FACULTY OF LAW, ECONOMICS AND FINANCE

CREA Discussion Paper 2012-05

Center for Research in Economic Analysis
University of Luxembourg

International equity and bond positions in a DSGE model with variety risk in consumption

available online : http://wwwfr.uni.lu/recherche/fdef/crea/publications2/discussion_papers/2011

Hamano Masashige, CREA, University of Luxembourg

June, 2012

For editorial correspondence, please contact: crea@uni.lu
University of Luxembourg
Faculty of Law, Economics and Finance
162A, avenue de la Faïencerie
L-1511 Luxembourg

UNIVERSITÉ DU LUXEMBOURG

International equity and bond positions in a DSGE model with variety risk in consumption*

Hamano Masashige†

15 December 2011

Abstract

This paper analyzes equity and bond positions in a two-country DSGE model where the number of varieties, i.e. extensive margin is endogenously determined. Households take care about not only the price of goods but also the variety of goods they consume. The welfare-based real exchange rate fluctuations matter in international consumption risk sharing. We investigate analytically and numerically the implication of "variety risk" induced by fluctuations in extensive margins. In numerical computation of zero-order steady state portfolios, we employ the Devereux and Sutherland method. We show that, with variety risk, home biased equity positions are further amplified compared to those obtained with the standard model in the literature. The result is shown to be robust with or without firm heterogeneity in marginal costs of production.

*I would like to thank Philippe Martin for his encouragement and guidance. I would also like to thank Marta Arespa Castelló, Nicolas Coeurdacier, Michael B. Devereux, Fabio Ghironi, Robert Kollmann, Isabelle Méjan, Pierre Picard, Ridwan Rusli and Simeon Vosen for providing helpful comments and discussions. I am grateful for all seminar participants at IXth and Xth Doctoral Meetings in International Trade and International Finance, 13th and 15th Annual International Conference on Macroeconomic Analysis and International Fiance and ZEW summer workshop 2010. Of course all remaining errors are my own.

†University of Luxembourg, CREA, L-1511 Luxembourg, Luxembourg. *E-mail*: masahige.hamano@uni.lu.

Keywords: *real exchange rate, home biased equity puzzle, firm entry, firm heterogeneity*

JEL classification: *F12, F41, F43*

1 Introduction

Recent decades have witnessed a significant increase of exchange in both goods and financial assets. For instance the share of imported goods in GDP has more than doubled in the U.S., from 4.8 % in 1972 to 11.7 % in 2001. On the other hand, cross-border asset holdings have grown to such an extent that gross external positions of major developed countries have exceeded their GDP (Lane and Milesi-Ferretti (2003, 2005, 2007)).

Specifically, the expansion of trade has been driven by a rise in the number of available varieties i.e. extensive margins. For the U.S. imported extensive margins have quadrupled over the same period,(Broda and Weinstein (2004, 2006)). Besides their direct contribution through the current account as investigated in Galstyan and Lane (2008), the expansion of trade in extensive margins impacts the choice of optimal asset portfolios as a means to international consumption risk sharing. When households display a preference for variety, they use financial assets to insure against not only fluctuations in the price of goods but also in extensive margins.. What are the implications of such a "variety risk" on international risk sharing and portfolio positions? The paper theoretically addresses the above issues.[1]

The recent literature on international risk sharing has been developed around the "home biased equity puzzle" (French and Poterba (1991), Tesar and Werner (1995), Sercu and Vanpee (2007)). Despite better financial integration that has fostered the large cross-border asset holdings, investors still hold a disproportionate share of local equities. Home biased equity positions are sizable and still persistent among all countries (Table 1).

[1]Significant fluctuations in extensive margins along the business cycle are also documented in Broda and Weinstein (2010): In the U.S. 9% of purchases of product in a year are for new products that have not been existed previously and the upward bias of the standard CPI is about 0.6 to 0.9 percentage point per year.

Table 1: Equity Home Bias in 2008 (source: Coeurdacier and Rey (2011))

Country	$\frac{\text{Domestic Market}}{\text{World Market}}$	$\frac{\text{Domestic Equity}}{\text{Total Equity}}$	Equity Home Bias
Australia	1.8	76.1	0.76
Brazil	1.6	99	0.98
China	7.8	99.2	0.99
Canada	2.7	80.2	0.80
Euro Area	13.5	57	0.625
Japan	8.9	73.5	0.71
South Africa	1.4	52	0.517
South Korea	1.4	89	0.88
Sweden	0.7	44	0.43
Switzerland	2.3	51	0.50
United Kingdom	5.1	54.5	0.52
United States	32.6	77.2	0.66
South Africa	32.6	88	0.88

How can we explain this robust empirical fact? In the standard theoretical models of international macroeconomics with differentiated goods, when equity positions are supposed to hedge against the real exchange rate risk induced by terms of trade fluctuations, a home biased equity position becomes optimal only when the elasticity of substitution between local and imported goods is smaller than unity (Uppal (1993), Kollmann (2006a), Kollmann (2006b), Obstfeld (2007), Coeurdacier (2009)). The reason is that, under low trade elasticity condition, when the price of domestic consumption increases with the terms of trade appreciation, dividends and equity returns from domestic firms could rise due to a strong income effect, thus providing a positive financial income flow to housholds. However, when the elasticity is higher than unity, the opposite is true: during times of expensive domestic consumption, dividends of domestic firms decrease and a foreign biased equity positions would be observed.[2]

[2]As argued in Cole and Obstfeld (1991), when the elasticity of substitution between local and imported

Furthermore, it has been noticed that the attempt to explain home biased equity positions through the real exchange rate risk faces a more fundamental counterfactual drawback: between the real exchange rate fluctuations and equity returns, there are no empirical regularities (van Wincoop and Warnock (2006)). This modeling problem has been subsequently overcome by introducing bonds or forward exchange positions whose nominal returns are correlated perfectly with the real exchange rate fluctuations. As a result equity positions are also offered the possiblity to hedge against a risk other than the real exchange rate, namely the non-financial labor income risk (Coeurdacier et al. (2007), Coeurdacier and Gourinchas (2008), Coeurdacier et al. (2010), Engel and Matsumoto (2009, 2009a).[3]

Our paper is built upon this strand of literature. We incorporate cross-border asset holdings in equities and bonds into a model where extensive margins are endogenously determined. In our benchmark model based on Ghironi and Melitz (2005) with firm heterogeneity in marginal costs of production, the available set of varieties would be different across countries. In a model of Hamano (2012) without firm heterogeneity the consumption patterns in the same set of varieties would be different across countries because of a home bias in consumption. For both models, the variety risk induced by extensive margins cannot be hedged with bond positions alone. This is because bond returns do not adress the "welfare-based" real exchange rate fluctuations embedded in extensive margins. And a part of variety risk is well treated with equity positions.

The hedging intuition in our paper is described as follows. Home biased equity positions principally appear as a hedge against labor income risk (Heathcote and Perri (2004)). A home biased position can be a good hedge when labor income and equity returns correlate negatively, conditional on bond returns (Coeurdacier et al. (2007), Coeurdacier and Gourinchas (2008), Coeurdacier et al. (2010), Engel and Matsumoto (2009, 2009a)). At the same time, however, when equity returns and extensive margins are conditionally correlated negatively, a home biased equity position is also a good hedge against the variety

goods is unity, the real exchange rate risk is perfectly insured with the terms of trade fluctuations. Financial assets are redundant.

[3]See Coeurdacier and Rey (2011) for detailed exposition about the recent development.

risk induced by extensive margins. The reason is that when extensive margins rise (decrease) and there is a depreciation (appreciation) of the real exchange rate on a welfare basis, a home biased equity position can provide a negative (positive) financial income transmitting abroad the welfare gains (losses) obtained with higher (lower) extensive margins. Such a negative correlation between extensive margins and equity returns is realized by an investment shock which holds on firm/variety setting up costs in free entry. As a result, in addition to those induced by the labor income risk we observe an amplification of home biased equity positions.

Optimal bond positions are also impacted by introducing extensive margins. When bond returns are correlated negatively (positively) with extensive margins conditional on equity returns, they add a further long (short) position for domestic bonds. Additional long or short positions can successfully provide appropriate income transfers with respect to welfare gains or losses stemming from changes in extensive margins. Our model hence endogenizes the exogenous preference shock analyzed in Coeurdacier et al. (2007) relying on free entry of variety representing firms.

We explore the above hedging mechanism analytically with a static budget constraint and numerically relying on the method developed by Devereux and Sutherland (2008).[4] This is the first paper to our best knowledge that applies their method to the model including firm entry by Ghironi and Melitz (2005). The result is shown to be robust with or without firm heterogeneity. Our paper hence adds further arguments why we continue to observe sizable home biased equity positions until recently. Expansion of trade in extensive margins would account for that.

The paper is close to Coeurdacier et al. (2010) in the spirit analyzing zero-order steady state equity and bond positions and their first-order dynamics in a DSGE model. A major difference with respect to their paper is that 'investments' take place in the form of new firm creations, not in the standard capital accumulation process. In terms of business cycle moments for principal macroeconomic variables, however, it is shown that both papers are quite similar.[5] Castello (2008) analytically finds a home biased equity position

[4] Tille and van Wincoop (2008) also develops an analogous solution techniques.

[5] Bui (2009) and Rahbari (2009) also extend Coeurdacier et al. (2010) by introducing nominal rigidities

using a static budget constraint in a model which is rather similar to ours but without explicitly treating the variety risk. Hence the portfolio positions found in her paper are similar to those in existing literature.

The structure of the paper is as follows. In the next section we present the model including firm heterogeneity in marginal costs of production. For the purpose of comparison, the model without firm heterogeneity is also presented. In section 3 the hedging intuition is investigated using the static budget constraint. Zero-order steady state portfolios are computed numerically in section 4 where we also report business cycle characteristics of the model. In the last section we conclude.

2 The model

The world consists of two countries, Home and Foreign. Foreign variables are denoted with asterisks. Each country is populated by one unit mass of atomic households who consume variety of goods and supply labor. There is international borrowing and lending using two types of financial assets, equities and bonds of each country. The number of firms which are heterogenous in their marginal costs of production is endogenously determined following Ghironi and Melitz (2005). We also present the model without firm heterogeneity.

2.1 Households

The Home representative household maximizes expected intertemporal utility, $E_t \sum_{s=t}^{\infty} \beta_t^{s-t} U_t$. The utility at time t depends on consumption and labor supply as follows

$$U_t = \frac{C_t^{1-\gamma}}{1-\gamma} - \chi \frac{L_t^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}},$$

where $\gamma \, (\geq 1)$ denotes relative risk aversion. χ (> 0) represents the degree of non satisfaction from supplying labor L_t and φ (≥ 0) denotes Frisch elasticity of labor supply.[6]

in incomplete asset markets setting.

[6]With $\varphi = \infty$ the marginal disutility in supplying one additional labor becomes constant, χ. When $\varphi = 0$ the marginal disutility becomes infinite and the labor supply becomes inelastic. With the specification

β_t is endogenous discount factor which evolves as

$$\beta_{t+1} = \beta_t \Upsilon(C_t), \text{ with } \beta_0 = 1.$$

We give a functional form of $\Upsilon(C_t)$ as $\Upsilon(C_t) = \overline{\beta} C_t^{-\upsilon}$ where $0 \leq \upsilon < \gamma$ and $0 < \overline{\beta} C^{-\upsilon} < 1$. Because $\Upsilon'(C_t) < 0$, this specification guarantees the stationarity of the model which includes net foreign asset dynamics as explained in Schmitt-Grohe and Uribe (2003).

The basket of goods C_t is defined as

$$C_t = \left[\alpha^{\frac{1}{\omega}} C_{H,t}^{1-\frac{1}{\omega}} + (1-\alpha)^{\frac{1}{\omega}} C_{F,t}^{1-\frac{1}{\omega}}\right]^{\frac{1}{1-\frac{1}{\omega}}},$$

where α $(> 1/2)$ is the home bias in consumption. ω (> 0) denotes the elasticity of substitution between local $(C_{H,t})$ and imported goods $(C_{F,t})$. $C_{H,t}$ and $C_{F,t}$ are defined over a continuum of goods Ω:

$$C_{H,t} = V_{H,t}\left(\int_{\zeta\in\Omega_t} c_t(\zeta)^{1-\frac{1}{\sigma}} d\zeta\right)^{\frac{1}{1-\frac{1}{\sigma}}}, \qquad C_{F,t} = V_{F,t}^{*}\left(\int_{\vartheta\in\Omega_t} c_t(\vartheta)^{1-\frac{1}{\sigma}} d\vartheta\right)^{\frac{1}{1-\frac{1}{\sigma}}},$$

where $V_{H,t} \equiv N_{D,t}^{\psi-\frac{1}{\sigma-1}}$ and $V_{F,t}^{*} \equiv N_{X,t}^{*\psi-\frac{1}{\sigma-1}}$. $N_{D,t}$ and $N_{X,t}^{*}$ stand for the number of domestic and imported goods respectively firms. At any given time t, only a subset of goods $\Omega_t \in \Omega$ is available. σ (> 1) denotes the elasticity of substitution among varieties. We assume conventionally $\sigma \geq \omega$. ψ (≥ 0) represents the marginal utility which stems from one additional variety in each basket (Benassy (1996)). Specifically, the preference becomes Dixit and Stiglitz (1977) when $\psi = \frac{1}{\sigma-1}$. When $\psi = 0$ there is no preference for variety.

2.1.1 Budget constraint

The budget constraint of the Home representative household expressed in terms of consumption basket in Home is given by

in the paper the marginal disutility is increasing.

$$C_t + s_{h,t+1} x_t^s \left(N_{D,t} + N_{E,t}\right) + s_{f,t+1} Q_t x_t^{s*} \left(N_{D,t}^* + N_{E,t}^*\right) + b_{h,t+1} x_t^b + b_{f,t+1} Q_t x_t^{b*}$$

$$= w_t L_t + s_{h,t} N_{D,t} \left(x_t^s + \widetilde{d}_t\right) + s_{f,t} N_{D,t}^* Q_t \left(x_t^{s*} + \widetilde{d}_t^*\right) + b_{h,t} \left(x_t^b + \frac{\widehat{P}_t}{P_t}\right) + b_{f,t} \left(Q_t x_t^{b*} + \frac{\widehat{P}_t^*}{P_t}\right). \tag{1}$$

The household finances entry costs of new entrants $N_{E,t}$ and $N_{E,t}^*$ as well as all producing firms $N_{D,t}$ and $N_{D,t}^*$ in Home and Foreign at time t by purchasing a share of mutual funds. $s_{h,t+1}(s_{f,t+1})$ denotes share holding of Home (Foreign) equities in $t+1$. x_t^s (x_t^{s*}) is the real share price of Home (Foreign) equities denominated with Home (Foreign) consumption goods.[7] We define the welfare-based real exchange rate, Q_t, as the relative price of Foreign in terms of Home consumption: $Q_t \equiv P_t^*/P_t$. $\widetilde{d}_t$ ($\widetilde{d}_t^*$) stands for the average real dividends earned by Home (Foreign) firms. $b_{h,t+1}$ ($b_{f,t+1}$) represents holdings of bonds into $t+1$ issued in Home (Foreign). x_t^b (x_t^{b*}) is the real price of bonds issued in Home (Foreign). Bonds give only nominal payoff in the form of nominal CPI in the next period. Such nominal CPI is denoted with $\widehat{P}_t$ ($\widehat{P}_t^*$) in Home (Foreign). w_t stands for real wages. L_t is labor supply by the household.

The number of firms is assumed to follow a motion coherent with the above budget constraint from one period to another as $N_{D,t+1} = (1-\delta)\left(N_{D,t} + N_{E,t}\right)$ where δ represents an exogenous depreciation rate, a so-called "death shock". Though they are financed in entry, new entrants produce and generate dividends only one period after. The similar motion holds for Foreign firms.

Symmetrically, for the Foreign representative household, the real budget constraint expressed in terms of Foreign consumption basket become as follows

[7] Equities in this paper take the form of mutual funds. This is for the sake of simplicity rather than a good description of the reality. A notable exception is Martin and Rey (2004) which analyze 'extensive margins' of assets.

$$C_t^* + s_{f,t+1}^* x_t^{s*}\left(N_{D,t}^* + N_{E,t}^*\right) + s_{h,t+1}^* Q_t^{-1} x_t^s\left(N_{D,t} + N_{E,t}\right) + b_{f,t+1}^* x_t^{b*} + b_{h,t+1}^* Q_t^{-1} x_t^b$$

$$= w_t^* L_t^* + s_{f,t}^* N_{D,t}^*\left(x_t^{s*} + \widetilde{d}_t^*\right) + s_{h,t}^* N_{D,t} Q_t^{-1}\left(x_t^s + \widetilde{d}_t\right) + b_{f,t}^*\left(x_t^{b*} + \frac{\widehat{P}_t^*}{P_t^*}\right) + b_{h,t}^*\left(Q_t^{-1} x_t^b + \frac{\widehat{P}_t}{P_t^*}\right). \tag{2}$$

Note that asset markets clear at all times according to

$$s_{h,t+1} + s_{h,t+1}^* = s_{f,t+1} + s_{f,t+1}^* = 1, \qquad b_{h,t+1} + b_{h,t+1}^* = b_{f,t+1} + b_{f,t+1}^* = 0. \tag{3}$$

2.1.2 Optimal choices

The representative household maximizes the expected intertemporal utility with respect to $s_{h,t+1}$, $s_{f,t+1}$, $b_{h,t+1}$, $b_{f,t+1}$, L_t and C_t subject to (1) for all periods. We start by definindg real returns on each asset as follows

$$r_{h,t}^s \equiv (1-\delta)\frac{x_t^s + \widetilde{d}_t}{x_{t-1}^s}, \quad r_{f,t}^s \equiv (1-\delta)\frac{x_t^{s*} + \widetilde{d}_t^*}{x_{t-1}^{s*}}\frac{Q_t}{Q_{t-1}},$$

$$r_{h,t}^b \equiv \frac{x_t^b + \frac{\widehat{P}_t}{P_t}}{x_{t-1}^b}, \quad r_{f,t}^b \equiv \frac{x_t^{b*} + \frac{\widehat{P}_t^*}{P_t^*}}{x_{t-1}^{b*}}\frac{Q_t}{Q_{t-1}}.$$

$r_{h,t}^s$, $r_{f,t}^s$, $r_{h,t}^b$ and $r_{f,t}^b$ are gross real returns of equities and bonds between $t-1$ and t respectively.

Using the above expressions, Euler equations for share holdings become[8]

$$1 = \Upsilon\left(C_t\right) E_t\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma} r_{h,t+1}^s, \quad 1 = \Upsilon\left(C_t\right) E_t\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma} r_{f,t+1}^s,$$

Those for bond holdings are given by[9]

[8] Those for the Foreign representative household become,

$$1 = \Upsilon\left(C_t^*\right) E_t\left(\frac{C_{t+1}^*}{C_t^*}\right)^{-\gamma} r_{h,t+1}^s \frac{Q_t}{Q_{t+1}}\ , 1 = \Upsilon\left(C_t^*\right) E_t\left(\frac{C_{t+1}^*}{C_t^*}\right)^{-\gamma} r_{f,t+1}^s \frac{Q_t}{Q_{t+1}}.$$

[9] Those of Foreign counterparts are,

$$1 = \Upsilon(C_t) E_t \left(\frac{C_{t+1}}{C_t}\right)^{-\gamma} r^b_{h,t+1}, \quad 1 = \Upsilon(C_t) E_t \left(\frac{C_{t+1}}{C_t}\right)^{-\gamma} r^b_{f,t+1}.$$

The optimal labor supply is given by

$$\chi (L_t)^{\frac{1}{\psi}} = w_t C_t^{-\gamma}$$

The optimal consumption for each domestic, imported basket and individual variety is found to be

$$C_{H,t} = \left(\frac{P_{H,t}}{P_t}\right)^{-\omega} \alpha C_t, \qquad C_{F,t} = \left(\frac{P_{F,t}}{P_t}\right)^{-\omega} (1-\alpha) C_t.$$

$$c_t(\zeta) = V_{H,t}^{\sigma-1} \left(\frac{p_t(\zeta)}{P_{H,t}}\right)^{-\sigma} C_{H,t}, \qquad c_t(\vartheta) = V_{F,t}^{*\sigma-1} \left(\frac{p_t^*(\vartheta)}{P_{F,t}}\right)^{-\sigma} C_{F,t}.$$

In particular, $p_t^*(\vartheta)$ denote the price of exported goods from Foreign. Price indices which minimize expenditure on each consumption basket are given by

$$P_t = \left[\alpha P_{H,t}^{1-\omega} + (1-\alpha) P_{F,t}^{1-\omega}\right]^{\frac{1}{1-\omega}},$$

$$P_{H,t} = \frac{1}{V_{H,t}} \left(\int_{\zeta \in \Omega_t} p_t(\zeta)^{1-\sigma} d\zeta\right)^{\frac{1}{1-\sigma}}, \qquad P_{F.t} = \frac{1}{V_{F.t}^*} \left(\int_{\vartheta \in \Omega_t} p_t^*(\vartheta)^{1-\sigma} d\vartheta\right)^{\frac{1}{1-\sigma}}.$$

Observe that the price indices fluctuate with extensive margins. They are defined on a "welfare-basis". The impact of extensive margins in price indices is greater, the higher the love for variety, ψ.

Finally, we choose the welfare-based consumer price index, P_t, as numéraire in Home and define real prices as $\rho_{H,t} \equiv \frac{P_{H,t}}{P_t}$, $\rho_{F,t} \equiv \frac{P_{F,t}}{P_t}$, $\rho_t(\zeta) \equiv \frac{p_t(\zeta)}{P_t}$ and $\rho_t^*(\vartheta) \equiv \frac{p_t^*(\vartheta)}{P_t}$.

Similar expressions hold in Foreign.

$$1 = \Upsilon(C_t^*) E_t \left(\frac{C_{t+1}^*}{C_t^*}\right)^{-\gamma} r^b_{h,t+1} \frac{Q_t}{Q_{t+1}}, \quad 1 = \Upsilon(C_t^*) E_t \left(\frac{C_{t+1}^*}{C_t^*}\right)^{-\gamma} r^b_{f,t+1} \frac{Q_t}{Q_{t+1}}.$$

2.2 Firms

2.2.1 Free entry and motion

The supply side of the model is almost identical to Ghironi and Melitz (2005). Firms are heterogenous in terms of firm specific labor productivity z_D which they draw upon entry from a given Pareto distribution $G(z_D)$. Different from them, however, we assume that entry costs are paid with capital goods as well as labor.[10] Firm creation needs a set up cost f_E, which is invariant across time. The production of f_E is achieved by the following Cobb-Douglas technology with labor, $l_{E,t}$, and capital goods, K_t, as inputs:

$$f_E = \left(\frac{Z_{E,t} l_{E,t}}{\theta}\right)^{\theta} \left(\frac{K_t}{1-\theta}\right)^{1-\theta},$$

where $Z_{E,t}$ denotes labor productivity which is specific for firm creation and identical across firms. θ $(1-\theta)$ is the share of labor (capital) in total entry costs. For simplicity, we assume that K_t has the same composition as C_t.

Cost minimization yields the following factor demands as

$$\frac{w_t l_{E,t}}{\mu_t f_E} = \theta, \qquad \frac{K_t}{\mu_t f_E} = 1-\theta, \tag{4}$$

where $\mu_t \equiv (w_t / Z_{E,t})^{\theta}$ stands for real entry costs.

In equilibrium, share price should be equal to entry costs providing the following free entry condition:

$$x_t^s = \left(\frac{w_t}{Z_{E,t}}\right)^{\theta} f_E. \tag{5}$$

Similar conditions hold in Foreign.

2.2.2 Firm averages

Following Melitz (2003) and Ghironi and Melitz (2005), heterogenous firms are summarized by defining two specific average productivity levels, $\widetilde{z}_D$ for all $N_{D,t}$ producing firms and $\widetilde{z}_{X,t}$ for $N_{X,t}$ exporters.

[10] This types of specification is proposed in Bilbiie et al. (2007).

We assume the following Pareto distribution:

$$G(z_D) = 1 - \left(\frac{z_{D\min}}{z_D}\right)^k,$$

where $z_{D\min}$ is the minimum productivity level and k $(> \sigma - 1)$ is a shaping parameter. With the above distribution, $\widetilde{z}_D$ and $\widetilde{z}_{X,t}$ are given by

$$\widetilde{z}_D \equiv \left[\int_{z_{\min}}^{\infty} z_D^{\sigma-1} dG(z_D)\right]^{\frac{1}{\sigma-1}} = z_{D\min}\left[\frac{k}{k-(\sigma-1)}\right]^{\frac{1}{\sigma-1}},$$

$$\widetilde{z}_{X,t} \equiv \left[\frac{1}{1-G(z_{X,t})}\int_{z_{X,t}}^{\infty} z_D^{\sigma-1} dG(z_D)\right]^{\frac{1}{\sigma-1}} = z_{X,t}\left[\frac{k}{k-(\sigma-1)}\right]^{\frac{1}{\sigma-1}},$$

where $z_{X,t}$ denotes a cutoff productivity level with which firms earn just zero profit in export market.

Based on these average productivity levels, we can define the average real profits from domestic sales, $\widetilde{d}_{D,t}$, and those from exporting sales, $\widetilde{d}_{X,t}$. Exporting firms are supposed to pay an amortized amount of fixed costs, f_X, in terms of effective labor in each period. Taking such costs into account, $\widetilde{d}_{D,t}$ and $\widetilde{d}_{X,t}$ are expressed as

$$\widetilde{d}_{D,t} = \left(\widetilde{\rho}_{D,t} - \frac{w_t}{Z_t\widetilde{z}_D}\right)\widetilde{y}_{D,t}, \qquad \widetilde{d}_{X,t} = \left(Q_t\widetilde{\rho}_{X,t} - \frac{w_t}{Z_t\widetilde{z}_{X,t}}\right)\widetilde{y}^*_{X,t} - \mu_{X,t}f_X,$$

where $\widetilde{\rho}_{D,t}$ and $\widetilde{\rho}_{X,t}$ denotes the average real domestic and exporting price respectively. $\mu_{X,t} \equiv \frac{w_t}{Z_t}$ denotes the real costs for exporting.

The average production for the domestic market, $\widetilde{y}_{D,t}$, and that for exports, $\widetilde{y}_{X,t}$, are given by the following technology:

$$\widetilde{y}_{D,t} = Z_t\widetilde{z}_D l_{D,t}, \qquad \widetilde{y}_{X,t} = Z_t\widetilde{z}_{X,t} l_{X,t},$$

where Z_t denotes the productivity shock that hit all firms. $l_{D,t}$ and $l_{X,t}$ represent the labor demand required for the production of $\widetilde{y}_{D,t}$ and $\widetilde{y}_{X,t}$, respectively.

Because of monopolistic competition intensive margins are determined by their demand implying $\widetilde{y}_{D,t} = \widetilde{c}_{D,t} + N_{E,t}\widetilde{k}_{D,t}$ and $\widetilde{y}_{X,t} = \widetilde{c}^*_{X,t} + N^*_{E,t}\widetilde{k}^*_{X,t}$. $\widetilde{c}_{D,t}$, $\widetilde{k}_{D,t}$ $\widetilde{c}^*_{X,t}$ and $\widetilde{k}^*_{X,t}$

are the average consumption and capital goods demands faced by Home firms in domestic and export market respectively. Knowing the optimal demand functions found in the previous section, firms in each domestic and exporting market maximize their profits and set prices on average as

$$\widetilde{\rho}_{D,t} = \frac{\sigma}{\sigma - 1} \frac{w_t}{Z_t \widetilde{z}_D}, \qquad \widetilde{\rho}_{X,t} = \frac{\sigma}{\sigma - 1} \frac{w_t}{Z_t \widetilde{z}_{X,t}} Q_t^{-1} \tag{6}$$

Using the above optimal pricing and the fact that $\rho_{H,t} = N_{D,t}^{-\psi} \widetilde{\rho}_{D,t}$ and $\rho_{H,t}^* = N_{X,t}^{-\psi} \widetilde{\rho}_{X,t}$, $\widetilde{d}_{X,t}$ and $\widetilde{d}_{D,t}$ can be rewritten as

$$\widetilde{d}_{D,t} = \frac{1}{\sigma} N_{D,t}^{\psi(\omega-1)-1} \widetilde{\rho}_{D,t}^{1-\omega} \alpha M_t, \qquad \widetilde{d}_{X,t} = \frac{Q_t}{\sigma} N_{X,t}^{\psi(\omega-1)-1} \widetilde{\rho}_{X,t}^{1-\omega} (1-\alpha) M_t^* - \mu_{X,t} f_X. \tag{7}$$

where $M_t \equiv C_t + N_{E,t} K_t$ and $M_t^* \equiv C_t^* + N_{E,t}^* K_t^*$ represent the total consumption and capital goods demand in Home and Foreign respectively.[11]

Finally, the average real profits among all Home firms, $\widetilde{d}_t$, are thus expressed as follows

$$\widetilde{d}_t = \widetilde{d}_{D,t} + \frac{N_{X,t}}{N_{D,t}} \widetilde{d}_{X,t}.$$

Similar conditions hold in Foreign.

2.2.3 Share of exporters and zero-profit export cutoff

Using the average exporting firm productivity and the Pareto density function defined previously, the share of exporters in the total number of domestic firms is given by

$$\frac{N_{X,t}}{N_{D,t}} = z_{\min}^k \left(\widetilde{z}_{X,t}\right)^{-k} \left[\frac{k}{k - (\sigma - 1)}\right]^{\frac{k}{\sigma-1}}. \tag{8}$$

As it has been explained, there exists a cutoff productivity level, $z_{X,t}$, with which firms earn just zero profit from export in such a way that

[11]With the free entry condition (5) and factor demands (4), we have $K_t/x_{h,t}^s = K_t^*/x_{f,t}^{s*} = 1 - \theta$. Specifically, when $\theta = 0$, M_t and M_t^* coincide to the aggregated demand, aggregated consumption plus investments.

$$\frac{Q_t}{\sigma} N_{X,t}^{\psi(\omega-1)-1} \rho_{X,t}^{1-\omega}\left(z_{X,t}\right)(1-\alpha) M_t^* - \mu_{X,t} f_X = 0.$$

where $\rho_{X,t}\left(z_{X,t}\right)$ denotes the price with cutoff productivity. Combined with (7), the above condition can be rewritten as

$$\widetilde{d}_{X,t} = \mu_{X,t} f_X \frac{\sigma-1}{k-(\sigma-1)}. \tag{9}$$

(8) and (9) determine the number of exporters, $N_{X,t}$, and the average productivity level of exporting firms, $\widetilde{z}_{X,t}$.

Similar conditions hold in Foreign.

2.3 Labor market clearing and net foreign asset dynamics

Supplied labor units L_t are demanded for the production of domestic, exporting goods, fixed costs for exporting and firm creation implying that

$$L_t = N_{D,t} l_{D,t} + N_{X,t}\left(l_{X,t} + l_{FX,t}\right) + N_{E,t} l_{E,t},$$

where $l_{FX,t}$ denotes the labor demand for fixed costs for exporting. Noting $l_{D,t} = (\sigma-1)\widetilde{d}_{D,t}/w_t$, $l_{X,t}+l_{FX,t} = (\sigma-1)\widetilde{d}_{X,t}/w_t + \sigma l_{FX,t}$, $l_{FX,t} = \mu_{X,t} f_X / w_t$ and $l_{E,t} = \theta x_t^s / w_t$, the above condition can be rewritten as follows

$$L_t = (\sigma-1)\frac{N_{D,t}\widetilde{d}_t}{w_t} + \sigma \frac{N_{X,t}\mu_{X,t} f_X}{w_t} + \theta \frac{N_{E,t} x_t^s}{w_t}.$$

Similar conditions hold in Foreign.

The model is completed by considering net foreign asset dynamics. Real net foreign assets for Home at the end of period t (NFA_{t+1}) is defined as the sum of net foreign equity and bond positions as follows

$$NFA_{t+1} \equiv s_{f,t+1} Q_t x_t^{s*}\left(N_{D,t}^* + N_{E,t}^*\right) - s_{h,t+1}^* x_t^s\left(N_{D,t} + N_{E,t}\right) + b_{h,t+1} x_t^b + b_{f,t+1} Q_t x_t^{b*}.$$

Thus the budget constraint can be rewritten as

$$NFA_{t+1} = NX_t + NFA_t r^b_{h,t} + \xi_{h,t}, \tag{10}$$

In the above net foreign asset dynamics, NX_t denotes net exports and ξ_t stands for "excess returns" between $t-1$ and t relative to those on Home CPI-indexed bonds, $r^b_{h,t}$. They are defined as

$$NX_t \equiv N_{D,t}\widetilde{\rho}_{D,t}\widetilde{y}_{D,t} + N_{X,t}Q_t\widetilde{\rho}_{X,t}\widetilde{y}_{X,t} - C_t - N_{E,t}K_t,$$

$$\begin{aligned}\xi_t \equiv\ & s_{f,t}Q_t x^{s*}_{t-1}\left(N^*_{D,t-1} + N^*_{E,t-1}\right)\left(r^s_{f,t} - r^b_{h,t}\right) \\ & - s^*_{h,t}x^s_{t-1}\left(N_{D,t-1} + N_{E,t-1}\right)\left(r^s_{h,t} - r^b_{h,t}\right) + b_{f,t}Q_t x^{b*}_{t-1}\left(r^b_{f,t} - r^b_{h,t}\right).\end{aligned}$$

We define real holdings at the end of period t as[12]

$$a^s_{h,t} \equiv s_{h,t+1}x^s_t\left(N_{D,t} + N_{E,t}\right), \quad a^s_{f,t} \equiv s_{f,t+1}Q_t x^{s*}_t\left(N^*_{D,t} + N^*_{E,t}\right),$$

$$a^b_{h,t} \equiv b_{h,t+1}x^b_t, \quad a^b_{f,t} \equiv b_{f,t+1}Q_t x^{b*}_t.$$

Thus excess returns ξ_t can also be rewritten in vector form as

$$\xi_t \equiv a'_{t-1}r_{x,t},$$

$$a'_{t-1} = \begin{bmatrix} a^s_{f,t-1} & -a^{s*}_{h,t-1} & a^b_{f,t-1}\end{bmatrix},$$

[12] Real holdings (expressed in terms of Home consumption basket) for the Foreign representative households are,

$$a^{s*}_{h,t} \equiv s^*_{h,t+1}x^s_t\left(N_{D,t} + N_{E,t}\right), \quad a^{s*}_{f,t} \equiv s^*_{f,t+1}Q_t x^{s*}_t\left(N^*_{D,t} + N^*_{E,t}\right),$$

and

$$a^{b*}_{h,t} \equiv b^*_{h,t+1}x^b_t\ , \quad a^{b*}_{f,t} \equiv b^*_{f,t+1}Q_t x^{b*}_t.$$

$$r'_{x,t} = \begin{bmatrix} r^s_{fx,t} & r^s_{hx,t} & r^b_{hx,t} \end{bmatrix},$$

where $r^s_{fx,t} \equiv r^s_{f,t} - r^b_{h,t}$, $r^s_{hx,t} \equiv r^s_{h,t} - r^b_{h,t}$ and $r^b_{hx,t} \equiv r^b_{f,t} - r^b_{h,t}$.

Provided the steady state portfolios which will be detailed below, finally the first-order system of the model contains 46 equations and 46 endogenously determined variables. The whole system including the steady state and its first-order equations is summarized in appendix.

2.4 The model without heterogeneity

It would be particularly useful to consider a model without firm heterogeneity for the purpose of comparison. Imposing the symmetry among firms, the supply side of the model collapses to the one analyzed in Hamano (2012). In such a case, all firms export and there is no non-tradeness arising from the cutoff productivity determination.

Specifically, we impose $\widetilde{z}_D = \widetilde{z}_{X,t} = \widetilde{z}^*_D = \widetilde{z}^*_{X,t} = 1$ and $f_X = f^*_X = 0$ in the benchmark system without loss of generality. All firms become symmetric and export without export fixed costs. As a result, we have $N_{D,t} = N_{X,t}$, $N^*_{D,t} = N^*_{X,t}$, $\widetilde{\rho}_{D,t} = Q_t\widetilde{\rho}_{X,t}$ and $\widetilde{\rho}^*_{D,t} = Q_t^{-1}\widetilde{\rho}^*_{X,t}$ and zero-profit export cutoff (9), real costs for exporting ($\mu_{X,t} = \mu^*_{X,t} = 0$), share of exporters (8) and pricing for export market (6) are no more needed. Without heterogeneity we have 8 less variables and 8 less equations compared to the benchmark model.

3 Intuition of portfolio with variety risk

A crucial difference from the standard model in the literature are endogenous extensive margins driven by free entry. Because consumption baskets fluctuate with extensive margins, households should insure against such variety risk by appropriately designed portfolios. Put another way, what really matters for risk sharing is the welfare-based real exchange rate. In this section, we analytically investigate the intuition of portfolio choice and how the optimal positions are affected by considering the variety risk induced by extensive margins.

3.1 Risk in the welfare-based real exchange rate fluctuations

What is the welfare-based real exchange rate risk? We denote first-order dynamics with sans-serif font. The first-order welfare-based real exchange rate fluctuations are decomposed into two parts as

$$\mathsf{Q}_t = \widehat{\mathsf{Q}}_t + \psi \mathsf{R}_{v,t},$$

where $\widehat{\mathsf{Q}}_t$ ($\equiv \widehat{\mathsf{P}}_t^* - \widehat{\mathsf{P}}_t$) denotes the first-order fluctuations in the empirical-based real exchange rate and $\mathsf{R}_{v,t}$ represents those in extensive margins, i.e. the variety risk. The above expression is a general one and $\widehat{\mathsf{Q}}_t$ and $\mathsf{R}_{v,t}$ take different forms depending on whether there is firm heterogeneity or not.

Specifically, without firm heterogeneity, Q_t is expressed as

$$\mathsf{Q}_t = (2\alpha - 1)\,\mathsf{TOT}_t + \psi\,(2\alpha - 1)\,\mathsf{N}_{D,t}^R,$$

where the terms of trade are defined as the price of imported in terms of exported goods as $\mathsf{TOT}_t \equiv \widetilde{\mathsf{p}}_{X,t}^* - \widetilde{\mathsf{p}}_{X,t}$. $\mathsf{N}_{D,t}^R \equiv \mathsf{N}_{D,t} - \mathsf{N}_{D,t}^*$ denote the relative number of varieties between Home and Foreign.

With firm heterogeneity it takes the form of

$$\mathsf{Q}_t = (2S_{ED} - 1)\,\mathsf{TOT}_t - S_{ED}\widetilde{\mathsf{z}}_{\mathsf{X},t}^R + \psi\left[(2S_{ED} - 1)\,\mathsf{N}_{D,t}^R + (1 - S_{ED})\left(\mathsf{N}_{D,t}^R - \mathsf{N}_{X,t}^R\right)\right]$$

where $\mathsf{N}_{X,t}^R \equiv \mathsf{N}_{X,t} - \mathsf{N}_{X,t}^*$ and $\widetilde{\mathsf{z}}_{\mathsf{X},t}^R \equiv \widetilde{\mathsf{z}}_{\mathsf{X},t} - \widetilde{\mathsf{z}}_{\mathsf{X},t}^*$. $S_{ED} \equiv \alpha\rho_H^{1-\omega}$ is the steady state share of domestic goods in consumption expenditure. As it is explained in Ghironi and Melitz (2005), the relative number of exporters $\mathsf{N}_{X,t}^R$ and the difference in average productivity for exporting firms $\widetilde{\mathsf{z}}_{\mathsf{X},t}^R$ further contribute to the real exchange rate fluctuations with firm heterogeneity.

By shutting down firm heterogeneity as explained in section 5, the above two expressions of the real exchange rate become the same. And in both cases when there is no love for variety in the preference ($\psi = 0$), the real exchange rate fluctuations are driven only by those in the empirical-based real exchange rate, $\widehat{\mathsf{Q}}_t$. When consumers display the preference for variety, however, this is no more the case: the welfare-based real exchange rate, Q_t, which includes the variety risk, $\mathsf{R}_{v,t}$ matters. How should equilibrium portfolio positions be designed against the welfare-based real exchange rate risk? We analytically explore the point in the following section.

3.2 Hedging intuition behaind portfolio position

The intuition of portfolio positions would be best described relying on the first-order "static budget constraint". This method consists of finding portfolio positions which support the complete asset market allocation. In our model using (1) and (2), the first-order static budget constraint is given by

$$\mathsf{P}_t+\mathsf{C}_t-(\mathsf{P}_t^*+\mathsf{C}_t^*) = S_W\left(\mathsf{w}_t^R+\mathsf{l}_t^R\right)+(2s-1)\left[S_D\left(\mathsf{N}_{D,t}^R+\widetilde{\mathsf{d}}_t^R\right)-S_I\left(\mathsf{N}_{E,t}^R+\mathsf{x}_t^{sR}\right)\right]+2b\left(\widehat{\mathsf{P}}_t-\widehat{\mathsf{P}}_t^*\right), \tag{11}$$

where S_W, S_D and S_I denote the steady state labor income, dividends and investments relative to the steady state nominal expenditure (see appendix for details). $\mathsf{w}_t^R+\mathsf{l}_t^R$, $\mathsf{N}_{D,t}^R+\widetilde{\mathsf{d}}_t^R$ and $\mathsf{N}_{E,t}^R+\mathsf{x}_t^{sR}$ represent the relative nominal labor income, dividends and investments between Home and Foreign, respectively. In writing the above static budget constraint, we also have used the asset market clearing conditions (3) assuming the symmetry of holdings across countries at the steady state: $s = s_{h,t} = s_{f,t}^*$ and $b = b_{h,t} = b_{f,t}^*$. When $s > 1/2$ there is a home biased equity position. When $b < 0$, Home issues Home CPI indexed bonds which gives a nominal payoff $\widehat{\mathsf{P}}_t$ and simultaneously lends in Foreign CPI indexed bonds. The left hand side of the above static budget constraint is the relative nominal spending and the right hand is the relative nominal income earned by non-financial and financial assets.

Unfortunately, however, the above first-order static budget constraint (11) fails to capture the original first-order "period-by-period" dynamics as in the net foreign asset (10). The point is related to the sunk nature of entry costs in our model. In the original dynamics, equity returns, $r^s_{h,t}$, $r^s_{f,t}$, $r^{s*}_{h,t}$ and $r^{s*}_{f,t}$ do not include investment costs paid in each period because they are sunk while they appear in the form of $S_I\left(\mathsf{N}^R_{E,t}+\mathsf{x}^{sR}_t\right)$ in the above static budget constraint.[13]

Nevertheless, we find that the static budget constraint is useful because it can describe hedging intuition at work in a simple way. We follow Coeurdacier and Gourinchas (2008) for that purpose. Plugging the first-order perfect risk sharing condition which holds under complete asset markets in (11), we have

$$\left(1-\frac{1}{\gamma}\right)(\mathsf{P}_t-\mathsf{P}^*_t)=S_W\mathsf{R}_{w,t}+(2s-1)\,\mathsf{R}_{e,t}+2b\mathsf{R}_{b,t}, \tag{12}$$

where $\mathsf{R}_{w,t}$, $\mathsf{R}_{e,t}$ and $\mathsf{R}_{b,t}$ are the redefinition of the labor income risk, relative equity and relative bond returns, respectively. The above expression indicates that under complete markets, households in Home spend more (less) when there is inflation (deflation) on a welfare basis ($\mathsf{P}_t-\mathsf{P}^*_t>0$ (<0)) because $\gamma\geq 1$. Any fluctuations in the welfare-based real exchange rate ($\mathsf{Q}_t\equiv\mathsf{P}^*_t-\mathsf{P}_t$), labor income ($\mathsf{R}_{w,t}$), equity ($\mathsf{R}_{e,t}$) and bond returns ($\mathsf{R}_{b,t}$) must be balanced by appropriately designed equity and bond positions, s and b.

Remembering that the welfare-based real exchange rate is decomposed in two parts as $\mathsf{Q}_t=\widehat{\mathsf{Q}}_t+\psi\mathsf{R}_{v,t}$, and bond returns become as $\mathsf{R}_{b,t}=-\widehat{\mathsf{Q}}_t$ by definition, let us posit the following first-order relations among these variables:

$$\begin{aligned}\mathsf{R}_{w,t}&=\varrho_w\widehat{\mathsf{Q}}_t+\tau_w\mathsf{Z}^R_{E,t},\\ \mathsf{R}_{v,t}&=\varrho_v\widehat{\mathsf{Q}}_t+\tau_v\mathsf{Z}^R_{E,t},\\ \mathsf{R}_{e,t}&=\varrho_e\widehat{\mathsf{Q}}_t+\tau_e\mathsf{Z}^R_{E,t},\end{aligned}$$

where ϱ_i for $i=\{w,e,v\}$ captures how the empirical real exchange rate $\widehat{\mathsf{Q}}_t$ is related to $\mathsf{R}_{w,t}$, $\mathsf{R}_{v,t}$ and $\mathsf{R}_{e,t}$. $\mathsf{Z}^R_{E,t}\equiv\mathsf{Z}_{E,t}-\mathsf{Z}^*_{E,t}$ represents the relative investment shock and τ_j for $j=\{w,v,e\}$ captures the impact of the investment shock on $\mathsf{R}_{w,t}$, $\mathsf{R}_{v,t}$ and $\mathsf{R}_{e,t}$. The above relations can be considered as a reduced form interaction which holds in the original general equilibrium.

[13] That means we cannot say when the first-order static budget constraint (11) holds for every period, the first-order of period-by-period dynamics of (10) hold as well for every period. See the appendix of Coeurdacier et al. (2010) for the case where the two coincide.

Substituting the above set of relations in (12), it is easy to solve the optimal portfolio positions which replicate the complete market allocation. We have

$$s = \frac{1}{2}\left[1 - S_w \frac{\tau_w}{\tau_e} - \psi\left(1 - \frac{1}{\gamma}\right)\frac{\tau_v}{\tau_e}\right],$$

$$b = \frac{1}{2}\left(1 - \frac{1}{\gamma}\right)\left[1 + \psi\left(\varrho_v - \varrho_e \frac{\tau_v}{\tau_e}\right)\right] + \frac{1}{2} S_w \left(\varrho_w - \varrho_e \frac{\tau_w}{\tau_e}\right).$$

We start by arguing for the optimal equity position, s. Following a positive investment shock, $\mathsf{Z}^R_{E,t} > 0$, when labor income ($\mathsf{R}_{w,t}$) rises while equity returns ($\mathsf{R}_{e,t}$) decrease as $\tau_w > 0$ and $\tau_e < 0$, it derives a home biased equity position ($s > 1/2$) from the second term in the square bracket. What derives the home biased equity position is a hedge against labor income risk as in Heathcote and Perri (2004). When $\tau_w > 0$ and $\tau_e > 0$, however, it derives a foreign biased equity position as in Baxter and Jermann (1997).

In addition, the third term may also contribute to a home biased position in our model. Again, following a positive investment shock, a further home biased position arises when $\tau_v > 0$ and $\tau_e < 0$ meaning that extensive margins ($\mathsf{R}_{v,t}$) rise while equity returns ($\mathsf{R}_{e,t}$) decrease as a result of a positive investment shock. Because nominal CPI-indexed bonds, $\widehat{\mathsf{Q}}_t$, cannot load on $\mathsf{R}_{v,t}$, it is hedged by a home biased equity position with which households transfer abroad the welfare gain stemming from higher extensive margins.

For the bond position, b, the first term captures the hedge against the welfare-based real exchange rate fluctuations. Specifically, the hedging against the empirical-based real exchange rate fluctuations is captured by 1 in the square bracket in this first term. When the empirical-based real exchange rate appreciates ($\widehat{\mathsf{Q}}_t < 0$), returns on the nominal CPI-indexed bond rise because they are perfectly correlated as $\mathsf{R}_{b,t} = -\widehat{\mathsf{Q}}_t$. Thus from this stand point, having a long position ($b > 0$) becomes a good hedge because it provides higher nominal income at the very timing of an expensive consumption.

But the above empirical-based real exchange rate appreciation may be accompanied by fluctuations in extensive margins, the variety risk. When extensive margins rise, $\mathsf{R}_{v,t} > 0$, with $\widehat{\mathsf{Q}}_t < 0$ (hence with $\mathsf{R}_{b,t} > 0$) as $\varrho_v < 0$, because higher extensive margins mean a real deflation on a welfare basis, a short bond position ($b < 0$) which transfers the welfare gain with nominal payoff becomes a good hedge. When $\varrho_v > 0$, however, the opposite is true: taking a long position ($b > 0$) becomes a good hedge against the variety risk. Coeurdacier et al. (2007) analyze the similar term in the form of an exogenous preference shock ("i-pod shock" in their terminology). In addition, bond position is used to hedge the variety risk which cannot be loaded by equity position (the term captured by $\psi \varrho_e \tau_v / \tau_e$ in the square bracket).

The second term in the bond position is a hedge against the labor income risk. The latter includes $S_w \varrho_w / 2$ which is induced by the empirical-based real exchange rate fluctuations and $S_w \varrho_e \tau_w / \tau_e$ is which cannot be loaded by equity position. Again following Coeurdacier and Gourinchas (2008), a more informative form of portfolio positions is derived:

$$s = \frac{1}{2}\left[1 - S_w \phi_{w,e} - \psi\left(1 - \frac{1}{\gamma}\right)\phi_{v,e}\right], \tag{13}$$

$$b = \frac{1}{2}\left(1 - \frac{1}{\gamma}\right)(1 + \psi \phi_{v,q}) + \frac{1}{2} S_w \phi_{w,q}. \tag{14}$$

where $\mathsf{R}_{w,t} = \phi_{w,q}\widehat{\mathsf{Q}}_t + \phi_{w,e}\mathsf{R}_{e,t}$ and $\mathsf{R}_{v,t} = \phi_{v,q}\widehat{\mathsf{Q}}_t + \phi_{v,e}\mathsf{R}_{e,t}$ in such a way that $\phi_{i,j}$ for $i, j = \{w, q, e, v\}$, $i \neq j$ has the interpretation of a *conditional* covariance to variance ratio on the loading of the another asset. Looking at the above expressions, the specificity of the portfolio position with extensive margins can be stated as follows. When the covariance between the variety risk and equity returns conditional on bond returns, $\phi_{v,e}$, is negative (positive), it provides more (less) home biased equity position. For the bond position, when the covariance between the variety risk and bond returns conditional on equity returns, $\phi_{v,q}$, is positive (negative), it adds a long (short) position.

Note specifically, if there were the counterfactual "welfare-based CPI-indexed bonds" which load perfectly on the welfare-based real exchange rate fluctuations as $\mathsf{R}_{b,t} = -\mathsf{Q}_t$, the above portfolio positions become identical to those presented in Coeurdacier and Gourinchas (2008) and Coeurdacier et al. (2010).

4 Portfolio and Macroeconomic dynamics

In this section, we solve numerically zero-order steady state portfolio positions in the model relying on the method developed by Devereux and Sutherland (2008). After finding the equilibrium portfolio, we investigate whether the intuition found in the previous section is correct by looking for conditional correlations. We also document macroeconomic dynamics implied by the theoretical models.

4.1 Calibration

Table 2: Baseline parameter values

γ	constant risk aversion	2
$\overline{\beta}$	discount factor	0.96
υ	convergence speed	0.01
φ	Frisch elasticity of labor supply	2
σ	elasticity of substitution among varieties	3.8
ω	between Home and Foreign goods	2
α	home bias in consumption	0.85
δ	death shock	0.10
θ	share of labor in entry costs	0.6
k	Pareto distribution	3.34
ψ	love for variety	$1/(\sigma-1)$

We calibrate with parameter values as in Table 2. The calibration is conducted on an annual basis. The value of constant risk aversion (γ), discount factor $\left(\overline{\beta}\right)$, Frisch elasticity of labor supply (φ), home bias in consumption (α) and the elasticity of substitution between local and imported goods (ω) are taken from Coeurdacier et al. (2010). These values are well in the range used in the standard RBC literature. The share of labor in entry costs (θ) is also taken from Coeurdacier et al. (2010) in which production is done by a Cobb-Douglas technology using capital and labor. The value of death shock (δ) is chosen so that it matches to the U.S. job destruction rate per year. The value of the elasticity of substitution for varieties (σ) is set following Ghironi and Melitz (2005) based on the empirical finding of Bernard et al. (2003) about U.S. plant and macro trade data. We set ψ, the love for variety at $1/(\sigma - 1)$ according to the Dixit-Stiglitz preference. Bernard et al. (2003) also document that the standard deviation of log U.S. plant sales is 1.67 and the proportion of exporting firm is 21%. We choose the value of Pareto distribution (k) and fixed costs for export at the steady state f_X so that they match to these empirical findings as Ghironi and Melitz (2005).[14] The parameter υ which governs the convergence speed of net foreign asset is set to 0.01. With such a small value, the endogenous discount factor takes the value of 0.96 at the steady state ($\Upsilon(C) = 0.96$) as well as $\overline{\beta}$.

The productivity process is taken from Coeurdacier et al. (2010). They estimate it for G7 countries. We define the vector of AR(1) process as $\mathsf{Z}_{t+1} = \Gamma\mathsf{Z}_t + \epsilon_t$ where $\mathsf{Z}_t = \begin{bmatrix} \mathsf{Z}_t, & \mathsf{Z}_t^* & \mathsf{Z}_{E,t}, & \mathsf{Z}_{E,t}^* \end{bmatrix}'$ and $\varepsilon_t = \begin{bmatrix} \varepsilon_{Z,t}, & \varepsilon_{Z,t}^* & \varepsilon_{Z_E,t}, & \varepsilon_{Z_E,t}^* \end{bmatrix}'$. The productivity transmission matrix of Γ and the variance covariance matrix of innovations denoted with Σ are given by[15]

$$\Gamma = \begin{bmatrix} 0.75 & 0 & 0 & 0 \\ 0 & 0.75 & 0 & 0 \\ 0 & 0 & 0.79 & 0 \\ 0 & 0 & 0 & 0.79 \end{bmatrix} \text{ and } \Sigma = \begin{bmatrix} 1.44 & 0.65 & 0 & 0 \\ 0.65 & 1.44 & 0 & 0 \\ 0 & 0 & 2.99 & 0.57 \\ 0 & 0 & 0.57 & 2.99 \end{bmatrix}. \tag{15}$$

4.2 Steady state portfolio

To find the zero-order steady state portfolios, we apply the Devereux-Sutherland method (Devereux and Sutherland (2008)). Up to the first-order approximation, the expected excess returns, $E_t r'_{x,t+1}$, are a zero mean i.i.d shock. It follows that all assets become perfect substitute up to the first-order, hence the indeterminacy of portfolio positions.

[14]The corresponding standard deviation in the theoretical model is given by $1/(k - \sigma + 1)$.

[15]As it is mentioned in Devereux and Sutherland (2008), when the number of assets and the number of shocks are identical in the model, the steady state portfolio positions become independent from variance-covariance matrix of shock, Σ. And this is the case for our model.

However, the problem is overcome by considering the second order approximation of the Euler equations on asset holdings which provide sufficient set of equations in first-order by which the steady state portfolio is pinned down. With net foreign asset dynamics (10), the first-order system of the model can be written in the state-space form providing necessary matrices for the solution of portfolio.[16]

With the above parameters, the steady state portfolios are found as in Table 3. The first and second column provides the equity and bond positions for the benchmark model with firm heterogeneity ($s = 1.67$ and $b = 0.76$) and those without firm heterogeneity ($s = 1.13$ and $b = 0.44$), respectively. In addition, Table 3 documents the portfolio obtained with the counterfactual welfare-based CPI indexed bonds ($s = 0.86$ and $b = 0.81$ with firm heterogeneity and $s = 0.99$ and $b = 0.38$ without firm heterogeneity). With or without firm heterogeneity, we observe an amplified home biased equity position compared to those obtained with variety indexed bonds.

Why do we find such an amplified home biased equity position? The analysis in the previous section sheds light on the mechanism. Besides the labor income risk, equity positions should load on the variety risk without variety indexed bonds. As it has been described in (13), a negative conditional correlation between extensive margins and equity returns is crucial to generate an amplified home biased equity position. Table 4 provides conditional correlations, $Corr_q\left(\mathsf{R}_{w,t}, \mathsf{R}_{e,t}\right)$, $Corr_q\left(\mathsf{R}_{v,t}, \mathsf{R}_{e,t}\right)$, $Corr_e\left(\mathsf{R}_{w,t}, \widehat{\mathsf{Q}}_t\right)$ and $Corr_e\left(\mathsf{R}_{v,t}, \widehat{\mathsf{Q}}_t\right)$ in original period-by-period dynamics for both models with and without heterogeneous firms.[17][18] In addition to negative conditional correlations between labor income and equity returns for both models ($Corr_q\left(\mathsf{R}_{w,t}, \mathsf{R}_{e,t}\right) = -0.51$ and -0.55), those between extensive margins and equity returns are also negative ($Corr_q\left(\mathsf{R}_{v,t}, \mathsf{R}_{e,t}\right) = -0.37$

[16]Precisely, matrices of R_1, R_2, D_1, and D_2 in Devereux and Sutherland (2008).

[17]We compute the first-order fluctuations in original period-by-period dynamics of relative wage $\mathsf{R}_{w,t}$, extensive margins $\mathsf{R}_{v,t}$ and equity returns $\mathsf{R}_{e,t}$. These variables are computed from $\mathsf{R}_{w,t} = \mathsf{w}_t + \mathsf{L}_t - (\mathsf{Q}_t + \mathsf{w}_t^* + \mathsf{L}_t^*)$, $\mathsf{R}_{v,t} = S_{ED}\mathsf{N}_{D,t}^R - (1 - S_{ED})\,\mathsf{N}_{X,t}^R$ and $\mathsf{R}_{e,t} = \mathsf{r}_{hx,t}^s - \mathsf{r}_{fx,t}^s$.

[18]Conditional correlations in Table 4 and second moments in Table 5 are computed using frequency-domain technique proposed in Uhlig (1998). First-order series are filtered by Hodrick-Prescott filter The smoothing parameter is set to 400.

and -0.30). In the original period-by-period dynamics, when a positive shock on firm set up efficiency $Z_{E,t}$ takes place, the number of varieties rises in Home while equity returns decrease $\left(r^s_{h,t} < 0\right)$ because the share price decreases $\left(x^s_{h,t} < 0\right)$ through free entry condition. And the resulting negative correlation such that $Corr_q\left(\mathsf{R}_{v,t}, \mathsf{R}_{e,t}\right) < 0$ provides an amplified home biased position as we see in Table 3.

Bond positions do not change dramatically with variety risk and they remain positive. As (14) indicates, it must add a short (long) position when the conditional correlation between extensive margins and the empirical-based real exchange rate is negative (positive) compared to that obtained with variety indexed bonds. For the model with firm heterogeneity the conditional correlation is negative ($Corr_e\left(\mathsf{R}_{v,t}, \widehat{\mathsf{Q}}_t\right) = -0.18$) and indeed it adds a short position ($b = 0.76 < 0.81$). For the model without firm heterogeneity the correlation becomes positive ($Corr_e\left(\mathsf{R}_{v,t}, \widehat{\mathsf{Q}}_t\right) = 0.29$) and a further long position appears with variety risk ($b = 0.44 > 0.38$). Conditional correlations between labor income and the empirical-based real exchange rate $Corr_e\left(\mathsf{R}_{w,t}, \widehat{\mathsf{Q}}_t\right)$ take the value of -0.59 for both models.

The specificities of the portfolio with variety risk would also be clearly seen by a sensitivity analysis against the elasticity of substitution between local and imported goods, ω. Because equity positions load on the (welfare-based) real exchange rate fluctuations with variety risk, they regain a non-linearity with respect to ω (left hand side panels in Figure 1) as it is the case when equity positions load on the real exchange rate risk without bonds (Coeurdacier and Gourinchas (2008), Coeurdacier (2009)). If there were variety indexed bonds, equity position are solely used to hedge against labor income risk and they exhibit a stability with respect to ω (right hand side panels in Figure 1).

Table 3: Steady state portfolios

	Nominal bonds		Variety indexed bonds	
	With hetero	Without hetero	With hetero	Without hetero
s	1.67	1.13	0.86	0.99
b	0.76	0.44	0.81	0.38

Table 4: Conditional correlations

	$Corr_q(\mathsf{R}_{w,t}, \mathsf{R}_{e,t})$	$Corr_q(\mathsf{R}_{v,t}, \mathsf{R}_{e,t})$	$Corr_e\left(\mathsf{R}_{w,t}, \widehat{\mathsf{Q}}_t\right)$	$Corr_e\left(\mathsf{R}_{v,t}, \widehat{\mathsf{Q}}_t\right)$
With hetero	-0.51	-0.37	-0.42	-0.18
Without hetero	-0.55	-0.30	-0.43	0.29

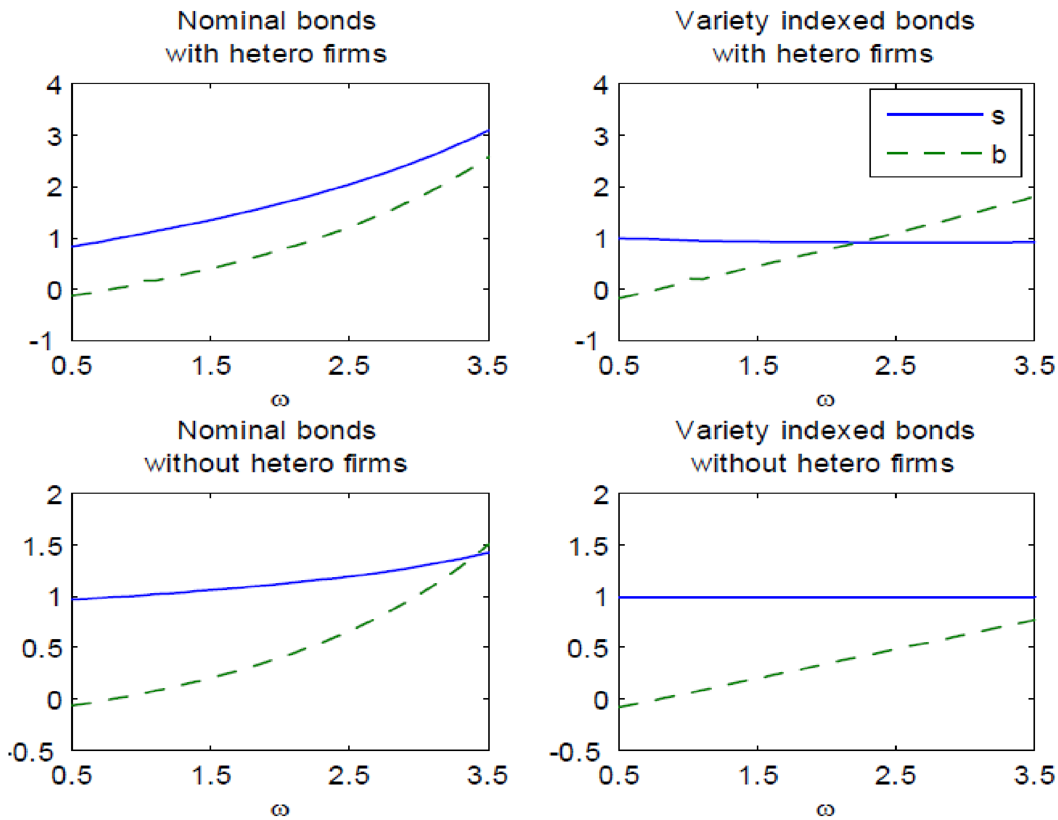

Figure 1: Sensitivity analysis with respect to ω

4.3 International business cycle

We now study the predictions of the model for the dynamics of key macroeconomic variables. Although the purpose of the paper is to investigate the portfolio positions with variety risk and not to reproduce a realistic business cycle matching the data, this sort of exercise would be useful shedding light on the shortcomings of the model and indicate the direction for the future research.

Table 5 provides the standard deviations and unconditional correlations of major macroeconomic variables for each model with and without firm heterogeneity. GDP Y_t and investments I_t are defined as $Y_t \equiv w_t L_t + N_{D,t}\widetilde{d}_t$ and $I_t \equiv N_E x^s_{R,t}$, respectively. Net exports are defined over GDP as NX_t/Y_t. Because variables denominated with the welfare-based CPI in the theoretical models contain unobservable fluctuations in extensive margins, they are abstracted in constructing real variables which correspond to the empirical data. Following Ghironi and Melitz (2005), any real variables X_t deflated with the welfare-based CPI, P_t, are transformed to those $X_{R,t}$ deflated with the empirical-based CPI, $\widehat{P}_t$, by $X_{R,t} \equiv P_t X_t/\widehat{P}_t$.

The table contains the actual business cycle data which come from Coeurdacier et al. (2010). The data are mean values of the G7 countries and annual basis for the period 1984-2004. We also report second moments implied by their model and Heathcote and Perri (2002) (complete markets) which are denoted with CKM and HP respectively in the table.

For the standard deviation of output, consumption, investments, hours worked and net exports, our models are successful in reproducing those in the data. They, however, fail to reproduce sizable fluctuations in the observed real exchange rate (0.80 with firm heterogeneity and 0.68 without compared to 4.05 in the data).

Our models perform as good as CKM and HP for the correlation with output in comparison of actual data. For cross country correlations, output is correlated less than consumption across countries (0.01 and 0.52 with heterogeneity and 0.10 and 0.72 without) while output is slightly more correlated than consumption in the data (0.49 and 0.46). The cross country correlations of investments and labor supply are negative (-0.15 and -0.15 with heterogeneity and -0.17 and -0.17 without) while they are positive in the data (0.27 and 0.43).

As argued in Heathcote and Perri (2002), since Backus et al. (1992) these (bad) characteristics, together with the low volatility of real exchange rate, are very common in the model which embodies a strong consumption risk sharing mechanism across countries. Introducing extensive margins as Ghironi and Melitz (2005) cannot help to improve in these dimensions.[19]

Table 5: Second moments

% std.dev. relative to output	Y_R	C_R	I_R	L	NX_R	$\widehat{Q}$
G7 data	2.07	0.74	6.89	0.91	0.55	4.05
Benchmark	1.51	0.42	4.72	0.65	1.00	0.80
Without hetero	1.51	0.41	4.27	0.66	0.97	0.68
CKM	1.87	0.41	4.42	0.70	0.57	0.74
HP	1.21	0.53	2.74	0.31	0.20	0.55
Correlation with output	C_R	I_R	L	NX_R	$\widehat{Q}$	
G7 data	0.78	0.85	0.83	−0.39	0.12	
Benchmark	0.59	0.84	0.66	−0.60	0.47	
Without hetero	0.57	0.86	0.66	−0.75	0.41	
CKM	0.38	0.71	0.61	−0.07	−0.22	
HP	0.96	0.96	0.97	−0.64	0.65	
Cross country correlation	Y_R	C_R	I_R	L		
G7 data	0.49	0.46	0.27	0.43		
Benchmark	0.01	0.52	−0.15	−0.15		
Without hetero	0.10	0.72	−0.17	−0.17		
CKM	0.17	0.58	−0.37	0.18		
HP	0.18	0.65	−0.29	−0.14		

5 Conclusion

We analyze zero-order steady state equity and bond positions in a two-country DSGE model where the number of varieties is endogenous. With variety risk in real exchange rate fluctuations, home biased equity positions are further strengthened. The result is shown to be robust with or without firm heterogeneity.

[19] Ghironi and Melitz (2005) also provide business cycle characteristics for their model with non-contingent bonds that are very similar to ours.

For future research, it would be important to empirically investigate to which extent the variety risk contributes to the home biased equity positions. One testable prediction of the paper is that the *welfare-based* real exchange rate would have a negative conditional correlation with equity returns. This would be in contrast with the result of van Wincoop and Warnock (2006) obtained with the empirical-based real exchange rate fluctuations. Another direction would be to investigate the valuation effect (Gourinchas and Rey (2007), Devereux and Sutherland (2010)) with extensive margins and reconsider the current account adjustment process with extensive margins analyzed in Corsetti et al. (2008).

References

Backus, D. K., P. J. Kehoe, and F. E. Kydland (1992, August). International real business cycles. *Journal of Political Economy 100*(4), 745–75.

Baxter, M. and U. J. Jermann (1997, March). The international diversification puzzle is worse than you think. *American Economic Review 87*(1), 170–80.

Benassy, J.-P. (1996, July). Taste for variety and optimum production patterns in monopolistic competition. *Economics Letters 52*(1), 41–47.

Bernard, A. B., J. Eaton, J. B. Jensen, and S. Kortum (2003, September). Plants and productivity in international trade. *American Economic Review 93*(4), 1268–1290.

Bilbiie, F., F. Ghironi, and M. Melitz (2007, Winter). Monetary policy and business cycles with endogenous entry and product variety. In *NBER Macroeconomics Annual 2007, Volume 22*, NBER Chapters, pp. 299–353. National Bureau of Economic Research, Inc.

Broda, C. and D. E. Weinstein (2006, May). Globalization and the gains from variety. *The Quarterly Journal of Economics 121*(2), 541–585.

Broda, C. and D. E. Weinstein (2010, June). Product creation and destruction: Evidence and price implications. *American Economic Review 100*(3), 691–723.

Broda, C. and D. W. Weinstein (2004, May). Variety growth and world welfare. *American Economic Review 94*(2), 139–144.

Bui, T. (2009). International portfolio dynamics in new open economy macroeconomics model. Technical report, Georgetown university.

Castello, M. A. (2008). International transmission, firm entry and risk sharing. Ph.d. dissertation, European university institute.

Coeurdacier, N. (2009, February). Do trade costs in goods market lead to home bias in equities? *Journal of International Economics 77*(1), 86–100.

Coeurdacier, N. and P.-O. Gourinchas (2008). When bonds matter: home bias in goods and assets. Working Paper Series 2008-25, Federal Reserve Bank of San Francisco.

Coeurdacier, N., R. Kollmann, and P. Martin (2007, September). International portfolios with supply, demand and redistributive shocks. NBER Working Papers 13424, National Bureau of Economic Research, Inc.

Coeurdacier, N., R. Kollmann, and P. Martin (2010, January). International portfolios, capital accumulation and foreign assets dynamics. *Journal of International Economics 80*(1), 100–112.

Coeurdacier, N. and H. Rey (2011). Home bias in open ecoomy financial macroeconomics. Technical report, London Business School.

Cole, H. L. and M. Obstfeld (1991, August). Commodity trade and international risk sharing : How much do financial markets matter? *Journal of Monetary Economics 28*(1), 3–24.

Corsetti, G., P. Martin, and P. Pesenti (2008, February). Varieties and the transfer problem: The extensive margin of current account adjustment. NBER Working Papers 13795, National Bureau of Economic Research, Inc.

Devereux, M. B. and A. Sutherland (2008, October). Country portfolios in open economy macro models. NBER Working Papers 14372, National Bureau of Economic Research, Inc.

Devereux, M. B. and A. Sutherland (2010, January). Valuation effects and the dynamics of net external assets. *Journal of International Economics 80*(1), 129–143.

Dixit, A. K. and J. E. Stiglitz (1977, June). Monopolistic competition and optimum product diversity. *American Economic Review 67*(3), 297–308.

Engel, C. and A. Matsumoto (2009a, July). The international diversification puzzle when goods prices are sticky: It's really about exchange-rate hedging, not equity portfolios. *American Economic Journal: Macroeconomics 1*(2), 155–88.

Engel, C. and A. Matsumoto (2009b, July). International risk sharing: Through equity diversification or exchange rate hedging? IMF Working Papers 09/138, International Monetary Fund.

French, K. R. and J. M. Poterba (1991, May). Investor diversification and international equity markets. *American Economic Review 81*(2), 222–26.

Galstyan, V. and P. R. Lane (2008, November). External imbalances and the extensive margin of trade. *Economic Notes 37*(3), 241–257.

Ghironi, F. and M. J. Melitz (2005, August). International trade and macroeconomic dynamics with heterogeneous firms. *The Quarterly Journal of Economics 120*(3), 865–915.

Gourinchas, P.-O. and H. Rey (2007, 08). International financial adjustment. *Journal of Political Economy 115*(4), 665–703.

Hamano, M. (2012). The consumption-real exchange rate anomaly with extensive margin. Working paper, University of Luxembourg.

Heathcote, J. and F. Perri (2002, April). Financial autarky and international business cycles. *Journal of Monetary Economics 49*(3), 601–627.

Heathcote, J. and F. Perri (2004). The international diversification puzzle is not as bad as you think. 2004 Meeting Papers 152, Society for Economic Dynamics.

Kollmann, R. (2006a, 01). A dynamic equilibrium model of international portfolio holdings: Comment. *Econometrica 74*(1), 269–273.

Kollmann, R. (2006b, February). International portfolio equilibrium and the current account. CEPR Discussion Papers 5512, C.E.P.R. Discussion Papers.

Lane, P. R. and G. M. Milesi-Ferretti (2003, February). International financial integration. CEPR Discussion Papers 3769, C.E.P.R. Discussion Papers.

Lane, P. R. and G. M. Milesi-Ferretti (2005, September). A global perspective on external positions. CEPR Discussion Papers 5234, C.E.P.R. Discussion Papers.

Lane, P. R. and G. M. Milesi-Ferretti (2007, November). The external wealth of nations mark ii: Revised and extended estimates of foreign assets and liabilities, 1970-2004. *Journal of International Economics 73*(2), 223–250.

Martin, P. and H. Rey (2004, December). Financial super-markets: size matters for asset trade. *Journal of International Economics 64*(2), 335–361.

Melitz, M. J. (2003, November). The impact of trade on intra-industry reallocations and aggregate industry productivity. *Econometrica 71*(6), 1695–1725.

Obstfeld, M. (2007.). International risk sharing and the costs of trade. Technical report.

Rahbari, E. (2009). Portfolios and business cycles in an open economy dsge model. Technical report, London Business School.

Schmitt-Grohe, S. and M. Uribe (2003, October). Closing small open economy models. *Journal of International Economics 61*(1), 163–185.

Sercu, P. and R. Vanpee (2007). Home bias in international equity portfolios: a review. Open Access publications from Katholieke Universiteit Leuven urn:hdl:123456789/175483, Katholieke Universiteit Leuven.

Tesar, L. L. and I. M. Werner (1995, August). Home bias and high turnover. *Journal of International Money and Finance 14*(4), 467–492.

Tille, C. and E. van Wincoop (2008, February). International capital flows. CEPR Discussion Papers 6705, C.E.P.R. Discussion Papers.

Uhlig, H. (1998). A toolkit for analysing nonlinear dynamic stochastic models easily. QM&RBC Codes, Quantitative Macroeconomics & Real Business Cycles.

Uppal, R. (1993, June). A general equilibrium model of international portfolio choice. *Journal of Finance 48*(2), 529–53.

van Wincoop, E. and F. E. Warnock (2006, December). Is home bias in assets related to home bias in goods? NBER Working Papers 12728, National Bureau of Economic Research, Inc.

A System

Price indices

$$\alpha \rho_{H,t}^{1-\omega} + (1-\alpha)\,\rho_{F,t}^{1-\omega} = 1$$

$$\rho_{H,t} = N_{D,t}^{-\psi} \widetilde{\rho}_{D,t} \qquad \rho_{F,t} = N_{X,t}^{*-\psi} \widetilde{\rho}_{X,t}^{*}$$

$$\alpha \rho_{F,t}^{*1-\omega} + (1-\alpha)\,\rho_{H,t}^{*1-\omega} = 1$$

$$\rho_{F,t}^{*} = N_{D,t}^{*-\psi} \widetilde{\rho}_{D,t}^{*} \qquad \rho_{H,t}^{*} = N_{X,t}^{-\psi} \widetilde{\rho}_{X,t}$$

Pricing

$$\widetilde{\rho}_{D,t} = \frac{\sigma}{\sigma - 1}\frac{w_t}{Z_t \widetilde{z}} \qquad \widetilde{\rho}_{X,t} = \frac{\sigma}{\sigma - 1}\frac{w_t}{Z_t \widetilde{z}_{X,t}} Q_t^{-1}$$

$$\widetilde{\rho}^*_{D,t} = \frac{\sigma}{\sigma - 1}\frac{w^*_t}{Z^*_t \widetilde{z}^*_D} \qquad \widetilde{\rho}^*_{X,t} = \frac{\sigma}{\sigma - 1}\frac{w^*_t}{Z^*_t \widetilde{z}^*_{X,t}} Q_t$$

Profits

$$\widetilde{d}_{D,t} = \frac{1}{\sigma} N_{D,t}^{\psi(\omega-1)-1} \widetilde{\rho}_{D,t}^{1-\omega} \alpha M_t$$

$$\widetilde{d}_{X,t} = \frac{Q_t}{\sigma} N_{X,t}^{\psi(\omega-1)-1} \widetilde{\rho}_{X,t}^{1-\omega} (1-\alpha) M^*_t - \mu_{X,t} f_X$$

$$\widetilde{d}_t = \widetilde{d}_{D,t} + \frac{N_{X,t}}{N_{D,t}} \widetilde{d}_{X,t}$$

$$\widetilde{d}^*_{D,t} = \frac{1}{\sigma} N_{D,t}^{*\psi(\omega-1)-1} \widetilde{\rho}_{D,t}^{*1-\omega} \alpha M^*_t$$

$$\widetilde{d}^*_{X,t} = \frac{Q_t^{-1}}{\sigma} N_{X,t}^{*\psi(\omega-1)-1} \widetilde{\rho}_{X,t}^{*1-\omega} (1-\alpha) M_t - \mu^*_{X,t} f^*_X$$

$$\widetilde{d}^*_t = \widetilde{d}^*_{D,t} + \frac{N^*_{X,t}}{N^*_{D,t}} \widetilde{d}^*_{X,t}$$

Definition of M

$$M_t = C_t + (1-\theta) N_{E,t} x^s_t$$

$$M^*_t = C^*_t + (1-\theta) N^*_{E,t} x^{s*}_t$$

Real cost for exporting

$$\mu_{X,t} = \frac{w_t}{Z_t}$$

$$\mu^*_{X,t} = \frac{w^*_t}{Z^*_t}$$

Free entry

$$x^s_t = \left(\frac{w_t}{Z_{E,t}}\right)^{\theta} f_E$$

$$x^{s*}_t = \left(\frac{w^*_t}{Z^*_{E,t}}\right)^{\theta} f^*_E$$

Number of firms

$$N_{D,t+1} = (1-\delta)\left(N_{D,t} + N_{E,t}\right)$$

$$N^*_{D,t+1} = (1-\delta)\left(N^*_{D,t} + N^*_{E,t}\right)$$

Optimal labor supply

$$\chi\left(L_t\right)^{\frac{1}{\psi}} = w_t C_t^{-\gamma}$$

$$\chi\left(L^*_t\right)^{\frac{1}{\psi}} = w^*_t C_t^{*-\gamma}$$

Labor Market clearing conditions

$$L_t = (\sigma-1)\frac{N_{D,t}\widetilde{d}_t}{w_t} + \theta\frac{N_{E,t}x^s_t}{w_t} + \sigma\frac{N_{X,t}\mu_{X,t}f_X}{w_t}$$

$$L^*_t = (\sigma-1)\frac{N^*_{D,t}\widetilde{d}^*_t}{w^*_t} + \theta\frac{N^*_{E,t}x^{s*}_t}{w^*_t} + \sigma\frac{N^*_{X,t}\mu^*_{X,t}f^*_X}{w^*_t}$$

Share of exporting firms

$$\frac{N_{X,t}}{N_{D.t}} = z^k_{\min}\left(\widetilde{z}_{X,t}\right)^{-k}\left[\frac{k}{k-(\sigma-1)}\right]^{\frac{k}{\sigma-1}}$$

$$\frac{N^*_{X,t}}{N^*_{D,t}} = z^k_{\min}\left(\widetilde{z}^*_{X,t}\right)^{-k}\left[\frac{k}{k-(\sigma-1)}\right]^{\frac{k}{\sigma-1}}$$

Zero-profit export cutoff

$$\widetilde{d}_{X,t} = \mu_{X,t} f_X \frac{\sigma - 1}{k - (\sigma - 1)}$$

$$\widetilde{d}^*_{X,t} = \mu^*_{X,t} f^*_X \frac{\sigma - 1}{k - (\sigma - 1)}$$

Definition of real returns (expressed in Home consumption basket)

$$r^s_{h,t} \equiv (1 - \delta) \frac{x^s_{h,t} + \widetilde{d}_{h,t}}{x^s_{h,t-1}}$$

$$r^s_{f,t} \equiv (1 - \delta) \frac{x^{s*}_t + \widetilde{d}^*_t}{x^{s*}_{t-1}} \frac{Q_t}{Q_{t-1}}$$

$$r^b_{h,t} \equiv \frac{x^b_{h,t} + \frac{\widehat{P}_t}{P_t}}{x^b_{h,t-1}}$$

$$r^b_{f,t} \equiv \frac{x^{b*}_t + \frac{\widehat{P}^*_t}{P^*_t}}{x^{b*}_{t-1}} \frac{Q_t}{Q_{t-1}}$$

Asset markets clearing conditions

$$a^s_{h,t} + a^{s*}_{h,t} = x^s_{h,t} \left(N_{D,t} + N_{E,t}\right)$$

$$a^s_{f,t} + a^{s*}_{f,t} = Q_t x^{s*}_t \left(N^*_{D,t} + N^*_{E,t}\right)$$

$$a^b_{h,t} + a^{b*}_{h,t} = 0$$

$$a^b_{f,t} + a^{b*}_{f,t} = 0$$

Euler Home and Foreign

$$C_t^{\gamma - \upsilon} E_t C_{t+1}^{-\gamma} = C_t^{*\gamma - \upsilon} E_t C_{t+1}^{*-\gamma} \frac{Q_t}{Q_{t+1}}$$

Euler shares

(Home)

$$1 = \overline{\beta} C_t^{\gamma - v} E_t C_{t+1}^{-\gamma} r_{h,t+1}^s$$

$$1 = \overline{\beta} C_t^{\gamma - v} E_t C_{t+1}^{-\gamma} r_{f,t+1}^s$$

Euler equation (bond)

(Home)

$$1 = \overline{\beta} C_t^{\gamma - v} E_t C_{t+1}^{-\gamma} r_{h,t+1}^b$$

$$1 = \overline{\beta} C_t^{\gamma - v} E_t C_{t+1}^{-\gamma} r_{f,t+1}^b$$

Definition of the expected excess returns

$$E_t r_{fx,t+1}^s = E_t \left[r_{f,t+1}^s - r_{h,t+1}^b \right]$$

$$E_t r_{hx,t+1}^s = E_t \left[r_{h,t+1}^s - r_{h,t+1}^b \right]$$

$$E_t r_{fx,t+1}^b = E_t \left[r_{f,t+1}^b - r_{h,t+1}^b \right]$$

Net foreign asset dynamics of Home

$$NFA_{t+1} = NX_t + NFA_t r_{h,t}^b + a_{f,t-1}^s r_{fx,t}^s - a_{h,t-1}^{s*} r_{hx,t}^s + a_{f,t-1}^b r_{fx,t}^b$$

Net export for Home

$$NX_t = \sigma \left(N_{D,t} \widetilde{d}_t + N_{X,t} \mu_{X,t} f_X \right) - M_t$$

B Steady state

Following Ghironi and Melitz (2005) we determine the steady state fixed costs for exporting f_X so that it gives 21% share of exporting firms as documented in Bernard et al. (2003). As it will be clear the solution procedure is a little bit more complicated than Ghironi and Melitz (2005) because of entry costs paid in consumption goods as well as labor services. We follow the solution procedure presented in the technical appendix of Ghironi and Melitz (2005) which is available on authors' web site. The goal here is to reduce the system into 3 equations and 3 unknown variables, N_D, w and f_X.

At the symmetric steady state, we assume without loss of generality that $Z = Z^* = f_E = f_E^* = z_{\min} = z_{\min}^* = 1$. Because of the symmetry we drop asterisks which denote foreign variables. It is noticed that $NFA = NX = 0$ and $Q = 1$ at the symmetric steady state. We choice the parameter χ so that the steady state labor supply becomes unity as $L = 1$.

Free entry condition gives $x^s = w^\theta f_E$ and the equation about the motion of firms gives $N_D = \frac{1-\delta}{\delta} N_E$. Knowing $\widetilde{d}_h = \widetilde{d}_D + \frac{N_X}{N_D} \widetilde{d}_X$ combined with Euler equations about share holdings, we get

$$\widetilde{d}_D + \frac{N_X}{N_D} \widetilde{d}_X = \frac{1 - \Upsilon (1-\delta)}{\Upsilon (1-\delta)} w^\theta f_E. \tag{16}$$

Now we rewrite $\widetilde{d}_D$ and $\widetilde{d}_X$. Knowing $\mu_X = w f_X$ from zero-profit export cutoff condition we have

$$\widetilde{d}_X = w f_X \frac{\sigma - 1}{k - (\sigma - 1)}. \tag{17}$$

With the above expression and using the steady state average domestic and exporting profits $\widetilde{d}_D$ and $\widetilde{d}_X$, $\widetilde{d}_D$ can be rewritten as

$$\widetilde{d}_D = \frac{S_{ED}}{S_{EX}} \frac{N_X}{N_D} \left[\frac{\sigma - 1}{k - (\sigma - 1)} + 1 \right] w f_X, \tag{18}$$

where $S_{ED} \equiv \alpha \rho_H^{1-\omega}$ and $S_{EX} \equiv (1-\alpha) \rho_F^{1-\omega}$, which are the steady state share on domestic and imported goods in total consumption. Noting $\rho_H = N_D^{-\psi} \widetilde{\rho}_D$, $\rho_F = N_X^{-\psi} \widetilde{\rho}_X$, $\widetilde{\rho}_D =$

$\frac{\sigma}{\sigma-1}\frac{w}{\widetilde{z}_D}$ and $\widetilde{\rho}_X = \frac{\sigma}{\sigma-1}\frac{w}{\widetilde{z}_X}$, S_{ED}/S_{EX} is turned out to be

$$\frac{S_{ED}}{S_{EX}} = \frac{\alpha}{1-\alpha}\left(\frac{N_D}{N_X}\right)^{-\psi(1-\omega)}\left(\frac{\widetilde{z}_X}{\widetilde{z}_D}\right)^{1-\omega}. \tag{19}$$

Plugging (18) and (17) into (16), we get the first equation:

$$\frac{\alpha}{1-\alpha}\left(\frac{N_X}{N_D}\right)^{1-\psi(\omega-1)}\left(\frac{\widetilde{z}_X}{\widetilde{z}_D}\right)^{1-\omega}\left[\frac{k}{k-(\sigma-1)}\right]+\frac{N_X}{N_D}\frac{\sigma-1}{k-(\sigma-1)} = \frac{1-\Upsilon(1-\delta)}{\Upsilon(1-\delta)}\frac{f_E}{f_X}w^{\theta-1}. \tag{20}$$

Next we derive the second equation. At the symmetric steady state we have $C + N_E x^s = w + N_D\widetilde{d}$ from the labor market clearing or aggregating the budget constraints of households. Eliminating C by $M = C + (1-\theta) N_E x^s$ and using $x^s = w^\theta f_E$ and (16) the above aggregated identity can be expressed as

$$\frac{M}{w} = 1 + \frac{1-\Upsilon+\Upsilon\delta(1-\theta)}{\Upsilon(1-\delta)}N_D w^{\theta-1} f_E, \tag{21}$$

The left hand side of the above equation can also be expressed using the zero-profit export cutoff condition (17) and the steady state expression of $\widetilde{d}_X$ as

$$\frac{M}{w} = \frac{1}{1-\alpha}N_X^{1-\psi(\omega-1)}\left(\frac{\sigma}{\sigma-1}\frac{w}{\widetilde{z}_X}\right)^{\omega-1}\frac{k\sigma}{k-(\sigma-1)}f_X. \tag{22}$$

Dividing the right hand side of both (21) and (22) by $N_D^{1-\psi(\omega-1)}$ and equating them we get the second equation as

$$\frac{1}{1-\alpha}\left(\frac{N_X}{N_D}\right)^{1-\psi(\omega-1)}\left(\frac{\sigma}{\sigma-1}\frac{w}{\widetilde{z}_X}\right)^{\omega-1}\frac{k\sigma}{k-(\sigma-1)}f_X = \frac{1}{N_D^{1-\psi(\omega-1)}} + \frac{1-\Upsilon+\Upsilon\delta(1-\theta)}{\Upsilon(1-\delta)}N_D^{\psi(\omega-1)}w^{\theta-1}f_E. \tag{23}$$

Finally the third equation can be obtained combining (19) and the steady state price index as

$$\frac{\left(\frac{\sigma}{\sigma-1}\frac{w}{\widetilde{z}_X}\right)^{\omega-1}}{N_D^{\psi(\omega-1)}} = \alpha\left(\frac{\widetilde{z}_X}{\widetilde{z}_D}\right)^{1-\omega} + (1-\alpha)\left(\frac{N_X}{N_D}\right)^{\psi(\omega-1)}. \tag{24}$$

There are 3 equations, (20), (23) and (24). $\widetilde{z}_D$ is given by Pareto distribution. $\frac{N_X}{N_D}$ is set to 0.21. This requires $\widetilde{z}_X = 2.94$ with baseline parameters. These non-linear equations are numerically solved for N_D, w and f_X. Especially the share of fixed cost for exporting in amortized entry cost becomes 11.9% with the baseline parameters.

Once found N_D, w and f_X, other variables are relatively easy to be found:

$$M = w + \frac{1-\Upsilon+\Upsilon\delta(1-\theta)}{\Upsilon(1-\delta)} N_D w^\theta f_E,$$

$$N_X = \frac{21}{100} N_D,$$

$$N_E = \frac{\delta}{1-\delta} N_D,$$

$$x^s = w^\theta f_E,$$

$$C = M - (1-\theta) N_E x^s,$$

$$\widetilde{\rho}_D = \frac{\sigma}{\sigma-1}\frac{w}{\widetilde{z}}, \;\; \widetilde{\rho}_X = \frac{\sigma}{\sigma-1}\frac{w}{\widetilde{z}_X},$$

$$\rho_H = N_D^{-\psi}\widetilde{\rho}_D, \quad \rho_F = N_X^{-\psi}\widetilde{\rho}_X.$$

The value of parameter χ is set by $\chi = wC^{-\gamma}$ so that $L = 1$.

B.1 Steady state shares

Here we define steady state shares which appear in calibrating the first-order set of equations. The share of domestic and imported goods in total expenditure are

$$S_{ED} \equiv \alpha \rho_H^{1-\omega} \text{ and } 1 - S_{ED} \equiv (1-\alpha)\, \rho_F^{1-\omega}.$$

The steady state share of fixed exporting costs, dividends on domestic, exporting and total sales relative to M are defined as

$$S_{FX}^M \equiv \frac{N_X w f_X}{M}, \;\; S_{DD}^M \equiv \frac{N_D \widetilde{d}_D}{M}, \;\; S_X^M \equiv \frac{N_X \widetilde{d}_X}{M}, \;\; S_D^M \equiv \frac{N_D \widetilde{d}_h}{M}.$$

Note in passage the share of dividends from domestic and exporting sales relative to total dividends are expressed respectively by

$$\frac{S_{DD}^M}{S_D^M} = \kappa S_{ED} \text{ and } \frac{S_X^M}{S_D^M} = \kappa\,(1 - S_{ED}),$$

where

$$\kappa \equiv \frac{\frac{1}{\sigma}}{\frac{1}{\sigma} - S_{FX}^M}.$$

The steady state share of investments, wage and consumption relative to M are defined as

$$S_I^M \equiv \frac{N_E x^s}{M}, \quad S_W^M \equiv \frac{w}{M}, \quad S_C^M \equiv \frac{C}{M}.$$

C First-order system

<u>Price indices</u>

$$\alpha \rho_{H,t} + (1-\alpha)\, \rho_{F,t} = 0$$

$$\rho_{H,t} = -\psi \mathsf{N}_{D,t} + \widetilde{\rho}_{D,t} \qquad \rho_{F,t} = -\psi \mathsf{N}^*_{X,t} + \widetilde{\rho}^*_{X,t}$$

$$\alpha \rho^*_{F,t} + (1-\alpha)\, \rho^*_{H,t} = 0$$

$$\rho^*_{F,t} = -\psi \mathsf{N}^*_{D,t} + \widetilde{\rho}^*_{D,t} \qquad \rho^*_{H,t} = -\psi \mathsf{N}_{X,t} + \widetilde{\rho}_{X,t}$$

<u>Pricing</u>

$$\widetilde{\rho}_{D,t} = \mathsf{w}_t - \mathsf{Z}_t \qquad \widetilde{\rho}_{X,t} = \mathsf{w}_t - \mathsf{Z}_t - \widetilde{\mathsf{z}}_{X,t} - \mathsf{Q}_t$$

$$\widetilde{\rho}^*_{D,t} = \mathsf{w}^*_t - \mathsf{Z}^*_t \qquad \widetilde{\rho}^*_{X,t} = \mathsf{w}^*_t - \mathsf{Z}^*_t - \widetilde{\mathsf{z}}^*_{X,t} + \mathsf{Q}_t$$

<u>Profits</u>

(Home)

$$\mathsf{N}_{D,t} + \widetilde{\mathsf{d}}_{D,t} = \psi (\omega - 1) \mathsf{N}_{D,t} + (1 - \omega) \widetilde{\rho}_{D,t} + \mathsf{M}_t$$

$$\mathsf{N}_{X,t} + \widetilde{\mathsf{d}}_{X,t} = \mathsf{Q}_t + \psi (\omega - 1) \mathsf{N}_{X,t} + (1 - \omega) \widetilde{\rho}_{X,t} + \mathsf{M}^*_t$$

$$\mathsf{N}_{D,t} + \widetilde{\mathsf{d}}_t = \kappa S_{ED} \left(\mathsf{N}_{D,t} + \widetilde{\mathsf{d}}_{D,t} \right) + (1 - \kappa S_{ED}) \left(\mathsf{N}_{X,t} + \widetilde{\mathsf{d}}_{X,t} \right)$$

(Foreign)

$$\mathsf{N}^*_{D,t} + \widetilde{\mathsf{d}}^*_{D,t} = \psi (\omega - 1) \mathsf{N}^*_{D,t} + (1 - \omega) \widetilde{\rho}^*_{D,t} + \mathsf{M}^*_t$$

$$\mathsf{N}^*_{X,t} + \widetilde{\mathsf{d}}^*_{X,t} = -\mathsf{Q}_t + \psi (\omega - 1) \mathsf{N}^*_{X,t} + (1 - \omega) \widetilde{\rho}^*_{X,t} + \mathsf{M}_t$$

$$\mathsf{N}^*_{D,t} + \widetilde{\mathsf{d}}^*_t = \kappa S_{ED} \left(\mathsf{N}^*_{D,t} + \widetilde{\mathsf{d}}^*_{D,t} \right) + (1 - \kappa S_{ED}) \left(\mathsf{N}^*_{X,t} + \widetilde{\mathsf{d}}^*_{X,t} \right)$$

<u>Definition of M</u>

$$\mathsf{M}_t = S^M_C \mathsf{C}_t + (1 - \theta) S^M_I \left(\mathsf{N}_{E,t} + \mathsf{x}^s_t \right)$$

$$\mathsf{M}^*_t = S^M_C \mathsf{C}^*_t + (1 - \theta) S^M_I \left(\mathsf{N}^*_{E,t} + \mathsf{x}^{s*}_t \right)$$

<u>Real cost for exporting</u>

$$\mu_{X,t} = \mathsf{w}_t - \mathsf{Z}_t$$

$$\mu^*_{X,t} = \mathsf{w}^*_t - \mathsf{Z}^*_t$$

Free entry

$$\mathsf{x}^s_t = \theta\left(\mathsf{w}_t - \mathsf{Z}_{E,t}\right)$$

$$\mathsf{x}^{s*}_t = \theta\left(\mathsf{w}^*_t - \mathsf{Z}^*_{E,t}\right)$$

Number of firms

$$\mathsf{N}_{D,t+1} = (1-\delta)\,\mathsf{N}_{D,t} + \delta\mathsf{N}_{E,t}$$

$$\mathsf{N}^*_{D,t+1} = (1-\delta)\,\mathsf{N}^*_{D,t} + \delta\mathsf{N}^*_{E,t}$$

Optimal labor supply

$$\mathsf{L}_t = \varphi\left(\mathsf{w}_t - \gamma\mathsf{C}_t\right)$$

$$\mathsf{L}^*_t = \varphi\left(\mathsf{w}^*_t - \gamma\mathsf{C}^*_t\right)$$

Labor Market clear

$$S^M_W\left(\mathsf{w}_t + \mathsf{L}_t\right) = (\sigma-1)\,S^M_D\left(\mathsf{N}_{D,t} + \widetilde{\mathsf{d}}_t\right) + \theta S^M_I\left(\mathsf{N}_{E,t} + \mathsf{x}^s_t\right) + \sigma S^M_{FX}\left(\mathsf{N}_{X,t} + \mu_{X,t}\right)$$

$$S^M_W\left(\mathsf{w}^*_t + \mathsf{L}^*_t\right) = (\sigma-1)\,S^M_D\left(\mathsf{N}^*_{D,t} + \widetilde{\mathsf{d}}^*_t\right) + \theta S^M_I\left(\mathsf{N}^*_{E,t} + \mathsf{x}^{s*}_t\right) + \sigma S^M_{FX}\left(\mathsf{N}^*_{X,t} + \mu^*_{X,t}\right)$$

Share of exporting firms

$$\mathsf{N}_{X,t} - \mathsf{N}_{D,t} = -k\widetilde{\mathsf{z}}_{X,t}$$

$$\mathsf{N}^*_{X,t} - \mathsf{N}^*_{D,t} = -k\widetilde{\mathsf{z}}^*_{X,t}$$

<u>Zero profit cutoff</u>

$$\widetilde{\mathsf{d}}_{X,t} = \mu_{X,t}$$

$$\widetilde{\mathsf{d}}^*_{X,t} = \mu^*_{X,t}$$

<u>Expected real returns</u>

$$\mathsf{E}_t\mathsf{r}^s_{h,t+1} \equiv \Upsilon(1-\delta)\,\mathsf{E}_t\mathsf{x}^s_{t+1} + [1-\Upsilon(1-\delta)]\,\mathsf{E}_t\widetilde{\mathsf{d}}_{t+1} - \mathsf{x}^s_t$$

$$\mathsf{E}_t\mathsf{r}^s_{f,t+1} \equiv \Upsilon(1-\delta)\,\mathsf{E}_t\mathsf{x}^{s*}_{t+1} + [1-\Upsilon(1-\delta)]\,\mathsf{E}_t\widetilde{\mathsf{d}}^*_{t+1} - \mathsf{x}^{s*}_t + \mathsf{E}_t\mathsf{Q}_{t+1} - \mathsf{Q}_t$$

$$\mathsf{E}_t\mathsf{r}^b_{h,t+1} \equiv \Upsilon\mathsf{E}_t\mathsf{x}^b_{t+1} + (1-\Upsilon)\,\psi\left[\mathsf{S}_{ED}\mathsf{N}_{D,t+1} + (1-\mathsf{S}_{ED})\,\mathsf{E}_t\mathsf{N}^*_{X,t+1}\right] - \mathsf{x}^b_t$$

$$\mathsf{E}_t\mathsf{r}^b_{f,t+1} \equiv \Upsilon\mathsf{E}_t\mathsf{x}^{b*}_{t+1} + (1-\Upsilon)\,\psi\left[\mathsf{S}_{ED}\mathsf{N}^*_{D,t+1} + (1-\mathsf{S}_{ED})\,\mathsf{E}_t\mathsf{N}_{X,t+1}\right] - \mathsf{x}^{b*}_t + \mathsf{E}_t\mathsf{Q}_{t+1} - \mathsf{Q}_t$$

<u>Euler Home and Foreign</u>

$$\gamma\mathsf{E}_t\left(\mathsf{C}_{t+1} - \mathsf{C}^*_{t+1}\right) - \mathsf{E}_t\mathsf{Q}_{t+1} = (\gamma - \upsilon)\,(\mathsf{C}_t - \mathsf{C}^*_t) - \mathsf{Q}_t$$

<u>Euler shares</u>

(Home)

$$\mathsf{E}_t\mathsf{r}^s_{h,t+1} = \gamma\mathsf{E}_t\mathsf{C}_{t+1} - (\gamma - \upsilon)\,\mathsf{C}_t$$

$$\mathsf{E}_t\mathsf{r}^s_{f,t+1} = \gamma\mathsf{E}_t\mathsf{C}_{t+1} - (\gamma - \upsilon)\,\mathsf{C}_t$$

Euler equation (bond)

(Home)

$$\mathsf{E}_t\mathsf{r}^b_{h,t+1} = \gamma\mathsf{E}_t\mathsf{C}_{t+1} - (\gamma - \upsilon)\,\mathsf{C}_t$$

$$\mathsf{E}_t\mathsf{r}^b_{f,t+1} = \gamma\mathsf{E}_t\mathsf{C}_{t+1} - (\gamma - \upsilon)\,\mathsf{C}_t$$

Realized excess returns

$$\begin{aligned}\mathsf{r}^s_{fx,t+1} = {} & \Upsilon(1-\delta)\left(\mathsf{x}^{s*}_{t+1} - \mathsf{E}_t\mathsf{x}^{s*}_{t+1}\right) + [1 - \Upsilon(1-\delta)]\left(\widetilde{\mathsf{d}}^*_{t+1} - \mathsf{E}_t\widetilde{\mathsf{d}}^*_{t+1}\right) \\ & -\Upsilon\left(\mathsf{x}^b_{t+1} - \mathsf{E}_t\mathsf{x}^b_{t+1}\right) + \mathsf{Q}_{t+1} - \mathsf{E}_t\mathsf{Q}_{t+1} - (1-\Upsilon)\,\psi\,(1 - \mathsf{S}_{ED})\left(\mathsf{N}^*_{X,t+1} - \mathsf{E}_t\mathsf{N}^*_{X,t+1}\right)\end{aligned}$$

$$\begin{aligned}\mathsf{r}^s_{hx,t+1} = {} & \Upsilon(1-\delta)\left(\mathsf{x}^{s}_{t+1} - \mathsf{E}_t\mathsf{x}^{s}_{t+1}\right) + [1 - \Upsilon(1-\delta)]\left(\widetilde{\mathsf{d}}_{t+1} - \mathsf{E}_t\widetilde{\mathsf{d}}_{t+1}\right) \\ & -\Upsilon\left(\mathsf{x}^b_{t+1} - \mathsf{E}_t\mathsf{x}^b_{t+1}\right) - (1-\Upsilon)\,\psi\,(1 - \mathsf{S}_{ED})\left(\mathsf{N}^*_{X,t+1} - \mathsf{E}_t\mathsf{N}^*_{X,t+1}\right)\end{aligned}$$

$$\begin{aligned}\mathsf{r}^b_{fx,t+1} = {} & \Upsilon\left(\mathsf{x}^{b*}_{t+1} - \mathsf{E}_t\mathsf{x}^{b*}_{t+1}\right) - \Upsilon\left(\mathsf{x}^b_{t+1} - \mathsf{E}_t\mathsf{x}^b_{t+1}\right) + \mathsf{Q}_{t+1} - \mathsf{E}_t\mathsf{Q}_{t+1} \\ & + (1-\Upsilon)\,\psi\,(1 - \mathsf{S}_{ED})\left[\left(\mathsf{N}_{X,t+1} - \mathsf{E}_t\mathsf{N}_{X,t+1}\right) - \left(\mathsf{N}^*_{X,t+1} - \mathsf{E}_t\mathsf{N}^*_{X,t+1}\right)\right]\end{aligned}$$

Net Foreign asset for Home (defined with respect to M)

$$\mathsf{NFA}_{t+1} = \mathsf{NX}_t + \frac{1}{\Upsilon}\mathsf{NFA}_t + \widetilde{a}^s_f\mathsf{r}^s_{fx,t+1} - \widetilde{a}^{s*}_h\mathsf{r}^s_{hx,t+1} + \widetilde{a}^b_f\mathsf{r}^b_{fx,t+1}$$

where by symmetry,

$$\widetilde{a}^s_f = \widetilde{a}^{s*}_h = \frac{a^s_f}{\Upsilon M} \text{ and } \widetilde{a}^b_f = -\widetilde{a}^b_h = \frac{a^b_f}{\Upsilon M}$$

Net export for Home (defined with respect to M)

$$\mathsf{NX}_t \equiv \sigma\mathsf{S}^M_D\left(\mathsf{N}_{D,t} + \widetilde{\mathsf{d}}_t\right) + \sigma\mathsf{S}^M_{FX}\left(\mathsf{N}_{X,t} + \mu_{X,t}\right) - \mathsf{M}_t$$