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Asymmetric Competition among Nation States. A differential game approach*

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Abstract

The aim of this paper is to analyse the impact of foreign investments on a small country's economy in a context of international competition. To that end, we model tax and infrastructure competition within a differential game framework between two unequal sized countries. The model accounts for the widely recognised characteristic that small states are more flexible in the political decision-making than bigger countries. On the other hand, we also acknowledge that small size is associated with limited institutional capacity in the provision of public goods. The model shows that the long run outcome of international competition crucially depends on the degree of capital mobility. We show in particular that flexibility mitigates a small economy's likelihood to collapse without eliminating its possible occurrence. Finally we highlight that the beneficial effect of flexibility of a small state increases with its inefficiency to provide public infrastructures and with the degree of international openness.

Keywords: Tax/infrastructure competition, differential games, open-loop/Markovian strategies.

JEL classification: H25, H73, O30, O43.

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1 Introduction

Small states generally suffer from very limited capital and labor resources both in amount and in variety. It is then natural that attracting foreign production factors can be an important way to fill in this gap. FDIs represent a potentially critical contributor to the development of small island developing states (Read, 2008). As a matter of fact, small economies tend to get more private capital from abroad as a ratio of total capital formation (Streeten, 1993). It is therefore not surprising that small states are highly open to international trade and capital flows (Alesina and Wacziarg, 1998). Indeed, using data from the World Bank, Figure 1 suggests that the share of FDI flows as a percentage of gross fixed capital formation is higher in small countries (populated by less than two million people) than in large countries (which population exceeds 30 million people). Furthermore, many small countries above the average line exhibit a high level of GDP per capital like Luxembourg, Malta, Cyprus or Estonia, whereas many small countries below the line have a lower level of per capita GDP. While this feature is quite clear in the cluster of small countries, it is difficult to find such a link in the cluster of larger countries. Iraq, Italy, Afghanistan, India and Spain are above the line, while USA, Ukraine, Nepal, and Greece among others, are situated below[1]. This feature appears clearly in Figure 2 where small countries presenting a high level of FDIs have also a high level of GDP per capita [2]. It follows that the role of FDIs seems more critical for the development of small nations, which thus have to compete internationally for mobile production factors. In this context it is interesting to investigate under which conditions such countries are viable in the long term and can even expand. To that end, we develop a dynamic framework to study how a small country tries to attract foreign capital by using two policy instruments, namely taxes and public infrastructures[3].

[1] This ambiguity of the role of FDIs on the economic performance of countries is already documentated in the literature (see for instance Alfaro et al. 2004).

[2] However, we have not controlled for other determinants of the per capita GDP level like for example, the availability of natural resources. Taking into account oil reserves and the recent increase in oil prices would explain the position of Qatar or Brunei in our figures.

[3] These public goods contribute to the domestic attractiveness of private capital since they are supposed to enhance private productivity. Illustrations can be transportation infrastructures, universities and public R&D investment, but also property rights enforcement, capital market regulations, labor and environmental regulations. It follows that

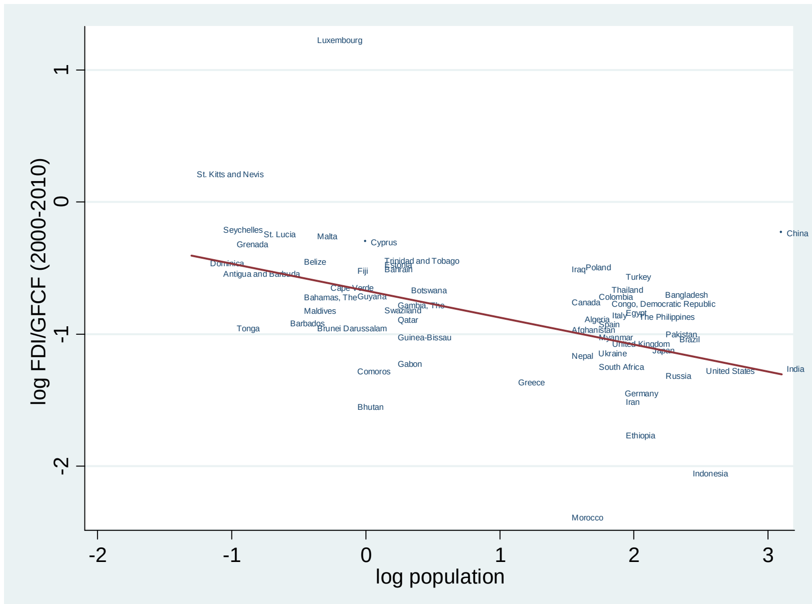

Fig 1

countries'attractiveness may also due to the quality of their institutions. In the Oxford Handbook of Entrepreneurship (2007), it is argued that the abundance of entrepreneurs in a country depends, among other factors, on the existence of regulations, property rights, accounting standards and disclosure requirements. Furthermore, in recent years there has been a surge of country and cross-country studies relating economic development to institutions, especially those affecting capital market development and functionality (La Porta et al. (1997) among others).

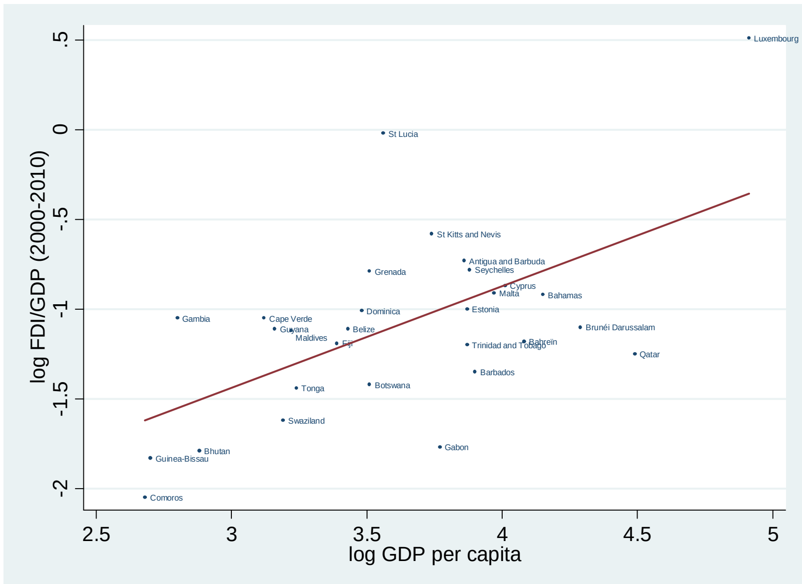

Fig 2

For the sake of simplicity, we focus on two competing countries of asymmetric size. The notion of size we use is defined with respect to the number of capital-owners who populate the different states. These capital owners are at the same time entrepreneurs and workers. Doing so, our model identifies country with economic size. The dynamic aspect of international competition is addressed by a differential game framework in which the strategic behavior of the small country differs from that of its larger rival. In fact, we account for the widely recognized characteristic that small states are more flexible in the political decision making than much bigger countries (see in particular Streeten, 1993). We thus assume that the small country adopts a Markovian feed-back behavior (the policy variables are continuously reset in response to the dynamics of the states of the world), whereas the larger country chooses an open-loop rule (the policy variables are set once for all at the initial period). We also acknowledge that small size is associated with handicaps. Indeed, small economies are generally characterized by limited institutional capacity in the provision of public goods (Commonwealth Secretariat, 2000) relative to big countries. We finally assume that the capital owners living in the two countries are heterogeneous in their attitude toward their attachment to home. For that reason, they are supposed to incur a cost of moving abroad which depends on their specific types. Additionally, their decision of relocating their capital is affected by capital taxation and by productivity-enhancing public infrastructures.

The main results of the paper can be summarized as follows. Two cases emerge. One in which international openness is high and another where international openness is low. The fundamental difference between these cases is that the small country only may collapse if capital mobility is high (high international openness). Furthermore, the higher mobility is the more likely the small economy will implode. However, higher efficiency in providing public infrastructures can partially countervail this effect and thus lessen the likeliness to collapse. In the second case, when capital mobility is low, international competition for capital can eventually lead to the decrease of the economic potential of the small economy without provoking its collapse. If capital mobility is very low, the model shows that international competition can only expand the small country's economy. We also assess to what extent flexibility is beneficial for the small country given that it suffers from limited institutional capacity. Comparing the Markovian and open-loop outcomes shows that flexibility mitigates a small economy's likelihood to collapse without eliminating its possible occurrence. Finally we show that the beneficial flexibility effect increases with the small country's inefficiency to provide public infrastructures and with the degree of international openness.

Our paper contributes to different strands of the existing literature. First, we provide a dynamic counterpart to previous static papers in which countries compete with two instruments. After Tiebout's seminal paper (1956), there has been a growing literature on the joint role of taxes and public inputs to attract mobile production factors. For example, Benassy-Quéré et al. (2007) provide an empirical analysis of the impact of taxes and public infrastructure on the allocation of private capital. They find that both corporate tax and public capital are significant in explaining inward FDIs. Pieretti and Zanaj (2011) propose a two-stage game in which a small and a large jurisdiction compete for capital using taxes and public goods as policy variables. These contributions are however static and thus unable to provide insights in long run outcomes. This is in particular true if the focus is on the long run behavior of small sized economies.

Differential games have been already applied to model oligopolistic competition (Dockner and Jorgensen, 1984, Karp and Perloff, 1993, Ceillini and Lambertini, 2004), but there exists only few applications to tax competition. For example, Coates (1993) deals with the issue of property tax competition and partially analyzes[4] the open-loop equilibrium of a dynamic game. Furthermore, we provide a differential game approach where the strategies of the players may differ and we show how important is the type of strategy for the long run features of the country-players. Dynamic interactions among jurisdictions to attract mobile factors have already been analyzed using the framework of repeated games. The main issue studied by this literature is the tax coordination problem between symmetric regions (Cardarella et al., 2002, Catenaro and Vidal, 2006, Itaya et al., 2008). The purpose of this paper is however different. It is not intended to model a repeated game between jurisdictions[5].

Unlike Alesina and Spolaore (1997) we regard the political size of a country as being exogenously given. We consider however that the economic magnitude expressed in terms of productive resources vary endogenously as a consequence of public policy and international competition. Like our model, the contribution of de la Croix and Dottori (2008) is also concerned with the collapse of a community. Indeed, to explain the tragedy of Easter Island, these authors show how a closed system can collapse as a result of a non-cooperative bargaining between clans. The context and the methodology of their paper is however different from ours since they use an overlapping generations model in which people live for two periods and compete in fertility rates.

The paper is organized as follows. The next section models the dynamic competition between two countries of asymmetric size. In Section 3, we derive the long run solutions. Section 4 analyzes the long run conditions of a small country. The importance of flexibility in small economies is assessed in section 5. Section 6 concludes.

2 The model

Suppose that the world is composed of two countries (regions) of unequal population size. Country size may be defined by its population, by its area, or by its national income (Streeten, 1993). We focus on the *population* aspect rather than on the spatial size. More precisely, size is defined with respect

[4]As mentioned by Cardarelli (2002).

[5]Repeated games are discrete games in which the same strategy is played in each period (Dockner et al., 2000). Differential games introduce state variables to describe the dynamic system whose evolution is modelled by a set of differential equations (Dockner et al., 2000).

to the number of capital-owners who populate the different states. These capital-owners are at the same time entrepreneurs and workers. Doing so, our model identifies country with economic size. Furthermore, capital-owners are free to relocate with their activity across countries at any point in time. At time $t = 0$, capital flows have not yet taken place, so population size in each country coincides with the number of natives.

At time $t = 0$, the population of jurisdictions is evenly distributed with unit density on the interval $[-S_1(0), S_2(0)]$. The small country extends from $-S_1(0)$ to the origin 0, and the rest of the world extends from 0 to $S_2(0)$. It follows that the small economy has a size $S_1(0)$ and the rest of the world a size $S_2(0)$ with $S_1(0) < S_2(0)$. We assume that the total number of firms is constant over time and normalized to one. We thus set, for any future time $t \geq 0$, $S_1(t) = S(t)$ and $S_2(t) = 1 - S(t)$.

Entrepreneurs Each citizen is endowed with one unit of capital which is combined with her labor to set up a firm. Hence, citizens are self-employed entrepreneurs. In the rest of the paper, we thus use firms and entrepreneurs indifferently. The firms are distributed on their respective sub-interval according to their disposition to set up a firm outside their home location. As in Ogura (2006), we assume that the population of entrepreneurs is heterogeneous in the degree of their attachment to home[6]. In our spatial setting we impose that the closer entrepreneurs are located to the extremes, the more they are attached to their current location. Conversely, the closer firms are to the border 0, the less they are attached to their territory and the easier they are able to relocate abroad[7]. This means that a firm of type $\alpha \in [-S_1(0), 0]$ located in the home country incurs a disutility of relocating abroad equal to kx, where x is the distance between 0 and α. The coefficient k represents the unit cost of moving capital abroad which can also be interpreted as the degree of international openness.

As in Pieretti and Zanaj (2011), we assume that each firm produces $q + a_i$ $(i = 1, 2)$ units of a final good, where q is the private component of (gross) productivity. The fraction a_i of the produced good depends on the public input supplied by the home (foreign) jurisdiction[8]. Notice that the product

[6] Heterogeneity in home attachment was first considered in the fiscal competition literature by Mansoorian and Myers (1993).

[7] For reasons of simplicity, we assume that relocation is only possible in the neighboring jurisdiction.

[8] The public input satisfies the local public good characteristic, which means that it is

$S_i \cdot (q + a_i)$ represents the total output produced in country $i = 1, 2$. This implies that $q + a_i$ is per capita output. The produced output is sold in a competitive (world) market at a given price normalized to one. We thus suppose that both countries have equal access to a common market. This also implies that the small jurisdiction does not suffer from a reduced home market. We further consider that the unit production cost is constant and equal to zero without loss of generality.

Furthermore, we adopt a temporal perspective of the above setting. For each period $t \in [\Delta t, +\infty)$ and for any $\Delta t > 0$, governments update their choices in terms of offered public goods and taxes[9].

Suppose now that an entrepreneur of type $x(t)$ initially located in the small country considers to stay at home or to invest her physical capital abroad. If she decides not to move, her profit is given by[10]

$$\pi_1(t) = q(t) + a_1(t) - T_1(t). \tag{1}$$

If she invests abroad, her profit becomes

$$\pi_2(t) = q(t) + a_2(t) - T_2(t) - k \cdot x(t).$$

It follows that the marginal entrepreneur x who is indifferent between investing abroad and staying at home verifies the condition

$$q(t) + a_1(t) - T_1(t) = q(t) + a_2(t) - T_2(t) - k \cdot x(t).$$

Consequently, we obtain

$$x(t, a_1, a_2, T_1, T_2) = \frac{a_2(t) - T_2(t)}{k} - \frac{a_1(t) - T_1(t)}{k}. \tag{2}$$

jointly used without rivalry by firms located in the same jurisdiction. It follows that the benefits and the costs of these good only accrue at the jurisdictional level. As in Zissimoss and Wooders (2008), we shall abstract from congestion costs . Taking account of congestion would complicate our framework without improving qualitatively the results. Moreover, if the public input represents immaterial goods as law and regulations (protecting intellectual property, specifying accurate dispute resolution rules,...), the absence of congestion is easily justified by the particular nature of these goods.

[9] Notice that we do not consider investments as sunk costs or we assume that the time period t is long enough to make the sunk costs of irrelevant.

[10] For sake of simplicity, we consider that q is such that the profit of each firm is positive for all equilibrium level of public goods and taxes.

In other words, the big country attracts capital ($x > 0$) from the small jurisdiction if the net gain of investing abroad, $a_2(t) - T_2(t)$, is higher than the net gain of staying at home, $a_1(t) - T_1(t)$, after taking into account the mobility cost kx. If $x < 0$,capital moves from the big jurisdiction to the small one.

The motion equation of the small country's size $S(t)$ is given by

$$\dot{S}(t) = -x = \frac{a_1(t) - T_1(t)}{k} - \frac{a_2(t) - T_2(t)}{k}. \tag{3}$$

We further assume that the preferences for the home location will change according to the relative attractiveness of the competing jurisdictions in the following way. For the firms that do not move, attachment to home will increase by x if the small economy is attractive to foreign investors ($x < 0$) and it will decrease if the foreign location attracts capital from the small country ($x > 0$). For the capital owners who relocate abroad, the attachment to the new location decreases with the attachment they had to the country they left.

Governments Adopting a public-choice perspective, we posit that the governments are tax revenue maximizers[11]. For this purpose, the above open countries compete simultaneously in taxes and public infrastructures to attract entrepreneurs. Given the policy mix chosen by the governments, firms decide where to locate. We suppose that the effective (net) tax revenue at the disposal of governments does not coincide with the amount of tax revenue collected. In the same spirit as Vaillancourt (1989) or Blumenthal and Slemrod (1992), tax collection is costly due to administering, monitoring and enforcing procedures (Kenny and Winer, 2006). If the marginal cost of collecting taxes is rising, then the net tax revenue $R(t)$ at time t is a convex function of the collected taxes. For tractability reasons, the net tax revenue will be given by, $R(t) = \sqrt{S_i(t)T_i(t)}$.

The instantaneous objective function of government $i (i = 1, 2)$ is thus given by

$$w_i(T_i(t), a_i(t)) = \sqrt{S_i(t)T_i(t)} - \frac{\beta_i}{2}a_i^2(t), \tag{4}$$

[11]This assumption should not be interpreted in the classical sense given by Brennan and Buchanan (1980) to Leviathan governments. We do not consider here that regulators are self-interested governments. We simply assume that the amount of collected taxes is used for public goods which do not directly affect the productivity of firms like green spaces, swimming pools, security bodies, etc.

where the second term is the cost of providing public inputs, assumed quadratic for tractability, whereas β_i is a country specific efficiency parameter. Indeed, the higher the value of β_i, the higher will be the unit and marginal costs of providing public infrastructures.

In this paper the key focus will be on the long run behavior of small states. For that purpose, we highlight two opposing features of small open economies.

First, according to the Commonwealth Secretariat (2000) the public sector of mini-states generally suffers from limited institutional capacity.[12] Moreover, small countries may face a certain difficulty to recruit a high-quality civil service given the limited pool of candidates in small states (Streeten, 1993). These facts can be translated into low efficiency and high unit costs in the provision of public services. To take account of these facts, we assume that $\beta_1 \equiv \beta > \beta_2$. Normalizing β_2 to 1, we impose $\beta > 1$.

Second, the small size can be considered as an asset (Kuznets, 1960; Easterly and Kray, 2000) given the economic success of many micro-states. Streeten (1993) suggests that collective action problems may be solved more easily in small countries[13], while the larger jurisdiction is not able or not willing to attain this degree of flexibility in decision making[14]. To capture this difference we assume that the large jurisdiction commits to a policy path adopted at the beginning of the game (open-loop strategy) whereas policy-makers of the small jurisdiction adopt a Markovian feed-back strategy.

This mixed representation offers a convenient way of modelling differences in decision flexibility (Dockner et al., 2000). Note that the mini-state, though being small in a political sense, can grow bigger as a result of sustained capital inflows. The small country's size could thus exceed a critical threshold that would urge the big country to react in a more aggressive way by adopting a

[12] The median wage bill of the public sector as a proportion of GDP is 31 percent in small states compared to a ratio 21 per cent in large developing countries (Commonwealth Secretariat and World Bank, 2000).

[13] These attributes facilitate greater single-mindedness and focus on economic policy-making and a more rapid and effective response to exogenous change (Armstrong and Read, 1995). Hence, in the present paper, we assume that the small economy updates its decision variables at each period t and is thus able to condition its actions on current observations.

[14] This could result from higher costs of social and political heterogeneity. Indeed after having reached policy consens, altering a previously decided policy package could be a very sensitive issue in a large country. Another reason may result from the extreme smallness of the mini-state which appears as unimportant to the large economy.

Markovian strategy too. In order to rule out such a behavioral change, we assume that the small country's economic magnitude will be bounded from above in a way that it never will be larger than the rival economy's size. Therefore, we impose that $S(t) \leq \overline{S} < \frac{1}{2}$ for any $t \geq 0$, where $\overline{S}$ is an upper bound. Without loss of generality, we shall assume that the bound is $\overline{S} = \frac{1}{2}$.

The dynamic objective-functions of the competing jurisdictions are respectively

$$J_1 = \max_{a_1, T_1} \int_0^{+\infty} e^{-rt} w_1(T_1(t), a_1(t)) dt, \tag{5}$$

$$J_2 = \max_{a_2, T_2} \int_0^{+\infty} e^{-rt} w_2(T_2(t), a_2(t)) dt, \tag{6}$$

where r is the discount rate of the public decision-makers which is supposed to reflect the degree of impatience of the population. Since there is no evidence that this rate should be dependent on a population's size, we accept that r is common to both jurisdictions.

3 Steady states and the long run policy mix

As explained above, we consider that the small jurisdiction adopts a Markovian strategy while the larger rival chooses an open-loop approach[15] in designing its optimal decision path. In the appendix we provide the full solution of this game. The steady state[16] size of the small country is

$$\hat{S} = \frac{(kr)^{-\frac{3}{2}}}{6\sqrt{2}} \left(\frac{\sqrt{2}}{\beta} - 1 \right) + \frac{2}{3}.$$

To guarantee that the small country remains smaller than $\overline{S} = \frac{1}{2}$ in the long term, we impose that $k < k^* = \left(\frac{1}{2}\right)^{\frac{1}{3}} \frac{1}{r}$ and $\beta > \underline{\beta} = \frac{\sqrt{2}}{1-\sqrt{2}(kr)^{\frac{3}{2}}}$. The long run

[15] In this present framework, we only pick up one heterogenous strategic Nash equilibrium (Definition is given in the Appendix.), rather than taking into account all of the possible strategies. Moreover, the strategy under study is not subgame perfect, though it is in deed one nondegenerate Markovian Nash equilibrium (See appendix for detail).

[16] This interior steady state is saddle point stable and we present its convergence path as well in the appendix.

policy mix of countries in terms of taxes and public infrastructure is

$$\widehat{a}_1 = \frac{1}{2\beta}\left(\frac{1}{kr}\right)^{\frac{1}{2}}, \qquad \widehat{T}_1 = kr\widehat{S},$$
$$\widehat{a}_2 = \frac{1}{2}\left(\frac{1}{2kr}\right)^{\frac{1}{2}}, \qquad \widehat{T}_2 = 2kr(1-\widehat{S}).$$

These values allow us to define the long run per capita $(\widehat{a}_i + q)$ output of country $i = 1, 2$. According to the above solutions, it is possible to show that the variable $\widehat{a}_i$ [17] augments with the long term size $\hat{S}$. Since $\hat{S}$ is positively related to FDI inflows, our model reproduces a stylized fact we highlighted in the introduction (see Figure 1), according to which the per capita output of small economies improves with inward foreign investments.

We also easily verify that $\widehat{a}_2 - \widehat{a}_1 = \frac{1}{4\beta}\left(\sqrt{2}\beta - 2\right)\sqrt{\frac{1}{kr}} > 0$ for $\beta > \sqrt{2}$ and $\widehat{T}_2 - \widehat{T}_1 = kr\left(2 - 3\widehat{S}\right) > 0$ given that $\widehat{S} < \frac{1}{2}$. In other words, the small economy will always be tax competitive but will never be attractive in terms of public infrastructures. This result is reminiscent of the literature on tax competition among economies of uneven size (Bucovetsky, 1991, Wilson, 1991, Kanbur and Keen, 1993, Trandel, 1994,) according to which the benefit of smallness translates into the ability to undercut the tax rates of larger countries. Contrary to another strand of the literature devoted to inter-jurisdictional competition in taxes and public infrastructures, our model does not generate equilibria in which the small economy taxes more than its larger rival (Hindriks *et al.* (2008), Pieretti and Zanaj, 2011). The reason is that the small country is at a disadvantage in providing public infrastructures due to the limited capacity of its public sector.

We further see that the less efficient the small country is in providing public infrastructures the more it is tax attractive and the less it is attractive in public infrastructures. Indeed, the gaps $\widehat{a}_2 - \widehat{a}_1$ and $\widehat{T}_2 - \widehat{T}_1$ rise with β. It is also interesting to notice that increasing international openness (lower k) has the same effect as rising β on both gaps. Thus, the higher capital mobility, the more the small country will be inclined to undercut its rival tax rates.

Finally, the long run solutions have to guarantee non-negative net budget constraints of both economies. For this reason one of the following two

[17] The steady-state value $\hat{a}_i$ written as a function of $\hat{S}$ is : $\hat{a}_i = 3kr(\hat{S}_i - \frac{2}{3}) + (\frac{1}{2kr})^{\frac{1}{2}}$. It follows that $\frac{\partial \hat{a}_i}{\partial \hat{S}_i} > 0$ is always true.

conditions must hold. Either (a) $k^{*} > k \geq \overline{k}$ with $\overline{k} = (\frac{1}{32})^{\frac{1}{3}}\frac{1}{r}$, or (b) k verifies $\underline{k} < k \leq \overline{k}$ with $\underline{k} = \left(\frac{1}{50}\right)^{\frac{1}{3}}\frac{1}{r}$ and β satisfies $\underline{\beta} < \beta \leq \overline{\beta}$ and $\overline{\beta} = \frac{1}{2\sqrt{2}-16(kr)^{\frac{3}{2}}}$. The budget constraint of the big country will be satisfied if $\widehat{w}_1 \geq 0$, since the conditions on parameters are less stringent for the big country than for the smaller rival [18].

Furthermore, since we imposed the condition $S(t) \leq \frac{1}{2}$, we must have that[19] $S = \frac{1}{2}$ if the efficiency parameter β satisfies[20] $\underline{\beta}(\underline{k}) \leq \beta \leq \underline{\beta}$ with $\underline{\beta}(\underline{k}) = \frac{\sqrt{2}}{1-\sqrt{2}(\underline{k}r)^{\frac{3}{2}}}$.

4 Will small states survive in the long run?

In this section, we focus our attention on the conditions under which the economic magnitude of the small economy will expand, shrink or even collapse. In the face of the current wave of globalization which implies increasing international mobility, we analyze the long run size variation of the small state according to capital mobility. Two cases should be considered:

Case 1 *High degree of international openness :* $\underline{k} < k < \overline{k}$.

In this case, the survival of the small economy depends crucially on its relative efficiency of providing public goods. Two sub-cases can be distinguished. One in which capital mobility is very high, i.e. $\underline{k} < k < k^{s}$ with[21] $k^{s} = \dfrac{1}{2\left[2+S(0)\right]\right)^{\frac{2}{3}}}\dfrac{1}{r}$ and a second one in which capital mobility is moderately high, i.e. $k^{s} < k < \overline{k}$. In the first sub-case, it is readily verified that the small economy expands in the long run, $\widehat{S} > S(0)$, if $\beta < \overline{\beta}$. However, if the small economy's relative efficiency in providing public infrastructures is too low, that is, $\beta > \overline{\beta}$, it will collapse. Furthermore, the closer the mobility cost approaches its lower bound $\underline{k}$, the more likely the small country will collapse. This occurs because the small economy has to lower its taxes to such an extent that it is no longer able to sustain its public

[18]It also appears that in either case of (a) or (b), we have $\widehat{S} \in \left[0, \frac{1}{2}\right)$.

[19]See proof in Appendix A.3

[20]More generally, we could impose that $S(t) \leq \overline{S} < \frac{1}{2}$. If so, $\underline{\beta}$ would depend on the upper bound $\overline{S}$. We thus get $\underline{\beta}(\overline{S}) = \frac{\sqrt{2}}{1+6\sqrt{2}(\overline{S}-\frac{2}{3})(kr)^{\frac{3}{2}}}$, which is decreasing in $\overline{S}$.

[21]It is readily verified that $k^{s} < \overline{k}$ if $0 < S(0) < \frac{1}{2}$.

expenditures ($\widehat{w}_1 = 0$). Strikingly two extreme scenarios can only emerge in the long run. Either the small economy expands or collapses. So if it shrinks it must collapse.

This extreme picture changes in the second sub-case. According to the values taken by β, the small economy can expand, collapse and shrink without collapsing. If $\beta < \beta^s$ with $\beta^s = \frac{\sqrt{2}}{1-6\sqrt{2}\left[\frac{2}{3}-S(0)\right](kr)^{\frac{3}{2}}}$, it will expand and if $\beta > \overline{\beta}$, it will collapse. For an intermediate efficiency value, i.e. $\beta^s < \beta < \overline{\beta}$, the small country will shrink but still survive. The following proposition can then be stated:

Proposition 2 *Assume that international openness is high. The small country can expand its economic size if it is not too inefficient in providing public infrastructures. Otherwise its size will shrink or even collapse in the long run.*

That means that in a world where capital can easily move, a small economy may have problems to survive even if it has the capacity to adapt more quickly to changing situations than bigger countries. In addition to tax competitiveness, governments also offer public inputs which increase firms' productivity. Relative efficiency in providing these infrastructures is therefore crucial (at least) for the survival of such small economies. The model shows that below a given threshold, rising capital mobility entices the small economy to cut its taxes to such an extent that its budgetary resources may vanish. It follows that especially micro-states can secure their existence in a global economy if their public sectors are sufficiently efficient in providing public infrastructures and their tax rate are not harmonized with those of larger countries. At best, this is a necessary condition for being attractive to foreign capital or at least to be able to survive.

Case 3 *Low degree of international openness: $k^* > k > \overline{k}$.*

Now, relative inefficiency of providing public goods is no longer a threat of collapsing since budget resources never run out. Formally, the limit value $\overline{\beta}$ tends to ∞ if k approaches $\overline{k}$. This contrasts importantly with the first case. The reason is that a low degree of financial openness makes capital more captive, which guarantees sufficient tax revenues to cover infrastructure costs. The worst case the small country can face is to suffer shrinkage of its economic size $(0 < \widehat{S} < S(0))$. This happens if $\widehat{k} > k > \overline{k}$ and $\beta > \beta^s$, with

$\widehat{k} = (\frac{1}{8[2-3S(0)])^2})^{\frac{1}{3}}\frac{1}{r}$. However, if mobility is very low, i.e. $k^* > k > \widehat{k}$, the small economy will attract foreign capital and will thus expand. We conclude with the following proposition:

Proposition 4 *Assume that international openness is low. The small country's economic size never collapses but may however shrink if the degree of international openness is not low enough. In either case, its survival is independent of its efficiency in providing public infrastructures.*

We provide a summary illustration of the different cases with respect to the parameter values of k and β in Figure 2

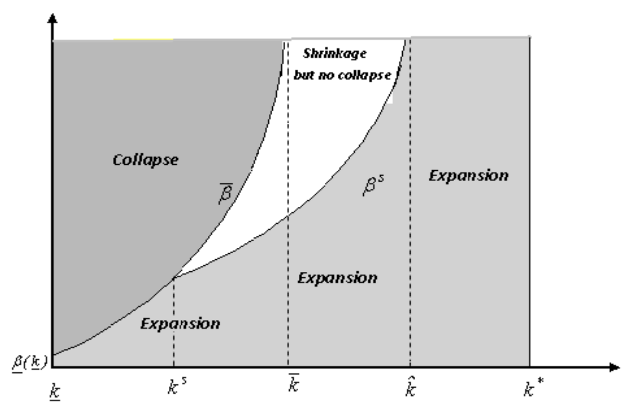

Figure 2

5 How important is flexibility for the small economy?

Now we wish to assess how beneficial for the small country flexibility is. Towards this end, we first calculate the long run size $\widetilde{S}$ of the small country if it chooses an open-loop behavior like its large rival. We thus obtain:

$$\widetilde{S} = \frac{(kr)^{-\frac{3}{2}}}{4}(\frac{1}{\beta} - 1) + \frac{1}{2}.$$

The flexibility effect can be caught by the difference $\widehat{S} - \widetilde{S}$, which is obtained by comparing the Markovian and open-loop outcomes. It is easy to check that this difference is always non negative. Hence, given the same parameters, the Markovian behavior adopted by the small country is preferable to the open-loop behavior. However, flexibility is not able to totally exclude collapse. It only makes its occurrence less likely.

Since we have $\frac{\partial(\widehat{S}-\widetilde{S})}{\partial\beta} > 0$, the small country's flexibility advantage increases with its inefficiency to provide public infrastructures. This effect is however weaker in the Markovian scenario[22]. In other words, flexibility acts as a countervailing force to inefficiency and the more a small country is inefficient in providing public inputs, the more valuable is flexibility for its long run survival.

Furthermore, higher capital mobility increases the relative advantage of flexibility since $\frac{\partial(\widehat{S}-\widetilde{S})}{\partial k} < 0$. Notice that higher mobility decreases the small economy's long term size but this occurs to a lesser extent in the Markovian scenario. It follows that flexibility countervails the negative effect of high capital mobility.

Finally, in the Markovian scenario the small country never collapses when the capital mobility is low enough ($k > \overline{k}$), whatever the level of inefficiency β. This condition becomes more restrictive if the small country adopts an open-loop behavior. Indeed, the absence of flexibility in policy making requires now that $k > \overline{\overline{k}} > \overline{k}$ with $\overline{\overline{k}} = (\frac{1}{4})^{\frac{1}{3}}\frac{1}{r}$.

6 Conclusion

In this paper, we investigate in particular whether a small open economy can survive in the long run when facing world competition. To this aim, we model the dynamic competition between two unequal sized economies. The policy makers of these two countries compete simultaneously in taxing mobile capital and offering public infrastructures. Firms choose to locate their capital in the country where their profits are maximized. We characterize the heterogenous behaviors of the two governments within a differential game framework, in which the small state adopts Markovian (flexible) behavior, while its larger rival commits to a pattern of strategies initially decided (open-loop behavior).

[22]In fact, the sensitivity to β of the small country's long run size is larger when the small country is not flexible, $\left|\frac{\partial\widehat{S}}{\partial\beta}\right| < \left|\frac{\partial\widetilde{S}}{\partial\beta}\right|$.

The results show that in case of high capital mobility the small economy will have problems to survive if it is weakly efficient in providing public infrastructure. When capital mobility is very low, the small state always expands despite its limited institutional capacity.

Further research is however needed. One reason is that the states are treated as mere tax revenue maximizers. This over-emphasizes the role of tax rates in the long run outcomes. It therefore would be interesting to analyze the case where governments are welfare maximizers who take into account the well-being of their populations. The present paper also models the private sector in an elementary way. Countries are undifferentiated in their ability to produce private goods and the production process is static. A future reflection should thus consider how international competition is able to impact the growth process of the competing economies when private productivity can differ between jurisdictions.

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A Appendix

We define as follows the notion of heterogenous strategic behavior which is used in Dockner et al. (2000)(Pages 87–92).

Definition 5 *A 2-tuple* (Ψ_1, Ψ_2) *of functions* $\Psi_1 : [0,1] \times [0,+\infty) \to \mathbb{R}_+^2$ *and* $\Psi_2 : [0,+\infty) \to \mathbb{R}_+^2$*, with* $\Psi_1 = (\Psi_{11}(S,t), \Psi_{12}(S,t)), \forall (S,t) \in [0,1] \times [0,+\infty)$ *and* $\Psi_2 = (\phi_{21}(t), \Psi_{22}(t))$*, is called a heterogenous Strategic Nash Equilibrium if, for each* $i = 1, 2$*, an optimal control path* $(a_i(\cdot), T_i)$ *of player* i *exists and is given by the Markovian Strategy for player 1:* $(a_1(t), T_1(t)) = (\Psi_{11}(S(t),t), \Psi_{12}(S(t),t)) = \Psi_1(S(t),t)$*, and open-loop strategy for player 2:* $(a_2(t), T_2(t)) = (\Psi_{21}(t), \Psi_{22}(t)) = \Psi_2(t)$*.*

The small open economy (the Markovian strategic player) takes the large country's (open loop) strategy $\Psi_2(t)$ as given, and hence, faces the following optimization problem:

$$\begin{cases} \max_{a_1,T_1} \int_0^\infty e^{-rt}\left[(S(t)T_1(t))^{\frac{1}{2}} - \frac{\beta}{2}a_1^2(t)\right], \\ \text{subject to } \dot{S}(t) = \rho\left(\frac{a_1(t)-T_1(t)}{k} - \frac{\Psi_{21}(t)-\Psi_{22}(t)}{k}\right). \end{cases} \tag{7}$$

The corresponding current-value Hamiltonian is

$$\mathcal{H}_1(T_1,S,a_1,\lambda_1) = \left[S^{\frac{1}{2}}(t)T_1^{\frac{1}{2}}(t) - \frac{\beta}{2}a_1^2(t)\right] + \lambda_1\left(\frac{a_1(t)-T_1(t)}{k} - \frac{a_2(t)-T_2(t)}{k}\right)$$

where λ_1 denotes is a costate variable.

The large economy faces the following problem:

$$\begin{cases} \max_{a_2,T_2} \int_0^\infty e^{-rt}\left[((1-S(t))T_2(t))^{\frac{1}{2}} - \frac{1}{2}a_2^2(t)\right], \\ \text{subject to } \dot{S}(t) = \rho\left(\frac{\Psi_{11}(S,t)-\Psi_{12}(S,t)}{k} - \frac{a_2(t)-T_2(t)}{k}\right). \end{cases} \tag{8}$$

The large country *conjectures* that the small economy's strategies are $\Psi_{12}(S,t) = \left(\frac{k}{2\rho\lambda_1(t)}\right)^2 S$ and $\Psi_{11}(S,t) = \frac{\rho}{\beta_1 k}\lambda_1(t)$, $\forall S \in [0,1]$ and $t \geq 0$. Thus, the current-value Hamiltonian of the large economy is defined as

$$\mathcal{H}_2(T_2,S,a_2,\lambda_2) = \left[(1-S(t))^{\frac{1}{2}}T_2^{\frac{1}{2}}(t) - \frac{1}{2}a_1^2(t)\right] + \lambda_2\left(\frac{a_1(t)-T_1(t)}{k} - \frac{a_2(t)-T_2(t)}{k}\right)$$

with λ_2 its costate variable.

The first order conditions yield the small economy's equilibrium choices $T_1(t) = \left(\frac{k}{2\rho\lambda_1}\right)^2 S(t)$, $a_1(t) = \frac{\lambda_1\rho}{k\beta}$. The costate variable verifies the equation $\dot{\lambda}_1(t) = r\lambda_1 - \frac{k}{4\rho\lambda_1}$ with the transversality condition $\lim_{t\to\infty} e^{-rt}\lambda_1(t)S(t) = 0$.

The optimal choices of the big economy are $a_2(t) = -\frac{\rho\lambda_2(t)}{k}$, $T_2(t) = \left(\frac{k}{2\rho\lambda_2(t)}\right)^2(1-S(t))$ with the costate equation

$$\dot{\lambda}_2(t) = r\lambda_2 - \frac{k}{4\rho\lambda_2} + \frac{k}{4\rho}\frac{\lambda_2}{\lambda_1^2}. \tag{9}$$

The associated transversality condition is $\lim_{t\to\infty} e^{-rt}\lambda_2(t)S(t) = 0$.

Moreover, we can readily check that the maximized Hamiltonian $H_1^*(S, \lambda_1)$ and $H_2^*(S, \lambda_2)$ are concave with respect to the state variables which guarantees that $a_i(t), T_i(t)$ $(i = 1, 2)$ are optimal paths. Therefore, the large country's conjecture about the rival's strategy is optimal. Hence, the solutions $\Psi_1(S, t)$ and $\Psi_2(t)$ for $S \in [0, 1]$ and $t \geq 0$ is one pair of Nash Equilibria.

A.1 Steady states

The long run solutions of the above dynamic system are given as follows:

Proposition 6 *At the Nash equilibrium, for any given parameters $\rho, k, r, \beta_i, i = 1, 2$, there is a potential interior steady state*

$$\widehat{S} = \frac{1}{6\sqrt{2}}\left(\frac{\rho}{kr}\right)^{\frac{3}{2}}\left(\frac{\sqrt{2}}{\beta} - 1\right) + \frac{2}{3}, \tag{10}$$

$$\widehat{a}_1 = \frac{1}{2\beta}\left(\frac{\rho}{kr}\right)^{\frac{1}{2}}, \ \widehat{T}_1 = \frac{kr}{\rho}\widehat{S}, \ \ \widehat{a}_2 = \frac{1}{2}\left(\frac{\rho}{2kr}\right)^{\frac{1}{2}}, \widehat{T}_2 = \frac{2kr}{\rho}(1 - \widehat{S}), \tag{11}$$

with the costate variables $\widehat{\lambda}_1 = \frac{1}{2}\left(\frac{k}{\rho r}\right)^{\frac{1}{2}}, \widehat{\lambda}_2 = -\frac{1}{2}\left(\frac{k}{2\rho r}\right)^{\frac{1}{2}}$. Notice that he steady state is a saddle point of the canonical system and it is one dimensional locally asymptotically stable.

A.2 Trajectories

The above analysis shows that there is a stable trajectory associated with the dynamic system. In this subsection, we explore the convergence path to make clear how the steady state is attained. Taking into account of the initial and transversality conditions, the FOCs yield the explicit trajectories

$$\lambda_1(t) = \frac{1}{2}\left(\frac{k}{\rho r}\right)^{\frac{1}{2}}, \ \ \lambda_2(t) = -\frac{1}{2}\left(\frac{k}{2\rho r}\right)^{\frac{1}{2}}.$$

The state trajectory becomes

$$S(t) = (S(0) - \widehat{S})e^{-3rt} + \widehat{S} \tag{12}$$

which is the optimal convergence path leading to the steady state. The convergence speed is $3r$.

A.3 State constraint $S(t) \leq \overline{S} < \frac{1}{2}$

Recalling that the small country's size is constrained ($S(t) \leq \overline{S} \leq \frac{1}{2}$), we adapt the Lagrangian function as follows

$$\mathcal{L}_1(T_1, S, a_1, \lambda_1) = \left[S^{\frac{1}{2}}(t) T_1^{\frac{1}{2}}(t) - \frac{\beta}{2} a_1^2(t) \right] + \lambda_1 \left(\frac{a_1(t) - T_1(t)}{k} - \frac{a_2(t) - T_2(t)}{k} \right) + \mu(S - \overline{S}).$$

The above first order conditions still hold, except the costate variable which now verifies the equation $\dot{\lambda}_1(t) = r\lambda_1 - \frac{k}{4\rho\lambda_1} + \mu$. Furthermore, we introduce the Kuhn-Tucker condition

$$\mu \left(S - \overline{S} \right) = 0.$$

In other words, we have, either $S < \overline{S}$ with $\mu = 0$ or $S = \overline{S}$ with $\mu \geq 0$. However, since the small economy's size is constrained by the upper-bound $\overline{S}$, we impose that $\mu = 0$ whenever $S = \overline{S}$.