frac{8}{25}\frac{2+3\pi_0}{\pi_0}$. A numerical illustration corresponding to the case π_0 is small is the following. Let $\pi_0 = 0.2$, $\bar{\lambda} = 505$, and $\underline{\lambda} = 100$. Then, $\frac{\bar{\lambda}}{\underline{\lambda}} = 5.05 \notin \left[\frac{8}{5}; 4.16\right]$, and $p_0^* = 62.529$ while $p_1^* = 252.5$. Another example corresponding to the case where $\pi_0 > \frac{1}{2}$ is $\pi_0 = 0.55$, $\bar{\lambda} = 135$ and $\underline{\lambda} = 100$. In this case, $\frac{\bar{\lambda}}{\underline{\lambda}} = 1.35 \notin \left[\frac{8}{5}; 2.12\right] U[4.63, \infty[$ while $p_0^* = 66.334$ and $p_1^* = 67.5$.Q.E.D. ■

The increase in the price announced in the second trial is the result of different forces. Consider first the situation where the probability assigned to the wide range distribution is low, i.e. $\pi_0 < \frac{1}{4}$. Were the difference between the two distributions not large, the seller would choose to lower the price in the second trial since, as shown above, π_1 is smaller than π_0, which reinforces the weight of the narrow range distribution in the relative likelihood between the two events. But the difference between the two distributions can be very large. Accordingly, this difference can be so large that it can countervail the weak probability that the wide range distribution is the good one. As a consequence, it can induce the seller to increase the price, in spite of the low probability assigned to the wide range distribution, to take advantage of a possible high-willingness to pay buyer in the second period.

Now consider the case when π_0 is not very different, but larger, than $\frac{1}{2}$. Then, the fact that both distributions are likely the same, a high value of π_1 would normally lead the seller to believe that the wide range distribution is the true one. Therefore, the optimal sequence can be increasing even when the two distributions are quite similar.

3 Optimal prices with a myopic seller

The forces defining the optimal choice of prices are dictated by the fact that the seller anticipates in period 0 what can happen in period 1. It is therefore interesting to compare the choice of a Bayesian seller with that of a myopic one who does not exploit the information resulting from the first trial.

3.1 Payoff maximization in the second period

Consider that the price p_0 is set in period 0 and that *no revision* is made concerning the distribution of buyers.

In the second period, the seller maximizes its payoff $\Pi_1(p_1)$ given by

$$\Pi_1(p_1) = p_1(1 - H_0(p_1))$$

where $H_1(p_1)$ is the same as $H_0(p_0)$.

$$H_0(\lambda) = H_1(\lambda) = \begin{cases} \pi_0 \frac{\lambda}{\bar{\lambda}} + (1-\pi_0)\frac{\lambda}{\underline{\lambda}} & \text{for } 0 < \lambda < \underline{\lambda} \\ \pi_0 \frac{\lambda}{\bar{\lambda}} + (1-\pi_0) & \text{for } \underline{\lambda} < \lambda < \bar{\lambda} \\ 1 & \text{for } \bar{\lambda} < \lambda \end{cases}$$

Hence the price selected in the second period is given by

$$p_1^* = \frac{1}{2}\frac{\bar{\lambda}\underline{\lambda}}{\underline{\lambda}\pi_0 + \bar{\lambda}(1-\pi_0)}, \tag{5}$$

which is the optimal price if the payoff is decreasing in $p_1 = \underline{\lambda}$, $1 - 2\underline{\lambda}(\frac{\pi_0}{\bar{\lambda}} + \frac{1-\pi_0}{\underline{\lambda}}) < 0 \Leftrightarrow \frac{\underline{\lambda}}{\bar{\lambda}} > 1 - \frac{1}{2\pi_0}$. The corresponding payoff is

$$\Pi_1(p_1^*) = \frac{1}{4}\frac{\bar{\lambda}\underline{\lambda}}{\underline{\lambda}\pi_0 + \bar{\lambda}(1-\pi_0)}$$

If the payoff is increasing in p_1, $p_1 = \underline{\lambda}$,i.e. $\frac{\underline{\lambda}}{\bar{\lambda}} < \frac{1}{2}$, then

$$p_1^{**} = \frac{\bar{\lambda}}{2} \tag{6}$$

is the optimal price, with corresponding payoff

$$\Pi_1(p_1^{**}) = \frac{\bar{\lambda}}{4}\pi_0.$$

It is easy to verify that $\Pi_1(p_1^*) \gtrless \Pi_1(p_1^{**})$ iff $\frac{\underline{\lambda}}{\bar{\lambda}} \lessgtr \frac{\pi_0}{1+\pi_0}$.

3.2 Payoff Maximization in the first period

The payoff in the first period writes as

$$\Pi_0 = (1 - H_0(p_0))p_0 + H_0(p_0)\Pi_1(p_1).$$

where the expression of $\Pi_1(p_1)$ shall be substituted either with $\Pi_1(p_1^*)$ or with $\Pi_1(p_1^{**})$ according as $\frac{\underline{\lambda}}{\bar{\lambda}} \lessgtr \frac{\pi_0}{1+\pi_0}$. The solution of first order conditions yields the price in period 1 as

$$p_0^* = \begin{cases} \frac{5}{8}\frac{\bar{\lambda}\underline{\lambda}}{\bar{\lambda}(1-\pi_0)+\underline{\lambda}\pi_0} & \text{for } 0 < \lambda < \underline{\lambda} \\ \frac{1}{8}\frac{\bar{\lambda}\underline{\lambda}\left(4\bar{\lambda}-4\bar{\lambda}\pi_0+5\underline{\lambda}\pi_0\right)}{\left(\bar{\lambda}(\pi_0-1)-\underline{\lambda}\pi_0\right)^2} & \text{for } \underline{\lambda} < \lambda < \bar{\lambda} \end{cases}. \tag{7}$$

Direct comparison of prices in the candidate optimal sequences of prices reveals that

Proposition 2 *Assume a myopic seller. The price of the house in the second period always decreases.*

Proof. We compare the expression of p_0^* in (7) with the expressions (5) and

(6) which shows that the price quoted in the second period is always small than the price in the first period. ■

This could be explained why most sellers probably do decrease prices when they face a rejection at their first trial. Of course, other explanations concur to reinforce this phenomenon, like the existence of a discount rate and/or the belief that a lower price is a piece of information that would be spread fastly among the potential buyers.

4 Conclusion

This note tackles a problem which has fascinated economists since a long period of time. It can be formulated in many contexts, differing by their assumptions and the difficulties of its treatment. In some sense we have chosen the simplest one since we have assumed that *(i)* one side of the market consists of a single agent behaving rationally, while the other one consist of a continuum of agents who behave myopically; *(ii)* only one buyer arrives at a time; and *(iii)* there are only two periods where the seller can meet a potential buyer. We feel that these assumptions might correspond to several situations encountered in the housing markets. Probably the most awkward among these assumptions is the third one. Generalizing our approach to an extended set-up accepting *a priori* an arbitrary number of periods with the existence of a discounting rate are objects for further research.

Of course our two other assumptions deserve attention. In particular, generalizing our approach to an extended set-up accepting a priori an arbitrary number of periods with a discounting rate constitutes a natural theme for further research on the topic studied in this note.

References

[1] BINMORE, K. (1980), "Nash Bargaining Theory: II" (London School of Economics I.C.E.R.D.).

[2] CHATT'ERJEE, K. and SAMUELSON, W. (1980), "The Simple Economics of Bargaining" (mimeo, Pennsylvania State University).

[3] CRAWFORD, V. (1981), "A Theory of Disagreement in Bargaining", Econometrica, 50, 607-638.

[4] FUDENBERG, D. and TIROLE J. : "Sequential Bargaining with Incomplete Information", The Review of Economic Studies, Vol. 50, No. 2 (Apr., 1983), pp. 221-247

[5] MYERSON, R. B.: "Optimal Auction Design," Mathematics of Operations Research, 6(1981), 58-73.

[6] RUBINSTEIN, A. (1982), "Perfect Equilibrium in a Bargaining Model", Econometrica, 50, 97-110.

[7] SOBEL, J. and TAKAHASHI. (1981), "A Multi-Stage Model of Bargaining" (D.P. 80-25*, University of San Diego, California).