FACULTY OF LAW, ECONOMICS AND FINANCE

CREA
Discussion
Paper

2009-12

Center for Research in Economic Analysis
University of Luxembourg

Fits and Misfits : Technological Matching and R & D Networks

available online : http://fdef.uni.lu/index.php/fdef_FR/economie/crea/discussion_papers/2008

Nicolas Jonard, CREA-University of Luxembourg
R. Cowan, BETA, Université Louis Pasteur
B. Sanditov, UNU-MERIT, University of Maastricht

December 7, 2009

For editorial correspondence, please contact : elisa.ferreira@uni.lu

University of Luxembourg
Faculty of Law, Economics and Finance
162A, avenue de la Faïencerie
L-1511 Luxembourg

Fits and Misfits: Technological Matching and R&D Networks

R. Cowan[1,2], N. Jonard[3] and B. Sanditov[2]

[1]BETA, Université Louis Pasteur.
[2]UNU-MERIT, University of Maastricht.
[3]Université du Luxembourg.

Keywords: network formation, small worlds
JEL classification: D85

December 7, 2009

Abstract

This paper presents an economic model of R&D network formation through the creation of strategic alliances. Firms are randomly endowed with knowledge elements. They base their alliance decisions purely on the technological fit of potential partners, ignoring social capital considerations and indirect benefits on the network. This is sufficient to generate equilibrium networks with the small world properties of observed alliance networks, namely short pairwise distances and local clustering. The equilibrium networks are more clustered than "comparable" random graphs, while they have similar characteristic path length. Two extreme regimes of competition are examined, to show that while the competition has a quantitative effect on the equilibrium networks (density is lower with competition), the small world features of the equilibrium networks are preserved.

1 Introduction

Interest in social and economic networks of various types has grown dramatically in the last two decades. The manifold empirical studies emphasize that networks often share two properties. First, they are small worlds — sparse networks with both short distances between pairs of agents, and strong local clustering. Second, they have skewed link distributions, often with a fat right tail. These properties have been observed in a wide variety of networks.[1] A challenge in the economic, mostly game theoretic literature on network formation has been to capture these properties in a sound model. What incentives do (a

[1]Movie actors, power grids, neural networks (Watt and Strogatz 1998); food webs (Williams et al. 2002); academic co-authorship in many disciplines (Newman 2004; Goyal et al. 2006), strategic alliances (Verspagen and Duysters 2004; Powell et al. 2005); patent co-invention (Balconi et al. 2004).

few) agents have to form shortcuts, and why should they cluster locally? Why do a few agents acquire many links while many others seem able to form only a few?

In this paper we develop a model of R&D networks in which firms seek to innovate, combining theirs and their partners' knowledge in order to produce new knowledge. The key assumption is that the decision to forge a strategic alliance is based on the extent to which the two participating firms' endowments both resemble and complement each other. Two aspects of the model combine to produce small world architectures.

First is the requirement that both similarities and complementarities are present for an alliance to be profitable. The demands of absorptive capacity imply that partnering firms must be similar to each other. As Granovetter (1976) points out, very often such "similarity" is transitive, and thus in the larger network we should expect clusters of densely interconnected firms. But the fact that firms must also complement each other implies that similarity cannot be "identity", and so is not perfectly transitive. The simultaneous presence of common and distinct traits will induce correlation in the pairing decision, yielding clustering at the local level. This tendency is strongest when similarities are much more important than complementarities in the innovation process, that is, when what we share matters more than how we differ.

Second is the fact that firms are located within a specific knowledge space. Thus links between firms are implicitly links between points in that space. The interaction between the decision rule, the structure and random location of firms in knowledge space will imply short distances in network space. This creates the second feature of small worlds, namely short pairwise distances.

Within the framework of this model we are able to derive analytically density, degree distribution and clustering, and provide both asymptotic and numerical results on path lengths. The results show that the search for partners with a good technological fit can explain network architecture, and specifically small world properties. Although the search for technological fit is the stated aim for strategic R&D alliance participants, it is often left out of empirical studies. One contribution of this paper is thus to show that it is essential that technological fit is included. We also contrast two extreme forms of competition. At one extreme every alliance is aimed at a unique innovation, and so if successful gives a positive value to its inventor. At the other extreme we invoke a patent race, in which all alliances are aimed at the same, patentable, innovation, implying that only one success will produce any revenue. We compare these extremes, showing that the strength of competition affects the density of the network but that the basic structures, small worlds with skewed link distributions, are preserved.

2 Small worlds

Mechanisms underlying a skewed degree distribution are thought to be reasonably well understood. One argument is that in most industries firms' size distribution itself is skewed. This suggests that a larger firm has more resources with which to create or maintain costly links than does a smaller one. Another popular argument is that skewness arises from a preferential attachment mechanism (Barabasi and Albert 1999): a firm with many links is often seen as an attractive partner to other firms either joining the network or revising their partnering decisions. This alone is enough to create a relatively "star-ish" network in which a small number of firms acquire many links while most have a few.

A bigger challenge has been to understand the genesis of small worlds.

The first relevant literature approaches the problem from an algorithmic perspective. Following the initial paper by Watts and Strogatz (1998), this literature is contributed to mostly by computer scientists and physicists. It seeks to design algorithms to construct small worlds, asks about the (often asymptotic) properties and functions of small world networks, and looks for them in large datasets (See Watts 1999; Amaral et al. 2000; Kleinberg 2000) Not surprisingly there are processes producing small worlds, and not surprisingly their economic underpinnings, if any, are thin (but then, that was not their purpose).

A second literature that provides much more solid foundations is the strategic management literature. Here, network formation and network structure are often discussed in terms of social capital. Clusters emerge through considerations of social control and because link formation often rests on third party referrals. On the other hand, structural hole arguments are used to explain how clique-spanning ties arise, initiated by firms aiming at accessing resources in distant parts of the industry, and at occupying strong brokerage positions. Together, these arguments explain the creation and maintenance of the small world structures that are often observed empirically. (These ideas have been widely discussed: see Coleman 1988; Burt 1992; Kogut et al. 1992; Uzzi 1997; Walker et al. 1997; Gulati and Gargiulo 1999; Ahuja 2000; Dyer and Nobeoka 2000; Rowley et al. 2000; Baum et al. 2003.)

In the context of strategic R&D alliances between firms, these arguments seem correct, but incomplete. Two parties to an alliance aimed at innovation of some kind must consider not only social capital but also whether their knowledge and competence complement each other. Indeed one of the standard explanations for why firms engage in (strategic, R&D) alliances, is to access technology or competence that they need but do not possess in-house.[2] We can imagine a space of technological competence, where any firm is located at a point representing its own expertise. If, in its choice of alliance partners, a firm looks to complement its expertise, it will search for partners that are relatively distant in this space. On the other hand though, considerations of absorptive capacity argue that partners to an alliance cannot be too far apart. If they are, their respective absorptive capacities will be too low, and the alliance will fail (Nooteboom 2000 or Grant 1996). This suggests that alliance success should have an inverted-U relationship with distance between the partners in the underlying knowledge space. There are now a small number of studies that seem to support this hypothesis (Mowery et al. 1998 and 1996; Ahuja and Katila 2001; Schoenmakers and Duysters 2006). This need for potential partnerships to achieve both some degree of commonality and some degree of complementarity is a key feature of the model we develop in this paper.

The third place where models for the formation of small worlds are provided is the economic literature. There, short pairwise distances tend to arise because of the presence of indirect benefits decaying with distance. And high clustering results from smaller costs (if possibly smaller benefits) of connecting to similar (nearby) nodes. Variants of Jackson and Wolinsky's (1996) communication model all exploit, to some degree, this feature. A network in which inter-node distances are, on average, long induces some value destruction,

[2] See for example Ahuja and Katila (2001), Mowerey et al. (1996) or Hagedoorn (1993).

which the creation of a shortcut can prevent. Jackson and Rogers (2005) locate nodes on islands. Costs of link formation are low if two nodes are on the same island, and high if they are on different islands. Together with the truncation of benefits, this structure creates arbitrarily dense islands with rare interconnections between islands at equilibrium. Carayol and Roux (2009) explore a framework in which nodes are located on a circle, and the cost of forming a link rises with geographic distance between nodes. Galleoti et al. (2004) also have a connections model where players' values and costs are heterogenous. In a world where society has groups and intra-group link are cheaper to form that inter-group links, they find that inter-connected stars are both socially efficient and stable, leading them to claim that centrality, center-sponsorship and small diameter are robust features of networks. In all of these models shortcuts (which bring down average inter-node distances) arise because a node seeks access to a distant resource, the value of which decays in transmission.

In comparison with the models above, our approach has 2 important differences. First, firms have richer properties. Each of them is defined as a set of attributes, drawn from a fixed universe (for example firms forming strategic R&D alliances, where attributes are pieces of knowledge, or competence in particular technologies). These attributes only create economic values when combined in the right proportion. Second, we keep the simplest structure with no indirect effects. Value (here, innovation) originates in links, and is not transmitted beyond its inventors. So shortcuts in the equilibrium network generated in our model will not result from firms' desire to shorten their distance to a valuable resource. The combination of firms' heterogeneity and their willingness to ally with similar yet complementary partners is enough to induce the formation of small worlds.

3 Definitions

3.1 Networks

The industry consists of a finite population $N = \{1, \ldots, n\}$ of firms. A *network* is a set g of ties (links) between unordered pairs of firms in N. Writing ij to represent the tie between firms i and j, $ij \in g$ indicates that i and j are linked in the network g. The network obtained by adding ij to an existing network g is denoted $g + ij$, while the network obtained by deleting ij from g is $g - ij$.

The *neighbourhood* of firm i consists of the firms to whom i is directly connected, denoted $N_i^g = \{j \neq i : ij \in g\}$. The size of the neighbourhood of i is the number of ties held by firm i, also called its *degree*, and is denoted $n_i^g = \#N_i^g$. The total number of links in the network is $E^g = \sum_{i \in N} n_i^g/2$, and thus the density of g is equal to $2E^g/n(n-1)$. A *singleton* is a firm i such that $n_i^g = 0$, i.e. a firm with no partner.

3.2 Strategies and equilibrium

Firms play a simultaneous link formation game. A *strategy* for firm i is a list of $(n-1)$ decisions $(s_{i,1}, \ldots, s_{i,i-1}, s_{i,i+1}, \ldots, s_{i,n})$, with $s_{i,j} \in \{0, 1\}$. If $s_{i,j} = 1$ then i proposes a partnership to j, whereas no partnership is proposed when $s_{i,j} = 0$. Link formation is bilateral, so in the network $g(\mathbf{s}) = g$ induced by the strategy profile $\mathbf{s}$, $ij \in g$ if and only if $s_{i,j}s_{j,i} = 1$, that is, both partners want the alliance and so propose to each other. Denote the payoff to firm i in network g as π_i^g (defined below). The network g is *pairwise stable* (Jackson and Wolinsky, 1996) if and only if

$$\min\{\pi_i^g - \pi_i^{g-ij}; \pi_j^g - \pi_j^{g-ij}\} \geq 0, \forall ij \in g$$

and

$$\min\{\pi_i^{g+ij} - \pi_i^g; \pi_j^{g+ij} - \pi_j^g\} < 0, \forall ij \notin g.$$

Links part of a stable network g should yield non-negative value to both partners, whereas links yielding negative value to at least one potential partner cannot be part of g.

3.3 Assessing small worlds

A small world is defined as a sparse network, displaying both local clustering and short pairwise distances. A sparse network is one in which density is low, that is, the number of edges divided by the number of possible edges is small. In empirical studies of small worlds, densities are typically lower than 5 to 10 percent.

There are two classic measures of clustering (see Newman 2003). In the first, clustering is measured by the proportion of neighbours of a node who are neighbours of each other. Computing that proportion for each node and taking the average yields the *clustering coefficient*. An alternative measure, which is more easily determined in our case, is the fraction of transitive triples in the whole network. *Transitivity* is defined as

$$\frac{\sum_i \#\{jk \in g : j, k \in N_i^g\}}{\sum_i \#\{jk : j, k \in N_i^g\}} = \frac{3 \times \text{ Nb of closed triangles in } g}{\text{Nb of connected triples in } g}.$$

The factor 3 appears in the numerator because each triangle involves 3 connected triples. On this definition, it is worth noting that the expected value of transitivity is the probability that jk exists conditional on ij and ik existing, or put another way, the probability of closing the connected triple formed by ij and ik by connecting j to k.

Contrasting with clustering, which is a local measure (of redundancy in ties), a measure of global structure is provided by the distribution of pairwise distances between nodes. A *path* of length $\ell + 1$ in g connecting i and j is a set of distinct nodes $\{i_1, \ldots, i_\ell\}$ such that $ii_1, i_1i_2, \ldots, i_\ell j \in g$. The *distance* separating i from j in g is the length of the shortest path in g between i and j. If no path exists between i and j, then by convention the distance is defined as infinite. Averaging all finite pairwise distances yields the *characteristic path length*. To cope with the issue of disconnected networks (which would introduce at least one infinite distance in the calculation) we restrict attention to distances between "reachable pairs".

A network is identified as a small world when clustering significantly exceeds that of a "comparable" random graph, whereas characteristic path length is of the same order. A benchmark often used in practice is an Erdös and Rényi (1960) random graph (ER), in which a tunable number of edges are allocated among randomly (and uniformly) selected pairs of nodes. The ER benchmark is made "comparable" to the graph under scrutiny by taking the same number of nodes and edges.

3.4 Innovation

Firms are the carriers of knowledge which they bring together in order to discover new knowledge. Knowledge is modelled as a set or list of discrete elements, where the set of all possible facts is $\{1, \ldots, w\}$. Firm i's knowledge portfolio is a w-position binary vector v_i, where $v_i^z = 1$ if i knows fact z and $v_i^z = 0$ otherwise.

In the introduction we argued that for joint innovation, both commonality and difference were important. Define v_{ij} as the commonality of i and j, i.e. the number of facts known to both: $v_{ij} = \#\{z : v_i^z = v_j^z = 1\}$. Also define s_{ij} as i's complementing of j, i.e. the number of facts known to i but not to j: $s_{ij} = \#\{z : v_i^z > v_j^z\}$. For successful innovation, i and j must both be similar to and complement each other. We model this by assuming success requires a commonality of δ and reciprocal complementarity of γ. The probability that the partnership ij innovates successfully is

$$r_{ij} = \begin{cases} 1 \text{ if } v_{ij} = \delta \text{ and } s_{ij} = s_{ji} = \gamma, \\ 0 \text{ otherwise.} \end{cases} \tag{1}$$

Successful partners must thus have the right technological fit, i.e. share competencies in δ technologies and have their competencies complement each other in γ technologies.[3]

3.5 Payoffs

The model represents firms' behaviour in a fast moving industry in which innovation is the motivating goal of firms, and post-innovation competition in the market place is parametrized in a simple manner. Innovative success is affected by the partner firms' technological fit, whereas profits from innovation are affected by the nature of competition. In what follows we examine two extremes of competition.

In the first, the benefits from each successful innovation are fully appropriated by its 2 inventors, regardless of the amount of R&D in the industry and the success of other partnerships. The alliance decision is based entirely on technological fit, and we refer to the resulting network as the knowledge or technology network: t. In the second case, the benefits from each successful alliance have to be shared with all other successful alliances (equivalently, only 1 alliance will succeed as in a standard patent race). Here market considerations are essential and we refer to the resulting network as the market network: m.

Profits in the two cases are written as

$$\pi_i^g = \frac{1}{1 + \alpha \left(E^g - 1\right)} \sum_{j \in N_i^g} r_{ij} - c n_i^g,$$

[3]These are very strong conditions on alliance success. There are two obvious ways they can be relaxed. Overlap could be "near" optimal overlap and still result in success. So the overlap rule could be changed to $\delta - \rho \leq v_{ij} \leq \delta + \rho$. Similarly, since firms do not need to share every possible piece of knowledge, the "reciprocal complements" rule could be changed to $s_{ij} \geq \gamma$ and $s_{ji} \geq \gamma$. (Notice that this also relaxes the strong reciprocity condition.) Both of these generalizations, when made individually, are relatively straightforward, and involve summing the functions given here. Relaxing them simultaneously is more intricate however.

where $g \in \{t, m\}$, $c > 0$ is the cost of forming or maintaining an alliance, paid by each of the partners to it, $\alpha \in \{0, 1\}$ captures the intensity of competition for innovative reward ($\alpha = 0$ for the knowledge case, and $\alpha = 1$ for the market case).

4 Equilibrium networks

In this section we develop the model on the assumption that any second stage competition is not relevant to the alliance decision-making process. In essence, every alliance is aimed at a unique innovation which is not in any way a substitute for any other innovation. This is a strong assumption, but one which is often implicitly made, both in theoretical and empirical network studies.

The expected profit for firm i in a (technology-driven) network t is thus

$$\pi_i^t = \sum_{j \in N_i^t} r_{ij} - cn_i^t. \tag{2}$$

As links convey no indirect benefits, the marginal value of both an existing and a non-existing link ij is simply $\pi_i^t - \pi_i^{t-ij} = \pi_i^{t+ij} - \pi_i^t = r_{ij} - c$. Technological fit being symmetric (see Equation 1), if the incremental value of a link is positive to one partner, it is to the other as well. Proposition 1 below states the results, omitting the straightforward proof.

Proposition 1 *When $c > 1$ the empty network is (uniquely) stable. When $c \leq 1$, for any values of δ and γ there is a unique equilibrium network, in which all firms with overlap of δ and reciprocal complementarity of γ form an alliance.*

4.1 Discussion of general structures

Proposition 1 provides a minimal description of the equilibrium networks. Before turning to the formal derivation of network statistics, we can describe heuristically some effects of parameter changes on network architectures.

We have chosen to model an industry in which there is a fixed total number w of relevant facts, with the intention of characterizing industries by how much of the relevant knowledge is necessary to innovate (the size of $\gamma + \delta$ relative to w). An interpretation is that in young industries, the amount of (relevant) knowledge known to the firm is low relative to the amount that can in principle be discovered. In a more mature industry by contrast, what remains to be discovered can be thought of as smaller, hence a higher $\delta + \gamma$ relative to w. Should we then expect in the model that alliances will be more common in a more mature, exploitative industry, or rather that active networking will be a distinct feature of young, innovative industries? This will depend of the relative importance of commonality and complementarity.

Start from the fact that a match between i and j requires that i knows exactly δ of j's $\delta + \gamma$ pieces of knowledge. The probability of this occurring is highest when the probability that i knows any particular fact is $\delta/(\delta + \gamma)$. This would mean that i knows (in expected terms) $w \cdot \delta/(\delta + \gamma)$ facts. But in the model firms know exactly $\delta + \gamma$ facts. So the probability that i and j fit is highest when $(\delta + \gamma)/w = \delta/(\delta + \gamma)$, and falling away as parameter values are such that we depart from the previous equality. If complementarity demands dominate ($\delta/(\delta+\gamma)$ small), networks will be dense when innovating firms have little knowledge ($(\delta+\gamma)/w$ also small) so that almost any pair of firms complement each other. Young industries with (relatively) unexplored technological opportunities will be a place of intense technological partnering. On the other hand, when commonality demands are high ($\delta/(\delta+\gamma)$ large), dense networks will emerge when total knowledge endowments are high, so that firms are relatively more similar to each other (a larger portion of the knowledge vector is common). Here the requirement of an important technological overlap will only permit intense networking in mature, exploitation-oriented industries.

Beside density, another important feature of alliance networks is transitivity. Consider first an industry in which innovation is driven by creation of novel combinations of existing knowledge, so that we might expect γ the demand for reciprocal complementarities, to be high. In the extreme $\gamma = (w-\delta)/2)$, and no triangle of firms can form. In this situation, the partners of any firm must all be identical to each other, and so cannot be partners to each other (they would bring no novelty at all to the alliance). In this case however, squares (cycles of period 4) can form: partners of i are mirror images of i and a mirror image has a cycle of period 2.[4] Note that in this situation firms ally with partners who are identical to each other, and thus possibly redundant from the point of view of innovation. The implication however is that a strong need for complementarity γ, everything else equal, should decrease transitivity in the network. At the other extreme, in an industry where innovation is not driven by creation of novel combinations of existing knowledge, bringing novelty to a partnership may be less important than having a good, easy working relationship. In this case commonality in knowledge would dominate the decision and the demand for reciprocal complementarities γ would be small relative to δ. In the extreme $\gamma = 0$, and the partners of any firm again are all identical to each other, and identical to the focal firm i. This time all possible triangles involving i and its neighbours will form, and the network will fracture into small, isolated cliques of identical firms, each component representing a sub-sector of the industry. Increasing δ in that case would increase the size of the cliques, and the grand coalition (the complete industry network) would obtain when $\delta = w$. The implication here is that when the need for complementarity γ is very small the network will display transitivity. The extent of the phenomenon will depend however on δ and w, via the effect on density which affects how many triangles can in principle form.

4.2 Density and degree distribution

From Proposition 1, we can derive the properties of the larger network of all alliances. Our interest is in how these properties are affected by the relative importance of commonality (δ) and complementarity (γ) in the innovative process.

The strict condition we place on alliance formation, namely that holding exactly $\delta+\gamma$ ideas is necessary to be part of any alliance, implies that firms endowed with different amounts of knowledge are doomed never to partner. In what follows we focus on a population of firms all of which command the right amount of knowledge. This strategy has two

[4] To be more precise, partners of i have δ facts in common with each other and with i, but in the remaining $w-\delta$ places in their knowledge vectors, they are mirror-images of i and identical to each other.

advantages. First, in an arbitrary population endowed with random amounts of knowledge, one effect of changing the parameters γ and δ would be to change the proportion of firms commanding $\gamma + \delta$ technologies. With a fixed total number of firms, this changes the number that can participate in the network. Thus the effects of complementarities and overlap are confounded with changes in the effective, network-participating, population size. Fixing the number of "alliance-ready" firms over the parameter space removes this effect. The second advantage is that empirical analyses of alliances tend to consider as their population only those firms who have ever had an alliance. Thus they exclude firms that, for whatever reason (perhaps they are too young to have enough knowledge to be attractive; or they command enough knowledge to be self-sufficient) have never had an alliance. We abstract from these issues by considering a population of n firms commanding $\delta + \gamma$ technologies and so which are, even if not participating in any alliances, at least "alliance-ready".

The starting element to derive the properties of the network of alliances is the probability q that two firms form a link. As each firms carries $\gamma + \delta$ ideas uniformly distributed over w positions, the probability that i and j form a partnership is simply the hypergeometric probability (of drawing exactly δ of $\delta+\gamma$ existing ideas in $\delta+\gamma$ drawings without replacement from a set of w possible objects), which is written

$$\Pr\{ij \in t | a_i = a_j = \delta + \gamma\} = q = \frac{\binom{\delta+\gamma}{\delta}\binom{w-\delta-\gamma}{\gamma}}{\binom{w}{\delta+\gamma}}. \tag{3}$$

Thus we can state the following result.

Proposition 2 *When all firms command $\delta+\gamma$ technologies and consider only technological fit, q is the (expected) density of the equilibrium network t, and the degree distribution of firms in the equilibrium network t is binomial with parameters $n-1$ and q:* $\Pr\{n_i^t = z\} = \binom{n-1}{z} q^z (1-q)^{n-z}$.

Figure 1 shows how expected density (q) responds to changes in parameters. Following the discussion in Section 4.1, density is presented as a function of the general knowledge level $(\delta + \gamma)/w$, and the relative importance of commonality versus complementarity, $\delta/(\delta + \gamma)$. In this space, a horizontal move represents the same relative change in δ and γ (so that the ratio $\delta/(\delta + \gamma)$ remains constant), and a vertical move represents changes of identical absolute sizes but opposite directions in δ and γ (so that $\delta + \gamma$ is unchanged).

As suggested earlier, we observe that density is highest along the ray defined by $(\delta + \gamma)/w = \delta/(\delta + \gamma)$. It is worth noting that the probability of matching falls rapidly as we move away, so even though all firms are a priori "alliance-ready", density remains close to 0 over significant portions of the parameter space.

4.3 Transitivity

On the definition given in Section 3.3, the expected value of transitivity (the expected proportion of transitive triples in the equilibrium network) is $\Pr\{jk|ij, ik\}$, the probability that jk forms conditional on ij and ik existing. From then we want to know whether the model produces excess transitivity compared to a random graph of the same density.

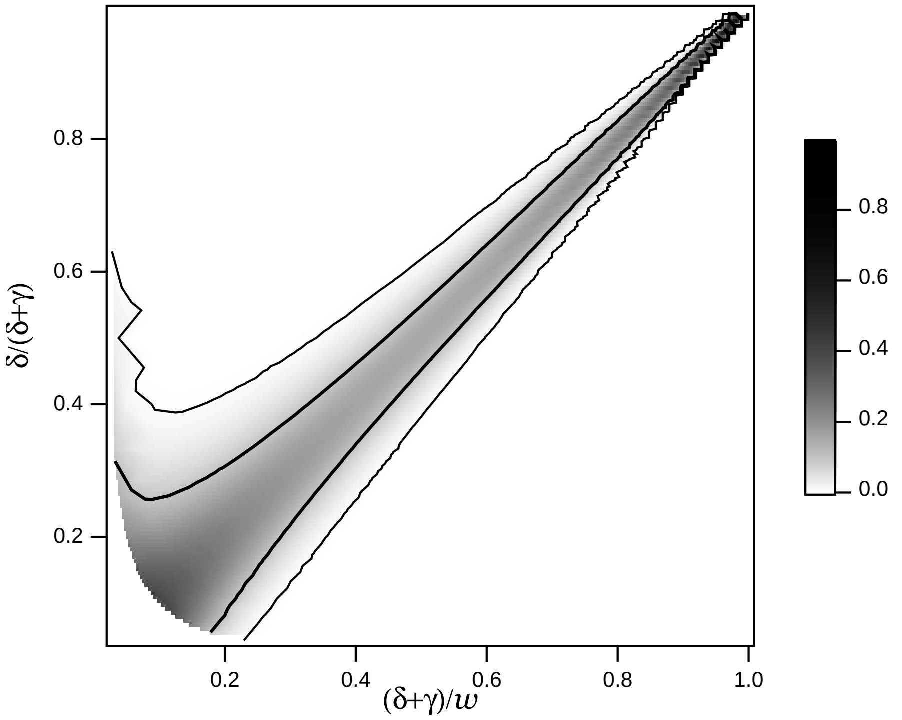

Figure 1: Density of the equilibrium network t as a function of $(\delta+\gamma)/\omega$ and $\delta/(\delta+\gamma)$, all firms command $\delta+\gamma$ technologies. Contours: 0.01, thin line; 0.1, thick line.

Noting that in a random graph edges form independently, the probability that the third edge of a triangle forms conditional on two of its edges already existing is simply the probability that an edge forms. This probability in turn is simply q, the density of the random graph. So rescaled transitivity (the extent to which triangles occur relative to a "comparable" random graph) is written $\Pr\{jk|ij,ik\}/q$, with q defined in Equation (3). A ratio larger (smaller) than 1 indicates the presence (absence) of excess clustering, relative to the random benchmark.

Proposition 3 *When all firms command $\delta+\gamma$ technologies, re-scaled transitivity in the equilibrium network is equal to*

$$\frac{\sum_{b=\underline{b}}^{\overline{b}} \binom{\delta}{b}\binom{\gamma}{b}^2\binom{w-\delta-2\gamma}{\gamma-b}\binom{w}{\delta+\gamma}}{\binom{\delta+\gamma}{\delta}^2\binom{w-\delta-\gamma}{\gamma}^2},$$

where $\underline{b} = \max\{0; 3\gamma+\delta-w\}$ *and* $\overline{b} = \min\{\gamma;\delta\}$*. Rescaled transitivity is significantly greater than one over much of the (δ,γ) space, and is larger for smaller values of the complementarity parameter γ.*

Proof Obtaining $\Pr\{ij,ik\}$ is straightforward: it is the probability of 2 independent events, $\Pr\{ij,ik\}=q^2$. The numerator $\Pr\{ij|ik,jk\}$ is slightly less direct to compute. This time, write firms i, j and k's knowledge portfolios as a $w\times 3$ matrix (firm i is the first column, etc.) and suppose there are $\delta-b$ rows in the matrix written 111, with $0\le b\le\delta$. Then the rest of the matrix is perfectly defined. There must be b rows of 110, b rows of 101 and b

rows of 011 to ensure the right overlap between all pairs of firms. Then $\gamma - b$ rows of 001, $\gamma - b$ rows of 010 and $\gamma - b$ rows of 100 so that the right complementarities between pairs are obtained. The remaining rows (of which there are $w - \delta - 3\gamma + b$) must be written 000. How many ways there are to array the 1s held by all 3 firms so that $\delta - b$ rows of the matrix are written 111 is a product of binomial coefficients, i.e.

$$\frac{w!}{(\delta - b)!b!^3\,(\gamma - b)!^3\,(w - \delta - 3\gamma + b)!}.$$

As the 1s held by all 3 firms can be arrayed in $\binom{w}{\delta+\gamma}^3$ ways, the probability that the knowledge matrix has $\delta - b$ rows of 111 is written

$$\frac{(\delta + \gamma)!^3\,(w - \delta - \gamma)!^3}{(\delta - b)!b!^3\,(\gamma - b)!^3\,(w - \delta - 3\gamma + b)!w!^2}.$$

Rearranging the factorials, representing them as binomial coefficients and invoking the law of total probability (the knowledge matrix can have from 0 to δ rows written 111) the final result obtains as

$$\Pr\{ij, ik, jk\} = \frac{\binom{\delta+\gamma}{\delta}\binom{w-\delta-\gamma}{\gamma}}{\binom{w}{\delta+\gamma}^2} \sum_{b=\underline{b}}^{\bar{b}} \binom{\delta}{b}\binom{\gamma}{b}^2\binom{w-\delta-2\gamma}{\gamma-b},$$

where $\underline{b} = \max\{0; 3\gamma + \delta - w\}$ and $\bar{b} = \min\{\gamma; \delta\}$, so that all binomial coefficients are well defined. Finally the expected value of transitivity is

$$\Pr\{jk|ij, ik\} = \frac{\sum_{b=\underline{b}}^{\bar{b}} \binom{\delta}{b}\binom{\gamma}{b}^2\binom{w-\delta-2\gamma}{\gamma-b}}{\binom{\delta+\gamma}{\delta}\binom{w-\delta-\gamma}{\gamma}}.$$

From there, rescaled transitivity obtains directly by computing the ratio of $\Pr\{jk|ij, ik\}$ to q, the density of the graph. ■

Figure 2 displays how transitivity (left panel) and the logarithm of rescaled expected transitivity (right panel) respond to changes in the general knowledge level $(\delta + \gamma)/w$, and the relative importance of commonality versus complementarity, $\delta/(\delta + \gamma)$. One contour of density (density of 0.01) is superimposed on the two images, to show the parts of the parameter space where networks are both reasonably dense and clustered.

The results visible in Figure 2 are consistent with the intuitions developed in Section 4.1. The results are that, first, small values of δ and γ sustain transitivity, as evidenced by the dark region close to the origin in the left panel of Figure 2. A move up along the vertical axis starting from the zone of high density corresponds to an increase in δ and the same decrease in γ, the net effect of which is a decline of transitivity. A move down the vertical axis form the high density zone also yields a response in the same direction of falling transitivity. The effect of density (how many triangles can form in principle) is thus dominating.

To detect the presence of excess transitivity, it is interesting to look at the right panel in Figure 2, which presents rescaled transitivity (the measure normalized by the amount of transitivity present in an equivalent random graph). Here the main observation is that

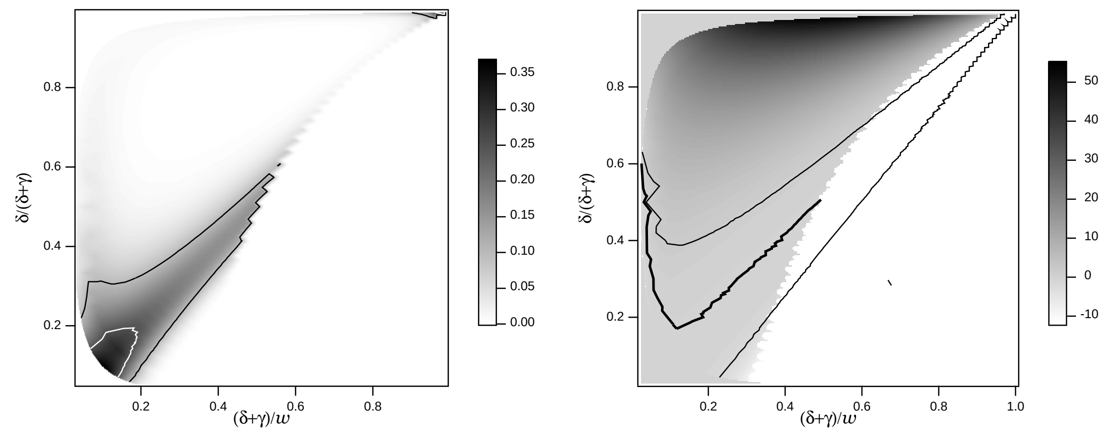

Figure 2: Left panel: Expected transitivity; white contour 0.25, black contour 0.1. Right panel: Logarithm of rescaled expected transitivity: thick contour line is rescaled transitivity=1; thin contour line is density=0.01. In the white area to the right of the figures, parameter values are such that triangles cannot form, and transitivity is not defined.

rescaled transitivity is monotonically increasing with $\delta/(\delta+\gamma)$. Once we correct for the effect of pure randomness on triangle formation, the picture is thus modified. When the dominant consideration in partnering is commonality of knowledge rather than complementarity, the networks that arise are much more locally coherent than equivalent random networks. Rescaled transitivity is greater than one, and often much greater than one.

4.4 Distances

The second defining feature of a small world is a short characteristic path length, possibly approaching that of a random graph. The analytical derivation of the distance distribution is not possible in the model, however we are able to provide the following asymptotic result.

Proposition 4 *As the number of firms in the industry increases, the diameter of the industry network approaches $\lfloor \delta/\gamma \rfloor + 2$. When the network is connected, the limit is approached from above.*

Proof Each firm in the industry is located at one point in a discrete knowledge space of size $\binom{w}{\delta+\gamma}$. All possible knowledge configurations are equally likely, so as the number of firms increases, the proportion of occupied nodes in the knowledge space approaches 1. Consider the limiting case of each point in knowledge space being occupied by at least one firm. We can construct a longest shortest path. Consider a source node that has all its knowledge in the first $\delta + \gamma$ positions, and a target node that has its knowledge in the final $\delta + \gamma$ positions. Reserve the first $\delta + \gamma$ positions, then divide the remaining $w - (\delta + \gamma)$ positions in sets of length γ starting from the right end, letting $\xi (< \gamma)$ denote the remainder of the integer division. In Table 1 we record the number of 1s in each group of cells as we move from the source node to the target node through the nodes on the path. The movement down the table can be seen as moving $\delta + \gamma$ 1s from the beginning of the vector to the end of it, moving at most γ at a time. Moving exactly γ will permit link formation between adjacent rows of the table.

Number of ordered positions	$\delta+\gamma$	ξ	γ	$\cdots$	γ	γ	$\cdots$	γ	γ	γ
Source firm	$\delta+\gamma$	0	$\cdots$	$\cdots$	0	0	$\cdots$	0	0	0
Firm 2	δ	0	$\cdots$	$\cdots$	0	0	$\cdots$	0	0	γ
Firm 3	$\delta-\gamma$	0	$\cdots$	$\cdots$	0	0	$\cdots$	0	γ	γ
$\cdots$			continue until $\delta-v\gamma<\gamma$, then							
Firm $v+2$	$\delta-v\gamma$	0	$\cdots$	$\cdots$	0	γ	$\cdots$	γ	γ	γ
Target firm	0	0	$\cdots$	0	γ	$\delta-v\gamma$	$\cdots$	γ	γ	γ

Table 1: Constructing a longest shortest path.

As a result, the path from source to target has length $v+2$, where v is the largest integer such that $\delta - v\gamma < \gamma$, i.e. $v = \lfloor \delta/\gamma \rfloor$, hence the result. The longest pairwise distance is obtained as the integer part of the ratio of knowledge commonality to complementarity increased by 2. This is the limiting case in which there is at least one firm at every point in knowledge space. As population size falls, points in the knowledge space are vacated. Considering any source-target pair, if a point on the constructed path is vacated, the path will be impossible to implement. If this does not disconnect the graph, the shortest path between source and target will increase. ■

What we observe here is that the short characteristic path lengths arise from the interaction of the knowledge space with the alliance-formation rule. In contrast to much of the literature, it does not arise from the effects of indirect benefits. In Jackson and Wolinsky's (1996) connections model and its descendants, nodes receive benefits from nodes at all finite distances in the graph, but benefits that decay with distance. This provides in incentive for each node to ensure that it has short paths to all other nodes, implying incentives for at least some nodes to create shortcuts. In our model, there are no indirect benefits: benefits to a firm arise only from innovation taking place along a single link.

For the finite case we treat the issue numerically. At each point in the (δ, γ)-(integer)plane, we generate 100 random networks and record the average distance among reachable pairs. Values of δ and γ such that there are no reachable pairs (i.e. when the graph is empty) do not contribute to the set of observations. Using the average density of those 100 networks, we also generate 100 random networks of that expected density, and record for them the average distance among reachable pairs. Re-scaled path length at each point in the (δ, γ)-plane is the ratio of those two path lengths. We then pool the data and build a frequency histogram, as shown in Figure 3.

The bulk of the distribution is concentrated around 1, indicating a structure with distances very similar to those of an equivalent random graph over the entire relevant parameter space.

It is worth emphasizing that the asymptotic result predicts a diameter which does not depend on w, the total number of pieces of knowledge in the industry, but solely on the relative importance of commonality and complementarity. Also note that the asymptotic diameter does not scale with n: the existence of a finite limit makes the network a small world in a very strong sense.

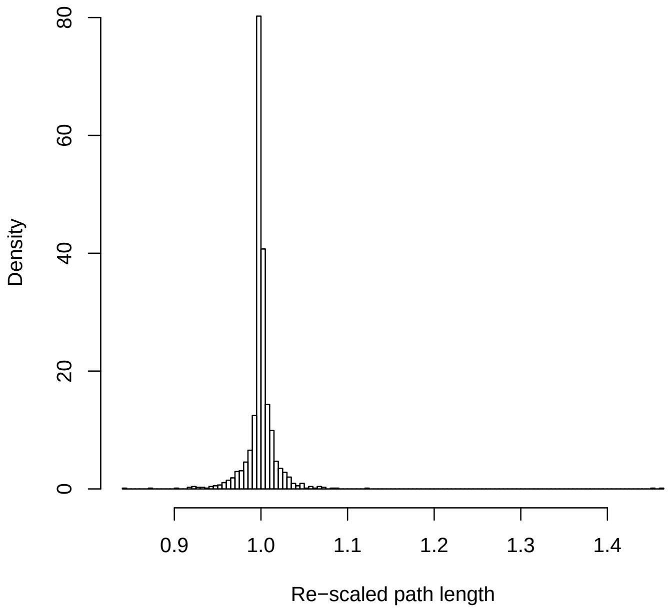

Figure 3: Frequency distribution of re-scaled distance between reachable pairs over the (δ, γ) plane, all firms command $\delta + \gamma$ technologies.

4.5 Small worlds?

While there is no consensus regarding a statistical definition of a small world, a commonly used statistic is the ratio of re-scaled transitivity to re-scaled characteristic path length. Ratios significantly larger than one indicate the presence of a small world. In Figure 4 we plot the values of this ratio against network density for all relevant (δ, γ) combinations. In addition, we use different markers depending on whether the point lies above or below diagonal in the $(\delta/\left(\delta+\gamma\right), \left(\delta+\gamma\right)/w)$-plane. We can observe two things from this figure. First, small worlds are only present when the network is not dense. This is in fact consistent with the general observation that small worlds (and many empirically observed networks) are quite sparse. Second we observe here the importance of complementarity versus commonality in determining network structure. When strong complementarity in partners' knowledge stocks is necessary for an alliance to succeed ($\delta/(\gamma+\delta)$ is small) few triangles form and transitivity is low: small worlds are not present. At the other extreme, when commonality in knowledge is necessary for success ($\delta/(\gamma+\delta)$ is large) small worlds do form, again provided the network is not too dense.

We observe that focussing the alliance decision on technological fit produces networks which are highly clustered relative to equivalent random graphs. These networks also have path lengths that are close to those in equivalent random graphs. The degree distribution does not display power law properties: it is binomial and so is skewed to the right, but with an exponential tail. All together, these results permit us to state that technological fit, which is the stated aim for strategic alliance participants, is a strong explanatory factor for observed, small world, network architectures. More sophisticated network motives, which

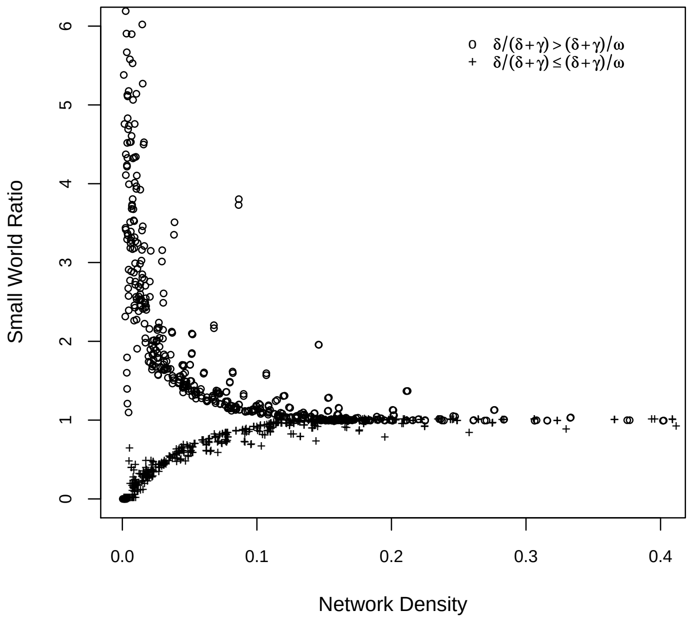

Figure 4: Ratio of re-scaled transitivity to re-scaled average path length as a function of network density, partitioned on $\delta/(\delta+\gamma) \lesseqgtr (\delta+\gamma)/w$.

would involve indirect effects and attempts by firms to manipulate the larger network beyond their immediate set of alliances are unnecessary.

5 Equilibrium networks with competition

The model just developed assumed no competition. Partner choice and equilibrium networks were restricted only by knowledge constraints. Now we turn to a case in which market considerations play a role. We examine a situation of extreme competition, in which one and only one alliance will be profitable ex post. All firms compete for the same discovery, and the first partnership that innovates obtains a patent and receives monopoly profits, while other firms get nothing. The essential issue in this section is how competition affects the equilibrium network, using the no-competition case as a benchmark. In effect, competition will add partner selection constraints, and we examine how they interact with the knowledge constraints discussed earlier, i.e. the relative importance of commonality (δ) versus complementarity (γ) in the innovative process.

5.1 Competition in networks

Actors in any alliance network are not only part of a network, they are also part of an industry which may involve inter-firm competition. This observation, while widely acknowledged, seldom appears in the empirical literature on alliances. In some industries, such as pharmaceuticals, the competition between partners in many alliances is quite attenuated (for example an alliance between a dedicated biotech firm and a big pharma firm may involve no competition on any market at all) while in others, such as micro-electronics or automobiles, partners in an alliance are often very direct competitors, even in the market for the outputs of the alliance. Theoretical papers tend to include a post network formation stage of competition. It is so in the literatures on coalition formation (see Bloch 1995; Joshi 2008 and references therein) and on more general R&D networks (Goyal and Joshi 2003 and 2006). Typically these papers find that the degree of competition is central in determining which network architectures are observed at equilibrium. Goyal and Joshi (2003) find that dominant group architectures (the equilibrium network consists of a complete sub-component plus singletons) are often stable structures. Small world features were not the object of their study though. Goyal and Joshi (2006) mention patent races as examples of "playing-the-field" network games (games with specific externalities across the links of a given player, and across links of different players). Our model however is quite different due to the constraints imposed by technological fit, in addition to those stemming from pairwise stability.

5.2 Strategies and equilibrium

We assume that firms are engaged in a patent race with no discounting. In essence, any innovation project will succeed so the project that has a positive return is merely the first one to succeed. We assume that projects undertaken are in every relevant respect identical, so the probability that any particular project is the first to succeed is simply one over the number of projects. Thus the expected profit function for firm i in the (market) network m is simply

$$\pi_i^m = \frac{n_i^m}{E^m} - cn_i^m.$$

Recall that there are no indirect benefits in the model, thus a link between misfits will never be observed in equilibrium as it creates no value and carries a cost. So $\pi_i^m = (1/E^m - c)\, n_i^m$, with the understanding that a link only forms between technologically fit players. Denoting A_i the number of alliances not involving i, $A_i^m = E^m - n_i^m$ in the network m, we have $\pi_i^m = f(n_i^m, A_i^m) - cn_i^m$, which depends only on the number of the firm's links and the (halved) aggregate number of links of other firms in the industry. In Goyal and Joshi's (2006) terminology, the game is a playing-the-field game. As they point out, the classical patent race is also a playing-the-field game. The difference between the game here and the example they consider is that in their case firms are the locus of innovation, whereas in our case it is the partnerships themselves. The aggregate revenue $f(z, A)$ is concave in own links, i.e. $f(z+1, A) - f(z, A)$ is decreasing with z, and satisfies the strategic substitutes property that $f(z+1, A') - f(z, A') < f(z+1, A) - f(z, A)$ when $A' > A$. The game thus represents a situation in which there is a positive externality across the links of a given firm, and a negative externality across the links of different firms.

If technological fit is not an issue, i.e. firms can form all the partnerships they wish to form, then (as in Goyal and Joshi, Proposition 3, p. 329) a symmetric network[5] with

[5]Note that for a symmetric network g of any degree d^* to be possible, the number of players n cannot be odd: $E^g = nd^*/2$ is guaranteed to be an integer number for all integer value of d^* only if n is even.

degree $d^* \in \{1, \ldots, n-2\}$ is pairwise stable if

$$\frac{2(n-2)}{n(nd^*+2)} < c < \frac{2(n-2)}{n(nd^*-2)}. \tag{4}$$

In general, however, other pairwise stable structures can exist for c in the right interval, and not all values of c fall in an interval which can take the form required by Condition 4. We do not attempt an exhaustive characterization of the equilibrium networks, and will turn instead to numerical experiments. We can however observe that in the full setup in which firms are both constrained by knowledge fit and pairwise stability, the following result obtains readily.

Proposition 5 *The equilibrium market network m is always less dense than the technology network t. In addition, if Condition 4 holds so that there exists a symmetric equilibrium network of degree d^*, then (a) if $\max\{n_i^t; i \in N\} \leq d^*$ then the technology network t is a market equilibrium network; (b) if $\min\{n_i^t; i \in N\} \geq d^*$ then one (of possibly many) equilibrium market networks is symmetric with each firm having exactly d^* alliances.*

Proof All potential links are in the technology network t. Either all are pairwise stable, or there are firms forced by profit maximization to drop a subset of their links in t, in which case there can only be fewer links in total.

Suppose c satisfies Condition 4, and $n_i^t \leq d^*, \forall i \in N$. No pair of firms can add a link. Does a firm want to sever a subset of its links? For any $i \in N$, $A_i = (\sum_{j \neq i} n_j^t - n_i^t)/2 < (n-2)\,d^*/2 = A$. Using the strategic substitutes property and concavity in own links, $\gamma(n_i^t, A_i^t) - \gamma(n_i^t - 1, A_i^t) \geq \gamma(n_i^t, A) - \gamma(n_i^t - 1, A) \geq \gamma(d^*, A) - \gamma(d^* - 1, A) > c$, i.e. no firm is willing to drop links.

Suppose c satisfies Condition 4, and $n_i^t \geq d^*, \forall i \in N$. The technology constraints are not binding and so the symmetric equilibrium of degree d^*, which we know to be stable, can be formed by firms. ■

Integrating the constraints of pairwise stability and knowledge fit makes the problem very complicated to solve mathematically. The reason is that some firms are bound by the pairwise stability constraint (those who have many partners in the knowledge network t) and some are bound by the knowledge matching constraint (those with few partners in the knowledge network t). We thus resort to a simple, edge-by-edge addition algorithm to randomly generate equilibrium networks starting from the technology network t. An equilibrium market network m will obtain as a subset of the set t of potential links.

Starting with an empty network $m_0 = \{\varnothing\}$,we draw links at random from the technology network t and incrementally add them to the market network. At step s, since m_{s-1} only contains links between fit partners, we pick a random link in $t - m_{s-1}$, check that it is marginally profitable and add it to m_{s-1} to form m_s, the current market network. We then recalculate $A_i^{m_s}, \forall i \in N$. At this stage, we make use of the strict quasi-concavity of the profit function to informally define the "best-response" of firm i as $b_i^{m_s} = b_i^{m_s}(A_i^{m_s}) = -A_i^{m_s} + \sqrt{A_i^{m_s}/c}$. As either $\lfloor b_i^{m_s} \rfloor$ or $\lceil b_i^{m_s} \rceil$ is the optimal number of partners for firm i in m_s, we check that no firm exceeds it best response. If such a firm is found, one of its links is randomly deleted to remove the violation. We repeat this process until no more links can be added. This creates a network that satisfies the constraints of knowledge fit and pairwise stability.

5.3 Density and degree distribution

Using the algorithm described above, or each (δ, γ) pair, we generate equilibrium networks and pool the observations to construct our estimated distribution. Industry size is set to $n = 100$, while the knowledge universe has size $w = 100$.

Figure 5 displays the degree distributions of the technology network and of the market network in 4 different environments. In each of them, we provide the degree distribution in the technology network t, and 3 degree distributions obtained for 3 different values of c, to illustrate the influence of cost on the properties of the equilibrium networks. Each distribution represents the average behaviour of 200 instances of the technology network and its attendant market network.

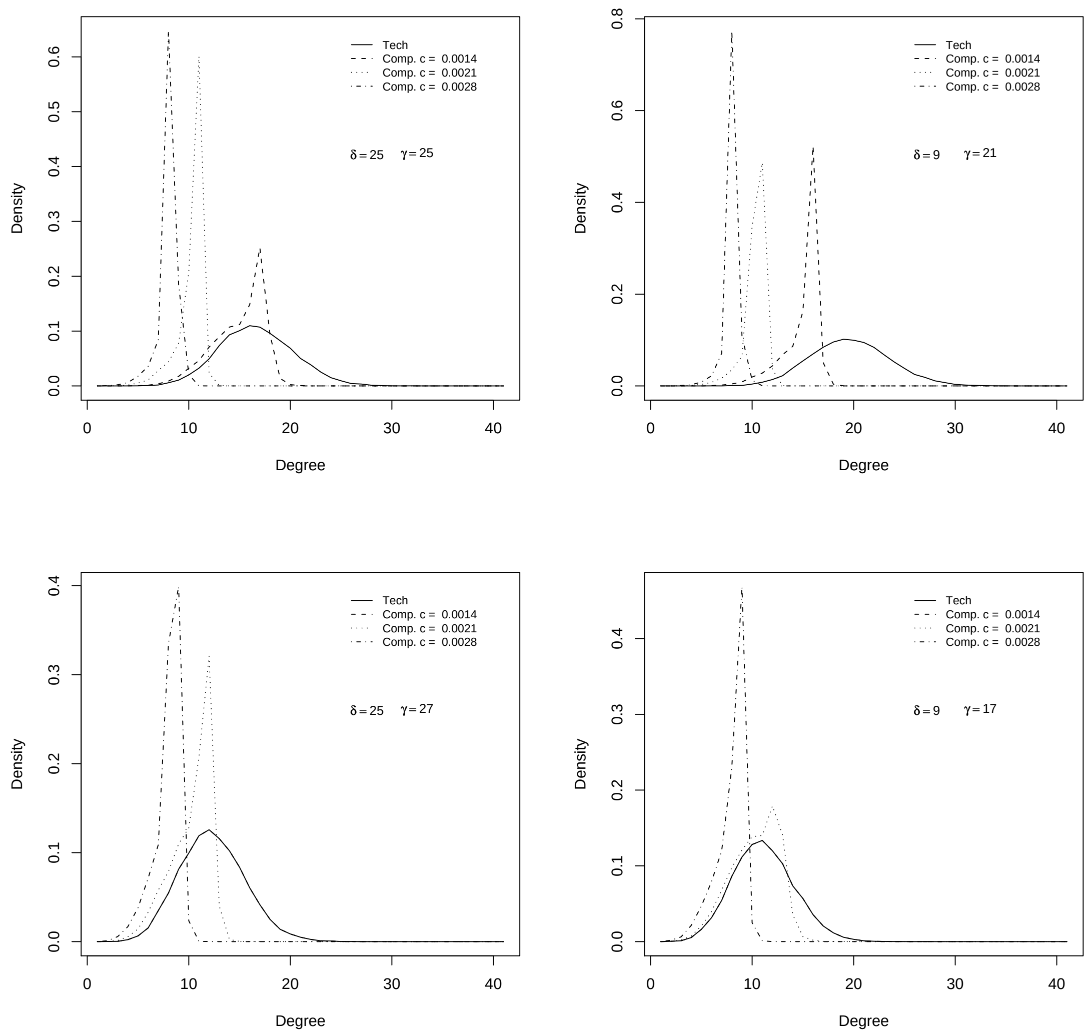

Figure 5: Degree distributions for different parameter values. Curves that cannot be seen lie on top of the technology network distribution.

The upper panels of Figure 5 correspond to areas of (δ, γ)-plane where the network is dense: $\delta/(\delta+\gamma) \approx (\delta+\gamma)/w$. The left panel corresponds to $(\delta, \gamma) = (25, 25)$, a context in which firms know much of the relevant knowledge in the industry, the demand on both commonality and complementarity. The right panel corresponds to $(\delta, \gamma) = (9, 21)$, a situation in which knowledge is more scarce and the demand on complementarity is strong relative to that on similarity.

The lower panels in Figure 5 correspond to off-diagonal situations and thus sparse industry networks. The lower left panel is a situation in which $\delta/(\delta+\gamma) < (\delta+\gamma)/w$, with $(\delta, \gamma) = (25, 27)$ while the right panel depicts a situation on the other side of the diagonal, with $(\delta, \gamma) = (9, 17)$.

In each of the panel we observe in principle three types of distributions: those whose mass is highly concentrated around a single value; those which mimic the technology distribution almost exactly; and those which mimic the technology distribution (less exactly) up to a certain value, and then concentrate mass at that value. The explanation is that there are in principle two types of firms: those whose networking behaviour is constrained by knowledge, and those whose behaviour is constrained by profit maximization. If the technology network is dense, most of the firms will be constrained by profit maximization, and so we will see the mass of the distribution concentrated around the profit-maximizing degree (in many of the 200 realizations, the symmetric equilibrium emerges). If the technology network is sparse, it provides the constraint, and in the extreme case, the competition network is the technology network, and so the degree distributions lie on top of each other. In the intermediate case, in the same network, some firms are constrained by knowledge fit, and others are constrained by profit maximization. Here we see a combination of the two effects.

The effect of c, the cost per alliance, is to change where the profit maximization constraint binds. That is, higher values of c imply that the optimal number of alliances will fall, so we see a departure from the technology degree distribution at lower values. When c is low, the profit-maximizing number of partnerships is high, so all firms will "take all the partners they can get", and so the degree network is identical to the technology network.

5.4 Small worlds

The technology network showed small world properties over significant parts of the parameter space. Adding competition does not destroy this result. Small worlds remain, and more specifically, small worlds remain in the parts of the parameter space where they emerged when competition was absent. To facilitate comparison with our benchmark case, we show ratios for the statistics in the technology case to those in the network case. For each point in the parameter space we compute the technology to competition ratio, and plot histograms of that ratio. We do this for raw and rescaled transitivity and average pairwise distance, as well as for the small world ratio itself.

The upper right panel of Figure 6 shows a frequency distribution of the ratio of transitivity in the two regimes, over the entire (δ, γ)-plane. Most values of this ratio are very close to one, as expected. The removal of some links drives transitivity down in a relatively mechanical way. The same obtains when rescaled transitivity is considered in the lower right panel, the ratio of the competition to no competition cases being centered around 1, though more dispersed than the ratio of raw measures.

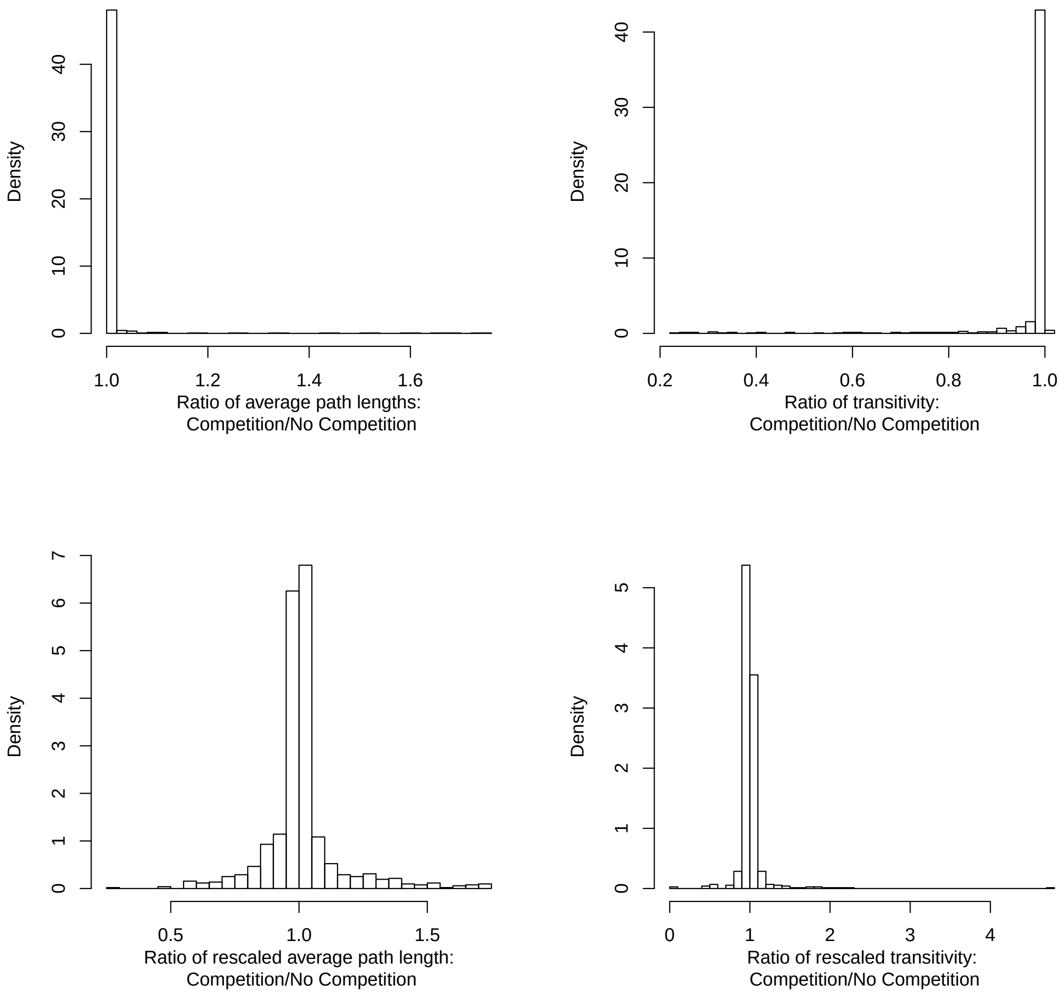

Figure 6: Comparing competition with no competition: ratios of path length and transitivity over the parameter space.

Turning to pairwise distances, we can observe in the upper left panel of the figure that competition also has essentially no effect on average path lengths. The explanation here is that there are many shortest paths between any pair of nodes. Thus the effect of introducing competition, which is to remove edges relative to the no-competition case, does not, except in relatively rare instances, remove all of the shortest paths. Thus competition has essentially no effect on average path length, and any effect it has is to increase path length. The effect would be marked only if competition brought density way below that the technology network. The same picture obtains when rescaled average pairwise distance is considered in the lower left panel, most of the mass of the distribution lying close to 1.

From this we can conclude that competition, even at its most punishing, does not have a big effect on network architecture insofar as small-world aspects are concerned. Density is smaller always, but clustering and short distances are preserved. And this in turn is

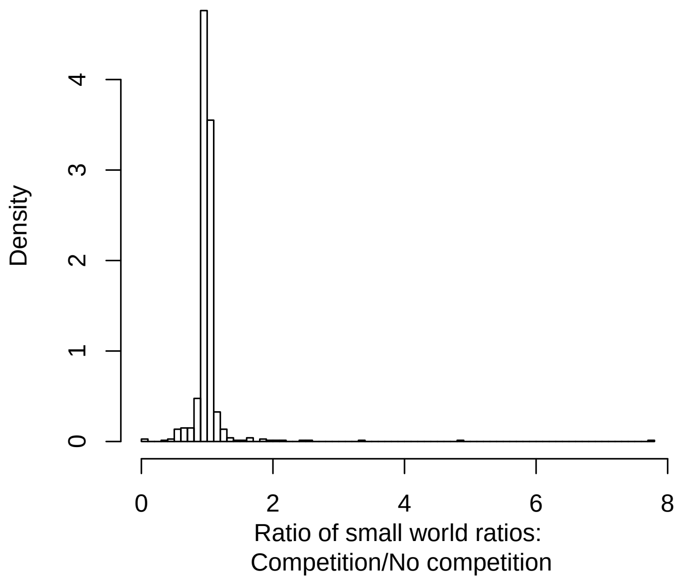

Figure 7: Comparing competition to no competition case: ratio of the small world ratio over the parameter space.

preserved in Figure 7 which computes the ratio of small-world ratios in the competition to no-competition cases: again the mass lies mostly around 1.

6 Conclusion

In this paper we have proposed and analyzed a simple model of alliance formation and joint innovation in which we have assumed that firms must reach a precise balance between commonality and complementarity in order to jointly innovate. Small worlds arise because firms have two essential characteristics. First, firms are heterogenous. They hold different technological endowments, and seek partners with whom to assemble these technologies in the right portfolio. This will imply random graph features, and specifically short distances, for the equilibrium network. But this is not enough yet. In addition to holding random knowledge endowments, firms act strategically. They take into account the cost and benefits from each potential alliance, and the behaviour of other firms in the network when it is relevant. This behaviour will create local correlation in the alliance decisions, and thus a tendency for the equilibrium to display excess clustering or transitivity. This tendency is strongest when similarities are much more important than complementarities in the innovation process, that is, when what we share matters more than where we differ. It is this combination of a random process of knowledge assignment and thus location in a knowledge space, and firms' strategic decision to ally only with profitable — here meaning relatively more similar than dissimilar — partners which creates small worlds in equilibrium.

It is worth emphasizing that we obtain small world results without assuming spillovers or any other indirect, possibly long-distance effect. So the mechanism at work to lower the diameter of equilibrium networks is fundamentally different from the more "traditional" incentive to create a shortcut to capture value created far away in the graph. Also the reason why there is clustering is a very simple and general one of "transitivity of attractions", a feature suggested but not formalized in Granovetter's (1976) seminal paper.

With this model we capture the idea that when a firm makes a strategic alliance it is interested in technological fit with its partners. With respect to the strategic management literature, it is worth emphasizing that we recover the properties of observed alliance networks — small worlds with skewed link distributions — ignoring entirely issues of social capital. This suggests that models of link formation (and hence network emergence) that ignore technology may be missing something important. Again, one thing we observe, though, is that the high values of clustering characteristic of small worlds are not present when γ is high. That is, when the requirement for reciprocal complementarities is very strong, clustered networks do not emerge. One conclusion to draw here is that in empirically observed networks, which tend to have high clustering, the need for this type of complementarity must be weak.

The second part of the model introduces competition following innovation. The basic idea is that of a patent race in which all alliances compete for the patent on a unique innovation. Introducing this aspect, which tends not to be present in empirical studies of network formation, has predictable effects, the most important one being to lower network density. By and large, though, the introduction of competition does not have a big effect on network structure. Here we can conclude that it may be safe to leave this feature out of our investigations without doing much damage to our understanding.

References

[1] Ahuja, G. (2000) "Collaboration networks, structural holes and innovation: A longitudinal study" *Administrative Science Quarterly*, 45, 425-455.

[2] Ahuja, G. and R. Katila (2001) "Technological acquisitions and the innovation performance of acquiring firms: A longitudinal study" *Strategic Management Journal*, 22, 197-220.

[3] Amaral, L.A., A. Scala, M. Bartelemy and H.E. Stanley (2000) "Classes of Small-World Networks" *Proceedings of the National Academy of Science*, 111, 49-52.

[4] Balconi, M., S. Breschi and F. Lissoni (2004) "Networks of inventors and the role of academia: An exploration of Italian patent data", *Research Policy*, 33, 127-145.

[5] Barabasi, A.L. and R. Albert (1999) "Emergence of Scaling in Random Networks", *Science*, 286, 509-512.

[6] Baum, J.A., A.V. Shipilov and T.J. Rowley (2003) "Where Do Small Worlds Come from?" *Industrial and Corporate Change*, 12, 697-725.

[7] Bloch, F. (1995) "Endogenous Structures of Association in Oligopolies", *Rand Journal of Economics*, 26, 537-556.

[8] Burt, R. S. (1992), *Structural Holes: The Social Structure of Competition,* Harvard University Press, Cambridge, Mass.

[9] Carayol and Roux (2009) "Knowledge flows and the geography of networks. A strategic model of small world formation", *Journal of Economic Behavior and Organization,* 71, 414-427.

[10] Coleman, J.A. (1988) "Social Capital in the Creation of Human Capital" *American Journal of Sociology*, 94, S95-S120.

[11] Cowan, R. and N. Jonard (2009) "Knowledge Portfolios and the Organization of Innovation Networks", *Academy of Management Review*, 34, 320-342.

[12] Dyer, J.H. and K. Nobeoka (2000) "Creating and Managing a High Performance Knowledge Sharing Network: The Toyota Case" *Strategic Management Journal*, 21, 345-367.

[13] Erdös, P. and A. Rényi (1960) "On the Evolution of Random Graphs", *Publications of the Mathematical Institute of the Hungarian Academy of Sciences*, 5, 17-61.

[14] Galeotti, A., Goyal, S. and J. Kamphorst (2006) "Network formation with heterogeneous players", *Games and Economic Behavior*, 54, 353-372.

[15] Goyal, S. and S. Joshi (2003), "Networks of Collaboration in Oligopoly", *Games and Economic Behavior*, 43, 57-85.

[16] Goyal, S. and S. Joshi (2006), "Unequal Connections", *International Journal of Game Theory*, 34, 319-349.

[17] Goyal, S., Van der Leij, M. and J.L. Moraga-González (2006) "Economics: An Emerging Small World", *Journal of Political Economy*, 114, 403-412.

[18] Grant, R.M. (1996) "Towards a Knowledge-Based Theory of the Firm" *Strategic Management Journal*, 17, 109-122.

[19] Granovetter, M. (1976) "The Strength of Weak Ties", *American Journal of Sociology*, 78, 1360-1380.

[20] Gulati, R. and M. Gargiulo (1999) "Where Do Inter-Organizational Networks Come from?" *American Journal of Sociology*, 104, 1439-1493.

[21] Hagedoorn, J. (1993) "Understanding the rationale of strategic technology partnering: inter-organizational modes of cooperation and sectoral differences", *Strategic Management Journal*, 14, 371-385

[22] Jackson, M. and B. Rogers (2005) "The economics of small worlds", *Journal of the European Economic Association*, 3, 617-627.

[23] Jackson, M.O. and A. Wolinsky (1996) "A Strategic Model of Social and Economic Networks" *Journal of Economic Theory*, 71, 44-74.

[24] Joshi, S. (2008) "Endogenous formation of coalitions in a model of a race", *Journal of Economic Behavior and Organization*, 65, 62-85.

[25] Kleinberg, J. (2000) "Navigation in a Small World" *Nature*, 406, 845.

[26] Kogut, B., W. Shan and G. Walker (1992) "Competitive Cooperation in Biotechnology: Learning through Networks?" in N. Nohria and R. Eccles (Eds.) *Networks and Organizations: Structure, Form and Action*, Harvard Business School Press, Boston.

[27] Mowery, D.C., Oxley, J.E. and B.S. Silverman (1996), "Strategic Alliances and Interfirm Knowledge Transfer" *Strategic Management Journal*, 17, 77-91.

[28] Mowery, D.C., Oxley, J.E. and B.S. Silverman (1998) "Technological Overlap and Interfirm Cooperation: Implications for the Resource-based View of the Firm" *Research Policy*, 27, 507-523.

[29] Newman, M. (2003) "The Structure and Function of Complex Networks", *SIAM Review*, 45, 167-256.

[30] Newman, M. (2004) "Coauthorship networks and patterns of scientific collaboration", *Proceedings of the National Academy of Science*, 101, 5200-5205.

[31] Nooteboom, B. (2000) *Learning and Innovation in Organizations and Economics* Oxford University Press.

[32] Powell,W.W., D.R. White, K.W. Koput and J. Owen-Smith (2005) "Network Dynamics and Field Evolution: The Growth of Inter-organizational Collaboration in the Life Sciences" *American Journal of Sociology*, 110, 1132-1205.

[33] Rowley,T. D. Behrens and D. Krackhardt (2000) "Redundant governance structures: An analysis of structural and relational embeddedness in the steel and semiconductor industries" *Strategic Management Journal*, 21, 369-386.

[34] Schoenmakers W. and G. Duysters (2006) "Learning in strategic technology alliances" *Technology Analysis and Strategic Management,* 18, 245-264.

[35] Uzzi, B. (1997) "Social Structure and Competition in Inter-firm Networks: The Paradox of Embeddedness" *Administrative Science Quarterly*, 42, 35-67.

[36] Verspagen, B. and G. Duysters (2004) "The small worlds of strategic technology alliances" *Technovation*, 24, 563?571.

[37] Walker, G., B. Kogut and W. Shan (1997) "Social Capital, Structural Holes and the Formation of an Industry Network" *Organization Science*, 8, 108-125.

[38] Watts, D. J. (1999). *Small Worlds: The Dynamics of Networks Between Order and Randomness*, Princeton, Princeton University Press.

[39] Watts, D.J. and S. Strogatz (1998) "Collective Dynamics of Small-World Networks", *Nature*, 393, 440-442.

[40] Williams, R.J., E.L. Berlow, J.A. Dunne, A.-L. Barabási and N.D. Martinez (2002) "Two Degrees of Separation in Complex Food Webs" *Proceedings of the National Academy of Science*, 99, 12913-12916.