

# CREA Discussion Paper 2012-14

Center for Research in Economic Analysis  
University of Luxembourg

## **On the long run economic performance of small economies**

*available online : [http://www.fr.uni.lu/recherche/fdef/crea/publications2/discussion\\_papers/2011](http://www.fr.uni.lu/recherche/fdef/crea/publications2/discussion_papers/2011)*

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December, 2012

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# On the long run economic performance of small economies\*

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November 17, 2012

## Abstract

In this paper, we analyze the long run economic performance of a small economy open to foreign investments. Policy instruments used to attract investments are taxes and attractive public infrastructures, whereas the policy choices of the rest of the world are taken as given. Applying the Pontryagin's maximum principle, we first show that there exists one long run optimal size of the small economy which is saddle-point stable. The transitional path is two-dimensional, if the small economy is patient enough. Then, we show that the share of tax income allocated to the infrastructure expenditures plays an important role in attaining such a steady state. However, a deviation from this policy path can lead to an eventual economic collapse.

**Keywords:** economic dynamics, spatial dynamic competition, public goods competition, foreign direct investments.

**JEL classification:** O30, O43, H25, H73, F13, F15.

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\*The authors gratefully acknowledge discussions with Raouf Boucekkine, Herbert Dawid, Pierre Picard and Myrna Wooders. The usual disclaimer applies.

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# 1 Introduction

It is well-known that small states suffer from very limited capital and labor resources both in amount and in variety. Their small home market size prevents them from exploiting scale and scope economies. It is therefore not surprising that small states are highly open to international trade and capital flows (Alesina and Wacziarg, 1998). Because of their smallness these countries are highly depending on forces outside their control, which could threaten their economic viability (Briguglio, 1995). However, the strong growth performance of some small states suggests that it is possible to at least partially offset this vulnerability and increase their resilience by means of appropriate endogenous policies (Armstrong and Read, 2002). Armstrong, De Kervenoael and R. Read (1998) show that one country's economic smallness does not necessarily have a negative effect on its performance. This can be explained by the fact that small countries develop policy abilities and use instruments to overcome their natural handicaps. For example, some of the richest countries in the world are small states such as Luxembourg.<sup>1</sup> This is an illustration of what Briguglio *et al.* (2009) call the "Singapore Paradox": a country highly vulnerable to exogenous shocks still manages to attain high economic performance.

Since domestic capital is relatively scarce in very small economies, it follows that attracting foreign investments is an important way to fill in this gap. Indeed, small states tend to get more private capital from abroad as a ratio of total capital formation (Streeten, 1993). Moreover, capital inflows may also be a critical contributor to the growth and development of small states (Read, 2008). Some empirical evidence shows a positive impact of FDI on economic growth and the possibility of spillover effects to local firms (Castellani and Zanfei, 2006).

The general focus of the paper is to analyse the economic viability of a very small economy in a globalized world. To this end, we assume that the policy-makers of the

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<sup>1</sup>Note that high vulnerability and good economic performance are not contradictory aspects of very small economies. Indeed, Iceland has been an example for good economic performance but has shown great fragility in the context of the latest financial crisis.

small country use a mix of two instruments in a dynamic optimization framework to promote the durability of their economy. More exactly, the policy-mix consists in attracting foreign capital through low taxes and/or high level of public goods, which enhance firms' productivity. Public goods can cover a wide range of infrastructures, services and regulations provided by the local and/or the central government are attractive to firms if they enhance their productivity.<sup>2</sup> Consequently, capital locates according to differentials in offered public good levels and tax differentials. Specifically, a country may not reduce its attractiveness by a unilateral increase in taxes if foreign investors are compensated by more infrastructure provision. Foreign investors are ready to pay higher taxes because infrastructure become more valuable to them (e.g. Haufler, 1998, Pieretti and Zanaj, 2011).

In this paper, the small economy is small enough to consider the rest of the world's choices as exogenously given. This does not mean that very small countries cannot grow in terms of productive capacity by attracting a high volume of foreign direct investments. What we actually assume is that their ability to grow bigger than large countries is limited.<sup>3</sup> For example, if smallness does not only rely on population size but also on a territorial criterion, the existence of limited usable land can be considered as a absorptive constraint for foreign investments.<sup>4</sup> Another limitation can result from the smallness of native population since it constraints the availability of administrative

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<sup>2</sup>In this context, we may consider transportation infrastructures, universities and public R&D investment, but also property rights enforcement, capital market regulations, labor and environmental regulations and the absence of red tape procedures. It follows that countries' ability to attract foreign investment may also be attractive for the quality of their institutions. In the Oxford Handbook of Entrepreneurship (2007), it is argued that the abundance of entrepreneurs in a country depends, among other factors, on the existence of regulations, property rights, accounting standards and disclosure requirements. Furthermore, in recent years there has been a surge of country and cross-country studies relating economic development to institutions, especially those affecting capital market development and functionality (La Porta et al. (1997) among others).

<sup>3</sup>Exceptions could however exist if we focus for example one particular sector like banking and finance.

<sup>4</sup>According to Armstrong and Read (1995), the small size constitutes a significant impediment to sustained economic growth, especially in the case of island and land-locked states.

and public resources which are necessary for increasing the provision of attractive infrastructures. Finally, a high ratio of foreign-owned firms may be perceived as a loss of economic independence and induce some resistance to new FDIs.

The literature has investigated the role of jurisdictions' size on their capacity to attract capital. Recent papers show that small economies can be attractive not only for tax reasons but also for their provision of public infrastructures (Justman et al., 2005, Zissimos and Wooders, 2008, Hindriks et al., 2008, Pieretti and Zanaj, 2011). This paper extends this literature by modelling the dynamics of a small economy's strategies to attract foreign investments. More precisely, we study a small state's intertemporal choice of optimal taxes which are used to afford public goods that enhance firms' productivity. Applying the Pontryagin's maximum principle (see, for example, Boucekkine *et al.* (2007)) we then characterize the potential steady states attainable by the small economy.

The main findings of the paper can be summarized as follows. We show that there exists three types of steady states. One in which the size of the initially small country attracts sufficiently external capital to grow as big as its upper limit. One in which the small economy is no more economically viable since it loses all its productive capital. Finally, an intermediate configuration in which the domestic economy survives while remaining small. In this scenario, there exists at least one intermediate steady state which exhibits saddle point stability. If the small economy does not undergo the optimal path which leads to one of these intermediate equilibria it may converge to the worst case and disappear. The survival of the small economies is thus an important public policy issue which implies an appropriate choice of initial conditions and a dynamic update of the tax policy.

The dynamic interactions among jurisdictions to attract mobile factors have already been analyzed using the framework of repeated games. The main issue studied by this literature is the tax coordination problem between symmetric regions (Cardarella *et al.*, 2002, Catenaro and Vidal, 2006, Itaya *et al.*, 2008). The purpose of this paper is however not to model a game between jurisdictions. We rather focus on the strategic choices of

a very small open economy facing exogenously given choices of the rest of the world. The world is thus divided into two unequal sized regions where size refers to the magnitude of the population, which coincides with the number of capital-owners who are simultaneously entrepreneurs and workers. Specifically, our paper tries to offer an insight into the dynamic policy behavior of a very small country trying to guarantee the long run survival of its economy. We therefore analyze, in an infinite time horizon, the dynamics of its size and its policy instruments for exogenous foreign levels of taxes and public goods. Most of these literature assumes a country's borders to be subject to the same analysis as any other human made institution.<sup>5</sup> In our paper however we consider that only the economic size can vary endogenously as a consequence of public policy but that its territorial magnitude remains unchanged.

In this paper, we first show that there exists one long run efficient (or optimal) size of the small economy which is saddle-point stable. The attainment of this efficient steady state is easier when the time preference is low enough, given a certain degree of international openness. Or alternatively, when international openness is high enough, given a certain rate of time preference. The reason is that, in both scenarios, the efficient transitional path is two-dimensional instead of one-dimensional. Interestingly, the initial conditions allowing the small economy to converge to this efficient state are guaranteed by the allocation of an appropriate share of tax income to the infrastructure expenditures. However, imposing the right tax-share attributable to infrastructure projects may prove politically difficult and divergence from the efficient path is not unlikely. Economic collapse could then be a possible end-state. Along this divergence path, the small economy would vainly try to retain capital by tax dumping, thus driving its infrastructure expenditures to zero. However, this scenario does not occur if the public benefit of infrastructures net of taxes provided by the rest of the world is negative.

The paper is organized as follows. The next section develops a dynamic model and

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<sup>5</sup>It is interesting to recall that since 1990, 33 new countries have been formed and, many of them are very small like Kosovo, Macedonia, Slovenia, East Timor, Marshall Islands, Namibia, Palau, etc. The dynamics of countries creation is thus a very current and lively process.

the optimal conditions via Pontryagin's Maximum Principle. Section 3 derives and analyses the steady states of the model. Finally section 4 concludes.

## 2 The model

The world is composed of two regions of unequal population size.<sup>6</sup> In the rest of the paper, we consider the smaller region as the small or the home country and the larger as the foreign country or the rest of the world indifferently. We assume that the members of both jurisdictions are at the same time entrepreneurs and workers and each of them owns one unit of productive capital. We thus assume that the endowment in human resources and physical capital grows in proportion to the human population. Furthermore, the size of a country is equivalent to the number of firms located in its territory. Our model identifies a country by the size  $S$  of its economy, whereas, the size of its area and of its native population will be given. In other words, the territorial and political size of the countries remain unchanged through time.

At time  $t = 0$ , these jurisdictions are represented on an interval  $[-S(0), S^*(0)]$ .<sup>7</sup> The size of the small country is  $S(0)$  and extends from  $-S(0)$  to 0 which corresponds to the border. The rest of the world has a size of  $S^*(0)$  with  $S(0) < S^*(0)$  and extends from 0 to  $S^*(0)$ . The firm-owners in both jurisdictions are evenly distributed on their respective sub-interval according to their disposition to invest outside their home location. As in Ogura (2006), we assume that the population of investors is heterogeneous in the degree of their attachment to home.<sup>8</sup>

In our spatial setting we assume that the closer firms are located to the extremes the more they are attached to their current location. Conversely, the closer firms are to the border 0, the less they are attached to their territory and the easier they are able

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<sup>6</sup>Country size may be defined by its population, by its area, or by its national income (Streeten, 1993). In our paper, we focus on the *population* aspect rather than on the spatial size.

<sup>7</sup>The substrict " \* " refers to the large (foreign) jurisdiction.

<sup>8</sup>Heterogeneity in home attachment was first considered in the fiscal competition literature by Mansoorian and Myers (1993).

to relocate abroad.<sup>9</sup> This means that a firm of type  $h \in [-S(0), 0]$  located in the home country incurs a disutility of relocating abroad which equal  $k \cdot x$ , where  $x = d(h, 0)$ , i.e. the distance between 0 and  $h$ . The coefficient  $k$  represents the unit cost of moving capital abroad which can also be interpreted as the degree of international integration.

Now assume that each population member of both jurisdictions owns one unit of capital which she combines with her labor to set up a firm to produce  $q + a_i$ ,  $a_i = a, a^*$ , units of a final good, where  $q$  is the private component of (gross) productivity. The fraction  $a_i$  of the produced good depends on the public input supplied by the home (foreign) jurisdiction.<sup>10</sup> This makes the country more attractive to foreign financial firms and increases the attachment to home of domestic financial firms, *ceteris paribus*. The produced output is sold in a competitive (world) market at a given price normalized to one. Assuming that both countries have equal access to a common market this assumption implies that the small jurisdiction does not suffer from a reduced home market. We further suppose that the unit production cost is constant and equal to zero without loss of generality.

We now adopt a temporal perspective of the above setting. Each period  $t \in [\Delta t, +\infty)$ , (for any  $\Delta t > 0$ ) governments update their choice in terms of offered public goods and taxes.<sup>11</sup> We assume that the total number of entrepreneurs,  $S(t) + S^*(t)$  will be constant over time  $t$  and normalized to one. Since firms may move, the relative size of both jurisdictions will change with  $t$ . In the following, we focus on the behavior of a small country. Hence, the economic size of the small country can however extend, but the

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<sup>9</sup>For reasons of simplicity, we assume that relocation if any is only possible in the neighboring jurisdiction.

<sup>10</sup>The public input satisfies the local public good characteristic, which means that it is jointly used without rivalry by firms located in the same jurisdiction. It follows that the benefits and the costs of these good only accrue at the jurisdictional level. As in Zissimoss and Wooders (2008), we shall abstract from congestion costs . Taking account of congestion would complicate our framework without improving qualitatively the results. Moreover, if the public input represents immaterial goods as law and regulations (protecting intellectual property, specifying accurate dispute resolution rules,...), the absence of congestion is easily justified by the particular nature of these goods.

<sup>11</sup>Notice that we assume there are no sunk cost on the investment or that our unit of time  $t$  is long enough to cancel the sunk cost of investment.

maximum amount of FDIs that the small country can host will be limited to such an extent that its economy always remain small compared to the large country. Therefore, we suppose that the home country's economy  $S(t)$  is small enough to consider the rest of the world's choices as exogenously given. In order to ensure the consistency of this assumption, the following state-space constraint appears in the dynamic optimization problem.

$$0 \leq S(t) < \bar{S} < \frac{1}{2} \text{ for } t \in [0, +\infty). \quad (1)$$

Providing firms with public infrastructures is costly. The public technology which serves to produce each period the public input is given by the function  $f(S(t), T(t))$  where  $T(t) \in [0, \hat{T}]$  denotes the tax levied on one unit of capital at time  $t$ , and  $\hat{T} \in (0, \infty)$  is a constant. The tax revenue is transformed by the country  $i$  into public infrastructures according to a production function which takes the general form  $f_i[S_i(t), T_i(t)]$ .

Supposing that the public good depreciates at a rate  $\delta > 0$ , we can write the following motion equations of public input

$$\dot{a}(t) = -\delta a(t) + f[S(t), T(t)], \quad (2)$$

$$\dot{a}^*(t) = -\delta a^*(t) + f^*[S^*(t), T^*(t)]. \quad (3)$$

For simplicity, we shall work with the following functions:  $f[S(t), T(t)] = \zeta S(t)T(t)$  and  $f^*[S^*(t), T^*(t)] = \zeta^* S^*(t)T^*(t)$  where  $\zeta_i$  ( $0 < \zeta_i < 1$  and  $\zeta_i = \zeta, \zeta^*$ ) represents a non-negative productivity factor which is specific to country  $i$ . Note that the coefficient  $\zeta$  can be interpreted as the ability of the small jurisdiction to transform taxes into infrastructures expenditures. This ability can be a technical feature of funding public-expenditures but it can also reflect priorities emanating from collective decisions.<sup>12</sup>

Assume now that an entrepreneur of type  $h(t)$  initially located in the small country considers to stay at home or to invest her/his physical capital abroad. If she/he decides

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<sup>12</sup>Indeed the effective (net) tax revenue collected by the governments does not coincide with the gross amount of tax revenue collected. Following Vaillancourt (1989) and Blumenthal and Slemrod (1992), tax collection is costly due to the administration, monitoring and enforcing procedures associated with it.

not to move, her/his profit is given by<sup>13</sup>

$$\pi(t) = q(t) + a(t) - T(t) \quad (4)$$

If she invests abroad, her/his profit becomes

$$\pi^*(t) = q(t) + a^*(t) - T^*(t) - k \cdot x(t)$$

Furthermore, consider that this capital-owner is indifferent between investing abroad and staying at home. Then it follows that

$$q(t) + a(t) - T(t) = q(t) + a^*(t) - T^*(t) - k \cdot x(t).$$

After setting  $b^*(t) = \frac{a^*(t) - T^*(t)}{k}$ , we obtain

$$x(t, a, a^*, T, T^*) = b^*(t) - \frac{a(t) - T(t)}{k}. \quad (5)$$

In other words, the foreign country attracts capital ( $x > 0$ ) from the small jurisdiction if the net gain of investing abroad, i.e.  $a^*(t) - T^*(t)$ , is higher than the net gain of staying at home,  $a(t) - T(t)$ , after taking into account the mobility cost  $kx$ .

The motion equation of the size variable  $S(t)$  of the small economy is then given by

$$\dot{S}(t) = -x = \frac{a(t) - T(t)}{k} - b^*(t), \quad (6)$$

with the initial condition  $\frac{1}{2} > \bar{S} > S(0) > 0$ .

Note that the relocation of a subset of firms at each period alters the ranking of firms' attachment to home. In the following, we adopt the following rule. For all  $\tilde{h}(t) \in [-S(t), S^*(t)]$ , we define  $\tilde{h}(t) = \tilde{i}(t - \Delta t) + x$ , where

$$\tilde{h}(t) = \begin{cases} h(t) \in [-S(t), O(t)] \\ h^*(t) \in [O(t), S^*(t)] \end{cases}$$

and  $O(t)$  stands for the origin at period  $t$ .

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<sup>13</sup>For sake of simplicity, we assume that  $q$  is such that the profit of each firm is positive for all equilibrium level of public goods and taxes.

We thus assume that the preferences for the home location will change according to the following rule. For the firms that do not move, attachment to home will increase by  $x$  if the small economy is attractive to foreign investors ( $-x < 0$ ) and it will decrease if the foreign location attracts capital from the small country ( $x > 0$ ). For the capital owners who move abroad, the higher their attachment to the country they leave, the lower the attachment to the new location will be.

In the rest of the paper we focus on the small jurisdiction. We analyze, in an infinite time horizon, the dynamics of its size  $S(t)$  and its policy instruments  $T(t)$  and  $a(t)$  for exogenous foreign levels of taxes  $T^*$  and public goods  $a^*$ . We thus consider that the rest of the world does not react to the small country's decisions. We also analyze the convergence of the variables  $S(t)$ ,  $T(t)$  and  $a(t)$  towards possible steady states.

Suppose that policy makers maximize the discounted linear-quadratic utility that depends on tax revenues,  $S(t) \cdot T(t)$ , net of the cost of producing public inputs  $\beta/2 a^2(t)$ , where  $\beta > 0$  represents an efficiency parameter. The objective-function of the small economy is given by

$$\max_{T(t)} W = \int_0^{\infty} e^{-rt} \left[ S(t)T(t) - \frac{\alpha}{2} (S(t)T(t))^2 - \frac{\beta}{2} a^2(t) \right] dt, \quad (7)$$

subject to

$$\dot{a}(t) = -\delta a(t) + f[S(t), T(t)], \quad (8)$$

$$\dot{S}(t) = -x = \frac{a(t) - T(t)}{k} - b^*(t),$$

$$0 \leq S(t) < \bar{S} < \frac{1}{2} \quad (9)$$

where  $\alpha$  represents a cost parameter of collecting taxes and the rate of time preference  $r$  which represents the degree of "impatience" of the home country's population. We assume furthermore that the linear-quadratic utility is increasing and concave with respect to the total tax, that is,  $\alpha$  is such that  $T < 1/\alpha$ .

We now characterize the inter-temporal optimal tax strategy chosen by the policy makers in the small country. Applying Pontryagin's maximum principle, we derive a canonical system of ordinary differential equations that has to be satisfied by the optimal trajectories. Since the Hamiltonian of the dynamic optimization problem is

concave with respect to the state variables, the Maximum principle provides not only necessary but also sufficient optimality conditions for interior solutions (see e.g. Theorem 4.2, Dockner *et al.*, 2000, Chiang, 2000, Hartl *et al.*, 1995, or Sethi and Thompson, 1981). Denote by  $\mu$  and  $\nu$  the costate variables corresponding respectively to the state variables  $S(t)$  and  $a(t)$ .

**Proposition 1** *For any state trajectory  $(S(t), a(t))$ , there exist piecewise absolutely continuous costates  $\mu(t)$  and  $\nu(t)$ , and two multipliers  $\theta_1(t) \geq 0$ ,  $\theta_2(t) \geq 0$ , such that the optimal choice variable  $T(t)$  satisfies*

$$T(t)S^2(t) = \frac{(1 + \zeta\nu)S - \frac{1}{k}(\mu - \theta_1 + \theta_2)}{\alpha}.$$

The costate equations become

$$\dot{\mu} = r\mu - \zeta T\nu - (1 - \alpha TS)T, \quad (10)$$

$$\dot{\nu} = (r + \delta)\nu + \beta a - \frac{1}{k}(\mu - \theta_1 + \theta_2). \quad (11)$$

Furthermore,

$$\dot{S} = - \left( b^* - \frac{a - T}{k} \right) \leq 0, \text{ if } S(t) = \frac{1}{2}; \theta_1(t) \left( \frac{1}{2} - S(t) \right) = 0, \theta_1 \geq 0, \quad (12)$$

$$\dot{S} = - \left( b^* - \frac{a - T}{k} \right) \geq 0, \text{ if } S(t) = 0; \theta_2(t)S = 0, \theta_2 \geq 0. \quad (13)$$

Finally, the transversality conditions  $\lim_{t \rightarrow \infty} e^{-rt}\mu S = 0$  and  $\lim_{t \rightarrow \infty} e^{-rt}\nu a = 0$  are satisfied.

In the following section, we characterize the potential steady states of the system and we analyze how the steady states can be attained.

### 3 Steady states and convergence

Steady states are defined as rest points of the dynamic equations (2), (6), (10) and (11) remembering that the decision variables ( $a^*$  and  $T^*$ ) of the rest of the world are given.

Due to the state space constraints, there are two types of possible long term solutions: *efficient* steady state(s) and *inefficient* (constraint) steady states. In the first case, the small country survives in the long run without attaining its maximum economic size ( $0 < \hat{S} < \bar{S}$ ) or it reaches its highest possible size ( $\hat{S} = \bar{S} < \frac{1}{2}$ ). In the inefficient case, the small economy can eventually collapse ( $\underline{S} = 0$ ) or it expands to the limit level  $\bar{S} < \frac{1}{2}$ . Note that in the first scenario, the upper-limit solution is efficient while it is not in the second scenario.

### 3.1 The small economy converges to an efficient size

The existence of an efficient steady state is crucial. Indeed, it means that there exists a policy mix consisting of taxes and public infrastructures able to guarantee the long term survival of a small economy. If we consider interior steady states  $\hat{S}$  ( $0 < \hat{S} < \bar{S}$ ), the boundary constraints are both not binding. Hence,  $\hat{\theta}_1 = 0$  and  $\hat{\theta}_2 = 0$  and the interior rest points of the dynamic system (2), (6), (10) and (11) are specified in the following proposition.<sup>14</sup> The following proposition states that there exists a unique efficient steady state for any parameter constellation.

**Proposition 2** *Assume foreign policy choices are  $a^*$  and  $T^*$ . There is always one steady state given by*

$$\hat{a} = \frac{\zeta(r + \delta)}{\alpha(r + \delta)\delta + \beta\zeta^2} (> 0), \hat{S} = \frac{\delta \hat{a}}{\zeta \hat{T}}, \text{ and } \hat{T} = \hat{a} - (a^* - T^*) \quad (14)$$

*and the two costate variables are*

$$\hat{\mu} = 0 \text{ and } \hat{v} = -\frac{\beta}{r + \delta} \hat{a} (< 0). \quad (15)$$

*This steady state is a saddle point of the canonical system (2), (6), (10) and (11). Moreover, it is one dimensional locally asymptotically stable, if  $r > \frac{\hat{T}}{k\hat{S}}$ . Otherwise, if  $r < \frac{\hat{T}}{k\hat{S}}$ , it is two dimensional locally asymptotically stable.*

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<sup>14</sup>The proof is given in the appendix.

The stability of the above interior steady state results from the existence of at least one negative eigenvalue in the 4-dimensional eigenvalue space. More precisely, this always guarantees the existence of at least one convergent path. We only have one negative eigenvalue if  $r > \frac{\hat{T}}{k\hat{S}}$  and two negative eigenvalues if  $r < \frac{\hat{T}}{k\hat{S}}$ . In the first case, the efficient transitional path is one dimensional whereas it is two-dimensional in the second case. Consequently, if time preference is low enough ( $r < \frac{\hat{T}}{k\hat{S}}$ ), reaching the efficient steady state will be eased by the fact that the convergence path is a plane. We can alternatively say that

**Corollary 1** *For a given value of time preference, the attainment of an efficient steady state is easier when international openness is high, namely  $k < \hat{T}/r\hat{S}$ .*

Furthermore, the small economy is able to move on the just described efficient path(s), if the policymakers are able to choose proper initial conditions  $a(0)$  and  $S(0)$ . To reach that goal, note that these initial values depend on the parameters of the model and in particular on  $\zeta$ . The parameter  $\zeta$ , which is the share of taxes allocated to infrastructure expenditures, can then serve as an important policy instrument for setting the appropriate initial conditions.

In addition to the above interior steady state, other interior solutions may appear for special parameter and coefficient combinations. We present these cases in the Appendix.

Let us now analyse the nature of the steady state solutions. First note that according to (15) it is optimal for the small state to equate the net (of taxes) amount of provided public goods  $\hat{a} - \hat{T}$  to that of the foreign economy  $a^* - T^*$ . This equality is necessary for migration to cease. It also appears that the amount of public infrastructure offered in the steady state does not directly depend on the rest of the world's decision variables.<sup>15</sup> This is not the case for the equilibrium tax rate  $\hat{T}$  and consequently for the equilibrium size  $\hat{S}$ . In other words, the equilibrium level of infrastructures provided by the small

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<sup>15</sup>According to (15), we could have chosen to express  $\hat{T}$  (or  $\hat{S}$ ) as an independent solution of the foreign decision instruments. In this case  $\hat{a}$  and  $\hat{S}$  (or  $\hat{a}$  and  $\hat{T}$ ) would depend on  $a^*$  and  $T^*$ .

state depends only on parameters specific to its economy, while the equilibrium tax level is both determined by domestic and foreign characteristics.<sup>16</sup> At first sight it is remarkable that the small economy is independent from international conditions in determining its long term public infrastructures. The underlying intuition is however straightforward. Assume the large economy becomes more attractive which means that  $(a^* - T^*)$  increases. In order to preserve its attractiveness, the small economy reacts by decreasing its tax rate  $\widehat{T}$  in a way to keep unchanged the provision of its infrastructure.

We can now turn to some comparative statics. We see that the steady state provision of public goods increases with the time preference  $r$  since  $\frac{\partial \widehat{a}}{\partial r} > 0$ . The reason is that the more the home country is impatient the more it will be reluctant to postpone to invest in public infrastructures. However, the impact of a higher share  $\zeta$  of tax revenue allocated to infrastructures by the small economy is ambiguous. Indeed, according to (15) we have  $\frac{\partial \widehat{a}}{\partial \zeta} > 0$  if  $\zeta < \bar{\zeta}$  ( $\bar{\zeta} = \sqrt{\frac{\alpha\delta(r+\delta)}{\beta}}$ ) and  $\frac{\partial \widehat{a}}{\partial \zeta} < 0$  if  $\zeta > \bar{\zeta}$ . Therefore, as long as  $\zeta < \bar{\zeta}$ , allocating more (increasing  $\zeta$ ) tax revenue to the provision of public inputs increases the equilibrium level of infrastructures and the equilibrium tax level.<sup>17</sup> In other words, the small country reacts by increasing infrastructure attractiveness and decreasing tax competitiveness. However, if the threshold  $\bar{\zeta}$  is exceeded, devoting a larger share of taxes to public inputs paradoxically decreases the equilibrium level of infrastructures. The reason is that increasing  $\zeta$  above  $\bar{\zeta}$  induces too high infrastructure expenditures for a given level of taxes. The small state then prefers to switch to the opposite regime of lowering its infrastructure attractiveness and augmenting its tax competitiveness (decreasing  $\widehat{T}$ ). Also note that the likeliness of this last regime choice increases with the inefficiency in providing infrastructures, because  $\bar{\zeta}$  decreases with  $\beta$ .

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<sup>16</sup>An exception could be the parameter  $r$  if it is interpreted as an international rate of interest, but this is clearly not the case in our model.

<sup>17</sup>Since  $\widehat{T} = \widehat{a} - (a^* - T^*)$ , and thus  $\frac{\partial \widehat{a}}{\partial \zeta}$  and  $\frac{\partial \widehat{T}}{\partial \zeta}$  are equal.

Equation (14) implies  $\widehat{S}\widehat{T} = \frac{\delta\widehat{a}}{\zeta}$ , which means that the tax income of the small country is not affected by changes in the net attractiveness ( $a^* - T^*$ ) of rest of the world. For example if  $a^* - T^*$  rises, the home country will react by decreasing its equilibrium tax rate  $\widehat{T}$ . This will attract new foreign firms and thus increase  $\widehat{S}$  to such an extent that the tax income  $\widehat{S}\widehat{T}$  remains unchanged.

Next we analyze the impact of a change<sup>18</sup> in  $\widehat{a}$  on the equilibrium size of the home economy. It is straightforward to show that  $\frac{\partial\widehat{S}}{\partial\widehat{a}}$  has the opposite sign<sup>19</sup> of  $a^* - T^*$ . Since condition  $\widehat{T} = \widehat{a} - (a^* - T^*)$  must hold in the steady state, both regions are equally *attractive* if the net amount of public goods offered by the small and large economies are either positive ( $a^* > T^*$  and  $\widehat{a} > \widehat{T}$ ). Conversely, both regions are equally *unattractive* if  $T^* > a^*$  and  $\widehat{T} > \widehat{a}$ . The derivative  $\frac{\partial\widehat{S}}{\partial\widehat{a}}$  is negative in the first case and positive in the second case. The impact of  $\widehat{a}$  on  $\widehat{S}$  can be interpreted in the following way. If both regions are equally attractive, entrepreneurs have (for given moving costs) a preference for the country which lowers its tax rate. However, if the small country increases the provision of public inputs it increases its tax rate (see equation (15)). It follows that capital flows out of the small country and the size  $\widehat{S}$  shrinks. If both regions are equally unattractive, entrepreneurs have (for given moving costs) a preference for the region which increases the provision of infrastructure. Hence, an increase in  $\widehat{a}$  results in a capital inflow and the size  $\widehat{S}$  expands consequently.

Until now we answered the question about the capacity of the small economy to survive in an efficient way. However, the small economy can be viable in different ways. Relative to its initial economic size it can shrink ( $\widehat{S} < S(0)$ ) or it can extend ( $\widehat{S} > S(0)$ ). In the latter case, it can reach its maximal attainable size if the parameter combination ( $\delta, \zeta, k, r$  and  $a^*, T^*$ ) is such that  $\widehat{S} = \overline{S}$

If the policy makers of the small country are willing and able to set the economy on the right path, their economy will survive in the long run, otherwise the economy will diverge. Divergence will end up in one of two inefficient steady states we analyze in the following section. The inefficient long run outcome can be either the upper-bound

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<sup>18</sup>This change can result from a shock on  $\zeta$  or  $r$ .

<sup>19</sup> $\frac{\partial\widehat{S}}{\partial\widehat{a}} = -\frac{1}{\zeta} \frac{\delta}{(\widehat{a} + T^* - a^*)^2} (a^* - T^*)$

solution  $\bar{S}$  or economic collapse ( $\underline{S} = 0$ ).

## 3.2 The small economy diverges to inefficient steady states

### 3.2.1 Divergence to the upper-bound

We saw above that there is one dimensional or two-dimensional trajectories that converge to the optimal steady state. All other trajectories lead to one of the two inefficient solutions,  $\underline{S} = 0$  and  $\bar{S} < \frac{1}{2}$ .<sup>20</sup>

Let us now consider the case in which the small country diverges to its upper bound  $S(t) = \bar{S}$ . Remember that we realistically assume that the the small country's economy is limited in its capacity to grow bigger than the rest of the world. Even if it reaches its upper bound size it will remain small enough to consider the rest of the world's choices as exogenously given.

If  $S(t) = \bar{S} < \frac{1}{2}$ , the constraint on the state variable becomes binding and the steady state value of  $\theta_1$  has to be positive and  $\theta_2 = 0$ . Hence, the condition (12), in Proposition 1, should hold. Whenever the small economy reaches its upper bound  $\bar{S}$ , additional FDIs can no more be absorbed and therefore,  $S(t)$  is constant or decreasing:  $\dot{S}(t) \leq 0$ . In the appendix, we show how the following steady state values are obtained

$$\bar{S} = \bar{S} < \frac{1}{2}, \quad \bar{a} = \frac{\zeta}{\zeta - 2\delta}(a^* - T^*), \quad \bar{T} = \bar{a} - (a^* - T^*). \quad (16)$$

Moreover, the costate variables become

$$\bar{\nu} = \frac{1}{r + \delta - \frac{\zeta}{2}} \left( \frac{1}{2} - \frac{\alpha\bar{T}}{4} - \beta\bar{a} \right) \quad \text{and} \quad \bar{\mu} = \frac{\bar{T}}{r} \left( 1 - \frac{\alpha\bar{T}}{2} + \zeta\bar{\nu} \right), \quad (17)$$

$$\bar{\theta}_1 = \left( 1 - \frac{2kr}{\rho\bar{T}} \right) \bar{\mu} > 0, \quad \text{and} \quad \bar{\theta}_2 = 0. \quad (18)$$

The above analysis leads to the following conclusion.

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<sup>20</sup>For more discussion on the saddle point stability see, for example, Azariadis (2000), de la Croix and P. Michel (2002), Eicher and Turnovsky (2001) or Galor (2007), and the references therein.

**Proposition 3** *The economy converges to its upper-limit size, given by (16), if (i)  $\zeta > 2\delta > 0$  and  $a^* > T^*$ , or (ii)  $0 < \zeta < 2\delta$  and  $a^* < T^*$ . The multiplier  $\theta_1$  is given by (18) and is strictly positive with  $\mu$  and  $\nu$  given respectively by (17).*

According to equation (16), we see that if the small country's economy converges to its limit-size  $\bar{S} < \frac{1}{2}$ , it has to tie its long term policy decisions to  $(a^* - T^*)$  according to a linear rule<sup>21</sup>. When the net benefit offered by the rest of the world is positive,  $a^* - T^* > 0$  (case a), it appears that the home country can converge to the upper-bound value  $\bar{S} < \frac{1}{2}$  if the share of tax revenue devoted to infrastructure expenditures,  $\zeta$ , is high enough ( $\zeta > 2\delta > 0$ ).

### 3.2.2 Collapse

Now we consider the case where the small economy could suffer from a possible economic collapse, i.e.,  $S(t) = 0$ . In this case, the constraint on the state variable must be binding in order to exclude a negative population value and the steady state values of  $\theta_1 = 0$  and  $\theta_2$  have to be positive. Hence, condition (13) in Proposition 1, should hold. Furthermore, once  $S(t)$  has attained the lower bound, it can no more decrease and it should be either constant or increasing, that is,  $\dot{S}(t) \geq 0$ , as it is shown in Proposition 1. In this case, we get

$$\underline{S} = 0, \underline{\theta}_1 = 0, \underline{\theta}_2 = -\mu, \quad (19)$$

$$\underline{a} = 0, \underline{T} = T^* - a^*, \quad (20)$$

$$\underline{\nu} = 0, \underline{\mu} = \frac{1}{r}(T^* - a^*) (< 0). \quad (21)$$

Hence, the following result obtains.

**Proposition 4** *Assume a policy mix of the rest of the world that satisfy  $T^* < a^*$ .<sup>22</sup> Then, the small state ends up in an economic collapse ( $\underline{S} = 0$ ) specified in (19). The multiplier  $\theta_2$  is strictly positive.*

<sup>21</sup>Indeed, from (21) we deduce  $\bar{T} = \frac{2\zeta}{\zeta - 2\delta}(a^* - T^*)$ .

<sup>22</sup>This is only necessary condition but not sufficient.

It follows that the economy of the small country may only collapse if the rest of the world is able to grant a positive net public benefit ( $a^* - T^* > 0$ ) to the firms. In other words,

**Corollary 2** *The small economy never collapses, would the rest of the world never be attractive ( $a^* - T^* < 0$ ).*

Furthermore, if the small country is caught on a path which leads it to the long run state depicted in the above proposition, it will try to retain capital by permanent tax underbidding. Eventually, the small country's tax and infrastructure expenditures will tend to vanish as it appears in equation (19).

## 4 Conclusion

Many authors recognize that small countries dramatically lack (quantitatively and qualitatively) fundamental productive resources. These deficiencies appear especially in the form of limited productive capital, entrepreneurs and human capital. In this paper, we merged these three types of production factors in one entity by assuming that capital owners, firm owners and workers are the same individuals bearing different mobility preferences and analyzed the policy-mix which can attract these lacking resources. Applying dynamic optimization techniques, we derived efficient and inefficient steady states and their stability conditions. We showed that the share of tax income attributed by the small economy to the provision of attractive infrastructures plays a key-role to evolve on an efficient convergence path. A neglect of this aspect can be deleterious for the the survival of a small economy.

In this paper the focus was on a very small economy which takes the policy decisions of the foreign world as given. This assumption is generally acceptable if the home country is small enough. In a future work, however, our framework should be able to model a non cooperative game between the small and the big jurisdictions. Accordingly, it would be of a great interest to show how the new modelling would change the likely occurrence of the small country's economic collapse.

# Appendix

## Proof of Proposition 1

The current value of the Hamiltonian corresponding to the underlying economy is

$$\mathcal{H}(T, S, a, \mu, \nu) = \left[ ST - \frac{\alpha}{2}(ST)^2 - \frac{\beta a^2}{2} \right] - \mu \left( b^*(t) - \frac{a(t) - T(t)}{k} \right) + \nu [-\delta a(t) + \zeta ST].$$

The following Lagrangian accounts for the state constraints<sup>23</sup> of the model

$$\mathcal{L}(T, S, a, \mu, \nu, \theta_1, \theta_2) = \mathcal{H}(T, S, a, \mu, \nu) + \theta_1 \left( b^*(t) - \frac{a(t) - T(t)}{k} \right) - \theta_2 \left( b^*(t) - \frac{a(t) - T(t)}{k} \right).$$

It is easy to see that  $\mathcal{H}(T, S, a, \mu, \nu)$  is concave with respect to the state variables  $S$  and  $a$ . Hence, the first order conditions are necessary and sufficient for the existence of an optimum. Deriving the first order conditions from the Hamiltonian we obtain (??) with respect to  $T$ , while we get (11) and (10) with respect to both state variables. The multipliers of the state boundary constraints check (12) and (13). QED.  $\square$

## Proof of Proposition 2

We first state the existence of possible interior steady state(s) in addition to that given in Proposition 2. Then we provide the proof.

**Proposition 5** *The following additional steady state(s) may appear:*

(II.1) *If  $a^* - T^* = -\frac{kr\delta}{4\zeta} (< 0)$ , there is a further interior steady state specified by*

$$\hat{T}_1 = \frac{kr\delta}{2\zeta}, \quad \hat{a}_1 = (a^* - T^*) + \hat{T}_1, \quad \hat{S}_1 = \frac{\hat{T}_1}{kr} \quad (22)$$

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<sup>23</sup>See, for example, Chiang, page 301-302.

and two costate variables <sup>24</sup>

$$\widehat{\nu}_1 = \frac{\alpha \widehat{S}_1^2 \widehat{T}_1 + \beta \widehat{a}_1 - \widehat{S}_1}{\zeta \widehat{S}_1 - r - \delta}, \quad \widehat{\mu}_1 = k[(r + \delta)\widehat{\nu}_1] + \beta \widehat{a}_1. \quad (23)$$

(II.2) If  $a^* - T^* = 0$ , the additional steady state is

$$\widehat{T}_2 = \frac{kr\delta}{\zeta} \quad (24)$$

and we obtain the remaining steady state variables by replacing the subscript 1 by 2 in (22) and (23).

(II.3) If  $a^* - T^* > 0$ , the second steady state is specified by

$$\widehat{T}_3 = \frac{kr}{2\zeta} \left[ \delta + \sqrt{\delta^2 + \frac{4\zeta}{kr} \delta (a^* - T^*)} \right] \quad (25)$$

and the others are the same as in (22) and (23) by replacing the subscript 1 to 3.

(II.4) If  $-\frac{kr\delta}{4\zeta} < a^* - T^* < 0$ , there are another two interior steady states where

$$\widehat{T}_{4,5} = \frac{kr}{2\zeta} \left[ \delta \pm \sqrt{\delta^2 + \frac{4\zeta}{kr} \delta (a^* - T^*)} \right] \quad (26)$$

and the remaining the others are the same as in (22) and (23) by replacing the subscript 1 by 4 and 5.

### Proof.

At the interior steady state,  $\theta_1 = 0, \theta_2 = 0$  and we can rewrite the first order condition as follows

$$\left\{ \begin{array}{l} T = \frac{S - \frac{\mu}{k} + \zeta S \nu}{\alpha S^2} \\ \dot{S} = -(b^* - \frac{a-T}{k}), \\ \dot{a} = \delta a + \zeta S T, \\ \dot{\mu} = r\mu - \zeta T \nu - (1 - \alpha S T) T, \\ \dot{\nu} = (r + \delta)\nu + \beta a - \frac{1}{k}\mu. \end{array} \right. \quad (27)$$

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<sup>24</sup>The condition  $\zeta \widehat{S}_1 - r - \delta + \xi \neq 0$  must hold.

We rewrite the first equation as follows

$$\frac{\mu}{kS} = 1 + \zeta\nu - \alpha ST. \quad (28)$$

Substituting (28) into the 3rd equation and arranging leads to the

$$\dot{\mu} = r\mu - \frac{T\mu}{kS}.$$

Hence,  $\dot{\mu} = 0$  leads to two cases:  $\widehat{\mu} = 0$  or  $r = \frac{\widehat{T}}{k\widehat{S}}$ .

We consequently have two groups of steady states: general one  $\widehat{\mu} = 0$  or/and special one where  $r = \frac{\widehat{T}}{k\widehat{S}}$ .

(I)  $\widehat{\mu} = 0$ .

It is easy to check that the interior steady states are given by (14) and (15). To determine their stability, we consider the corresponding Jacobian

$$J_I = \begin{pmatrix} \frac{1}{k\alpha} & \frac{1+\zeta\widehat{\nu}}{\widehat{S}^2} & \frac{1}{k} & \frac{1}{\alpha k^2 \widehat{S}^2} & -\frac{\zeta}{\alpha k \widehat{S}} \\ 0 & -\delta & & -\frac{\zeta}{\alpha k \widehat{S}} & \frac{\zeta^2}{\alpha} \\ 0 & 0 & -\frac{1}{k\alpha} & \frac{1+\zeta\widehat{\nu}}{\widehat{S}^2} + r & 0 \\ 0 & \beta & & -\frac{1}{k} & r + \delta \end{pmatrix} \\ = \begin{pmatrix} \frac{\widehat{T}}{k\widehat{S}} & \frac{1}{k} & \frac{1}{\alpha k^2 \widehat{S}^2} & -\frac{\zeta}{\alpha k \widehat{S}} \\ 0 & -\delta & -\frac{\zeta}{\alpha k \widehat{S}} & \frac{\zeta^2}{\alpha} \\ 0 & 0 & -\frac{\widehat{T}}{k\widehat{S}} + r & 0 \\ 0 & \beta & -\frac{1}{k} & r + \delta \end{pmatrix}.$$

It is easy to show that the eigenvalues of the Jacobian are given by

$$e_1 = \frac{\widehat{T}}{k\widehat{S}} > 0, \quad e_2 = r - \frac{\widehat{T}}{k\widehat{S}} > 0 \text{ ( or } < 0 \text{ )},$$

$$e_{3,4} = \frac{r}{2} \pm \frac{1}{2} \sqrt{r^2 + 4 \left[ \frac{\beta\zeta^2}{\alpha} + (r + \delta)\delta \right]}.$$

Hence,  $e_3 > 0$  and  $e_4 < 0$ , which guarantees one dimensional convergence to the steady state. The other part of the convergence depends on  $e_2$  is negative or not, that is, the relation of  $r$  with respect to the other parameters and exogenous variables.

(II)  $r = \frac{\widehat{T}}{k\widehat{S}}.$

In this case, we have  $S = \frac{T}{kr}$ .  $\dot{S} = 0$  leads to  $a = (a^* - T^*) + T$  and  $\dot{a} = 0$  gives  $\zeta ST = \delta a$ . Combining these conditions, we obtain

$$\frac{\zeta}{kr} T^2 - \delta T - \delta(a^* - T^*) = 0,$$

which yields to two roots

$$T = \frac{kr}{2\zeta} \left[ \delta \pm \sqrt{\delta^2 + \frac{4\zeta}{kr} \delta(a^* - T^*)} \right]$$

and if  $\Lambda = \delta^2 + \frac{4\zeta}{kr} \delta(a^* - T^*) > 0$ , both roots are real. Furthermore, depending on  $\Lambda$  is larger or smaller than  $\delta^2$ , we have different conditions which leads to positive  $T$ s and which serve as the other steady states.  $\square$

### Proof of Proposition 3

In the upper-corner solution case,  $S = \frac{1}{2}$  and, hence,  $\theta_2 = 0$  and  $\theta_1 > 0$ . According to the complementary slackness conditions, we must have  $\bar{a} - \bar{T} = a^* - T^*$ , that is,  $\bar{T} = \bar{a} - (a^* - T^*)$ .

From  $\dot{a} = 0$ , we obtain

$$\bar{a} = \frac{\frac{\zeta}{2}(a^* - T^*)}{\frac{\zeta}{2} - \delta}. \quad (29)$$

The solution  $\bar{a}$  is be positive if and only if  $\delta > \frac{\zeta}{2}$  and  $a^* < T^*$ , or  $0 < \delta < \frac{\zeta}{2}$  and  $a^* > T^*$ .

The condition  $\dot{\mu} = 0$  leads to

$$r\bar{\mu} = \frac{\bar{T}}{2} \frac{1}{k} (\bar{\mu} - \bar{\theta}_1) \quad (30)$$

or

$$\bar{\theta}_1 = \left( \frac{2kr}{\bar{T}} - 1 \right) \bar{\mu}. \quad (31)$$

Similarly,  $\dot{\nu} = 0$  leads to

$$(r + \delta)\bar{\nu} = \frac{1}{k}(\bar{\mu} - \bar{\theta}_1) - \beta\bar{a}. \quad (32)$$

On the other hand, (??) can be rewritten as

$$\alpha\bar{S}\bar{T} = 1 - \frac{1}{k} \frac{\bar{\mu} - \bar{\theta}_1}{\bar{S}} + \zeta\bar{\nu} \quad (33)$$

which gives

$$-\frac{1}{k}(\bar{\mu} - \bar{\theta}_1) = \frac{\alpha\bar{T}}{4} - \frac{1}{2} - \frac{\zeta\bar{\nu}}{2}.$$

Combining with (32), it follows

$$\bar{\nu} = \frac{1}{r + \delta - \frac{\zeta}{2}} \left[ \frac{1}{2} - \beta\bar{a} - \frac{\alpha\bar{T}}{4} \right] \quad (34)$$

and

$$\bar{\mu} = \frac{\bar{T}}{r} \left( 1 + \zeta\bar{\nu} - \frac{\alpha\bar{T}}{2} \right). \quad (35)$$

Hence, we obtain a complete solution for the steady state  $\bar{S} < \frac{1}{2}$ . The above solutions are meaningful if and only if  $\bar{\theta}_1 > 0$ .  $\square$

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