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Optimal Foreign Investment Dynamics in the Presence of Technological Spillovers *

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Abstract

In this paper we present a dynamic model of a firm which decides whether to outsource parts of its production to a less developed economy where wages and the level of technology are lower. Outsourcing reduces production costs but is associated with spillovers to foreign potential competitors. Spillovers over time increase productivity of firms in the foreign country and make them stronger competitors on the common market. The paper analyzes the inter-temporally optimal behavior of the firm and shows that two outcomes are possible in the long-run. There is one steady state where the firm invests a positive amount in the foreign country and there is a continuum of steady states with no investment. The paper then derives conditions such that it is optimal for the firm to invest in the foreign country and different types of optimal dynamic investment patterns are characterized. In addition, using numerical dynamic optimization methods, the effect of the speed of technology adoption and of the wage differential on total labor income in the home country, is studied taking into account the transition dynamics.

JEL classification: F21, D92, C61

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1 Introduction

With the decline of communism in the nineties of the last century, Central and Eastern European economies have become more open and more attractive for foreign capital. At the same time, the high growth rates of Asian countries such as China or India have also attracted foreign investors more frequently. Although most of foreign direct investment (FDI) is still undertaken in industrialized countries such as the US, Great Britain, France, Italy and the Netherlands, just to mention a few, more and more direct investment flows to the newly industrializing countries (NICs) in Central and Eastern Europe and to Asian economies. According to the UNCTAD 2007 World Investment Report [26] the highest increase in FDI inflows from 2005 to 2006 among all regions occured in South-East Europe with a growth of 74%. One important motivation for firms to invest in NICs is to get access to these markets, however, recent empirical evidence suggests that cost considerations have gained importance relative to market entry motives for foreign investment and are now in many cases the main factor influencing firms location decisions (see Kinkel and Lay [16])[1].

From a political perspective FDI is highly appreciated in the countries where the investment is undertaken. An important aspect in this respect is the expectation that FDI does not only generate income for local firms and workers, but also raises the productivity of local producers in the industry where investment takes place and also in vertically related indistries. Hence many NICs hope that spill-overs will reduce the gap to the technoligcial frontier and that may help to make the whole economy benefit from new the investments. Several channels of spillovers have been discussed in the literature, most prominently the demonstration effect, labor turnover (both inducing horizontal spillovers) and vertical linkages (see e.g. Saggi [23]). Although the empirical evidence concerning the existence of positive horizontal spillovers from FDI is mixed (see e.g. Görg and Greenaway [8]), recent empirical studies find positive horizontal spillovers from FDI using firm level data from Hungary (Halpern and Murakozy [12]), Romania (Smarzynska and Spatareanu [25]) and from 17 emerging market economies (Gorodnichenko et al. [9]).

[1] See e.g. Dunning [3] for a discussion of different motives for FDI.

So, while the firm that undertakes FDI may gain in the short-run from lower costs in NICs, spill-over effects can raise productivity of (potential) competitors in foreign countries. In particular in industries where markets have become globalized such improvements of the competitiveness of foreign producers might severely affect future revenues of the firm. Therefore, a firm thinking about FDI faces a trade-off between short-run cost advantages and loosing its competitive edge on the (global) market in the long-run.

A dynamic view is needed to analyze the trade-off described above, and in this contribution we present a theoretical model of a firm that faces the corresponding inter-temporal optimization problem. In contrast to other contributions within that line of research (see our literature review below), we assume that firms in the home country and in the foreign country compete on the same market. Most existing theoretical work relies on the market opening motive for FDI and assumes a local market in the foreign country. In our framework the firm under consideration operates on an oligopolistic market and may move some of its production by investing in physical capital in a NIC. Such investment, however, goes along with positive spill-overs which raise the productivity of competitors of the firm in the NIC. The incentive to invest in the foreign country are cost reductions due to lower wages there.

While most contribution in the literature do not explicitly study the dynamics of the model, our goal with this paper is to pay special attention both to the long-run outcome of our model as well as to its transition dynamics. In particular, we are interested in the question how the transient and the long-run outcome is affected by economic factors like the initial productivity and wage gap between the regions or the spillover rate. A main finding of our study in this respect is that there are two classes of co-existing steady-states, one with and one without FDI, and that small changes in the economic factors mentioned above might induce that the long-run outcome jumps from one type to the other. The insight that there might be a non-continuous shift between an FDI and a non-FDI regime is a new finding in this area of research that is based on our dynamic approach. This insight has important implications for economic policy design. The best way to reach certain policy goals might be to influence the system such that the initial state moves into the basin of attraction of a steady state that is desired by the policy-maker. Although we do not explicitly consider the role of a policy maker in our model, we give some indication of such type of analysis by examining the impact of key parameters on current and accumulated labor income and on accumulated firm profits. In terms of transient investmend dynamics our dynamic approach allows us to address an issue raised by empirical observations about the dynamic patterns of foreign investment. Using data from German manufacturing firms Kinkel and Maloca [17] report that about 17% of the firms who have moved (parts of) their production abroad in 2000/2001 re-transfered it back to Germany within the follwing five years. Obviously such a pattern of investment followed by disinvestment might be due to incorrect expectations about production conditions in the foreign country or actual changes in these conditions, but, as we show in this study, under certain conditions it might also correspond to intertemporally optimal behavior in the presence of technological spillovers. In particular, we demonstrate that such non-monotone investment patterns are optimal if the initial technology gap between the two countries is sufficiently large.

In order to characterize the optimal investment paths and their dependency from parameters and initial conditions we combine an analytical and a numerical approach. Steady-state outcomes can be fully described by analytical means and we can also analytically provide some characterization of the basins of attraction of the co-existing steady states. To gain additional insights about the transient dynamics and the sensitivity with respect to key parameters of the firm's and workers income streams under optimal investment a numerical dynamic programming algorithm with grid adaptation in the state-space is used.

Considering the theoretical literature dealing with FDI one realizes that most contributions are static multi-stage models or focus on the steady sate of dynamic models (a survey of theoretical models is presented by Cheng [4] and Saggi [23]). Several papers study the entry mode decision between FDI and exporting in multi-stage oligopoly settings (see e.g. Petit and Sanna Randaccio [21], Glass and Saggi [7], Mattoo et al. [19]) or multi-sector models (e.g. Helpman et al. [15]). A contribution which presents a truly dynamic model analyzing FDI strategies under technology differences is the paper by Das [2]. She presents a model where multinational firms transfer technology to their subsidiaries in foreign countries and where firms in the host country benefit from the technology transfer. In this respect our model will be related to the one presented by Das [2]. However the motivation and the main issues of our paper are very different since neither cost reduction motives or interregional wage differences nor global competition effects are issues in [2]. The focus there is on the intertemporally optimal price and quantity path of the subsidiaries that are only active in the (local) foreign market. Also the question whether the multinational firm should invest in FDI is not in the scope of that paper.

Other dynamic models that deal with technology transfer as a result of FDI in an oligopolistic market are the papers by Wang and Blomström [27], Lin and Saggi [18] and Petit et al. [22]. The focus of these contributions is rather on the level of technology to be transferred (Wang and Blomström [27]), on the timing of FDI (Lin and Saggi [18]) and on the question of how the choice choice between export and foreign direct investment affects the incentive to innovate (Petit et al. [22]).

The rest of the paper is organized as follows. In the next section we present the theoretical model. In section 3 we analyze the long-run behavior of the investment policy of the firm and in section 4 we study transitional dynamics. Section 5 concludes and points to possible extensions of the model. Appendix A briefly describes an example of our model with linear demand and all proofs are given in Appendix B.

2 The model

We consider a dynamic two-country model, where country H ('home-country') is a developed industrialized country whereas country F ('foreign country') is a newly industrializing country. We consider a partial industry model where n firms from the two countries compete on an oligopolistic market. This common market can be seen as a joint market in a liberalized trade region like the EU or as the market in either of the two countries, where producers from abroad offer their goods through exports. Using such a specification we abstract from any 'market entry' motivations for FDI but concentrate on pure cost considerations. Among the competitors m firms are located in country F, and $n-m$ $(n>m)$ are based in country H. We consider a scenario where all these $n-m$ firms currently produce their entire output at home, but one of the firms in country H considers to invest in country F in order to build up production capacities there. Firms are ordered in a way such that firm 1 is the potential foreign investor, firms with label $i=2,\ldots,n-m$ are located in country H and do not invest abroad and firms with label $i=n-m+1,\ldots,n$ are located in country F. We denote by $Q_i(t)$ the output of firm i at time t.

Firms produce using labor as the only variable production input. Production capacities of a firm in a country are determined by its capital stock there. Output per input unit in the two countries is given by $A_H(t)$ and $A_F(t)$ with $A_H(t)>A_F(t)$. If a firm from country H produces in country F, productivity reads $A_{HF}(t)$ where $A_F(0)<A_{HF}(0)<A_H(0)$. Since our focus is on the effects of technological spill-overs generated by FDI on the evolution of the technology gap between the two countries, we abstract from technological change in the developed country and assume that A_H and A_{HF} are constant over time, whereas $A_F(t)$ may change over time due to spill-over effects. In both countries labor is supplied at wage rates w_H and w_F, where $w_H>>w_F$. We assume that wage rates are fixed and constant over time where a reason for such wage rigidities might be that industry wages in the two countries are determined by collective bargaining and associated institutional inflexibilities. The assumption of constant wages seems particularly strong for w_F since within our framework labor productivity in that country will rise over time (relative to that in country H). Our focus is however on the positive effect of spill-overs on the competitiveness of firms in the developing country and therefore we assume for simplicity that also the wage in country F is constant over time. Our analysis would not be qualitatively altered if we allowed for rising wages in country F with a growth rate below the one of A_F. The observation that wages grow much slower than productivity has been made in many newly industrializing countries.

Unit production costs read $c_i=w_i/A_i(t), i\in\{H,F\}$ for a firm producing in its home country and $c_{HF} = w_F/A_{HF}$ for a firm from country H producing in country F. We assume that firms in country H can reduce their unit production costs if they produce in the foreign country, i.e.

$$\frac{w_H}{A_H} > \frac{w_F}{A_{HF}}. \tag{1}$$

In order to produce abroad, firm 1 has to invest to build up production capacities in country F. We denote by $I(t) \in \mathcal{R}$ foreign investment of firm 1 and by $K_F(t)$ the capital stock of firm 1 in country F at time t. It should be noted that we also allow for negative investment, and due to the spillover-effects described below disinvestment might in principle be optimal for firm 1. The capital accumulation equation is given by

$$\dot{K}_F(t) = I(t) - \delta K_F(t), \tag{2}$$

where δ is the depreciation rate of capital. The maximal quantity the firm can produce in country F is given by $Q_1^F(t) = A_{HF}K_F(t)$. Gross investment costs of the firm are assumed to be given by $\beta I + \gamma I^2$, where β and γ are positive constants. This is a standard formulation in investment models with capital adjustment costs, where β denotes the price of capital (see e.g. Dockner et al. [5]).

Foreign direct investments of country H firms in country F generate technological spill-overs, which lead to a reduction of the productivity gap between the two countries. Because of these spill-overs, the investment of the home-country firm in the developing country will increase the technology level there, which raises the productivity of the firms in the developing country. Therefore, we posit that the change in technology level in the foreign country is given by,

$$\dot{A}_F(t) = \lambda K_F(t) \left(A_{HF} - A_F(t)\right). \tag{3}$$

The speed of absorption is determined by the absorption rate $\lambda > 0$ and by cumulated past foreign investment, $K_F(t)$. The higher the capital stock in the foreign country, and the larger the productivity gap, the faster is absorption for a given level of λ. From an empirical point of view there has been some debate whether the size of spillovers indeed increases with the size of the technological gap. Our assumption of a positive relationship between spillovers and the size of the gap is for example backed by empirical findings in Griffith et al. [10].

2.1 Oligopoly Production

Firms compete on an oligopolistic market for a homogenous good. The price at time t is determined by an inverse demand function $P(Q)$, where we assume that $P(.)$ is strictly decreasing with $\lim_{Q\to 0} P(Q) > w_H/A_H, \lim_{Q\to\infty} P(Q) = 0$, twice continuously differentiable on $\mathcal{R}^+$ and that $QP(Q)$ is strictly concave in Q. Furthermore, we assume that the market is sufficiently large such that firm 1 from country H always fully employs its capacity in country F, i.e. $Q_1(t) > Q_1^F(t)$. In addition, we posit that all firms have sufficient production capacity in their home country to produce the oligopoly output and that there are no constraints on labor supply. Given these assumptions marginal production costs of firms from country H are given by w_H/A_H regardless of their level of foreign investment. Producer from country F have constant marginal costs w_F/A_F.

In every period price and output quantities are given by equilibrium values for oligopoly markets with heterogenous production costs. Based on results by Murphy et al.[20] we can conclude that the Cournot equilibrium exists and is unique under our assumptions (see Lemma 3 in the Appendix). We denote by $Q_H^*(A_F)$ and $Q_F^*(A_F)$ the equilibrium quantities, where the dependence on A_F is stressed because A_F is the only determinant of the equilibrium quantities that is not fixed over time. The corresponding market price is denoted by $P^*(A_F)$ and the profits of firm 1 at time t are given by

$$\pi_1^*(K_F(t), A_F(t)) := Q_H^*(A_F(t)) \left(P^*(A_F(t)) - \frac{w_H}{A_H} \right) + K_F(t) A_{HF} \left(\frac{w_H}{A_H} - \frac{w_F}{A_{HF}} \right). \quad (4)$$

The second term gives the cost savings of firm 1 due to its production outsourcing to country F.

Depending on the ratio of marginal costs of the producers in the two countries, both types of producers may sell positive quantities in equilibrium or the whole market may be served by producers from a single country. We denote by $\tilde{P}$ the equilibrium price in a Cournot duopoly with (n-m) competitors that all have constant marginal costs c_H. In order to be able to sell positive quantities firms in country F must have marginal costs below this level. Put differently,

$Q^*_F(A_F) > 0$ if and only if

$$A_F > \underline{A}_F := \frac{w_F}{\tilde{P}}.$$

Due to our assumptions about the inverse demand function we have $\underline{A}_F < A_{HF}$. We do not consider scenarios where producers from country H are driven out of the market, i.e. we assume that A_{HF} is chosen such that for $c_L = \frac{w_L}{A_{HF}}$ the equilibrium quantity for producers from country H is still positive. Straightforward calculations then yield the following characterization of the dependency of the profits of firm 1 from the two state variables.

Lemma 1

(i) The profit function function π^*_1 is continuously differentiable with respect to A_F and K_F on $(0, \underline{A}_F) \cup (\underline{A}_F, A_{HF}] \times [0, \infty)$.

(ii) For $A_{HF} \geq A_F > \underline{A}_F$ the profit function π^*_1 is strictly decreasing with respect to A_F, for $A_F \leq \underline{A}_F$ it is constant in A_F.

(iii) The profit function π^*_1 is strictly increasing with respect to K_F for all $K_F \geq 0$.

2.2 The optimal investment paths

We assume that all firms maximize their infinite horizon discounted profit streams. However, all firms except the potential foreign investor, firm 1, make only quantity decisions which do not directly influence any future payoffs. Therefore, as long as current quantity choices do not influence future actions of the competitors these firms face no inter-temporal decision problem. Here we only consider equilibria where current actions are independent from past quantity choices, which implies that quantities are chosen according to the standard (static) Cournot equilibrium. The objective of firm 1 is to choose an investment strategy $I(t)$ that maximizes

$$J_1 = \int_0^\infty e^{-\rho t} \left[\pi^*_1(K_F(t), A_F(t)) - \beta I(t) - \gamma I(t)^2\right] dt \tag{5}$$

subject to (3), the capital accumulation equation (2), the state constraint

$$K(t) \geq 0 \quad \forall t \geq 0$$

and the initial conditions $A_F(0) = A_F^{ini} < A_{HF}$ and $K_F(0) = 0$. The parameter $\rho > 0$ indicates the time preference of the firm. Obviously, the relevant range of capital stocks is bounded from above by the stock $\bar{K}$ corresponding to the monopoly output on the market and therefore the state-space of the problem is compact with $(K_F, A_F) \in [0, \bar{K}] \times [\epsilon, A_{HF}]$ with some small $\epsilon > 0$.

In what follows we will characterize the inter-temporally optimal investment strategy for firm 1. Applying Pontryagin's maximum principle a canonical system of ordinary differential equations can be derived that has to be satisfied by the optimal trajectories[2]. Since the Hamiltonian of the dynamic optimization problem of firm 1 is not concave with respect to the state the Maximum principle provides only necessary but no sufficient optimality conditions (see e.g. Sethi and Thompson [24]).

Lemma 2 *For any state trajectory* $(K_F(t), A_F(t))$ *that corresponds to an optimal investment strategy of firm 1 there exist piecewise absolutely continuous co-states* $\mu_1(t), \mu_2(t)$ *and a multiplier* $\nu(t) \geq 0$ *such that*

$$I(t) = \frac{\mu_1 - \beta}{2\gamma},$$

in addition to (3) and (2) the costate equations

$$\dot{\mu_1} = (\delta + \rho)\mu_1 - \mu_2\lambda(A_{HF} - A_F) - A_{HF}\left(\frac{w_H}{A_H} - \frac{w_F}{A_{HF}}\right) - \nu \quad (6)$$

$$\dot{\mu_2} = (\lambda K_F + \rho)\mu_2 - Q^{*\prime}(A_F)P'(Q^*)Q_H^*(A_F)\left(2 + \frac{P''(Q^*(A_F))}{P'(Q^*(A_F)}Q_H^*(A_F)\right) \quad (7)$$

as well as

$$\nu K = 0$$

hold and the transversality conditions $\lim_{t\to\infty} e^{-\rho t}\mu_1 K_F = 0$, $\lim_{t\to\infty} e^{-\rho t}\mu_2 A_F = 0$ *are satisfied. In addition, at any time* τ *where the trajectory hits the boundary* $K = 0$ *the co-state* μ_1 *might have a jump with* $\lim_{t\to\tau-}\mu_1(t) \leq \lim_{t\to\tau+}\mu_1(t)$.

[2] In what follows we omit the time index t.

In the following section we characterize the potential steady states of the system under optimal investment, and analyze how the initial technology gap $(A_{HF} - A_F)$ determines which steady state is reached.

3 Steady state analysis

All steady states for the considered model have to be rest points of the differential equations (2), (3), (6) and (7). There exist two different types of steady states, the *catch-up steady state*, where firm 1 invests abroad and where eventually the productivity of firms based in country F reaches the productivity level of the foreign investor, and the *no-investment steady states* where firm 1 has zero capacity in the foreign country and the productivity of firms in country F does not catch up to the level A_{HF}. A necessary condition for the existence of a catch-up steady-state is that the price of capital is not too large. In particular we assume in what follows that

$$\beta < \frac{A_{HF}}{\rho + \delta} \left(\frac{w_H}{A_H} - \frac{w_F}{A_{HF}} \right). \tag{8}$$

The next two propositions provide precise characterizations of the two types of steady states[3].

Proposition 1 *Given assumptions* (1) *and* (8)*, there exists a saddle point stable catch-up steady state with* $\hat{A}_F = A_{HF}$ *and*

$$\hat{K}_F = \frac{A_{HF}}{2\delta\gamma(\rho + \delta)} \left(\frac{w_H}{A_H} - \frac{w_F}{A_{HF}} \right) - \frac{\beta}{2\gamma\delta} > 0.$$

It should be noted that in the steady state, $\hat{\mu}_1 > 0$ and $\hat{\mu}_2 < 0$ holds. This is intuitive since $\hat{\mu}_1$ and $\hat{\mu}_2$ are the shadow prices of the capital stock in the developing country and of technology in the foreign country, respectively, where the first has a positive effect while the second has a negative effect on the profits of the home country firm.

Proposition 1 states that as long as unit costs for firm 1 in country F are below the unit costs at home there are initial values for the productivity gap $(A_{HF} - A_F)$ and for the foreign capital

[3] In the following of this paper, $\hat{x}$ denotes the steady state value of variable x.

stock such that it is optimal for firm 1 to have positive production capacities in country F. However, this does not necessarily imply that it is optimal for firm 1 to start foreign investment if currently there is none. It turns out that the absorption rate plays a crucial role in that respect. Define

$$\bar{\lambda} = -\frac{\rho(\rho+\delta)}{A_{HF} - \underline{A}_F} \left(\frac{A_{HF}}{\rho+\delta} \left(\frac{w_H}{A_H} - \frac{w_F}{A_{HF}} \right) - \beta \right) / \frac{\partial \pi_1^*(0, \underline{A}_F)}{\partial A_F}$$

as the threshold for the spillover intensity. The next proposition shows that as long as the intensity is above this level it depends on the productivity gap $A_{HF} - A_F$ whether it is optimal for firm 1 to start investing in country F.

Proposition 2 *For $\lambda > \bar{\lambda}$ there exists a value $\bar{A}_F \in (\underline{A}_F, A_{HF})$ such that*

(i) $I(0, A_F) > 0$ for all $A_F \notin [\underline{A}_F, \bar{A}_F]$.

(ii) For all initial values $A_F \in (\bar{A}_F, A_{HF}]$ the catch-up steady-state is reached in the long run.

(iii) There exists a subset $\mathcal{A}$ of the interval $[\underline{A}_F, \bar{A}_F]$ with positive measure such that for each $\widehat{A}_F \in \mathcal{A}$ there is a steady state where $\widehat{I} = \widehat{K} = 0$. For $\widehat{A}_F \in int(\mathcal{A})$ the steady state is neutrally stable.

The optimal investment depends non-monotonously on the initial technology gap. If the gap is very large or very small it is optimal to invest abroad, however in an intermediate range there exists a continuum of steady states for $K = 0$ where no investments take place. Whereas no single steady state in this continuum is asymptotically stable, the connected set of equilibria has a positive basin of attraction. If in a steady state with $\hat{A}_F \in int(\mathcal{A})$ a small perturbation to a positive capital stock occurs, this does not induce the firm to start investing in the foreign country and the foreign capital stock will return to zero. During this process small technological spill-overs occur leading to a small increase in A_F. Accordingly, the system will end up again at a steady state in the continuum of no-investment steady-states. However the brief perturbation has lead to a slight increase in the value of A_F. If the initial technology gap is so large that potential competitors in the foreign country are not able to gain positive market shares in equilibrium (i.e. $A_F(0) < \underline{A}_F$), then firm 1 can realize cost savings without jeopardizing future price levels and market shares. The firm can choose an investment pattern such that a positive capital stock is kept in the foreign country as long as the gap is sufficiently large, but all capital is retrieved once the level of technology in the foreign country approaches the threshold $\underline{A}_F$ where market entry of foreign firms would become feasible. Hence, trajectories with positive investment followed by disinvestment may be optimal and positive initial investment does not always imply that the catch-up steady state is reached in the long run. An implication like this is however true if the initial technology gap is small ($A_F(0) > \bar{A}_F$). In such a case capital stock is not only built-up but also kept in the foreign country in the long run

These considerations are illustrated in figure 1 where we show state trajectories under optimal investment for different initial states[4]. The two different basins of attraction can be clearly distinguished and the boundary between the basins is indicated by the bold dashed line. It should be noted that for initial conditions that lie directly on the boundary of the basins the firm has two investment paths that yield identical discounted payoffs, where one of the two implies convergence to the catch-up steady state whereas on the other path the foreign capital stock is reduced to zero[5]. We can see from this figure that the basin of attraction of the no-investment steady-states includes the states where the initial productivity gap is large and positive initial investment is optimal. The figure illustrates a situation where the set of no-investment steady-states is just a single interval on the $K = 0$ line. In order to guarantee that there are unique switching points from positive investment to no-investment and back to positive investment additional regularity assumptions about the market demand have to be made.

Proposition 3 *If* $\frac{\partial^2 \pi_1^*(0, A_F)}{\partial A_F^2} \geq 0$ *and* $\lambda > \bar{\lambda}$ *then* $\mathcal{A} = [\underline{A}_F, \bar{A}_F]$. *This condition is in particular satisfied for a linear demand function.*

[4] This illustration is based on the parameter setting we will use in our numerical analysis in the next section.

[5] Recently some literature has emerged studying the properties of the boundary between different basins of attraction in high-dimensional optimization problems, so called DNS curves (see e.g. Haunschmied et al. [14]).

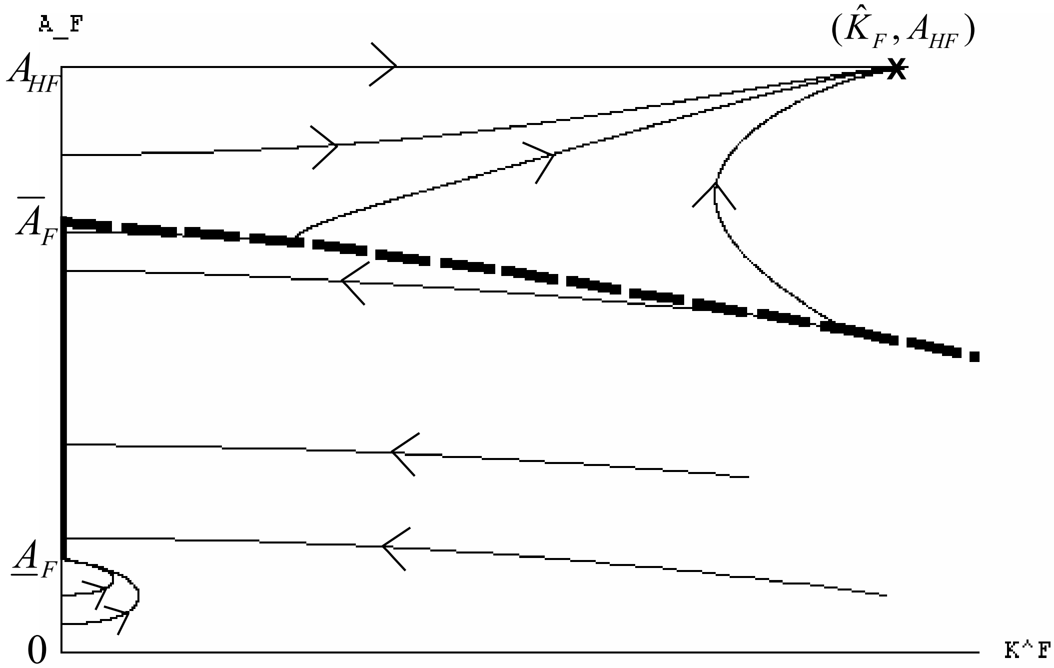

Figure 1: Dynamics of the productivity and of the foreign capital stock in country F under the optimal investment strategy. The catch-up steady state is indicated by an 'X', the continuum of no-investment steady states by the bold line at $K_F = 0$.

Before we discuss the transient dynamics of the optimal investment path let us briefly examine the impact of some crucial model parameters on the equilibrium constellation.

A decrease of the absorption rate λ does not affect the foreign capital stock in the catch-up steady state. However, it decreases $\bar{A}_F$ thereby increasing the range of initial productivity gaps which lead to convergence to that steady state. The threshold $\bar{A}_F$ collides from above with $\underline{A}_F$ as λ becomes small. The slower the technology transfer from firm 1 to firms in country F is, the larger becomes the initial productivity gap that prevents firm 1 from foreign investment. Recalling that investment in country F increases the level of technology of the firms there and, thus, the competitiveness of these firms, this result is very plausible. If this process is slow, the negative inter-temporal effect of investing in the foreign country is small, making investment profitable for firm 1.

Turning to the impact of the wages in the home country H on the steady-state constellation it becomes clear from the expression for $\hat{K}_F$ that the capital stock firm 1 holds in country F in the catch-up steady-state decreases if w_H is lowered. Lower wages at home lead to less transfer of production abroad. Furthermore, a decrease of w_H has several effects on investment incentives. First, the cost savings that are realized when production is moved abroad go down, which decreases investment incentives. Second, marginal costs of production at home are decreased which changes the equilibrium price and therefore also alters the marginal effect of an increase of A_F on firm profits. Investment increases the speed of A_F growth and therefore this effect also influences investment incentives. In general it is not clear how these effects compare to each other, but for special cases, like a linear demand function, it can be shown that the direct cost-saving effect dominates. Also, a decrease in w_H makes it harder for foreign firms to sell positive quantities on the common market and the range of A_F values where investment is positive because no foreign firm is in the market becomes larger.

We summarize these considerations in the following corollary.

Corollary 1

(i) If the absorption rate λ is decreased the threshold $\bar{A}_F$ becomes smaller whereas $\underline{A}_F$ stays constant. For sufficiently small λ we have $\bar{A}_F < \underline{A}_F$. Then optimal investment is always positive for $K = 0$ and for all initial values of $A_F \in [0, A_{HF}]$ the catch-up steady-state is reached in the long run.

(ii) For a linear demand function and $\lambda > \bar{\lambda}$, both thresholds $\bar{A}_F$ and $\underline{A}_F$ go up if w_H is decreased.

An implication of the second part of this corollary is that if $A_F(0)$ is only slightly above the threshold $\bar{A}_F$ a small decrease in domestic wages might reduce the long run foreign capital stock of firm 1 from $\hat{K}_F$ to zero. In the following section we will demonstrate with a numerical example that this second effect of decreasing w_H might imply that a decrease in domestic wages might lead to a substantial increase of the total labor income earned by workers in the considered industry in country H.

4 Transition dynamics, labor-income and the role of domestic wages

Having characterized the steady states of the trajectories under optimal investment and their basins of attraction, we will now consider properties of the transient dynamics before the steady states are reached and evaluate the corresponding evolution of the profits of producers and the factor income of labor in country H. To that end we rely on a numerical analysis[6] of a version of the model with linear demand The inverse demand function is given by

$$P(t) = \bar{P} - \sum_{i=1}^{n} Q_i(t), \tag{9}$$

where $\bar{P}$ is a positive constant.

The default parameter values used in the numerical calculations are given in table 1. The ratio of wages in the two countries is four to one and the monetary unit is normalized in a way that the wage in country F is $w_F = 1$. Unit costs of production for firm 1 in country F ($w_F/A_{HF} = 0.5$) are well below the unit costs at home ($w_H/A_H = 1$) and, therefore, assumption 1 is satisfied.

In figure 2 we show trajectories of the foreign capital stock (a), foreign productivity (b), investment (c), profits (d), production quantities (e) of firm 1 and the price of the good (f) for initial values $K_F(0) = 0, A_F(0) = 1.7$. Since $\bar{A}_F < 1.7$ for this parameter setting, these initial values are in the basin of attraction of the catch-up steady state. Accordingly foreign

[6] The numerical analysis was carried out using a computer code provided by Lars Grüne, which uses a numerical dynamic programming algorithm with flexible grid size in the state-space. See Grüne and Semmler [11] for details.

$n = 4$	$m = 3$
$A_H = 4$	$A_{HF} = 2$
$w_H = 4$	$w_F = 1$
$\delta = 0.06$	$\lambda = 0.4$
$\bar{P} = 5$	$\rho = 0.03$
$\beta = 0$	$\gamma = 500$

Table 1: Standard parameter setting

capital is increasing over time and foreign productivity converges towards the productivity level of the technology transferred by firm 1[7]. It can be seen that also foreign investment and the profit of firm 1 are monotonously increasing. It should be noted however that for different parameter settings (in particular for larger values of the absorption rate λ) non-monotonous profit trajectories can arise with an initial dip during the time-interval where the increase in foreign productivity triggered by foreign investment is particularly fast. As concerns production, we observe that the total quantity produced by firm 1 is slightly decreasing over time, whereas the quantity produced abroad grows significantly. The slight decrease in total production of firm 1 is due to a decrease over time of the price of the produced good. The downward trend of the price is a result of the increasing productivity of firms in country F, which induces a reduction of their unit costs.

Whereas figure 2 illustrates the case where the foreign investment is optimal for the firm because of the small size of the technological gap, in figure 3 we show the state dynamics under optimal investment if the initial technological gap is so large that the foreign competitors cannot offer positive quantities on the market. In that case the foreign investment is a short-run phenomenon and production in country F is terminated once the technological gap has been reduced to a level where foreign competitors could enter the market. For our parameter setting the threshold $\underline{A}_F = 0.33$ and it can be seen in figure 3(b) that this is exactly the long run level of A_F under optimal investment of firm 1. As briefly discussed in the introduction a foreign

[7] The dashed lines in figures 2(b) and 3(b) indicate the productivity level of firm 1 in country F.

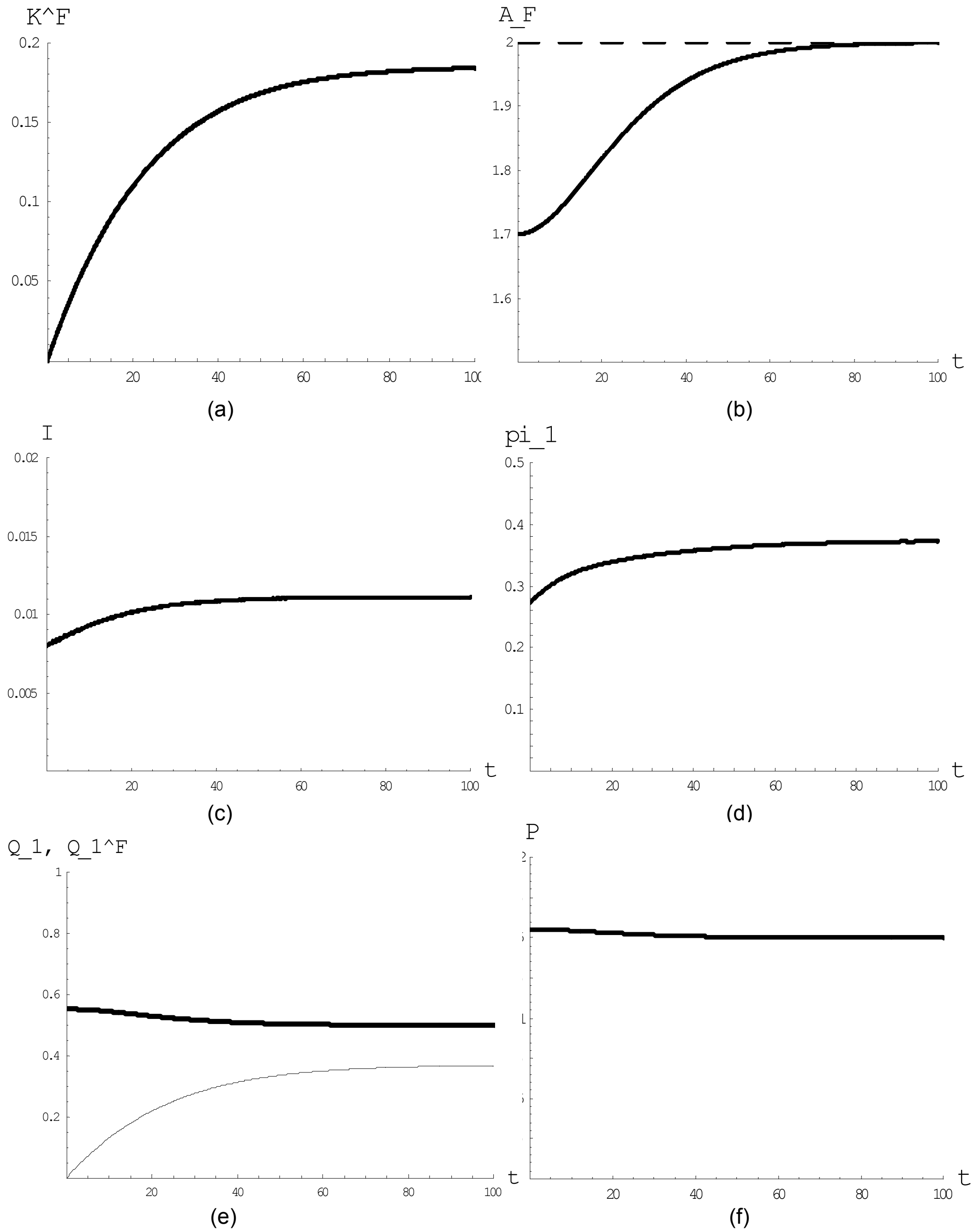

Figure 2: Trajectories for the standard parameter setting and initial values $K_F(0) = 0, A_F(0) = 1.7$: (a) foreign capital stock; (b) foreign productivity; (c) foreign direct investment; (d) profits of firm 1; (e) output quantity (bold line) and quantity produced in country F (slim line) by firm 1; (f) price of the good.

investment pattern of investment followed by disinvestment can be observed by a substantial percentage of firms that outsource parts of their production. According to our analysis such behavior is not necessarilly induced by myopic planning or incorrect expectations, but might be due to perfectly rational intertemporal considerations of the investor. It exploits the (short-run) potential for production cost reduction without generating additional competition in the (global) market.

In the remainder of this section we will focus on the impact of changes in the initial productivity gap $A_{HF} - A_F(0)$ and in the wage level in country H on the dynamics generated by optimal investment behavior of firm 1. The focus on domestic wages is motivated by a lively debate in several European countries about the effects of relatively high domestic wages on the transfer of production abroad and on domestic labor income. In many industries in these countries wages are determined by collective bargaining in a rigid institutional setting and - as assumed in our model - have been very inflexible in previous years.

To discuss the impact of changes in these parameters we consider not only firm profits in country H but also the income earned by workers in the considered industry in that country. Looking at these two variables allows to get some indication of welfare effects in country H in the framework of our partial equilibrium model. It should however be kept in mind that these considerations do not take into account the disutility of labor for workers and also the effects of price changes on consumer surplus in country H. Since we do not explicitly model the labor market and domestic demand in country H, such effects are not captured in our model.

The (nominal) labor income of workers in country H is given by

$$LI(t) = \frac{w_H Q_H^*(A_F(t)) - K_F(t) A_{HF}}{A_H}.$$

Assuming that workers discount future income with the same rate firms discount future profits, the discounted accumulated labor income under the optimal foreign investment strategy reads

$$ALI(A_F^{ini}; w_H) = \int_0^\infty e^{-\rho t} \frac{w_H Q_H^*(A_F^*(t)) - K^{F*}(t) A_{HF}}{A_H} dt,$$

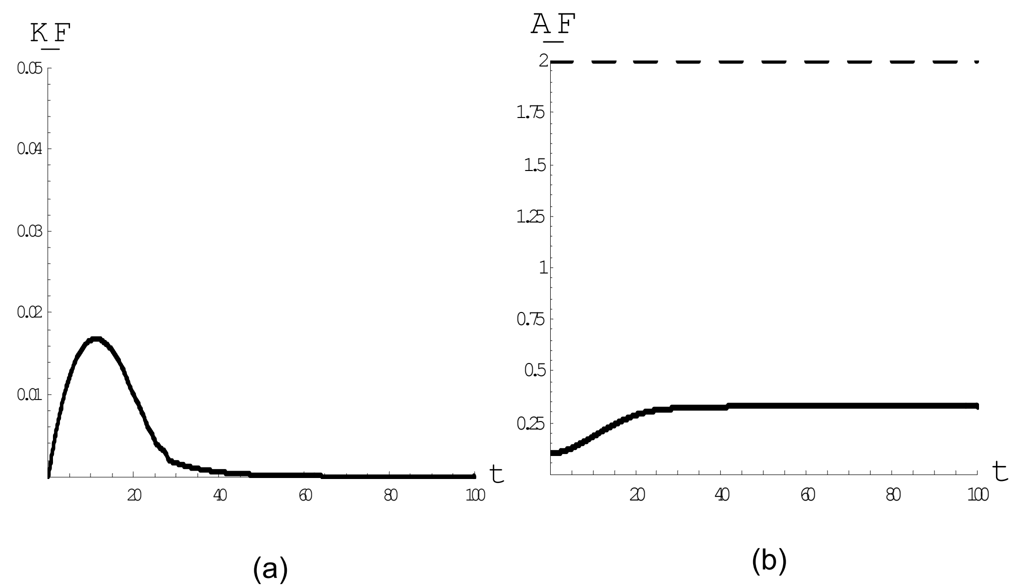

Figure 3: Trajectories for the standard parameter setting and initial values $K_F(0) = 0, A_F(0) = 0.1$: (a) foreign capital stock; (b) foreign productivity.

where (K^{F*}, A_F^*) are the state trajectories under the optimal investment path for initial conditions $K_F(0) = 0, A_F(0) = A_F^{ini}$.

In figure 4(a) the trajectory of labor income for the standard parameter setting underlying figure 2 is depicted. Labor income decreases over time. This is due to the fact that firm 1 reduces the quantity produced at home thereby reducing the amount of domestic labor that is employed. This effect is stronger than the negative effect of production outsourcing on prices and, therefore, labor income in country H decreases. The dotted line in figure 4(a) indicates the evolution of labor income if the domestic wage is reduced from $w_H = 4$ to $w_H = 3.5$. A wage decrease has a negative direct effect on labor income (assuming fixed output) but two positive indirect effects, a static and a dynamic one. First, wage reduction reduces marginal production costs and hence increases the quantity supplied by firm 1 in equilibrium. Second, as discussed in the previous section, a decrease in w_H reduces the foreign investment of firm 1 and therefore increases the quantity produced in country H during the transient phase and in the long-run steady state. It can be seen from figure 4 a that initially the static negative and positive effects more or less cancel and labor income stays almost the same. However, after a relatively short time interval the dynamic positive indirect effect becomes dominant and labor income at the steady state is larger for $w_H = 3.5$ than for $w_H = 4$. A decrease in domestic wage increases the discounted accumulated labor income: $ALI(1.7; 3.5) = 14.95 > ALI(1.7; 4) = 10.13$. It is quite obvious that a decrease in domestic wages also increases the discounted profit of the domestic firm: $J_1 = 14.36$ for $w_1 = 3.5$ versus $J_1 = 11.35$ for $w_1 = 4$.

The positive effect of a wage decrease for $A_F^{ini} = 1.7$ is based on the impact of w_H on the steady state level of K_F. In the previous section we have pointed out that a wage decrease also has a second important effect, namely the increase of $\bar{A}_F$. If due to this increase the threshold $\bar{A}_F$ passes the initial foreign technology level A_F^{ini}, the effects of a wage decrease can be much more substantial than observed above. The initial conditions $K_F = 0, A_F = A_F^{ini}$ suddenly become an equilibrium and it is optimal for firm 1 not to engage in any direct foreign investment. There is a discontinuous change in the optimal investment strategy, which becomes constant at zero.

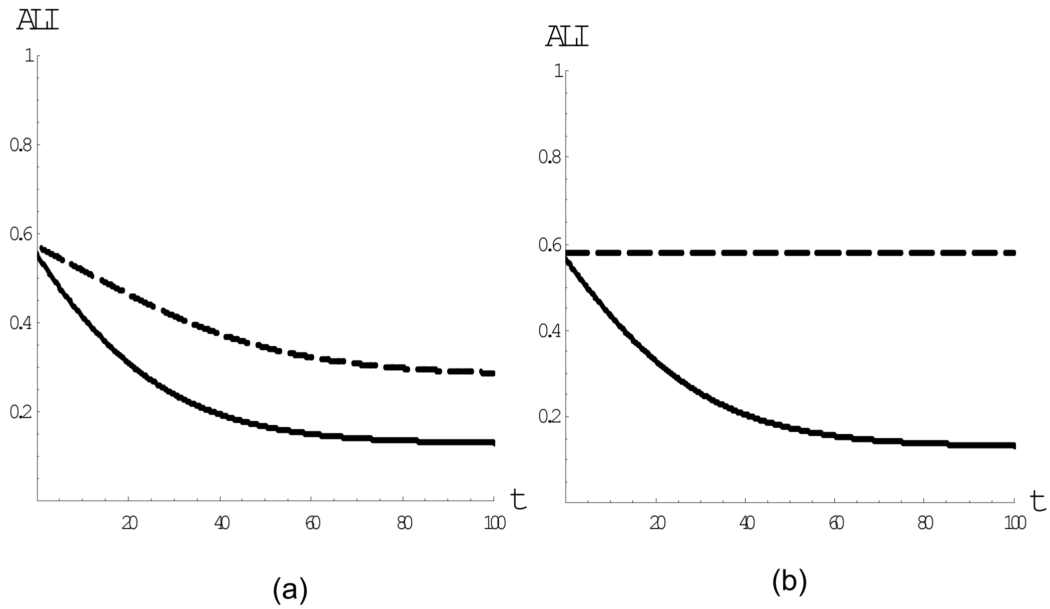

Figure 4: Labor income in country H for $w_H = 4$ (solid line) and $w_H = 3.5$ (dashed line): (a) $A_F^{ini} = 1.7$; (b) $A_F^{ini} = 1.65$.

Output quantities and prices stay constant over time and so does the labor income of domestic workers in that industry. We illustrate such a scenario in figure 4(b), where the evolution of labor income under $w_H = 4$ and $w_H = 3.5$ is depicted for a case where the initial technology gap is larger than above ($A_F^{ini} = 1.65$). The gains of a wage decrease are much more substantial than in the case shown in panel (a), which is also reflected in the larger gap of discounted accumulated labor income: $ALI(1.65; 3.5) = 19.356 > ALI(1.65; 4) = 10.54$.

The outcomes obtained so far suggest that in scenarios where firm 1 invests positive amounts in the foreign country, decreasing wages induce increasing accumulated labor income with a discontinuous jump upwards at the wage rate where the firm stops to invest abroad. Without foreign direct investment the only remaining effects of a wage decrease on labor income is the negative direct and the static positive indirect effect. As can be seen in figure 5b, the direct effect is always dominant in the range $\bar{A}_F > A_F^{ini}$. In that range, a wage increase induces an increase

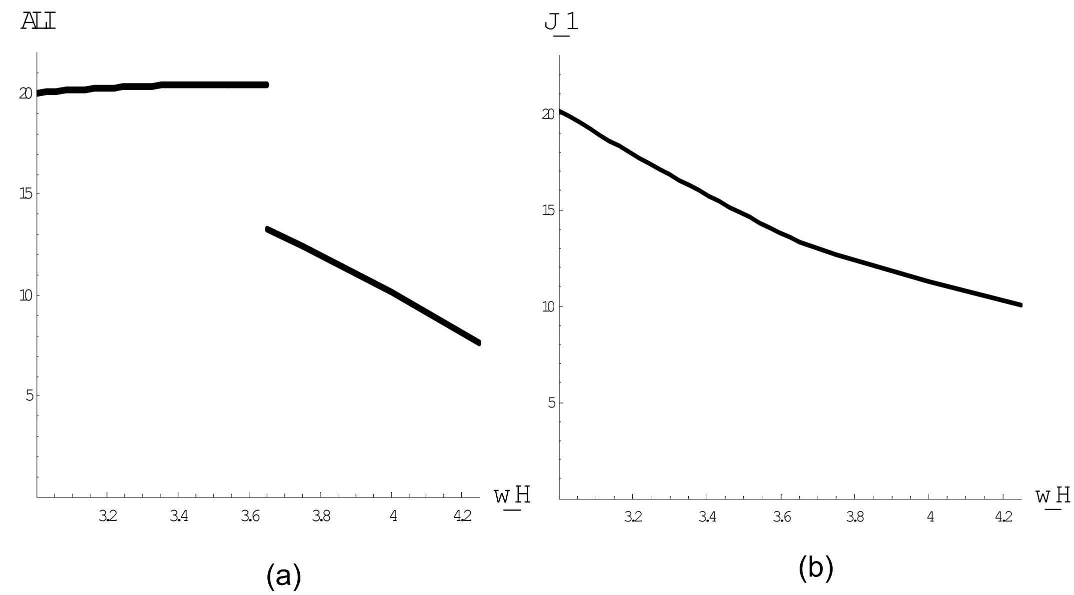

Figure 5: Accumulated firm profits (a) and accumulated labor income (b) in country H as a function of domestic wages for $A_F^{ini} = 1.7$.

in labor income. At the wage level where $\bar{A}_F = A_F^{ini}$ there is a downward jump in accumulated labor income and in the range of wages with positive foreign investment ($\bar{A}_F < A_F^{ini}$) there is a qualitative change in behavior. In that range accumulated labor income decreases with increasing domestic wages. Accordingly, accumulated labor income is maximized at the highest wage level where the firm does not invest abroad, i.e. at a wage level where $\bar{A}_F = A_F^{ini}$. Note that this wage level decreases if A_F^{ini} goes up and accordingly the initial technological gap goes down. Accumulated profits of firm 1 are a decreasing function of domestic wages with a slight kink at the wage level where the transition from a no-investment steady-state to a catch-up scenario occurs.

The discontinuity of the optimal investment strategy and the labor income which we discussed with respect to the parameter w_H of course also occurs with respect to changes of the other model parameters, in particular with respect to the initial technology level in the foreign country or the technology absorption rate. In figure 6 we show accumulated firm profits and accumulated labor income as a function of A_F^{ini} for a domestic wage of $w_H = 4$. Workers and the domestic firm gain from an increase in the initial technological gap. The jump of accumulated labor income at $\bar{A}_F$ can be clearly seen. Note also that for values of $A_F < \underline{A}_F$ labor income slightly increases as the technological gap gets smaller. This is due to the fact that as long as the foreign firms are not in the market the production quantity of firm 1 is unaffected by A_F but the foreign investment of the firm becomes smaller the closer the foreign firms get to the point where they can compete on the market.

5 Conclusion

In this paper, we have presented a dynamic model of a firm which competes with local and foreign firms in an oligopolistic market. The firm has an incentive to invest and produce in a developing country because of lower unit costs there, but raises the technology level of its potential competitors in the developing economy by investing, due to spill-overs of investment.

Our analysis shows that there are two possible types of long run outcomes, a catch-up scenario, where the domestic firm invests abroad and the productivity of the foreign competitors in the long run matches the productivity of the domestic subsidiary in country F, and a no-innovation scenario where the domestic firm decides against FDI. In the absence of an initial capital stock in the foreign country positive investment is optimal if the technological gap is either very large or very small. Furthermore, we have shown that an implication of the co-existence of

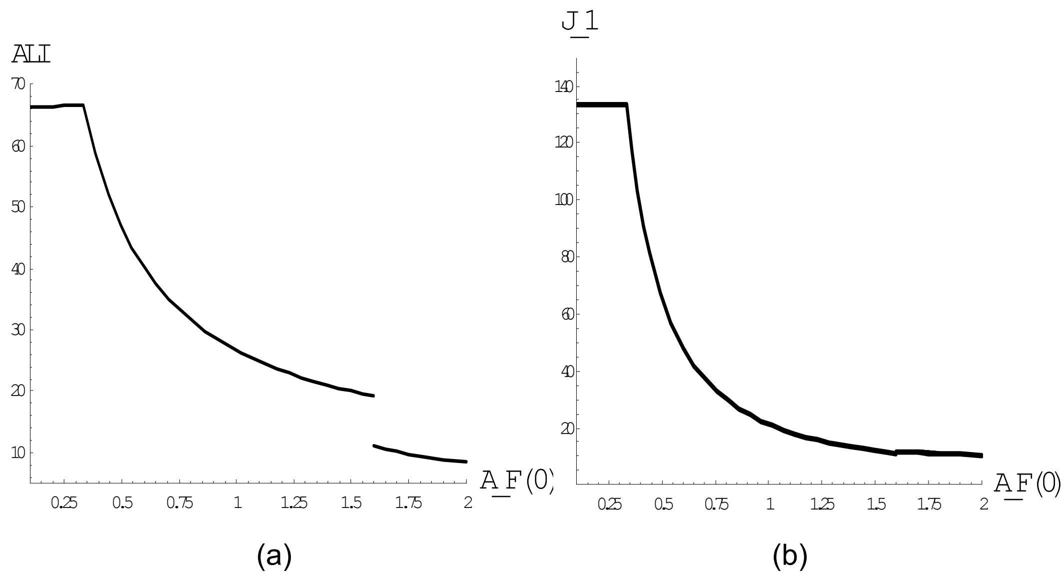

Figure 6: Accumulated firm profits (a) and accumulated labor income (b) in country H as a function of initial foreign productivity for $w_H = 4$.

these two scenarios is a non-continuous dependence of key variables like long-run foreign capital, long-run prices or accumulated domestic labor income on parameters of the model. We have focused on the impact of changes in the initial productivity gap or in the domestic wage level and have shown that a slight decrease in the domestic wage level or a slight increase of the initial productivity gap might have substantial positive effects on the domestic labor income by inducing the transition from a catch-up scenario to a no-investment scenario. Furthermore, depending on the initial level of the productivity gap a marginal decrease of the wage level, which does not lead to a transition between scenarios, might have negative (in the no innovation scenario) or positive (in the catch-up scenario) effects on domestic labor income. If we assume that even without FDI the productivity gap will slowly close over time, these findings suggest that the qualitative effect of changes in the wage level might change radically over time. Clearly, this has implications for wage policy if domestic labor income is of concern. Finally, we have shown under which circumstances patterns of positive foreign investment followed by disinvestment correspond are in accordance with intertemporally optimal behavior of the investing firm.

The fact that we consider a scenario where only one of the firms in country H has the opportunity to invest abroad might be seen as a severe restriction of our analysis. However, this is not the case. A straight-forward extension of our model where all m firms in coutry H can invest leads to a differential game model and it is easy to show that no matter whether symmetric open-loop or stationary Markovian Nash equilibria are considered[8] there is again a unique catch-up steady state where each investor holds exactly the steady-state foreign capital stock calculated in our single investor setup. Furthermore, for all values of the technology gap where a single potential investor should abstain from FDI there is an (open-loop or stationary Markovian Nash) equilibrium where all potential investors abstain from investing[9]. The intuition for this result is straight-forward. In a scenario with several potential investors deviation of a single firm from the zero investment strategy might induce positive investments of the other potential investors which has a negative impact on the deviators profit. Accordingly, incentives to start positive foreign investments in a setup with multiple potential investors are smaller that in the scenario considered in this paper, where no other firm can invest abroad. Therefore, the main qualitative findings derived for the single investor case carry over to a scenario with multiple potential investors.

As to future research, several extensions of the model should be considered. In the current analysis the level of the technology associated with the direct foreign investment was considered as given. It would be interesting to study optimal investment behavior allowing for an endogenous determination and dynamic changes of the technological level the firm uses in the foreign

[8]See Dockner et al. [5] for an introduction to differential games and to these equilibrium concepts.

[9]The exact formulation of the differential game extension of our model and the proofs for these two claims are available from the autors upon request.

country. Furthermore, the choice of R&D investments of the firms in the foreign country might be incorporated into the analysis. There is empirical evidence and much discussion in the literature about the fact that a firm's ability to absorb technological spillovers depends positively on its R&D activities (see e.g. Cohen and Levinthal [1]). Taking into account such effects would make the spillover intensity λ an endogenous variable and generate strategic effects between the competitors in the different countries.

Appendix A: Example with linear demand

In case demand is of the linear form (9) standard calculations show that equilibrium quantities are given by

$$Q_H^*(A_F) = \begin{cases} \frac{\bar{P}-(m+1)\frac{w_H}{A_H}+m\frac{w_F}{A_F}}{n+1} & A_F > \underline{A}_F \\ \frac{\bar{P}-\frac{w_H}{A_H}}{n-m+1} & A_F \leq \underline{A}_F \end{cases}$$

$$Q_F^*(A_F) = \begin{cases} \frac{\bar{P}-(n+1-m)\frac{w_F}{A_F}+(n-m)\frac{w_H}{A_H}}{n+1} & A_F > \underline{A}_F \\ 0 & A_F \leq \underline{A}_F, \end{cases}$$

where $\underline{A}_F = \frac{(n-m+1)w_F}{\bar{P}+(n-m)\frac{w_H}{A_H}}$.

Inserting into the profit function for firm 1 gives

$$\pi_1^*(K_F, A_F) = Q_H^*(A_F)^2 + K_F A_{HF}\left(\frac{w_H}{A_H} - \frac{w_F}{A_{HF}}\right),$$

which implies

$$\frac{\partial \pi_1^*(K_F, A_F)}{\partial A_F} := \begin{cases} -\frac{2m}{n+1} Q_H^*(A_F)\frac{w_F}{A_F^2} & A_F > \underline{A}_F \\ 0 & A_F < \underline{A}_F \end{cases}$$

Appendix B

Lemma 3 *Given our assumptions about demand and cost structure there exists a unique equilibrium in the Cournot oligopoly game.*

Proof: Murphy et al. [20] show that the following conditions are sufficient for existence and uniqueness of an equilibrium in a Cournot oligopoly game:

(a) The inverse demand function $p(Q)$ is continuously differentiable and strictly decreasing with respect to Q.

(b) For every firm i the cost function is continuously differentiable, non-decreasing and convex with respect to Q_i.

(c) The function Q p(Q) is is strictly concave with respect to Q.

(d) There exists a vector of quantities $(Q_1^{mon}, \ldots, Q_n^{mon})$ that maximizes the joint profit of all producers.

(e) There exists a vector of quantities $(Q_1^{sur}, \ldots, Q_n^{sur})$ that maximizes total surplus on the market.

Conditions (a) to (c) follow directly from our assumptions. To see that condition (d) is satisfied it should be first noted that due to the linear cost structure joint market profits are maximized if all output is produced by one single firm which has the lowest unit costs. Accordingly, we only have to consider the monopoly problem of that single firm. Since all unit costs are strictly positive, the assumption $\lim_{Q\to\infty} P(Q) = 0$ implies that the monopolist's profits are decreasing in Q for sufficiently large Q and this implies the existence of a solution to the monopolist's problem. Concerning (e) we can again concentrate on the case where all output is produced by a single firm i. Assumptions $P(0) > c_H, P'(Q) < 0$ and $\lim_{Q\to\infty} P(Q) = 0$ imply that the surplus maximization problem has a (unique) solution characterized by $P(Q) = c_i$. Therefore there exists a unique equilibrium in the Cournot game we are considering. □

Proof of Lemma 1:

(i) Since $P(Q)$ is twice continuously differentiable, it follows directly from the first order conditions that equilibrium quantities are continuously differentiable with respect to A_F and K_F on $(0, \underline{A}) \cup (\underline{A}_F, A_{HF}] \times [0, \infty)$. Hence, also the profit function of Firm 1 is continuously differentiable with respect to these variables on that domain.

(ii) Direct calculation yields under consideration of the envelope theorem that

$$\begin{aligned}\frac{\partial \pi_1^*(K_F, A_F)}{\partial A_F} &= {Q_H^*}'(A_F)\left(P^*(A_F) - \frac{w_H}{A_H}\right) + Q_H^*(A_F)P'(Q^*)\left((n-m){Q_H^*}'(A_F) + m{Q_F^*}'(A_F)\right) \\ &= (n-m-1){Q_H^*}'(A_F)P'(Q^*)Q_H^* + m{Q_F^*}'(A_F)P'(Q^*)Q_H^* \\ &= ({Q^*}'(A_F) - {Q_H^*}'(A_F))P'(Q^*)Q_H^*, \qquad (10)\end{aligned}$$

where $Q^* = (n-m)Q_H^* + mQ_F^*$.

To determine the signs of the derivatives of the equilibrium quantities with respect to A_F we implicitly differentiate the first order conditions. Assume that $A_{HF} \geq A_F > \underline{A_F}$. Then firms from both countries offer positive quantities on the market, where all producers from the same country offer the same quantities. Therefore, equilibrium quantities are characterized by the first order conditions

$$\begin{aligned}P'(Q^*(A_F))Q_H^*(A_F) + P(Q^*(A_F)) &= \frac{w_H}{A_H} \qquad (11) \\ P'(Q^*(A_F))Q_F^*(A_F) + P(Q^*(A_F)) &= \frac{w_F}{A_F}, \qquad (12)\end{aligned}$$

Total differentiation with respect to A_F gives

$$\begin{aligned}P''(Q^*(A_F))Q_H^*(A_F){Q^*}'(A_F) + P'(Q^*(A_F)){Q_H^*}'(A_F) + P'(Q^*(A_F)){Q^*}'(A_F) &= 0 \\ P''(Q^*(A_F))Q_F^*(A_F){Q^*}'(A_F) + P'(Q^*(A_F)){Q_F^*}'(A_F) + P'(Q^*(A_F)){Q^*}'(A_F) &= -\frac{w_F}{A_F^2},\end{aligned}$$

and we obtain

$$\begin{aligned}{Q_H^*}'(A_F) &= -{Q^*}'(A_F)\left(1 + \frac{P''(Q^*(A_F))}{P'(Q^*(A_F)}Q_H^*(A_F)\right) \qquad (13) \\ {Q_L^*}'(A_F) &= -\frac{w_F}{A_F^2 P'(Q^*(A_F))} - {Q^*}'(A_F)\left(1 + \frac{P''(Q^*(A_F))}{P'(Q^*(A_F)}Q_F^*(A_F)\right). \qquad (14)\end{aligned}$$

Adding $(n-m)$ times (13) to m times (14) gives

$${Q^*}'(A_F) = -\frac{mw_F}{A_F^2 P'(Q^*(A_F))} - n{Q^*}'(A_F) - {Q^*}'(A_F)\frac{P''(Q^*(A_F))}{P'(Q^*(A_F))}Q^*(A_F),$$

which implies

$${Q^*}'(A_F) = -\frac{mw_F}{A_F^2 P'(Q^*(A_F))\left(n + 1 + \frac{P''(Q^*(A_F))}{P'(Q^*(A_F)}Q^*(A_F)\right)}. \qquad (15)$$

The assumption that $QP(Q)$ is strictly concave implies that $2P'(Q)+QP''(Q) < 0$ and therefore the bracket in the denominator is positive. Because of $P'(Q) < 0$ we get overall that $Q^{*\prime}(A_F) > 0$. Taking this into account we obtain by inserting (13) into (10) that

$$\frac{\partial \pi_1^*(K_F, A_F)}{\partial A_F} = Q^{*\prime}(A_F)P'(Q^*)Q_H^*(A_F)\left(2 + \frac{P''(Q^*(A_F))}{P'(Q^*(A_F))}Q_H^*(A_F)\right) < 0. \tag{16}$$

If $A_F < \underline{A}_F$ all firms in country F produce zero in equilibrium and the equilibrium quantities of firms in country H are therefore independent from A_F. Accordingly,

$$\frac{\partial \pi_1^*(K_F, A_F)}{\partial A_F} = 0.$$

(iii) Follows directly from $w_H/A_H > w_F/A_{HF}$. □

Proof of Lemma 2:

The Hamiltonian for the investment problem of firm 1 is given by

$$H(A_F, K, \mu_1, \mu_2, \nu) \quad = \quad \pi_1^*(K_F, A_F) - \beta I - \gamma I^2 + \mu_1(I - \delta K_F) + \mu_2 \lambda K(A_{HF} - A_F)$$

and taking into account the state-constraint we obtain the Lagrangian

$$L(A_F, K, \mu_1, \mu_2, \nu) \quad = \quad \pi_1^*(K_F, A_F) - \beta I - \gamma I^2 + \mu_1(I - \delta K_F) + \mu_2 \lambda K(A_{HF} - A_F) + \nu K.$$

According to Pontryagin's maximum principle for every optimal investment path there must exist piecewise absolutely continuous co-states $\mu_i(t)$ and a piecewise continuous multiplier $\nu(t) \geq 0$ such that investment at each t maximizes L, the co-states satisfy the equations

$$\dot{\mu}_1 = \rho\mu_1 - \frac{\partial L}{\partial K_F}$$
$$\dot{\mu}_2 = \rho\mu_2 - \frac{\partial L}{\partial A_F},$$

for the multiplier $\nu K = 0$ holds for all t and the transversality conditions are satisfied. Furthermore, for for each time τ_i of discontintuity of μ_1 there exists an $\eta(\tau_i) \geq 0$ with

$$\lim_{t \to \tau_i +} \mu_1(t) = \lim_{t \to \tau_i -} \mu_1(t) + \eta(\tau_i)$$

(see e.g. Sethi and Thompson [24] and Hartl et al. [13]). Because in our problem no movement of the state on the $K = 0$ line is possible, the problem is time autonomous and has an infinite time horizon, it is obvious that an optimal trajectory can never leave the boundary once it has hit it. Therefore, the only possible dicontinuity of μ_1 may occur at the time when the optimal trajectory hits the $K = 0$ line. Direct calculation together with (16) yields the Lemma. □

Proof of Proposition 1

Suppose that $K_F \neq 0$ holds. Setting $\dot{A}_F = 0$, $\dot{K}_F = 0$, $\dot{\mu}_1 = 0$, $\dot{\mu}_2 = 0$, we obtain the other variables of this steady state as

$$\begin{aligned} \hat{A}_F &= A_{HF}, \\ \hat{\mu}_1 &= \tfrac{A_{HF}}{\delta+\rho}\left(\tfrac{w_H}{A_H} - \tfrac{w_F}{A_{HF}}\right), \\ \hat{I} &= \delta \hat{K}_F = \tfrac{\hat{\mu}_1 - \beta}{2\gamma}, \\ \hat{\mu}_2 &= \tfrac{1}{\rho+\lambda\hat{K}} \tfrac{\partial \pi_1^*(\hat{K}, A_{HF})}{\partial A_F} \\ \hat{\nu} &= 0. \end{aligned} \tag{17}$$

Not that due to assumption (8) we have $\hat{\mu}_1 > \beta$ and therefore foreign investment and foreign capital at this steady state are indeed positive. The value for $\hat{K}$ can be obtained by direct calculations. Furthermore, Lemma 1(ii) implies that $\hat{\mu}_2 < 0$.

To show that this fixed point of the canonical system is indeed a steady-state of the optimal policy we observe that for $(K, A_F) = (0, A_{HF})$ optimal investment is positive (which will be proven in Proposition 2), which implies that the considered fixed point is the only candidate for a steady-state on the line $A_F = A_{HF}$. For $A_F(0) = A_{HF}$ we have $A_F(t) = A_{HF}$ $\forall t$ and the relevant state-space reduces to one dimension. Standard results imply that every control problem with a one-dimensional compact state space must have at least one steady-state and therefore the fixed point of the canonical system characterized above must indeed be a steady-state under the optimal investment rule.

To determine stability of this steady state we consider the Jacobian at the steady state, which is given by

$$J_I = \begin{pmatrix} -\delta & 0 & \dfrac{1}{2\gamma} & 0 \\ (A_{HF} - \widehat{A}_F)\lambda & -\widehat{K}^f & 0 & 0 \\ 0 & \lambda\mu_2 & \delta + \rho & -(A_{HF} - \widehat{A}_F)\lambda \\ \lambda\mu_2 & -\frac{\partial^2 \pi_1^*(\hat{K}, A_{HF})}{\partial A_F^2} & 0 & \lambda\widehat{K}_F + \rho \end{pmatrix}.$$

It is easy to show that the eigenvalues of the Jacobian are given by

$$e_1 = -\delta < 0, \;\; e_2 = \delta + \rho > 0, \;\; e_3 = \frac{A_H w_F - A_{HF} w_H}{2A_H \delta\gamma(\delta + \rho)} < 0, \;\; e_4 = \rho - \frac{\lambda(A_H w_F - A_{HF} w_H)}{2A_H \delta\gamma(\delta + \rho)} > 0,$$

with the inequalities due to our assumption (1). Since there are two negative and two positive eigenvalues, the fixed-point is a saddle point of the canonical system and therefore the steady state is locally asymptotically stable. □

Proof of Proposition 2

We start by considering the necessary conditions that have to be satisfied at a steady-state with $\hat{K}_F = 0$. In any such steady state $\hat{I} = 0$ must hold, which implies $\hat{\mu}_1 = \beta$. Taking that into account we obtain from $\dot{\mu}_1 = 0$ that

$$\hat{\mu}_2 = \frac{1}{\lambda(A_{HF} - A_F)} \left((\rho + \delta)\beta - A_{HF} \left(\frac{w_H}{A_H} - \frac{w_F}{A_{HF}} \right) - \hat{\nu} \right).$$

Due to $\hat{\nu} \geq 0$ this is equivalent to

$$\hat{\mu}_2 \leq q_1(A_F) := -\frac{\rho + \delta}{\lambda(A_{HF} - A_F)} \left(\frac{A_{HF}}{\rho + \delta} \left(\frac{w_H}{A_H} - \frac{w_F}{A_{HF}} \right) - \beta \right).$$

Furthermore, $\dot{\mu}_2 = 0$ requires

$$\mu_2 = q_2(A_F) := \frac{1}{\rho} \frac{\partial \pi_1^*(0, A_F)}{\partial A_F}.$$

This implies that a steady-state with $\hat{K} = 0$ can only exist for values of A_F where $q_1(A_F) \geq q_2(A_F)$.

It is easy to see that q_1 is strictly decreasing in A_F with $\lim_{A_F \to A_{HF}-} q_1(A_F) = -\infty$. Furthermore, taking into account that there exist $0 < \underline{Q} < \bar{Q}$ with $Q^*(A_F) \in [\underline{Q}, \bar{Q}]$ for all $A_F \in [\underline{A}_F, A_{HF}]$ and $P(.) \in C^2[\underline{Q}, \bar{Q}], Q^*(.) \in C^1(\underline{A}_F, A_{HF}]$ we can conclude from (16) that q_2 is bounded from below on $(\underline{A}_F, A_{HF}]$. Therefore, for values of A_F sufficiently close to A_{HF} we have $q_1(A_F) < q_2(A_F)$ and no steady-state with $\hat{K} = 0$ can exist. Assuming that q_1 and q_2 intersect in $[\underline{A}_F, A_{HF})$, denote by $\bar{A}_F = \max_{\underline{A}_F \leq A_F < A_{HF}}[A_F : q_1(A_F) = q_2(A_F)]$ the largest intersection point between q_1 and q_2 in that interval. A sufficient condition for the existence of such an intersection point is $q_1(\underline{A}_F) > q_2(\underline{A}_F)$ and it is easy to see that this inequality is equivalent to $\lambda > \bar{\lambda}$. Accordingly, under the condition $\lambda > \bar{\lambda}$ there exists a set $\mathcal{A} \subseteq [\underline{A}_F, A_{HF})$ with positive measure such that for all $A_F \in \mathcal{A}$ we have $q_1(A_F) \geq q_2(A_F))$ and there is a fixed point of the canonical system with $\hat{K} = 0, \hat{\mu}_1 = \beta, \hat{\mu}_2 = q_2(A_F), \hat{\nu} = \lambda(A_{HF} - A_F)(q_1(A_F) - q_2(A_F)) \geq 0$.

Considering the range $A_F \in (0, \underline{A}_F)$, it follows directly from Lemma 1(ii) that $q_2(A_F) = 0 > q_1(A_F)$. Therefore, there is no steady-state with $K = 0$ and optimal investment is positive for $K = 0$ and $A_F \in (0, \underline{A}_F)$.

Point (i) of the proposition follows directly from the arguments above. To show point (ii), it should be noted that because of $\dot{A}_F \geq 0$ no optimal cycles can occur. Therefore all optimal trajectories have to converge to some steady state and for $A_F(0) > \bar{A}_F$ the only candidate is the catch-up steady-state.

Finally, we have to show (iii). For any state $(A_F, 0)$ with $A_F \in \mathcal{A}$ there is a co-state vector and a non-negative multiplier such that the state-costate-multiplier vector is a fixed point of the canonical system. However, since for our problem in general the Hamiltonian is not concave with respect to the states the maximum principle provides necessary but not sufficient optimality conditions. Therefore, we still have to show that positive investment is not optimal for initial states $(0, A_F)$ with $A_F \in \mathcal{A}$. Assume that there exists some value $\tilde{A}_F$ and some value $\hat{\beta}$ of the parameter β where $q_1(\tilde{A}_F) > q_2(\tilde{A}_F)$ holds, but positive investment is optimal at the state

$(0, \tilde{A}_F)$. This means that $V(0, \tilde{A}_F) > \pi_1^*(0, \tilde{A}_F)/\rho$, where V denotes the value function of the problem for firm 1. If we continuously increase the parameter β then $V(0, \tilde{A}_F)$ decreases continuously and for sufficiently large β no investment is optimal for $A_F = \tilde{A}_F$. This implies that for some $\tilde{\beta} > \hat{\beta}$ we must have

$$V(0, \tilde{A}_F) = \pi_1^*(0, \tilde{A}_F)/\rho. \tag{18}$$

It is well known that $V(0, \tilde{A}_F) = H(0, \tilde{A}_F, \tilde{\mu}_1, \tilde{\mu}_2)/\rho$ where $(\tilde{\mu}_1, \tilde{\mu}_2)$ are the initial values of co-states corresponding to the optimal trajectory yielding the value $V(0, \tilde{A}_F)$. Inserting the optimal investment rule into the Hamiltonian gives

$$V(0, \tilde{A}_F) = \left(\pi_1^*(0, \tilde{A}_F) + \frac{(\tilde{\mu}_1 - \beta)^2}{4\gamma}\right)/\rho,$$

which together with (18) implies $\tilde{\mu}_1 = \beta$ and $I(0, \tilde{A}_F) = 0$ for $\beta = \tilde{\beta}$, but $I(0, \tilde{A}_F)$ is strictly positive for all values of β below $\tilde{\beta}$. Accordingly, for all $\beta < \tilde{\beta}$ we must have $\tilde{\nu} = 0$ and by continuity for $\beta = \tilde{\beta}$ there must exist a fixed point of the canonical system for $A_F = \tilde{A}_F, K_F = 0, \mu_1 = \beta$ and $\nu = 0$. It follows from our arguments above that any fixed point with such properties must be at the intersection of $q_1(A_F)$ and $q_2(A_F)$. Increasing β shifts q_1 upwards but does not affect q_2 and therefore we have $q_1(\tilde{A}_F) > q_2(\tilde{A}_F)$ for $\beta = \tilde{\beta}$. This contradicts the observation that q_1 and q_2 must intersect at $\tilde{A}_F$ and therefore our assumption that positive investment is optimal at $(0, \tilde{A}_F)$ must have been wrong. Accordingly, any state $(0, A_F)$ with $A_F \in \mathcal{A}$ must indeed be a steady-state of the optimal policy.

To analyze the stability of these steady states, note that for every steady state with $\hat{A}_F \in int(\mathcal{A})$ optimal investment is zero for all states with $K_F = 0$ in the neighborhood of the steady state. Assume that there exists a sequence of states $\left\{\tilde{K}_{F,s}, \tilde{A}_{F,s}\right\}_{s=0}^{\infty}$ such that $\lim_{s\to\infty}(\tilde{K}_{F,s}, \tilde{A}_{F,s}) = (0, \hat{A}_F)$, $\tilde{K}_{F,s} > 0 \;\forall s$ and $\lim_{s\to\infty}(I(\tilde{K}_{F,s}, \tilde{A}_{F,s}) - \delta\tilde{K}_{F,s}) \geq 0$. Denoting the corresponding values of the co-states and multiplier by $\tilde{\mu}_{i,s}, \tilde{\nu}_s$, we must have $\tilde{\nu}_s = 0$ for all s. Assuming without restriction of generality that the sequence of co-states converges[10] we denote the limit

[10] Obviously, we can restrict attention to a compact subset of the co-state space and therefore there must alway exist a convergent subsequence we can consider.

by $(\tilde{\mu}_1, \tilde{\mu}_2) = \lim_{s\to\infty}(\tilde{\mu}_{1,s}, \tilde{\mu}_{2,s})$. Obviously, we must have $\frac{\tilde{\mu}_1-\beta}{2\gamma} \geq 0$ and by continuity of the Hamiltonian we get $H(0, \hat{A}_F, \tilde{\mu}_1, \tilde{\mu}_2)/\rho \geq \pi_1^*(0, \hat{A}_F)/\rho$. If this inequality is strict we get a contradiction to the fact that $(0, \hat{A}_F)$ is a steady state. If the inequality holds as equality, we must have $\tilde{\mu}_1 = \beta, \tilde{\nu} = 0$ and $(0, \hat{A}, \tilde{\mu}_1, \tilde{\mu}_2)$ can only be a fixed point of the canonical system if $q_1(\hat{A}_F) = q_2(\hat{A}_F)$, which contradicts that $\hat{A}_F \in int(\mathcal{A})$. This implies that no such sequence exsits and accordinlgy there must exist a neighborhood $\mathcal{U}$ of $\hat{A}_F$ such that for all $(K_F, A_F) \in \mathcal{U}$ we have $\dot{K}_F = I(K_F, A_F) - \delta K_F < -\epsilon$ for some $\epsilon > 0$. Hence, after any sufficiently small deviation of the state from $(0, \hat{A}_F)$ the optimal trajectory returns to the line $K = 0$ and a steady-state in the interior of $\mathcal{A}$ therefore is neutrally stable. $\square$.

Proof of Proposition 3:

If $\frac{\partial^2 \pi_1^*(0, A_F)}{\partial {A_F}^2} \geq 0$ then $q_2(A_F)$ is non-decreasing on $[\underline{A}_F, \bar{A}_{HF}]$. For $\lambda > \bar{\lambda}$ we have $q_2(A_F) < q_1(A_F)$ and due to the fact that $q_1(A_F)$ is strictly decreasing and $\lim_{A_F\to A_{HF}} q_1(A_F) = -\infty$ there must be a unique intersection point of $q_1(A_F)$ and $q_2(A_F)$ in $(\underline{A}_F, A_{HF})$. This implies directly that $\mathcal{A} = [\underline{A}_F, \bar{A}_F]$.

In case of a linear demand, we obtain from the expressions given in Appendix A that

$$\frac{\partial^2 \pi_1^*(K_F, A_F)}{\partial A_F^2} = \frac{2m^2}{(n+1)^2} \frac{w_F^2}{A_F^4} + \frac{4m}{n+1} Q_H^*(A_F) \frac{w_F}{A_F^3} > 0$$

holds for all $A_F > \underline{A}_F$. $\square$.

Proof of Corollary 1:

To prove point (i) of the corollary we have to observe that $\lim_{\lambda\to 0} \sup[q_1(A_F)|A_F \in [\underline{A}_F, A_{HF})] = -\infty$ and therefore for sufficiently small λ we have $q_1(A_F) < q_2(A_F)\ \forall A_F \in [0, A_{HF})$ and therefore there exists no steady-state with $\hat{K} = 0$.

The observation that $\underline{A}_F$ is decreasing in w_h for linear demand follows directly from the expression given in Appendix A. Concerning $\bar{A}_F$, we observe first that Q_H^* increases for decreasing w_H and therefore $\frac{\partial \pi_1^*(K_F, A_F)}{\partial A_F}$ decreases for decreasing w_H. Therefore $q_2(A_F)$ shifts downwards for decreasing w_H on the interval $[\underline{A}_F, A_{HF}]$. On the other hand, the curve $q_1(A_F)$ shifts upwards if w_H becomes smaller. Taking into account the arguments in the proof of proposition 3 this implies that the unique intersection point between q_1 and q_2 moves to the right. $\square$.

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