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Carbon Capture and Storage and Transboundary Pollution: A Differential Game Approach*

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Abstract

We study the strategic behavior of two countries facing transboundary pollution under a differential game setting. In our model, the reduction of both pollution and CO_2 concentration occur through the creation of pollution sinks, rather than through the adoption of cleaner technologies. To our knowledge, this is the first formal attempt to model carbon capture and storage. Furthermore, we provide the explicit short-run dynamics for this game with symmetric open-loop and a special Markovian Nash strategies. Furthermore, we analyze and compare these strategies and the games' steady states along some balanced growth paths. Our results

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show that if the initial level of pollution is relatively high, state dependent emissions reductions can lead to higher overall environmental quality, hence, feedback strategy leads to less social waste.

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JEL Classification: Q58, Q55, Q52, C73.

1 Introduction

According to the International Energy Agency (2010), energy related CO_2 emissions have increased by 4.9 and 1.0 per cent in non-OECD and OECD countries respectively during the period 2006-2007, with no perspective of a slowdown in the coming decade. Although this energy quest is partly driven by the emergence of formerly developing countries, energy consumption in developed countries remains at high and slightly increasing levels. The largest share of this increase in energy demand is absorbed by new coal, gas, and oil fired power plants, oil sands and more recently unconventional gas sources.[1]

As a consequence, non-renewable energies will be used over a longer period, and hence, there are incentives to use energy more efficiently. Unfortunately, improvements in energy use can only be applied progressively, as they often entail a switch in technology and ensuing adoption costs. Conversely, capture technologies could be applied at a large scale in a shorter period, mainly for two reasons: (i) first, the only requirement is to fit coal-fired power plants with capture technologies; (ii) second, capture technologies can be implemented on large energy supply sites at lower cost than current low emissions technologies.[2]

Carbon capture and storage (CCS) have been at the forefront lately, as it encompasses a broad set of technologies, and it essentially consists in capturing CO_2 emissions from large point sources and storing it in geological formations. In the recent past CCS has raised increasing interest among scientists and policy makers. In 2007 for instance, CCS was accepted as a climate change mitigation possibility within the Kyoto Protocol, on top of national regulations (see IEA (2010)). However, the international coordination and the implementation of large scale transboundary policies remains largely an unaddressed issue, dealt at the national or regional level so far. This is notably true for the development of large scale CCS demonstration sites.

[1]Unconventional gas sources refers to gas which has not been exploited to its full extent, essentially unprofitable. These gas sources include: (i) deep natural gas, trapped in very deep deposits underground, (ii) tight natural gas, stuck in tight formations, trapped in impermeable rocks or non-porous sandstone formations, and (iii) shale gas, caught in shale rock formations, deep underground. See Smith (1980) for more details about unconventional gas sources.

[2]The International Energy Agency (2010) noted that carbon capture and storage is fundamental in a least-cost carbon abatement mix. If carbon capture and storage technologies are not implemented, the overall costs to limit to 2^0C global mean temperature would rise by 70%.

International cooperation on CCS regulation seems unavoidable, given the international nature of climate change. In the coming years, CCS will therefore certainly be part of a mix of solutions to mitigate climate change, as it permits non-negligible reductions of CO_2 emissions.

In the present paper, we use a dynamic framework to analyze the strategic behaviour of countries when pollution is borderless and mitigation policies are nationally financed. This framework corresponds to the CCS case nowadays. Our framework focuses on non-renewable resources, *i.e.* we do not consider neither the possibility of adopting technologies based on renewable resources nor energy saving technological progress. This is consistent with the view that CO_2 emissions due to the usage of non renewable resources will still be an important issue in the future.

Furthermore, the transboundary nature of pollution is of tremendous importance when dealing with CO_2 emissions. A number of contributions have tackled this issue before: Copeland and Taylor (1994, 1995) study the relationship between trade and transboundary pollution. In their 1994 paper, they find that "... free trade increases world pollution..." and in their 1995 paper, they go some steps further pointing at some policies to deal with global pollution. Hatzipanayotou *et al.* (2002, 2005) offer different abatement policies, while Alemdar and Ozyildirim (2002) study the relationship among transboundary pollution, knowledge spillovers and growth in a North-South model. However, all these studies ignore the possibility of having recourse to CCS, and only analyze long term issues. In the present work, we take both aspects into account.

Our framework partly relies on Dockner *et al.* (1993, 2000)'s model of a two player dynamic game of international pollution control, which characterizes cooperative and non-cooperative strategies of a government maximizing the discounted stream of benefits of the representative consumer. Special attention is paid to the existence, multiplicity and properties of stationary steady states via Hamilton-Jacob-Bellman equations. We however depart from this previous work in several ways. First, the stationary equilibrium analysis does not account for short-run dynamics, which are of importance given we are interested in CCS. Implementing CCS requires to complement existing technologies in the short run. We therefore introduce a transitory dynamic analysis using Pontryagin's maximum principle. Second, contrary to Dockner *et al.* (1993), we present two alternative non-cooperative scenarios characterized by open-loop strategies or Markovian strategies. In the absence of supranational institutions enforcing environmental regulations, our scenarios describe some alternatives available to policy makers, the multilateral negotiations involved and the optimal strategies. Notice that optimal strategies are adaptable in time and depend on the state of pollution.

Further, we can describe analytically the short run dynamics. Adoption of new, more environmental friendly technologies is lengthy and involves considerable inertia due to high fixed costs of present technologies. Therefore, we allow CCS mechanisms to reduce pollution. In terms of the modelling strategy, we suppose that pollution is predetermined and that agents decide the amount of emissions to reduce. As a corollary, pollution reduction does not imply a direct reduction of output in the production process. Following Aghion and Howitt (1998), we introduce a potential rate of regeneration in the equation for environmental quality. In the existing literature, most studies ignore the possibility that different countries may be endowed with different absorption capacities of pollution. We believe it is of tremendous importance to take into account this possibility since the self-regeneration capacity of nature acts as a pollution sink.

Countries with open-loop strategies commit once and for all to a trajectory of reduction of emissions. We treat as open-loop strategies situations in which it is difficult or too costly to check commitments. A unique equilibrium solution of the dynamic system is found, which is interior under certain assumptions. With certain weak parameter restrictions, there exists a balanced growth path where pollution and efforts displayed to absorb emissions grow at the same rate as the emissions. Contrary to most of the previous findings and in line with Wirl (1994), we prove that Markovian strategies are not always socially less desirable. Indeed, allowing countries to revise their effort to reduce emissions every period leads to lower overall levels of pollution, provided levels of pollution are sufficiently high.

The paper is organized as follows: in Section 2, we briefly introduce the basics of CCS economics, and the challenges it will pose to future multilateral rounds of negotiations. Section 3 presents the model while Sections 4 and 5 explain the theoretical results for different strategies. Section 6 concludes.

2 Carbon capture and storage: the economics, policies and regulations [3]

The tremendous growth performance of a number of economies and the ensuing energy famine thereof has led to a situation where one new coal-fired power plant opens every day worldwide, most of which are located in China as well as a number of other emerging economies. However coal plays a significant role in countries like Poland,and Germany as well, since their decision to step progressively out of nuclear energy. All these facts indicate that increases of energy supply will not come from zero-emissions sources in the medium run. In this regard, CCS appears as a transition technology for the coming decades. Indeed, CCS technologies would build up on traditional resources, taking into account climate change contingencies.

Whenever a fossil fuel is burned, it generates CO_2. It is usually emitted in the atmosphere, contributing to climate change. CCS is a technology to capture, compress, transport and store CO_2 emissions permanently. So far, there is no other known technology able to capture CO_2 emissions. However, CCS technology is relatively costly: estimates show that fitting a power plant with additional CCS technology might increase its cost by about 50 per cent for the installation of the capture equipment. On top of that, the capture process, the transport and storage systems have to be built and operated, increasing the running costs further.[4] Inferring the cost structure on CCS is not straightaway, as the returns to scale depend on the interlink of costs of capture, transport and storage, each of which highlighting potential economies of scale at given emissions levels (see Bielicki, 2008).

Incentives to produce cleaner CO_2 emitting energy is very limited, either at the supplier level or at the country level. However, a number of steps has been undertaken to foster CCS regulation at the national and international level. Probably the most noticeable step has been the integration of CCS as a climate change mitigation instrument within the Kyoto Protocol in 2007, although so far, no agreement has been found regarding the inclusion of CCS in the Clean Development Mechanism.[5] A further issue facing CCS regulation is the classification of CO_2

[3]This section is largely based on IPCC (2007) and MIT (2007)

[4]See more detail as to the CCS cost in Heal and Tarui (2008).

[5]The Clean development Mechanism allows developed countries to invest in emissions reductions in less developed countries, and can earn certified emissions reduction credits to the former country

emissions as industrial by-product or as waste product. This has important consequences as industrial projects are in general subject to less stringent regulations than waste disposal projects. In the London Convention,[6] CO_2 from capture processes is defined as industrial waste, which then opens the path to further legislation allowing its storage in sub-seabed geological formations. There is however no mention to CO_2 transport across borders in this Convention.

There are a number of further regulatory and policy initiatives at the regional level. The Oslo-Paris Convention consists of the European Union, fifteen governments of the western coasts and catchments of Europe. They cooperate on issues related to marine environment protection. In the US, the Enhanced Oil Recovery has a long experience in transporting, injecting and storing CO_2.[7]

Despite existing regulations, many issues remain legally unframed. Among others, let us mention questions related to permissions of exploration and storage, risk regulation, assessments of the effects of projects, measures in case of leakage, responsibility transfer and transport-related regulation. The lack of a complete regulation on CCS leaves many countries in a situation of self-organization. In the next section, we present a model where countries act strategically to determine their level of emissions abatement. They pre-commit simultaneously to an entire path of abatement (open loop strategies), or condition their action to the current period's level of pollution (Markovian strategies).

3 The model

Suppose there are two countries, i and j, with a common border. Both produce the same consumption good with pollution as a byproduct. These two countries share the same pollution state, $x(t)$, measured in gigatonnes of carbon (GtC), whose evolution in time is given by the

group.

[6]The London Convention dealt with the prevention of marine pollution by wastes dumping and other matters. It is in force since 1975 and was largely amended and modernized in 1996.

[7]see Marston and Moore (2008).

following equation[8]

$$\dot{x}(t) = E(t) - \beta(u_i + u_j) - \delta x(t), \;\; t \geq 0, \tag{1}$$

where the initial condition $x(0) \geq 0$ is given and parameter $\delta \in [0, 1]$ measures the pollution absorption rate of nature. The total emissions stock is the sum of each country's stock $E(t) = E_i(t) + E_j(t)$ and it is a known positive function of pollution emissions. In the long run $E(t)$ may be constant, increasing or decreasing at a constant rate. u_i and u_j measure the quantity of emissions not released in the atmosphere by country i and j, respectively. In our context, emission absorption refers to the capability a country has to limit its emissions of pollutants. We understand this capability as the infrastructures resulting from specific investments. CCS qualifies in this regard. Indeed, in equation (1) emissions reduction is reached through an extension of the existing technology (*i.e.* a CCS technology), rather than a change in the existing technology. The positive coefficient β measures the unit effect of emissions reduction. Notice that if abatement is zero, pollution will increase much faster. Furthermore, if emissions are over the absorption rate of nature itself, pollution will increase without bound.

Facing the emissions reduction problem, the two countries need to choose their rates of emissions absorption u_l, $l = i, j$, that maximize their utility

$$\max_{u_l} \int_0^T e^{-r_l t} \left(-\frac{c_l x^2(t)}{2} - \frac{u_l^2}{2} - b_l x u_l \right) dt + S(x(T)), \;\; l = i, j, \tag{2}$$

subject to the state constraint (1), where $r_l \in [0, 1)$ is the time preference. The first term $\frac{c_l x^2(t)}{2}$ measures the direct cost of a polluted environment, such as health care, natural disaster lost, or purifying water.[9] $\frac{c_l}{2}$ is the unit adjustment cost, a positive constant. The second term, $\frac{u_l^2}{2}$, represents the adjustment cost of emissions reduction where the unit adjustment cost is normalized to one. Furthermore, we assume that reducing one emissions unit costs $b_l u_l$, with b_l, the state emissions reduction cost, which is a positive constant. Emissions reduction total cost is $\frac{u_l^2}{2} + b_l x u_l$, that is, the sum of the purchasing cost, $b_l x u_l$, and adjustment cost, $\frac{u_l^2}{2}$. When $b_l = 0$, the problem boils down to the case of Dockner and Long (1993) where the reduction

[8] Long (1992), Dockner and Long (1993), Aghion and Howitt (1998), Schumacher and Zou (2008) and Stokey (1998) employ similar pollution accumulation process.

[9] Kumar and Rao (2001) study the economic benefits of air quality improvement in Panipat Thermal Power Station Colony in India. They find that medical care cost and health status are significantly determine peoples' willingness to pay for the reduction of air pollution and improving the air quality.

cost is state independent. We assume that $b \geq 0$ because as mentioned by Misiolek and Elder (1989) "pollution abatement costs are dependent on emissions levels", and therefore, "marginal abatement costs are positive and increasing with abatement efforts". In the present setting, the marginal reduction cost is $u_l + b_l x$.[10] In line with the literature, we assume that the direct cost from pollution is more important than the reduction cost, that is, $c > b^2 \geq 0$. $S(x(T))$ is a given positive function for $T \leq \infty$ satisfying $\lim_{T\to\infty} S(x(T)) = 0$ and $S_x < 0$.

In contrast with Dockner et all (1993), countries emissions are exogenously given while emissions reduction is endogenously chosen.

4 Solution

First, we study some open-loop strategy.[11] At the beginning of the game, both countries commit to reduce emissions levels $u_l = u_l(t)$ for each period in time, independent of the state of the world. We compute the explicit solution path and analyze the effect of these terminal conditions. then, we analyze the model's steady state and its balanced growth path. Finally, the effect of b_l on emissions reductions and on the pollution state was studied.

Then, we study one (among many) Markovian Nash equilibrium via the Pontryagin maximum principle where the emissions reduction strategies change according to the state of pollution with $u_l(t) = u_l(x(t), t)$.

In the next section, we compare the two strategies in terms of their long-run steady states and their balanced growth paths.

[10] Harford (1991, 1993) studies some other state-dependent pollution control enforcement.

[11] As mentioned in *The Economist*, May 31, 2007, "Germany ... is trying to get the world to agree on what to do when the Kyoto Protocol on curbing greenhouse gases runs out in 2012. America, which dislikes the tough targets that the Europeans want the world to sign up to, is proposing separate negotiations between the world's big emitters..." If America agreed on the proposal of the Europeans, then both Europe and America play a kind of open-loop strategies. If withdrawals happen, as America's withdraw from the Kyoto agreement in 2001, then they would play with Markovian strategies.

4.1 Open-loop strategies

In the sequel of this paper, we focus on symmetric solutions. The two countries are identical and commit to a reduction strategy which only depends on time t. Then country i's problem is to find a trajectory $\{u_i(t)\}_{t=1}^{\infty}$ to maximize her utility, subject to the state constraint (1). Let us define the open-loop Nash Equilibrium:

Definition 1 *A couple (ϕ_i, ϕ_j) of functions $\phi_l : [0, +\infty) \to \mathbb{R}_+$, $l = i, j$, is called an open-loop Nash Equilibrium if, for each $l = i, j$, an optimal control path $u_l(\cdot)$ exists and is given by the open-loop strategy $u_l(t) = \phi_l(t)$.*

Country i's Hamiltonian function, given country j's optimal strategy as ϕ_j, can be defined as[12]

$$H_i(x, u_i, \lambda_i, t) = \left(-\frac{c_i x^2(t)}{2} - \frac{u_i^2}{2} - b_i x u_i \right) + \lambda_i \left[E(t) - \beta(u_i + \phi_j(t)) - \delta x(t) \right].$$

Following Pontryagin's maximum principle, we have

$$u_i(t) = -b_i\, x + \beta \lambda_i(t), \tag{3}$$

and the costate variable, which serves as the shadow value of emissions reduction, is

$$\dot{\lambda}_i(t) = r_i \lambda_i(t) - \frac{\partial H_i}{\partial x} = (r_i + \delta)\lambda_i + c_i x + b_i u_i, \tag{4}$$

as well as the transversality condition

$$\lim_{t \to \infty} e^{-r_i t} \lambda(t) x(t) = 0 \text{ if } T = \infty, \quad \text{or } \lambda(T) = S_x(x^*(T)) \text{ if } T < \infty. \tag{5}$$

[12]Note that there is an exogenous function $E(t)$ which may depend on time t, especially during the short run. The Hamilton Jacobi Bellman (HJB) equation shows up as a real partial differential equation with a term V_t. Given there is no boundary (or transversality) condition, it is more difficult to find solutions in general. Besides, the solution is no longer stationary in most of the cases. From this point of view, Pontryagin's maximum principle provides richer results. For more details about the advantage of using Pontryagin's Maximum principle, see Chow (1997).

Under the assumption of the existence of a symmetric equilibrium, we obtain the following system

$$\begin{cases} u = -(bx(t) + \beta\lambda(t)), \\ \dot{x} = E(t) + (2b\beta - \delta)x + 2\beta^2\lambda, \\ \dot{\lambda} = (c - b^2)x + (r + \delta - b\beta)\lambda, \end{cases} \tag{6}$$

with the initial condition of x and the transversality condition on λ.

If emissions are constant over time in the long-run, $E = \overline{E}$, there are two possibilities in the long term when $b\beta = r + \delta$. In the first situation, the country absorbs all new emissions, and the stock of pollution falls down to zero: $\overline{x} = 0$, $\overline{\lambda} = -\frac{\overline{E}}{2\beta^2}$ and $\overline{u} = \frac{\overline{E}}{2\beta}$. In the second situation, the country faces an indeterminacy. Indeed, any combination of $(\overline{x}, \overline{\lambda})$ is a possible steady state for given emissions $\overline{E}$ as long as $\overline{\lambda} = -\frac{\overline{E} + (2r+\delta)\overline{x}}{2\beta^2}$.

While if $b\beta \neq r + \delta$, there exists one long-run positive saddle path stable steady state $\overline{x} = \frac{\overline{E}(r+\delta-b\beta)}{D}$ and $\overline{\lambda} = -\frac{(c-b^2)\overline{E}}{D}$, if and only if $r + \delta - b\beta$ and $D = 2\beta^2(c - b^2) - (2b\beta - \delta)(r + \delta - b\beta)$ are both positive and $c > b^2$. Otherwise, the long run steady state is unstable. We impose the following assumption to ensure stability:

Assumption 1 *Suppose* $c > b^2$, $r + \delta > b\beta$, $D = 2\beta^2(c - b^2) - (2b\beta - \delta)(r + \delta - b\beta) > 0$.

Hence, the explicit solution can be given as follows, for any $t \geq 0$,

$$\begin{aligned} x(t) &= \frac{1}{\xi_2 - \xi_1}\left[(2b\beta - \delta - \xi_1)e^{\xi_2 t}\left(x(0) + \int_0^t e^{-\xi_2 s}E(s)ds\right)\right. \\ &\quad \left. - (2b\beta - \delta - \xi_2)e^{\xi_1 t}\left(x(0) + \int_0^t e^{-\xi_1 s}E(s)ds\right)\right] \\ &\quad + \frac{2b\beta - \delta - \xi_2}{\xi_2 - \xi_1}\left(e^{\xi_1 t} - e^{\xi_2 t}\right)\lambda(0), \end{aligned} \tag{7}$$

and

$$\begin{aligned} \lambda(t) &= \frac{(2b\beta - \delta - \xi_1)}{\xi_2 - \xi_1}\left[\left(e^{\xi_1 t} - e^{\xi_2 t}\right)x(0) + \int_0^t \left(e^{\xi_1(t-s)} - e^{\xi_2(t-s)}\right)E(s)ds\right] \\ &+ \frac{\Lambda(t)}{\xi_2 - \xi_1}\lambda(0), \end{aligned} \tag{8}$$

with

$$\begin{aligned} \lambda(0) &= \frac{\xi_2 - \xi_1}{\Lambda(T)}\lambda(T) \\ &-\frac{(2b\beta-\delta-\xi_1)}{\Lambda(T)}\left[\left(e^{\xi_1 T}-e^{\xi_2 T}\right)x(0)+\int_0^T\left(e^{\xi_1(T-s)}-e^{\xi_2(T-s)}\right)E(s)ds\right], \end{aligned} \tag{9}$$

and

$$\Lambda(t) = (2b\beta-\delta-\xi_1)e^{\xi_1 t}-(2b\beta-\delta-\xi_2)e^{\xi_2 t},$$

$$\xi_1 = \frac{r+b\beta-\sqrt{(r+b\beta)^2+4D}}{2}(<0),\ \ \xi_2 = \frac{r+b\beta+\sqrt{(r+b\beta)^2+4D}}{2}(>0).$$

Combining all results in the above analysis, we conclude

Proposition 1 *If $D > 0$ and for any given terminal condition $S(x(T))$, the dynamic system (6) has a unique solution which is given by (7) and (8) with (9). Furthermore, if in the long-run $E(t)$ is a constant and Assumption 1 holds, then there exists a unique interior positive steady state of pollution stock, which is a saddle point.*

Suppose now that in the long-run, $E(t)$ is not constant but grows at constant rate $g \in \mathbb{R}$.

We define a balanced growth path as the path where all the endogenous variables grow at constant rates, which can be negative, zero or positive. Hence, variable X can be expressed as $X(t) = \widehat{X}e^{g_X t}$ along its balanced growth path, with $\widehat{X}$ the level where the balanced growth path starts and g_X, its growth rate.

Then the dynamic canonical system (6) shows that there is a balanced growth path along which the pollution stock x, its shadow value λ and emissions reductions u grow at the same rate as emissions. Furthermore, some simple calculation yields that along the balanced growth path

$$\widehat{x} = \frac{(g+b\beta-(r+\delta))E(0)}{F(b)},\ \ \widehat{u} = -\left(b+\frac{\beta(c-b^2)}{g+b\beta-(r+\delta)}\right)\widehat{x}, \tag{10}$$

provided that $g < (r+\delta)-b\beta$ and $F(b) = (g+\delta)(g-(r+\delta))+b\beta(2r+3\delta-g)-2c\beta^2$. We can compute how $\widehat{x}$ and $\widehat{u}$ change with b, the unit state-dependent cost:

$$\frac{\partial\widehat{u}}{\partial b} = -\frac{E(0)(g+\delta)}{F^2(b)}\left[(r+\delta-g)^2-\beta^2 c\right]\begin{cases} >0, \text{ if } \dfrac{(r+\delta-g)^2}{\beta^2}<c, \\ =0, \text{ if } \dfrac{(r+\delta-g)^2}{\beta^2}=c, \\ <0, \text{ if } \dfrac{(r+\delta-g)^2}{\beta^2}>c. \end{cases}$$

The above expression reads that when the damage cost is high, $c > \frac{(r+\delta-g)^2}{\beta^2}$, higher b leads to higher reduction of emissions. In the last of the above three cases, when c is relatively low, higher b yields lower reduction of emissions. The latter result might seem strange at first sight and we come back to this point after obtaining the effect of b on the pollution level. Similarly,

$$\frac{\partial \widehat{x}}{\partial b} = \frac{2E(0)\beta}{F^2(b)}\left[(r+\delta-g)^2 - \beta^2 c\right] \begin{cases} < 0, \text{ if } \dfrac{(r+\delta-g)^2}{\beta^2} < c, \\ = 0, \text{ if } \dfrac{(r+\delta-g)^2}{\beta^2} = c, \\ > 0, \text{ if } \dfrac{(r+\delta-g)^2}{\beta^2} > c. \end{cases}$$

Again, the last statement above might appear counterintuitive at first sight. It states that under given conditions, higher b leads to a higher balanced growth path of the pollution level. In fact, this happens only if $\frac{(r+\delta-g)}{\beta} > \sqrt{c}$. That is, only if the agent is patient enough (large r) and/or the nature's regeneration level is high (large δ). The sum of r and δ, net of the emissions rate g discounted by the efficiency of emissions reductions β, is larger than the direct unit cost of pollution. We believe this is an unlikely situation with almost no pollution. In this case, an increase in the pollution level increases the efficiency of emissions reductions. $\widehat{u}$ would decrease and $\widehat{x}$ increase. More realistically, we assume that $(r+\delta-g)^2 < \beta^2 c$ and hence, $\frac{\partial \widehat{u}}{\partial b} > 0$ and $\frac{\partial \widehat{x}}{\partial b} < 0$.

We summarize the above analysis in the following proposition:

Proposition 2 *Suppose emissions $E(t)$ grow at a constant rate g and parameters verify $b^2 < \frac{b(r+\delta-g)}{\beta} < c$, then there exists a balanced growth path for system (6). Along this path, all endogenous variables, x, λ and u, grow at the same rate g and their levels are given in (10). Moreover, both $\widehat{x}$ and $\widehat{u}$ are increasing functions of $E(0)$. Provided $(r+\delta-g)^2 < \beta^2 c$, $\widehat{u}$ increases with b while $\widehat{x}$ decreases.*

Wrapping up our results for hight enough c, the direct pollution cost, we have proven that environmental quality improves and emissions fall when emissions reductions are settled once and for all.

In the next subsection, we study one type of state-dependent strategies. The level of emissions reduction is revised every period according to the state of the environment. In this case, countries can adapt their emissions of pollutants according to the contemporaneous environmental situation.

4.2 A Markovian Nash Equilibrium [13]

Suppose now that both countries can change their emissions reduction strategies depending on the pollution state. Define the Markovian Nash equilibrium as follows

Definition 2 *A couple of functions* (Ψ_i, Ψ_j), $\Psi_l : [0, +\infty)\times[0, +\infty) \to \mathbb{R}^2_+$ *with* $\Psi_l = \Psi_l(x, t), \forall(x, t) \in [0, +\infty)\times[0, +\infty)$ *for* $l = i, j$, *is a Markovian Nash Equilibrium if an optimal control path* u_l *of country* l *exists and is given by the Markovian Strategy for country* l*:* $u_l(t)) = \Psi_l(x(t), t)$, *for* $l = i, j$.

Example 4.1 in Dockner *et al.* (2000), players choose a mixture of open-loop and Markovian strategies via *conjecturing* the rival's potential optimal strategy. We adapt this conjecturing method and prove that indeed conjecturing offer one (among maybe many) Markovian strategy.

Country i's Hamiltonian function, taking country j's optimal strategy as given, is:

$$\mathcal{H}_i(x, u_i, \Psi_j(x, t), t) = \left(-\frac{c_i x^2(t)}{2} - \frac{u_i^2}{2} - b_i x u_i\right) + \lambda_{m,i}\left[E(t) - \beta(u_i + \Psi_j(x, t)) - \delta x\right],$$

where $\lambda_{m,i}$ is country i's costate variable. The first order condition for u_i gives $u^*_{m,i} = -(bx_m(t) + \beta\lambda_{m,i}(t))$ and the costate equation is

$$\dot{\lambda}_{m,i} = r\lambda_{m,i} - \frac{\partial \mathcal{H}_i}{\partial x} - \frac{\partial \mathcal{H}_i}{\partial u_j}\frac{\partial \Psi_j}{\partial x},$$

with country j's *expected optimal strategy* $\Psi_j(x, t) = -(bx + \beta\lambda_{m,j}(t))$. Thus, shadow value checks

$$\dot{\lambda}_{m,i} = (r + \delta - 2b\beta)\lambda_{m,i} + (c - b^2)x_m.$$

Country i's maximized Hamiltonian function reads

$$\mathcal{H}^*_i(x, \lambda_{m,i}, \Psi(x, t), t) = \mathcal{H}_i(x, \lambda_{m,i}, u^*_{m,i}, \Psi_j(x, t), t)$$

$$= -\tfrac{1}{2}(c - b_i^2)x^2 + (2b\beta - \delta)x\lambda_{m,i} + \tfrac{\beta^2}{2}\lambda^2_{m,i} + \beta^2\lambda_{m,i}\lambda_{m,j} + \lambda_{m,i}E(t),$$

which is strictly concave in x provided $c > b_i^2$. Provided that the *expected optimal response* of country j's feedback strategy, country i's objective is optimized by the above first order

[13] We only study one particular Markovian Nash equilibrium, there may be others. Notice that the Markovian Nash equilibria here are not necessarily sub-game perfect.

conditions. Thus, the above first order conditions constitute a non-degenerate Markovian Nash equilibrium.

Taking following *symmetric solution* with $u_{m,i} = u_{m,j} = u_m$ and $\lambda_{m,i} = \lambda_{m,j} = \lambda_m$, the optimal choice u_m, the state equation x_m and the costate variable λ_m, follow:[14]

$$\begin{cases} u_m = -(bx_m(t) + \beta\lambda_m(t)), \\ \dot{x}_m = E(t) + (2b\beta - \delta)x_m + 2\beta^2\lambda_m, \\ \dot{\lambda}_m = (c - b^2)x_m + (r + \delta - 2b\beta)\lambda_m, \end{cases} \tag{11}$$

with the initial condition for x known and the transversality condition for λ_m provided by

$$\lim_{t\to\infty} e^{-r_i t}\lambda_m(t)x_m(t) = 0 \text{ if } T = \infty, \quad \text{or } \lambda_m(T) = S_x(x^*(T)) \text{ if } T < \infty. \tag{12}$$

It is easy to check that there exist a positive nontrivial saddle path stable steady state for the pollution stock if and only if the following Assumption 2 holds.[15]

Assumption 2 $c > b^2$, $r + \delta - 2b\beta > 0$, $D_m = 2\beta^2(c - b^2) - (2b\beta - \delta)(r + \delta - 2b\beta) > 0$.

Under Assumption 2, system (11) has two eigenvalues $\xi_{m,i} = \dfrac{r \pm \sqrt{r^2 + 4D_m}}{2}, i = 1, 2$, with $\xi_{m,1} < 0$ and $\xi_{m,2} > 0$. Using the same arguments as before, we can get the explicit trajectory solution of system (11) replacing ξ_i by $\xi_{m,i}$.

To close this section, we state a result on the long-run stationary emissions of pollution:

Proposition 3 *Suppose emissions $E(t)$ grow at a constant rate g and parameters verify $2b^2 < \frac{b(r+\delta-g)}{\beta} < c$. There exists a balanced growth path for system (11) along which all endogenous variables, x_m, λ_m and u_m, grow at the same rate g. That is, $x_m = \widehat{x}_m e^{gt}$, $\lambda_m = \widehat{\lambda}_m e^{gt}$ and $u_m = \widehat{u}_m e^{gt}$, where $\widehat{x}_m = E(0)\frac{g-(r+\delta+2b\beta)}{G(b)}$ with $G(b) = (g + \delta)(g - (r + \delta)) + 2b\beta(r + 2\delta) - 2\beta^2(c + b^2)$ and $\widehat{u}_m = -\widehat{x}_m \frac{(g-(r+\delta))b+\beta(c-b^2)}{g-(r+\delta)+2b\beta}$.*

[14] The difference between the open-loop and Markovian strategy is the feedback effect which is presented by the extra term, $b\beta\lambda_m$, in the λ_m equation.

[15] If Assumption 2 does not hold, then either there is no positive steady state or the positive steady state is unstable.

5 Long-run Comparison- Feedback effects

In this section we study the feedback information when $E(t)$ is either constant or grows at a constant rate.

First, Assumptions 1 and 2 are not the same when $E(t)$ is a constant. If Assumption 1 holds and Assumption 2 fails, there is a positive steady state with open-loop strategies while there is no steady state with Markovian strategies. Similarly, if Assumption 1 fails and Assumption 2 holds, there is a positive steady state with Markovian strategies but not under open-loop strategies. In this section, we focus on the situation in which both open-loop and Markovian feedback strategies have positive pollution steady states.

In order to compare the effect of different strategies in the long run, let us assume that $E(t)$ is constant in the long run and that Assumption 2 holds in both open-loop and our special Markovian strategies. Combining Assumptions 1 and 2, we can impose

Assumption 3 *$c > b^2$, $\frac{\delta}{2\beta} < b < \frac{r+\delta}{2\beta}$, and $D_m > D > 0$.*

Assumption 4 *$c > b^2$, $b < \frac{\delta}{2\beta}(< \frac{r+\delta}{2\beta})$, and $D > D_m > 0$.*

Under Assumptions 3 or 4, both open-loop and Markovian strategies lead to a positive saddle path stable steady state, though the steady state values may not coincide. In fact, the long run steady state values of the pollution stock, $\overline{x}$ and $\overline{x}_m$ check

$$\frac{D_m}{r+\delta-2b\beta}\overline{x}_m = \overline{E} = \frac{D}{r+\delta-b\beta}\overline{x}.$$

Hence,

$$\frac{\overline{x}_m}{\overline{x}} < 1, \text{ provided } b < \sqrt{c} \tag{13}$$

from which we can deduce the following proposition:

Proposition 4 *Suppose pollutant emissions are constant $E(t) = \overline{E}$. Under Assumptions 3 or 4 the optimal Markovian feedback strategy leads to a lower level of long-run stock of pollution compared to the open-loop strategy. Furthermore, the Markovian strategy leads to faster convergence to its relatively lower steady state value for pollution stock, that is, $|\xi_1| < |\xi_{m,1}|$.*

Proposition 4 states that the special Markovian feedback strategy leads to a lower long-run pollution level than the open-loop commitment. The Markovian strategy offers information of the state of the economy at every time so that every country notices that reducing current pollutant emissions will reduce costs in the long term. Hence, Markovian strategies lead to a lower public bad. Our finding is in line with Wirl (1994) who shows a similar result: Markovian strategies yield less social waste.[16] Proposition 4 also shows that countries should take into account both the environmental cost, which may not be direct, and the state dependent cost of emissions reductions.

Besides, Markovian feedback strategies usually lead to free riding in the sense that there is lower "public goods in feedback than in open loop" and hence higher "public bad in feedback". However, this is not necessarily true in our framework. We have shown that if the environmental cost is high, $\sqrt{c} > b$, then it is better to increase the reduction of emissions. People would be more willing to pay to improve the environment rather than paying the health care bill. At the same time, players with open-loop strategies will stick to their commitments ignoring real-time information.

Further, comparing the two negative eigenvalues, ξ_1 and $\xi_{m,1}$, we have proven that the convergence speed is larger under feedback strategies, $|\xi_1| < |\xi_{m,1}|$. Again, depending on the cost coefficients b and c, the feedback strategy may provide countries with more information. This additional information leads to faster convergence to the lower steady state of the pollution level. Therefore, the Markovian Nash equilibrium can improve the long-run situation by giving the players chance to adjust their behaviors.

Now, let us consider the balanced growth case and suppose $E(t) = E(0)e^{gt}$. Careful calculation shows that

$$\frac{\widehat{x}}{\widehat{x}_m} = \frac{(g-(r+\delta)+b\beta)G(b)}{(g-(r+\delta)+2b\beta)F(b)}$$

and

$$\frac{\widehat{u}}{\widehat{u}_m} = \frac{\widehat{x}}{\widehat{x}_m}\,\frac{[g-(r+\delta)+2b\beta][(g-(r+\delta)b)+c\beta]}{[g-(r+\delta)+b\beta][(g-(r+\delta)b)+c\beta+\beta b^2]}.$$

As a consequence, we conclude that [17]

[16] See also Braden and Bromley (1981), Hoel (1991), and references therein.

[17] Detailed calculations are shown in the appendix.

Proposition 5 *Suppose that the conditions in Proposition 3 hold. Then along the balanced growth path, we have* $\widehat{x} > \widehat{x}_m$ *and* $\widehat{u} > \widehat{u}_m$ *with* $b > 0$*. If* $b = 0$*, then* $\widehat{x} = \widehat{x}_m$ *and* $\widehat{u} = \widehat{u}_m$*.*

This result shows that if the state dependent cost has a role, $b > 0$, then countries reduce emissions more under Markovian strategies than under open-loop strategies, *i.e.* $\widehat{u} > \widehat{u}_m$. Since the level of pollution under open-loop is always higher than under Markovian strategies $\widehat{x} > \widehat{x}_m$, the effect of emissions reductions is not efficient. Nonetheless, when the cleaning-up process is state independent, that is $b = 0$, the Markovian strategies coincide with the open-loop strategies along the balanced growth path. Therefore, the policy maker should study the pollution cost and whether it is or not state dependent. Only then she should choose between open-loop commitment or Markovian strategies.

6 Conclusion

We have presented an analysis starting from a strategic setting where two countries have to decide about their effort to reduce pollutant emissions through capture and storage mechanisms. Both countries benefit from these reductions, whereas the effort of reduction is private. Except offering explicit short run trajectory dynamics, two particular strategies are analyzed. First, both countries decide upon the emissions reduction once and for all independently of the evolution of the environment. Countries choose open-loop strategies and one could assimilate this strategy to a situation where both parties sign a treaty binding during a fixed time period, with commitments of emissions reductions. We show that in this case total emissions of pollutants decrease and the environmental quality improves when pollution cost is high enough. In the second case, the two countries play in Markovian strategies. Contrary to the common belief (see Dockner *et al.* (1993, 2000)), we find that if pollution is initially high, then Markovian strategies lead to a cleaner environment than opne-loop strategies. Intuitively, the countries tend to reduce emissions more when they can adjust their efforts from one period to another. From a policy point of view, governing carbon capture and storage mechanisms can do better in flexible mechanisms.

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A Appendix

A.1 Proof Proposition 1

Let us write system (6) in matrix form. Let

$$X(t) = \begin{pmatrix} x(t) \\ \\ \lambda(t) \end{pmatrix}, \quad A = \begin{pmatrix} 2b\beta - \delta & 2\beta^2 \\ \\ c - b^2 & r + \delta - b\beta \end{pmatrix}, \quad A_0 = \begin{pmatrix} E(t) \\ \\ 0 \end{pmatrix}.$$

Then, the above linear system (6) is equivalent to $\dot{X}(t) = AX + A_0$.

With Assumption 1, it is easy to see that matrix A has two eigenvalues

$$\xi_1 = \frac{r + b\beta - \sqrt{(r+b\beta)^2 + 4D}}{2}(< 0), \;\; \xi_2 = \frac{r + b\beta + \sqrt{(r+b\beta)^2 + 4D}}{2}(> 0).$$

Hence, the explicit solution of system (6) (or its matrix form) is given by

$$X(t) = e^{tA}X_0 + \int_0^t e^{(t-s)A}A_0(s)ds,$$

where $X(0) = \begin{pmatrix} x(0) \\ \lambda(0) \end{pmatrix}$, $x(0) = x_0$ is initial condition, $\lambda(0)$ is undetermined and will be determined by transversality condition. Exponential term

$$e^{tA} = V^{-1} \begin{pmatrix} e^{\xi_1 t} & 0 \\ \\ 0 & e^{\xi_2 t} \end{pmatrix} V$$

in which matrix V^{-1} is the inverse of eigenvector matrix V, and which are given by

$$V = \begin{pmatrix} -2\beta^2 & -2\beta^2 \\ \\ 2b\beta - \delta - \xi_1 & 2b\beta - \delta - \xi_2 \end{pmatrix}, \quad V^{-1} = \frac{1}{\det(V)} \begin{pmatrix} 2b\beta - \delta - \xi_2 & 2\beta^2 \\ \\ -(2b\beta - \delta - \xi_1) & -2\beta^2 \end{pmatrix}.$$

Hence, the explicit solution can be given as following.

$$\begin{aligned} x(t) &= \frac{1}{\xi_2 - \xi_1}\left[(2b\beta - \delta - \xi_1)e^{\xi_2 t}\left(x(0) + \int_0^t e^{-\xi_2 s}E(s)ds\right)\right. \\ &\quad \left. - (2b\beta - \delta - \xi_2)e^{\xi_1 t}\left(x(0) + \int_0^t e^{-\xi_1 s}E(s)ds\right)\right] \\ &\quad + \frac{2b\beta - \delta - \xi_2}{\xi_2 - \xi_1}\left(e^{\xi_1 t} - e^{\xi_2 t}\right)\lambda(0), \end{aligned} \tag{14}$$

and

$$\lambda(t) = \frac{(2b\beta-\delta-\xi_1)}{\xi_2-\xi_1}\left[\left(e^{\xi_1 t}-e^{\xi_2 t}\right)x(0)+\int_0^t\left(e^{\xi_1(t-s)}-e^{\xi_2(t-s)}\right)E(s)ds\right] + \frac{\Lambda(t)}{\xi_2-\xi_1}\lambda(0) \tag{15}$$

with

$$\lambda(0) = \frac{\xi_2-\xi_1}{\Lambda(T)}\lambda(T) - \frac{(2b\beta-\delta-\xi_1)}{\Lambda(T)}\left[\left(e^{\xi_1 T}-e^{\xi_2 T}\right)x(0)+\int_0^T\left(e^{\xi_1(T-s)}-e^{\xi_2(T-s)}\right)E(s)ds\right] \tag{16}$$

and $\Lambda(t) = (2b\beta-\delta-\xi_1)e^{\xi_1 t} - (2b\beta-\delta-\xi_2)e^{\xi_2 t}$. This finishes the proof.

A.2 Proof Proposition 5

It is easy to see that

$$\frac{\widehat{x}}{\widehat{x}_m} = \frac{(g-(r+\delta)+b\beta)G(b)}{(g-(r+\delta)+2b\beta)F(b)}$$

Define $A = (g-(r+\delta)+b\beta)G(b)$ and $B = (g-(r+\delta)+2b\beta)F(b)$. Then

$$\begin{aligned} A-B &= (g-(r+\delta))(G(b)-F(b)) + b\beta(G(b)-2F(b)) \\ &= (g-(r+\delta))b\beta(g+\delta-2b\beta) + b\beta[-(g+\delta)(g-(r+\delta)) - 2b\beta(r+\delta-g) + 2\beta^2(c-b^2)] \\ &= 2b\beta^3(c-b^2), \end{aligned}$$

that is, $A > B$ if $b > 0$, and hence, $\widehat{x} > \widehat{x}_m$; and if $b = 0$, $A = B$ and $\widehat{x} = \widehat{x}_m$.

The proof is finished since the same arguments are also true for $\frac{\widehat{u}}{\widehat{u}_m} \geq 1$.