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Jean Gabszewicz, CORE, Université Catholique de Louvain
Didier Laussel, GREQAM, Université de la Méditerranée
Tanguy van Ypersele, GREQAM, Université de la Méditerranée
Skerdilajda Zanaj, University of Luxembourg

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For editorial correspondence, please contact : elisa.ferreira@uni.lu

University of Luxembourg
Faculty of Law, Economics and Finance
162A, avenue de la Faïencerie
L-1511 Luxembourg

Market games and successive oligopolies

Jean Gabszewicz[*], Didier Laussel[†],
Tanguy van Ypersele[‡] and Skerdilajda Zanaj[§]

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Abstract

In this paper we first introduce an approach relying on market games to examine how successive oligopolies operate between downstream and upstream markets. This approach is then compared with the traditional analysis of oligopolistic interaction in successive markets. The market outcomes resulting from the two approaches are analysed under different technological regimes, decreasing *vs* constant returns.

1 Introduction

The traditional theory of successive markets, based on Spengler (1950), assumes that downstream firms behave as monopolists in their own output market, but as price takers when buying their input[1]. This assumption implies a specific sequentiality in the firms decisions: in the second stage, the downstream monopolist selects the output level, *conditional on the input price*. This choice generates a demand function for the input. The effective input price then obtains in the first stage from the equality between the downstream monopolist' s demand and the upstream monopolist's input supply decision. This supply decision is assumed to maximize the upstream monopolist's profit on the demand function of the downstream monopolist, while taking the price of his/her own input as given. This constitutes the traditional approach to present the property of double marginalization in the bilateral monopoly framework. It is also the framework often used in the literature of the effects of vertical aggreements.

The present paper firstly introduces an alternative approach to examine how successive oligopolies do operate between downstream and upstream markets.

[*]CORE, Université catholique de Louvain, Belgium
[†]GREQAM, Université de la Méditerranée, France
[‡]GREQAM, Université de la Méditerranée, France
[§]CORE, Université catholique de Louvain, Belgium

[1]This is the framework used also in Gabszewicz and Zanaj (2007).

This alternative approach relies on the notion of *strategic market game.* Strategic market games have been developped by Shapley (1977), Shapley and Shubik (1977) or Dubey and Shubik (1977) as an alternative to Cournot oligopoly models. In strategic market game models, traders send simultaneously bids and supplies of goods to the market mechanism. The ratio of bids and supplies exchanged for a good determine its terms of trade. In our paper, the downstream firms bid a quantity of money, which is used to purchase a quantity of input, necessary for the production of output. This choice generates an amount of money to be shared among the upstream oligopolists in proportion to their input production. We propose the following sequence of decisions. In the second stage, n downstream oligopolists play a Cournot game in the downstream market, and bid each a quantity of *money* he/she is willing to offer to get a share of the total input supplied by the m upstream firms in the first stage. In the first stage, the upstream oligopolists choose non cooperatively the amount of input they supply, in order to maximize the amount of money obtained from their input sales.

The two approaches essentially differ by the fact that, in the latter, the input price does not obtain through the market clearing condition while it does in the former. Here, the input price, expressed in monetary units, is equal to the ratio between the total amount of bids offered by the downstream firms, and the total amount of input supplied by the upstream ones. As discovered below, this approach leads to different market outcomes than those observed in the traditional approach, when successive oligopolies operate through the usual price mechanism. Consequently, it naturally invites to contrast the differences between the regime resulting from the just described market game approach, -call it the *market game regime*-, and the regime, which relies on the traditional theory of successive markets; call the latter the *market regime.* In particular, it invites to compare the size of the double marginalization resulting from each of these alternative regimes.

In the prototype examples considered, the comparison between the two regimes leads to the following conclusions: *(i)* the two regimes generate different market outcomes, with a different double marginalization under the market game and the market regimes; *(ii)* even if the market outcomes are different, the two regimes converge to the same outcome under unlimited entry in the input sector, both in the case of decreasing and constant returns; *(iii)* as in the traditional theory, and in spite of different outcomes, the market game approach does not prevent that, under decreasing returns, free entry in both markets also entails that the usual tendency for the input price to adjust to its marginal cost no longer holds, while it still does under constant returns.

The paper is organized as follows. The next section presents the model, while section 3 explores the subgame perfect equilibrium considering two prototypes of technologies for downstream firms. Section 4 concludes.

2 The model

Consider two markets, the downstream and upstream markets, with n *downstream* firms $i, i = 1, ...n$, in the first producing the output, and m *upstream* firms j, $j = 1, ...m$, in the second, producing the input, and selling it in exchange of *money* . The n downstream firms face a demand function $\pi(Q)$ in the downstream market, with Q denoting aggregate output. Firm i owns technology $f_i(z)$ to produce the output, with z denoting the quantity of the sole input used in the production process. The m *upstream* firms each produce the input z at a total cost $C_j(z), j = 1, .., m$.

This situation gives rise to a two stage sequential game. In the first stage, the active players are the m upstream firms with input supply strategies s_j. They aim at maximizing the amount of money they obtain from their input sales. The players in the second stage game are the n downstream firms with *money bidding strategies* b_i. They aim at maximizing their profit by obtaining through their bids the quantity of input required to produce their Cournot equilibrium quantity in the downstream market. The two markets are linked to each other *via* the production function f_i, namely,

$$f_i(z_i) = f_i(\frac{b_i S}{\Sigma_{k=1}^n b_k}),$$

where $\frac{b_i S}{\Sigma_{k=1}^n b_k}$ constitutes the fraction of total input supply S, obtained by firm i through its bidding strategy b_i. To illustrate this approach, we can imagine that downstream firms borrow money from the Central Bank to pay their input and have to reimburse it after the sales of the output: The firm's effective payoff is then given as the difference between profits and the amount of money borrowed from the Bank.

Given a total input supply S, the payoff in the second stage game for the i_{th} firm at the vector of strategies (b_i, b_{-i}) obtains as

$$\Pi_i(b_i, b_{-i}; S) = \pi\left(f_i(\frac{b_i S}{\Sigma_{h=1}^n b_h}) + \Sigma_{k\neq i} f_k(\frac{b_k S}{\Sigma_{h=1}^n b_h})\right) f_i(\frac{b_i S}{\Sigma_{h=1}^n b_h}) - b_i.$$

Given these payoffs, and a total supply S in the input market, the best reply, $b_i(b_{-i}(S))$ of firm i in the second stage game obtains as a solution (whenever it exists) to the problem

$$\underset{b_i}{Max}\ \Pi_i(b_i, b_{-i}; S).$$

A Nash equilibrium in the second stage game, conditional on a total input supply S, is a vector of strategies $(b_1^*(S), ..., b_n^*(S))$ such that, for all i, $b_i^*(S) = b_i(b_{-i}^*(S))$.

In the first stage game, upstream firms select their supply strategies s_j, $j = 1, .., m$. Given a n-tuple of supply strategies $(s_1, ..., s_j, .., s_m)$ and a vector of downstream firms' bids $(b_1, ..., b_n)$ in the second stage game, the amount of money received by firm j obtains as

$$\Gamma_j(s_j, s_{-j}) = \frac{\Sigma_{k=1}^n b_k}{\Sigma_{h=1}^m s_h} s_j - C_j(s_j),$$

which constitutes the payoff function of the j_{th}-upstream firm in the first stage game, conditional on the vector of bids $(b_1, ..., b_n)$ chosen by the downstream firms in the second stage. Denote by $(s_1^*, ..., s_m^*)$ a Nash equilibrium in the first-stage game, conditional on the vector of bids $(b_1, ..., b_n)$. A subgame-perfect equilibrium is a $(n+m)$-tuple of strategies $(b_1^*, ..., b_n^*; s_1^*, ..., s_m^*)$ such that (i) $(b_1^*, ..., b_n^*; S^*)$ is a Nash equilibrium conditional on S^* in the second-stage game, with $S^* = \Sigma_{h=1}^m s_j^*$; (ii) $(s_1^*, ..., s_n^*)$ is a Nash equilibrium in the first stage game conditional on the vector of bids $(b_1^*, ..., b_n^*)$.

3 Exploring subgame perfect equilibria

It is difficult to analyze subgame perfect equilibria at the full level of generality underlying the above model. This is why in view of comparing the present analysis with the traditional one, we explore the properties of subgame equilibria by looking at two prototype examples as in Gabszewicz and Zanaj (2006). The first corresponds to a situation in which downstream firms are endowed with a decreasing returns technology while the second is characterized by constant returns. Furthermore, we assume in both examples a linear demand function in the downstream market, as in Salinger (1988), Gaudet and Van Long (1996). We also assume that firms operating in the upstream (resp. downstream) market are all identical. Entry and competition are analyzed through the asymptotic properties of the subgame-perfect equilibrium when the number of firms in the markets is increased by expanding the economy, as in Debreu and Scarf (1963). The two examples are now successively considered.

3.1 Decreasing returns

The n *downstream* firms are assumed to face a linear demand $\pi(Q) = 1 - Q$ in the downstream market. They share the same technology $f(z)$ to produce the output, namely

$$q = f(z) = z^{\frac{1}{2}}.$$

The m upstream firms each produce the input z at the same linear total cost $C_j(s_j) = \beta s_j$, $j = 1, .., m$.

When the m upstream firms have selected a total amount of input $S = \Sigma_{h=1}^n b_h$ in the first stage game, the payoffs of the i_{th} downstream firm in the second stage game conditional on S, at the vector of bidding strategies (b_i, b_{-i}), writes as

$$\Pi_i(b_i, b_{-i}; S) = (1 - \left(\frac{b_i S}{b_i + B'}\right)^{\frac{1}{2}} - \Sigma_{k \neq i} \left(\frac{b_k S}{b_i + B'}\right)^{\frac{1}{2}}) \left(\frac{b_i S}{b_i + B'}\right)^{\frac{1}{2}} - b_i,$$

with $B' = \Sigma_{h \neq i} b_h$. Using first order conditions, the symmetric Nash equilibrium in the second stage game conditional on S obtains as

$$b_i^*(S) = b^*(S) = \frac{(n-1)(\sqrt{\frac{S}{n}} - S)}{2n}, i = 1, .., n.$$

In the first stage, the payoff Γ_j of the j_{th} upstream firm at the vector of strategies (s_j, s_{-j}) writes as

$$\Gamma_j(s_j, s_{-j}) = \frac{\Sigma_{k=1}^n b_k}{s_j + S'} s_j - \beta s_j.$$

with $S' = \Sigma_{h \neq j} s_h$ and $S = \Sigma_{h=1}^n b_h$. At the symmetric subgame perfect equilibrium, we know that

$$\Sigma_{k=1}^n b_k = nb^*(S) = \frac{(n-1)(\sqrt{\frac{S}{n}} - S)}{2}.$$

Plugging this expression in the payoff $\Gamma_j(s_j, s_{-j})$, we get

$$\Gamma_j(s_j, s_{-j}) = \frac{(n-1)(\sqrt{\frac{s_j + S'}{n}} - (S' + s_j)}{2(s_j + S')} s_j - \beta s_j.$$

From the first-order necessary and sufficient conditions and symmetry, we get that

$$s_j^* = s^* = \frac{(2m-1)^2(n-1)^2}{4nm^3(2\beta + n - 1)^2}, j = 1, .., m,$$

which constitutes the individual supply of input by each upstream firm at the subgame perfect equilibrium.

Substituting this value in $b^*(S^*)$, with $S^* = ms^*$, we get

$$b^*(S^*) = \frac{(n-1)^2 (2m-1)(4m\beta + n - 1)}{8n^2m^2 (2\beta + n - 1)^2},$$

which is the bidding strategy played by each downstream firm at the Nash equilibrium conditional on $S^* = ms^*$ in the second stage game. Acccordingly, the $(n+m)$-vector $(b^*(S^*), .., b^*(S^*); s^*, .., s^*)$ *constitutes, under decreasing returns, the symmetric subgame perfect equilibrium of the sequential game.*

In order to compare the double marginalization arising at the subgame perfect equilibrium described above, with the one arising in the traditional approach, we compute the output price in the downstream market resulting from the symmetric subgame equilibrium we have just identified. At this equilibrium, the level of production of each downstream firm is equal to $\left(\frac{b_i S}{b_i + B'}\right)^{\frac{1}{2}}$ with $b_i = b^*(S^*), B' = (n-1)b^*(S^*)$ and $S = S^* = ms^*$, namely, $\sqrt{\frac{(n-1)^2(2m-1)^2}{4n^2m^2(2\beta+n-1)^2}}$, or $\frac{(n-1)(2m-1)}{2nm(2\beta+n-1)}$. Substituting this value in the demand function $\pi(Q) = 1 - Q$, we get the output price π^* corresponding to the subgame perfect equilibrium, namely,

$$\pi^* = \frac{4m\beta + n - 1}{2m\,(2\beta + n - 1)}$$

A direct comparison between π^* and the output price obtained at equilibrium using the traditional approach, namely, $\frac{4m\beta+n-1}{2(2m-1)}$[2], shows that

Proposition 1 *Under decreasing returns, the double marginalization observed in the market mechanism model of successive oligopolies exceeds the one arising at the symmetric subgame perfect equilibrium in the market game model.*

Intuitively one would expect the opposite result, which is in fact the case when firms in the output market use constant returns (see below Proposition (4)). In the market games mechanism, downstream firms can strategically affect both input and output prices. This coordination of market power in two markets should lead to lower double margins with respect to the margins in the market mechanism model, where price decisions are made by agents who have market power in only one of the markets. This reasoning resembles to Spengler idea of how margins arise in the case of two independent monopolies as compared to the case when the decision is made by only one monopoly created through vertical integration.

Our intuitive expectations are based on linear technological transformation of input to output. The truth is that when this technological transformation is non linear, as in the case of the Cobb-Douglas we are considering, the market power of firms is shaped by the technological proportion between the input and output. With constant returns such proportion is equal to a constant and is neutral to the market power of firms. By contrast, downstream firms' position is weakened due to the fact that under decreasing returns each level of output requires a higher amount of input, as compared with the constant returns case. Moreover, with decreasing returns, such proportion is increasingly higher than one, as the quantity of output produced is increased.

A surprising outcome obtained at equilibrium in the traditional model is that, under decreasing returns, when both the number of upstream and downstream firms tend simultaneously and in the same proportion to infinity, the equilibrium input price *does not* converge to the upstream firms' marginal cost, but exceeds it by an amount which decreases with the ratio of the number of firms in each market ($\frac{n}{m}$). In the market game model, the counterpart of the input price is the ratio $\frac{\Sigma_{k=1}^{n} b_k}{\Sigma_{h=1}^{m} s_h}$ between the total money bids of downstream firms and the total input supply proposed by the upstream ones. At equilibrium this ratio is equal to $\frac{(4m\beta+n-1)}{2(2m-1)}$. Multiplying by k each value of n and m in this expression, we get

$$p^*(k) = \frac{4km\beta + kn - 1}{4km - 2},$$

[2] see Gabszewicz and Zanaj (2007) for these calculations

which does not tend to the input marginal cost β when k tends to infinity. Thus we may state the following

Proposition 2 *Under decreasing returns, and as in the market mechanism model, when both the number of upstream and downstream firms tend simultaneously to infinity, the equilibrium input price does not converge to upstream firms' marginal cost, but exceeds it by an amount which decreases with the ratio of the number of firms* $\frac{n}{m}$.

It follows then that the surprising property of the input price, which does not converge to its marginal cost, depends on the downstream technology rather then the market power in the input market. As long as the marginal productivity of the input tends to infinity, the subgame perfect equilibrium input price must exceed the marginal cost, regardless on who is deciding the input price.

Another observation revealed in Gabszewicz and Zanaj (2007) is that under decreasing returns, the profit of the downstream firms does not always increase with the number of upstream firms, in spite of the increase in competition resulting from entry in the upstream market. Substituting the values $b^*(S^*)$ and s^* in the payoff Π_i of each downstream firm at equilibrium, it is easily checked that, whatever the positive value of β, the derivative $\frac{\partial \Pi_i}{\partial m}$ is always positive: as intuitively expected, in this setup, *more competition in the upstream market here always entails an increase in profit for the downstream firms.* Indeed, here again two effects can be disentangled, a direct one on the input price and a indirect one on the output price. But with the market game mechanism, the direct effect always dominates because downstream firms do not take the input price as given any longer.

3.2 Constant returns

Assume now that downstream firms still face a linear demand $\pi(Q) = 1 - Q$ but now use a constant returns technology to produce the output:

$$f(z) = \alpha z,\ \alpha > 0.$$

The profits $\Pi_i(b_i, b_{-i}; S)$ of the i_{th} downstream firm at the vector of strategies (b_i, b_{-i}) and S now obtains as

$$\Pi_i(b_i, b_{-i}; S) = (1 - \alpha \frac{b_i}{b_i + B'} S - \alpha \sum_{k \neq i} \frac{b_k}{b_i + B'} S) \alpha \frac{b_i S}{b_i + B'} - b_i$$

with $B' = \sum_{h \neq i} b_h$.

Solving the maximization problem of a downstream firm and using symmetry, we get at equilibrium

$$b^*(S) = \frac{(1 - S\alpha)(n-1) S\alpha}{n^2}$$

Hence, the payoff Γ_j of an upstream firm at the first stage of the game, after substituting for b^*, obtains as

$$\Gamma_j(s_,s_{-j}) = \frac{\left(1-\left(s_j+S'\right)\alpha\right)\left(n-1\right)\left(s_j+S'\right)\alpha}{\left(s_j+S'\right)}s_j - \beta s_j.$$

Maximising $\Gamma_j(s_,s_{-j})$, yields at the symmetric equilibrium

$$s^* = \frac{\left(n\alpha-\alpha-n\beta\right)}{\left(n-1\right)\left(m+1\right)\alpha^2}.$$

Hence the optimal quantity of money $b^*(S^*)$ that each downstream firm bids obtains by substituting s^* in $b(s_j, s_{-j})$, namely,

$$b^*(S^*) = \frac{\left(\alpha-n\alpha-mn\beta\right)\left(\alpha-n\alpha+n\beta\right)m}{\alpha^2 n^2\left(n-1\right)\left(m+1\right)^2}.$$

As in the previous section, we may calculate the input price as the ratio $\frac{\Sigma_{k=1}^{n} b_k}{\Sigma_{h=1}^{m} s_h}$, and we get

$$p^* = \frac{n\left(\alpha+m\beta\right)-\alpha}{\alpha\left(m+1\right)n}.$$

Accordingly, the $(n+m)$-vector $(b^*(S^*), .., b^*(S^*); s^*, .., s^*)$ *constitutes, under constant returns, the symmetric subgame perfect equilibrium of the sequential game.*

In order for this vector to be an equilibrium, it is also required that the values $b^*(S^*)$ and s^* to be both positive. These two inequalities are both simultaneously satisfied if and only if the condition

$$\alpha \geq \frac{n}{\left(n-1\right)}\beta$$

holds. This condition coincides with the condition, which guarantees that both upstream and downstream firms make positive profits. Notice that this condition is slightly stronger than the condition required to be satisfied in the traditional model, which simply boils down to $\alpha \geq \beta$. The reason for this strengthening should be found in the indirect strategic power that downstream firms exert in the upstream market : they influence the amount of input sales via their money bids, and this influence fades away when the number n of downstream firms increases. In the traditional model, this influence does not exist since downstream firms take the input price as given when buying the input[3].

The two approaches - the market mechanism approach, or the market games approach, - mainly differ according to how downstream firms' total production costs are introduced in the model. In the market mechanism approach, these costs depend on the input price and the quantity of input invested in production. In the market game approach, total costs do not depend on the quantity

[3] A similar condition appears in another, but close, context (see Gabszewicz and Michel (1997)). These authors analyze the oligopoly equilibrium of a market game with exchange and production.

of input invested in the production, but reduces to a lump-sum amount corresponding to the bid offered by the downstream in exchange of its input share. Nothing prevents to deduce from it a notion of *average* cost (and marginal cost in the case of constant returns) simply by dividing the bid by the number of output units produced. Using this notion, it easy to show that, when the economy is replicated $k-$times, increasing thereby simultaneously the number of downstream and upstream firms, both the input and output prices converge to their competitive values, β and $\frac{\beta}{\alpha}$, respectively. We summarize the above in the following

Proposition 3 *Under constant returns and whatever the regime, market or market game, both the input and output prices tend to their competitive counterparts when the economy is replicated at the same speed in the upstream and the downstream markets.*

Indeed, when downstream firms use constant returns technology, even though at the limit the individual input demand tends to zero, the marginal productivity is constant. Therefore, the tendency of the input price to its marginal cost is reached, regardless of who fixes the input price when the number of firms is fixed.

Finally, it is interesting to compare the size of the double marginalization effect under the two regimes when downstream technology is constant returns. This can easily be done by directly comparing the equilibrium per downstream firm output production levels into the two regimes. We obtain the following

Proposition 4 *Under constant returns, the double marginalization observed in the market game regime exceeds the one arising at the symmetric subgame perfect equilibrium in the market regime.*

Notice the crucial role played by the technology linking the input and the output markets as explained above for Proposition (1): with constant returns, the double marginalization effect is larger under the market game regime than under the market regime, while the reverse holds under decreasing returns!

4 Conclusion

In this paper, we investigate entry and vertical integration in successive markets when the technology linking these markets is made explicit and the concept of market game is used to describe the economic outcome of the downstream and upstream firms' interaction. We have differentiated the effects of entry in these markets according to the nature of the technology: constant and decreasing returns, making explicit several properties which differ in each of these cases. Moreover, we have highlighted how double marginalization is influenced by the technology used to produce the output. This shows that the effects of vertical integration, which depend on double marginalization, will depend on the type of technology used by the downstream firms.

This exploration of industry equilibria departs from the existing literature because it does not start from the assumption of price taking agents in the demand side of the markets. In particular, it does not assume that downstream firms behave as price-takers in the upstream market, an awkward assumption because it is difficult to justify the fact that an economic agent behaves strategically in one market but not in the other. A reasonable treatment thus requires downstream firms behaving strategically simultaneously in the downstream and upstream markets. This is what we provide in this paper since the firms are strategic in both stages of the game, i.e, in the downstream and upstream markets.

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