

CREA Discussion Paper

2008-08

Center for Research in Economic Analysis
University of Luxembourg

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available online : http://fdef.uni.lu/index.php/fdef_FR/economie/crea/discussion_papers/2008

Thierry Bréchet, CORE, Université Catholique de Louvain & Louvain School of Management
Pierre M. Picard, University of Luxembourg

October, 2008

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Thierry Bréchet*, Pierre M. Picard[‡]

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Abstract

This paper presents a market design for the management of noise pollution created by aircraft traffic around airports. A local market for noise licenses allows noise generators to compensate noise victims. We show that the market allows the market designer to achieve his/her optimal allocation of flights provided that he/she does not over-weight the benefit of economic activity compared to the disutility of noise pollution. The fact that local representatives of noise victims may be strategic players does not fundamentally

*CORE and Louvain School of Management, Chair Lhoist Berghmans in Environmental Economics and Management, Université catholique de Louvain. Corresponding author.

[†]CREA, University of Luxembourg and CORE, Université catholique de Louvain, email: picard@uni.lu.

[‡]The authors thank participants to the Environmental Workshop at CORE, University of Lille 1, NARSC Toronto 2006, Economic Seminar of MEDAD and Workshop on Airport Noise Regulation at ECARES. This research benefited from financial support from *Research in Brussels*, IRSIB Région de Bruxelles-Capitale and ECARES, Free University of Brussels. They are also grateful to Paul Belleflamme, Jan Bruekner, Denny Ellerman, Paul Madden, Yann Menière, Jon Nelson and Kurt Van Dender for fruitful comments on preliminary versions of the paper. The paper is also available as CORE DP 2007/45.

alter this finding. Because of the market auctioneer's information constraints, noise licenses are likely to distribute windfall gains to residents, which alter the urban structure in the long run.

1 Introduction

The air transport industry has become a key factor of economic activity. Air transport generates significant social and economic benefits related to trade, employment, investment, tourism and leisure opportunities. However, like most human activities, air transport also generates less welcome external effects, among which noise pollution is probably the most salient one.¹ Noise-induced disturbances constitute a hot and topical societal problem for all major airports. Indeed, there seem to be no clear solutions to the issues about how to accommodate the residents suffering from noise damages and about how to accordingly determine the number and the distribution of aircraft movements around the airports.

Intriguingly, the economic literature has neglected to discuss the design of policy instruments that could efficiently balance social cost of noise pollution and economic benefits of airport economic activity. While many discussions have focused on technological improvements (quieter airplanes, alternative land-off and kick-off procedures) or on the definition of noise standards, they have not addressed the question of social optimality.² It is well-known that command-and-control policies do not lead to social optimality in the context of asymmetric information, which is relevant to airport regulation since information about residents' noise disutility is difficult to collect (we shall come back to that point). Even though, in some situations, regulators have been able to create noise abatement incentives by imposing different fees in function of the (theoretical) noise pressures of aircraft categories,

¹Another topical external impact is its contribution to climate change (air transport contributes to 3 percent of world greenhouse gases emissions). To cope with this problem, air transport sector will soon be included in the emission targets negotiated under the Kyoto protocol and in the European Emission Trading Scheme (EU-ETS) which regulates carbon dioxide emissions.

²See for instance Janic (1999) and Brueckner (2003). Brueckner and Girvin (2007) discuss the optimal taxation of aircrafts given a fixed global quota of noise emissions but do not consider the optimal determination of this global quota of noise emission.

they have been unable (or unwilling) to calibrate those fees to the actual noise disutility of the residents surrounding airports. In fact, the efficiency of such fees has not been established. Still, even if the optimal fee were implemented, it would neither solve the problem of spatial distribution of aircraft movements around airports, nor the issue of efficient compensations to the noise victims.

It is often argued that residents are already compensated for the social cost of noise damage by lower housing rents and prices. Yet, the fact that residents are compensated does not imply that social costs are internalized. Whereas such costs can be shifted from tenants to landlords who are bound to offer lower house rents, they are generally not shifted to the firms that generate the noise externality. Except in the rare situations where airports acquire the surrounding properties,³ the social costs of noise pollution are not internalized and there exists a need to design economic instruments that organize such an internalization.

The debate about the internalization of social costs is well-known to economists. Noting the reciprocal nature of harmful effects, Coase (1960) suggested to define appropriate property rights over the source of those effects and showed that regulation can be accomplished effectively and efficiently by a market. The present paper applies this idea in the context of noise pollution and airport activity. We propose *noise licenses* as the means of the negotiation between residents and airline companies. In order to compensate the residents for noise damage the property rights will be assigned to them. By selling those rights on a market, the residents will express their willingness to accept noise. The contribution of our paper is to show how a local market for noise licenses allows to implement the socially optimal number of flights and their spatial distribution among routes.

The market for noise licenses is organized as follows. Residents are organized

³To our knowledge, Flughafen Dusseldorf is one airport which has pursued a policy of house purchase in noisy areas. Some airports have been relocated in housing free areas (*e.g.* Oslo, Montreal-Mirabel).

by zones that sell noise licenses to airline companies. Each noise license consists in a right for one aircraft to fly over a specific zone during some time period. The supply of licenses is set by the zones and bounded only by technical feasibility.⁴ A (neutral) auctioneer collects the bids of zones' supplies and airline companies' demands, determines the price that clears the market and redistributes the licenses according to the bids. In equilibrium, the price of noise licenses is equal to both the marginal profit of the additional flight from/to the airport and to the disutility of additional noise disturbance in the more disturbed areas. Like any other market for tradeable licenses, this market for noise licenses offers a distinct advantage over traditional instruments like command-and-control, urban planning or fees. Once the market is adequately designed it reaches an equilibrium in which social benefit of the airlines' activity is balanced with its social costs. Because residents have full rights to issue noise licenses, they are never worse-off. At the same time, the social costs of noise pollution are internalized through the purchase of noise licenses by airline companies. This makes void the debate about airport noise pollution and local governments need no longer be involved in cumbersome information collection/studies on noise damage or in political arbitrage between supporters of environmental quality and airport economic activity. Thus, a local market where airline companies and airport residential neighbors can trade noise licenses provides governments with an efficient instrument to correct the aircraft noise externalities.

Designing a market for licenses in the context of noise pollution raises many theoretical challenges. The first one relates to the complementarity of zones over the routes. This complementarity results from the fact that, to fly over a route an aircraft needs to buy the licenses of all the zones of that route. We show that zones always supply a positive number of noise licenses and that no zone is willing to block the airplane traffic. The impact of a specific zone on the license price is limited by

⁴The maximal number of licenses is equal to the maximal number of movements the airport can sustain during the specified time period.

the presence of competing zones located on different routes. The second challenge relates to the spatial dimension of our market. In particular, we analyze how the design of zones and routes shapes the market equilibrium. We show that there exists a large class of feasible designs and that, under some conditions, it is possible for the market designer to replicate the socially optimal number and spatial distribution of flights with an appropriate design. The third challenge relates to the possibility for planes to fly over different jurisdictions where local governments' objectives naturally differ. We show that an optimal design is still possible, provided that the jurisdiction hosting the airport is the market designer. The fourth challenge relates to the possibility for zones' representatives to be strategic. Interestingly, the presence of such behaviors does not threaten the whole airport activity, it only reduces the set of parameters for which the planner's solution can be implemented. The last challenge is to capture the long run effects of the market for noise licenses on the urban structure. The market for noise licenses is likely to increase the population density in the noisy areas. **Related literature** There exists a vast literature on

the evaluation of the impacts of airport activities on property values in residential areas. This literature mainly focuses on the environmental impact of airports on neighboring residents and economic activity. McMillen (2004), Nelson (2004) or Schipper (2004) propose recent updates. On the one hand, Schipper (2004) shows that the medium value of environmental costs in a set of 35 European airport areas is \$0.0241 (0.0201 Euro) per passenger-km, the noise costs counting for 75 per cent.⁵ Numerous empirical studies have confirmed that aircraft noise influences property values around airports. Using the hedonic approach, Baranzini and Ramirez (2005) have recently used housing market data to infer the noise impact on housing rents in Geneva, Switzerland. They show that the impact of all sources of noise on housing

⁵This means an environmental cost of about \$2400 (2000 Euros) per 100-seat aircraft flight over 1000 km.

rents is about 0.7 per cent per acoustic decibels⁶ and about 1 per cent when considering exclusively airplane noise in the airport area. Interestingly, this measure does not significantly change with noise measuring procedures and with the institutional structure of the housing market (private *versus* government ownership). On the other hand the local economic benefits of airports are generally significant, covering tax revenues and direct and indirect employment opportunities. As soon as airport expansion is to be discussed, these issues become much trickier. Opening of new runways typically exacerbate the dilemma between noise concern and economic benefits. According to the US Federal Aviation Administration,⁷ 18 of the 31 large hub airports in the US plan to add runways in the next decade. On the other hand, Brueckner (2003) estimates that the O'Hare expansion would raise service related employment in the Chicago area by 185,000 jobs. Yet, again, this literature on costs and benefits of airport activity and noise pollution does not address the problem of internalization of the externality between aircrafts, airports and residents.

One may also wonder about the ability or the willingness of governments and relevant institutions to implement the optimal number and spatial distribution of aircraft movements. In 2001, the International Civil Aviation Organization (ICAO) Assembly endorsed the concept of a “balanced approach” to aircraft noise management.⁸ This consists of identifying the noise problem at an airport and then analyzing the various measures available to reduce noise through the exploration of four elements, namely *(i)* reduction at source (quieter aircraft), *(ii)* land-use planning and management, *(iii)* noise abatement operational procedures and *(iv)* operating restrictions. This organization aims to address the noise problem in the most cost-effective manner. In a recent directive, the European Commission also

⁶A unit of acoustic decibel (dBA) is a (logarithmic) measure of the sound pressure and therefore of the intensity of a sound perceived by humans. Because a sound is perceived differently according to its frequency, sound pressures are corrected by a “A weighting filter”.

⁷Source: US Department of transportation.

⁸See *www.icao.int*.

advocates a similar “balanced approach” to aircraft noise management (Directive 2002/30/EC, European Commission, 2002). The European Commission’s aims are however broader than those of ICAO as the Commission seeks to limit or reduce the number of people significantly affected by the harmful effects of noise, and to achieve maximum environmental benefit in the most cost-effective manner. The cost-effectiveness of the policy is clearly claimed but the choice of the instrument to implement the policy remains open. This is the very purpose of this article to propose an appropriate one.

Finally, our contribution with a market for noise licenses also relates to environmental economics.⁹ Although tradeable permits or licenses have been promoted as policy instruments for environmental issues since Dales (1968), they were generally regarded as impractical despite their theoretically attractive properties (cost-efficiency). Yet, since the 1990s, many instances of markets for tradeable permits have been successfully implemented, most notably for sulfur dioxide pollutant in the US power industry and for carbon dioxide in the EU (see Ellerman (2005) for an introductory note on pollution permits and Tietenberg (2006) for a comprehensive overview of theory and experiences). All these market experiences are inspired by Montgomery (1972) who formally showed that under a global emission constraint, competitive markets of tradeable permits yield cost efficient allocations of pollution abatement whatever the distribution of permits amongst polluters. In such a market, the global emission cap coincides with the total number of emission permits, those being emitted by the regulator (grandfathered or auctioned) and trade occurring among polluters.

Hence, the environmental economic literature generally discusses the efficiency

⁹The vocabulary used in the literature is sometimes loose, here. As noted by Montero (2007), the word *licenses* usually refers to permits or allowances in water and air pollution control, while rights are used in water supply management and quotas in fisheries management. We will use licenses throughout the paper.

properties of allocation of pollution quotas in a secondary market where a government has no information about the firms' cost structure and distribute permits (or quotas) to polluters, like in Montgomery (1972). Designing rules for initial allocations of pollution permits (or quotas) has remained a critical issue of information revelation until the particular auction mechanism recently proposed by Montero (2007). However, such mechanisms cannot be applied to our context because the government has no more information about individual's noise damage function than about each airline company' business structure.

In contrast to this literature, we consider the design of a primary local market for noise licenses that includes, like in Coase (1960), both local victims (the residents) and polluters (the aircraft companies). We give a particular focus on a design that makes the airport activity acceptable to neighboring residents. Since the latter are victims of aircraft noise disturbances, this naturally implies that property rights on noise (or quietness) are granted to those residents. They are then free to transfer those rights to aircraft companies by the means of *noise licenses*. Hence, the regulator does not impose any arbitrary global noise quota. By contrast, under some conditions, we will show that there exists market designs that implement the socially optimal number and allocation of air traffic amongst aircraft routes, and that it does not require the regulator's intervention after the creation of the market.

This article offers an contribution that is both policy-oriented and methodological. It presents an original and efficient policy instrument to regulate noise pollution around airports. It proposes a new application of the concept of tradeable licenses to the issue of noise exposure, with an emphasis on the spatial dimension of the problem.

The article is organized as follows. Section 2 presents the setting while Section 3 derives and discusses our proposal for a market for noise licenses. Section 4 proposes the optimal market design in the standard case of price taking zones. In Sections 5 and 6, two extensions are considered, one where aircrafts fly over independent

jurisdictions, and the other where zones' representatives behave strategically. In all cases we will see that the optimal solution can be implemented, but under specific conditions. Section 7 develops the short run and long run equilibria and shows how the market for noise licenses shapes the city structure. The conclusion follows.

2 The setting

2.1 The airport

Let us consider a civil airport located in the neighborhood of a large city. Air traffic is organized along several routes that airplanes may take when they land and take off. Landing and take-off routes are determined by exogenous technical characteristic, in particular the direction of wind. Yet, within the same set of technical parameters, there exist several route possibilities. For example, after the take-off, aircrafts may remain at low altitude, or go up, and they may go right or left. We denote $\bar{\mathcal{R}}$ and \bar{R} the set and the number of all technically possible routes ($\bar{R} = \#\bar{\mathcal{R}} \geq 1$). We denote \mathcal{R} ($\mathcal{R} \subseteq \bar{\mathcal{R}}$) and R ($R \leq \bar{R}$) the set and number of routes that are actually used. Along a route $r \in \mathcal{R}$, airplanes generate noise pollution that varies according to their altitude and acceleration. Because we are particularly interested in the problem of spatial distribution of flights, we set aside the issue of aircrafts heterogeneity. Let t be the distance from the airport on a given route and let y_r be the number of planes on route r .

2.2 The residents

The residents are homogenous with respect to their disutility for noise pollution but they differ according to their distance t from the airport. Thus, a resident i located on route r is endowed with an individual utility function equal to $U_r^i(t, y_r) = I^i - d_r(t, y_r)$, where I^i is her income and $d_r(t, y_r)$ is her disutility from noise exposure.

This disutility firstly depends on the distance from the airport: in general, the closer to the airport, the worse are the noise damages. It secondly depends on the number of flights. For simplicity we assume that $d_r(t, y_r) = \delta_r(t)y_r^2/2$ where $\delta_r(t)$ is a location-noise disutility parameter on that route. The parameter $\delta(t)$ reflects the loss of utility suffered by a resident when an aircraft flights over the location t on route r . So, $\delta(t)$ is typically larger in locations where aircraft have lower altitude, boost engine power and/or use flaps. This parameter depends on the profile of the route, which we take as given.¹⁰ Figure 1 depicts an example of an airport with several routes as well as an example of a profile of location-noise disutility parameter $\delta_r(t)$. The model generalizes to any increasing and convex disutility function d_r .

The population size at location t on route r is $n_r(t)$. Let $\beta_r(t) = \delta_r(t)n_r(t)$ be the total disutility parameter at location t on route r and let T_r be the distance at which noise pollution is no longer considered.¹¹ One then computes the aggregate disutility parameter on route r as $B_r \equiv \int_0^{T_r} \beta_r(t)dt$. Figure 2 provides an example of the distribution of the total disutility parameter on a route. It is important to observe that whereas individuals located close to the airport have higher disutility $\delta(t)$, the total disutility parameter $\beta(t)$ may be higher elsewhere because of a larger population density there.

Insert Figure 1 and Figure 2 about here

¹⁰The profile of a route includes the altitude, direction, speed and acceleration of aircraft that are prescribed in landing and taking-off procedures.

¹¹Residential areas with low or zero noise pollution are not considered for noise contention issues. Typically, this applies to areas with an annual average noise level lower than 55 decibels (L_{dn}). We also consider homogenous noise pressures amongst aircraft categories. When aircraft are heterogenous in noise pressures, the parameter $\delta_r(t)$ relates to the ‘average’ noise disutility at location t on route r .

2.3 Airline companies

Airplanes are supposed to belong to independent profit maximizing companies. Each flight's profitability varies with its characteristics such as travelers demand, flight distance, indivisibility, etc. There thus exists vertical differentiation of flights which we summarize by the following profit function $\pi(x) = \bar{\pi} - x$ where x is the index of the flight $x \in [0, \bar{\pi}]$, P_r the price of the route taken and $\bar{\pi} > 0$ the profit of the most profitable flight. Under this assumption, all flights are profitable. We assume for simplicity that x is uniformly distributed on the interval $[0, \bar{\pi}]$. The model can easily be generalized to other profit functions.¹²

2.4 The first-best

It is instructive to derive the allocation of flights under a command and control policy where a planner determines the number and the distribution of flights over possible routes under perfect information. The planner maximizes a weighted sum of profits and resident utility. Its problem is to find the set of routes \mathcal{R} and the allocation of flights $\{y_r\}$ such that

$$\max_{\mathcal{R}, \{y_r\}} W = \gamma \int_0^y (\bar{\pi} - x) dx - \sum_{r \in \mathcal{R}} \int_0^{T_r} d_r(t, y_r) n_r(t) dt \quad \text{s.t.} \quad y = \sum_{r \in \mathcal{R}} y_r$$

where γ is the weight the planner puts on profit. It is zero when profits do not accrue to local residents. It can be larger than one when the airport activity generates consumer surplus or additional peripheral activity.¹³ When some share of consump-

¹²Incentives for noise abatement may be introduced by requiring airlines to purchase shares of noise licenses that are proportional to the aircraft noise pressure or, in practice, set in accordance to the noise 'quota count' defined for each aircraft.

¹³For example, suppose that demands for travel are linear and that airline companies have constant marginal costs and monopoly power on each city pair. Then, consumer surplus is equal to half of the profit generated by firms. In this case, a planner considering all profits and consumer surplus would set $\gamma = 1 + 1/2 = 3/2$.

tion and profits accrues to individuals that do not belong to the jurisdiction of the planner, the latter would set a smaller γ .

The first order conditions read $\gamma(\bar{\pi} - y) = y_r B_r, \forall r \in \mathcal{R}$. The planner's solution yields the following total number of flights and its distribution amongst routes:

$$y^o = \frac{\bar{\pi}}{1 + [\gamma \sum_{r \in \mathcal{R}} B_r^{-1}]^{-1}} \quad \text{and} \quad \frac{y_r^o}{y^o} = \frac{B_r^{-1}}{\sum_{s \in \mathcal{R}} B_s^{-1}} \quad (1)$$

The optimal number of flights y^o increases with the aircraft profitability and decreases with the aggregate noise pollution on routes. The number of flights on a route falls as the aggregate noise pollution on this route increases.

Plugging this solution into the planner's objective, we get $W = \bar{\pi}y^o/2$ which increases in y^o . Since y^o rises with the number of routes R , the planner chooses to use all routes: $\mathcal{R}^o = \bar{\mathcal{R}}$. This result naturally follows from the assumption that disutility is convex and, importantly, it does not mean that the number of flights will be equal on all routes, as shown in equation (1).

This solution would correspond to a command-and-control solution where the regulator would fix the number of flights on each route. However, command-and-control policy requires full information on residents' preferences, airlines profitability and distribution of noise pollution. Furthermore, even under full information such a policy would not provide the regulator with the funds necessary to compensate residents. Moreover, this solution can not be decentralized by a tax system. Indeed, whereas a tax equal to $\bar{\pi} - y^o$ allows the planner to implement the total number of flights, there exists no tax instrument that allows it to allocate flights amongst routes. Furthermore, the redistribution of the proceeds of the fees to the residents is impossible without knowing their preferences. So a tax system would leave unsolved major parts of the problem (flight allocation and compensation).

Thus, neither command-and-control nor fees seem adequate to optimally regulate noise pollution around airports. Alternative policy instruments are required. In the following we discuss the implementation of this optimal allocation of flights with

a market for noise licenses. Such a market would not only implement the optimal flight's allocation but would also compensate residents for noise damage.

3 A market for noise licenses

We consider a market solution such as the one proposed by Coase (1960). That is, we define the property rights to benefit from a quiet environment and we give them to the residents.¹⁴ Residents are allowed to transfer those rights to airline companies by selling noise licenses. Montgomery (1972) has proved that, under perfect competition such a market yields the socially efficient solution. We make Montgomery's (1972) assumption that firms and zone representatives are price takers. We also assume the presence of a 'neutral auctioneer' who collects bids until he/she finds an equilibrium.

The main feature of our market for noise licenses is its spatial dimension. Residents are distributed over the space under aircraft routes and airline companies are required to buy the licenses to take a route. A natural question arises about the possible organization of residents in this market. In this paper we assume that residents are organized in neighborhoods, which we here call 'zones'. Zones are heterogeneous with respect to the noise exposure and to the size of the disturbed population. In each zone a representative agent acts in the market for noise licenses on behalf of his/her co-residents. Such an agent may be a representative of a residents' association or of a municipality. The design of this market therefore includes the choice of routes, the definition of the zones and the role of the market auctioneer. Figure 1 and 2 depict the case of an airport with various routes that each includes three zones.

Regarding the definition of routes and zones we have to introduce some notation. Let \mathcal{T}_r denotes a partition of the interval $[0, T_r]$ of route $r \in \mathcal{R}$ into several zones,

¹⁴It is well-known that, under the Coase Theorem, the optimal solution is reached whatever the allocation of property rights. Distributional effects will differ, however.

and let $Z_r = \#\mathcal{T}_r \geq 1$ be the corresponding number of zones. Hence \mathcal{T}_r is equal to the set $\{[0, t_{r,1}], \dots, (t_{r,z-1}, t_{r,z}], \dots, (t_{r,Z_r-1}, T_r]\}$. We then denote by z the index of the zone $(t_{r,z-1}, t_{r,z}]$ and we denote by $\mathcal{Z}_r = \{1, \dots, Z_r\}$ the set of such indices for route r .

3.1 The supply and demand for noise licenses

In each zone the representative is allowed to sell noise licenses at a price p_{rz} . To have one flight passing over the zone $z \in \mathcal{Z}_r$ during a specified time period, an airline company must buy one license at that price.¹⁵ Hence to take the route r the airline company must purchase the bundle of zones' licenses for a total the price of $P_r \equiv \sum_z p_{rz}$. We assume that the zone's representative is utilitarian and considers the following aggregate utility function:

$$U_{rz}(y_{rz}) \equiv \int_{t_{r,z-1}}^{t_{r,z}} U_r^i(t, y_{rz}) n_r(t) dt$$

One can write

$$\int_{t_{r,z-1}}^{t_{r,z}} d_r(t, y_{rz}) n_r(t) dt = \frac{1}{2} \beta_{rz} y_{rz}^2$$

where

$$\beta_{rz} \equiv \int_{t_{r,z-1}}^{t_{r,z}} \beta_r(t) dt$$

is a parameter measuring the total disutility *over the zone*. Notice that aggregate disutility depends both on individual utility function and population size in that zone. Given that the amount of money raised by the zone is equal to $p_{rz} y_{rz}$, the representative's utility is given by

$$U_{rz}(y_{rz}) = p_{rz} y_{rz} - \frac{1}{2} \beta_{rz} y_{rz}^2$$

¹⁵The attributes of the specified time period can be day/night, or smaller time periods, working days/week-ends. In case of several time periods, airlines redistribute their flight activity according the price of noise licenses in each time period. For the sake of exposition, this paper focuses on a market defined over a single time period.

For the moment, we assume that representatives are price takers. So, each representative maximizes this function by choosing the amount of licenses to sell and considering the price p_{rz} as a given. The first order condition is equal to $p_{rz} = \beta_{rz} y_{rz}$, which yields the following supply of licenses

$$y_{rz}^S(p_{rz}) = p_{rz}/\beta_{rz} \quad (2)$$

The demand for noise licenses comes from airline companies. We assume that flights are managed by independent profit maximizing companies. The latter have to purchase a bundle of noise licenses to take a route. The profit function of the flight $x \in [0, \bar{\pi}]$ is now $\pi(x) = \bar{\pi} - x - P_r$ where P_r is the cost of taking route r . The demand for licenses is therefore given by

$$y_r^D(P_1, \dots, P_R) = \begin{cases} 0 & \text{if } P_r > P_s \\ [0, \bar{\pi} - P_r] & \text{if } P_r = P_s \\ \bar{\pi} - P_r & \text{if } P_r < P_s \end{cases}$$

where $r \neq s \in \mathcal{R}$.

3.2 The equilibrium

We now define the role of the auctioneer in the market for noise licenses. The latter collects supply and demand bids from residents and firms and is in charge of finding an equilibrium price. As for any competitive market we assume that the only information the auctioneer gets is the number of license bids of residents and firms on each route. As a consequence, the auctioneer is not allowed to discriminate across zones. In our setting, this has a particular implication: the auctioneer must propose the same price for all zones located on a given route. Formally, $p_{rz} \equiv P_r/Z_r$ $\forall r \in \mathcal{R}, \forall z \in \mathcal{Z}_r$.

The licenses supplied by different zones over a same route are perfect complement for airline companies. At a given set of prices, the total supply is given by the most

restrictive use of the route in every zone. That is,

$$y_r^S(P_r/Z_r) = \min_{z \in \mathcal{Z}_r} y_{rz}^S(P_r/Z_r)$$

Finally, the instruments in that market design include the set of open routes, $\mathcal{R} \subseteq \overline{\mathcal{R}}$, and the definition of zones, $\{\mathcal{I}_r\}_{r \in \mathcal{R}}$.

Definition 1 *An equilibrium in the market for noise licenses consists in a set of prices P_r^* and a number of flights y_r^* such that, given the value of the policy instruments $\{\mathcal{R}, \{\mathcal{I}_r\}\}$, the market clears, that is, $y_r^D(P_1^*, \dots, P_R^*) = y_r^S(P_r^*/Z_r)$, $\forall r \in \mathcal{R}, \forall z \in \mathcal{Z}$.*

Such an equilibrium implies that there exists no allocation of routes that, at prevailing prices, would be preferred by any airline company or by any zone's representative. It also means that no resources are lost, given that the auctioneer makes no profit.

In this market equilibrium, the zone with the highest total disutility offers the smallest number of licenses and thus it determines the number of flights over a route. We call this zone the *critical zone*. This can be seen by using the supply function defined above,

$$\min_{z \in \mathcal{Z}_r} y_{rz}^S(P_r/Z_r) = \min_{z \in \mathcal{Z}_r} \{P_r/(Z_r \beta_{rz})\} = P_r/(\overline{\beta}_r Z_r)$$

where

$$\overline{\beta}_r \equiv \max_{z \in \mathcal{Z}_r} \beta_{rz}$$

is a parameter measuring the total disutility over the critical zone. Equating this supply function with demand yields the following proposition.

Proposition 1 *The equilibrium in the market for noise licenses exists and is unique. It is such that,*

$$P_r^* = P^* \equiv \frac{\overline{\pi}}{1 + \sum_{r \in \mathcal{R}} (\overline{\beta}_r Z_r)^{-1}} \quad \text{and} \quad y_r^* = \frac{\overline{\pi}}{1 + \left(\sum_{r \in \mathcal{R}} (\overline{\beta}_r Z_r)^{-1} \right)^{-1}} \quad (3)$$

Given that $y_r^S = P^*/(\bar{\beta}_r Z_r)$ we also have that,

$$\frac{y_r^*}{y^*} = \frac{(\bar{\beta}_r Z_r)^{-1}}{\sum_{s \in \mathcal{R}} (\bar{\beta}_s Z_s)^{-1}} \quad (4)$$

Proof. See appendix. ■

The design of the market for noise licenses may have an impact on the equilibrium price and number of flights. It is clear that adding new routes reduces the price of noise licenses and raises total aircraft activity ($\mathcal{R} \subset \mathcal{R}' \Rightarrow P^* > P^{*'} \text{ and } y^* < y^{*'}$). The way the design of zones shapes the equilibrium is manifold: it does not only depend on the number of zones on each route but also on the definition of critical zones. Suppose indeed that the market designer adds a zone on route r , without altering any critical zones. Therefore, we have that Z_r increases while $\bar{\beta}_s, \forall s \in \mathcal{R}$, remain constant. From Proposition 1, it follows that, in equilibrium, the price of routes increases and the total number of flights decreases. Also, flights are re-allocated away from route r . This needs not to be the case for different routes designs. Suppose indeed that the market designer adds a new zone on route r while keeping the noise damages equal in every zone: $\beta_{rz} \equiv \bar{\beta}_r \equiv B_r/Z_r \forall z$. This implies that $\bar{\beta}_r Z_r$ remains constant and that neither the price of routes nor the total number of flights change as Z_r increases. By the same token, the allocation of flights over routes remains unchanged. This is because a rise in the number of zones is associated (i) to a fall in the population of each zone and (ii) to a reduction in its share of the noise license price on the route, P_r/Z_r . The first effect reduces the disutility of zone representatives who, at a given price, augment their supply of noise licenses. The second effect reduces the revenues of zone representatives who restrict their supply. In equilibrium the two effects exactly balance.¹⁶ Finally, it is easy to construct examples where the market designer adds new zones but reduces the size of critical zones so that the equilibrium license price falls and the number of flights increases.

¹⁶These effects may not balance if zone representative were not utilitarian. Yet, the assumption of utilitarian zone representatives constitutes an interesting benchmark.

Actually, in the design of zones, the number of zones and the size of the critical zones are, to some extent, substitute instruments.¹⁷

The properties of the market for noise licenses crucially depend on the complementarity of noise pollution on each route. Indeed, noise pollution is a complementary bad for all zones on the same route since aircrafts must fly over all those zones. Typically, this may lead to a “tragedy of the commons” where agents do not internalize the global effect of their decisions. In our context, each residents’ representative owns the right to issue noise licenses and does not internalize the benefits and damages that other zones bear. One may conjecture that the number of licenses and flights is inefficient. Another way to see this is to observe that noise licenses offered by different zones on a same route are complementary goods. Therefore the market for noise licenses may be suspected to be subject to Cournot’s conjecture about under-provision of complementary goods. In this conjecture, independent suppliers of complementary goods would set inefficiently high price levels because they would not internalize the effect of their decisions on others. In our setting, those effects exist but are under the control of the market designer. By defining the zones, the latter is able to tune this effect and is also able to set the level of flight activity. The market designer neutralizes those effects by designing homogenous zones ($\beta_{rz} \equiv \bar{\beta}_r \equiv B_r/Z_r$). In this case the creation of new zones is balanced with a smaller disutility in the zones, which leaves prices and flight activity unchanged.

Because the division of routes into heterogenous zones allows the market designer to tune the total air traffic, it constitutes an important aspect of the design of a market for noise licenses. In the following we show that this market allows the implement the social planner’s solution.

¹⁷In practice, it may be unrealistic to consider a very large number of zones on a route of a given size.

4 An optimal market design

The market designer has a considerable degree of freedom in the divisions of routes into zones. It then is natural to ask whether and when an appropriate market design leads to the social planner's outcome in terms of route allocation and total number of flights. In our market for noise licenses the planner has the ability to choose two sets of instruments: the routes and the definition of zones. In this section we show the condition under which the market designer is able to implement the social planner's optimal solution by an adequate choice of these instruments. To this end, we compare the first best allocation with the competitive equilibrium. We do this, first, in the general setting. Then, we simplify the market design by imposing that all zones have equal length in all routes.

4.1 The generic case

The instruments for the market design include the set of routes $\mathcal{R} \in \overline{\mathcal{R}}$ and the definition of zones $\{\mathcal{T}_r\}$. We say that the planner can implement the optimal number and allocation of flights with a market for noise licenses if and only if there exists a set of instruments $(\mathcal{R}, \{\mathcal{T}_r\})$ such that $y_r^* = y_r^o \forall r \in \mathcal{R}^o$. The efficient route allocation will be obtained if and only if $\frac{y_r^o}{y^o} = \frac{y_r^*}{y^*}$, that is, by using (1) and (4),

$$\frac{\sum_{s \in \mathcal{R}} (\overline{\beta}_s Z_s)^{-1}}{\sum_{s \in \mathcal{R}^o} B_s^{-1}} = \frac{(\overline{\beta}_r Z_r)^{-1}}{B_r^{-1}} \quad \forall r \in \mathcal{R} \quad (5)$$

The efficient activity level will be reached if and only if $y^* = y^o$, that is, by using (1) and (3),

$$\frac{\sum_{s \in \mathcal{R}} (\overline{\beta}_s Z_s)^{-1}}{\sum_{s \in \mathcal{R}^o} B_s^{-1}} = \gamma \quad (6)$$

These two equalities imply that

$$\overline{\beta}_r Z_r = \frac{1}{\gamma} B_r, \quad \forall r \in \mathcal{R} \quad (7)$$

Condition (7) determines the trade-off in the design of the market for noise licenses. The market designer is able to implement the planner's outcome if and only if the total damage over the critical zone $\bar{\beta}_r$ times the number of zones Z_r is equal to $1/\gamma$ times the total damage over all zones B_r . To fulfill this condition, the market designer can either adapt the size of the critical zone or the number of zones. Indeed, the value of $\bar{\beta}_r Z_r$ is a function of the design of zones which is bounded below by B_r when there is only one zone ($Z_r = 1$) and which can be made equal to any real number above B_r by an appropriate choice of design of zones (see the proof of the following proposition). From this argument, it comes that the market designer is able to find a design of zones that verifies condition (7) only if $\gamma \leq 1$.

Finally, plugging expression (7) in (5) yields the equality

$$\sum_{s \in \mathcal{R}} B_s^{-1} = \sum_{s \in \mathcal{R}^o} B_s^{-1}$$

which implies that $\mathcal{R} = \mathcal{R}^o$. Since $\mathcal{R}^o = \bar{\mathcal{R}}$, the planner allows residents located on any feasible routes to supply noise licenses: $\mathcal{R} = \bar{\mathcal{R}}$.

The following proposition summarizes the necessary and sufficient condition for implementation.

Proposition 2 *The social planner's optimal allocation of flights can be implemented by a market for noise licenses if and only if $\gamma \leq 1$. In this case, the market designer opens all feasible routes ($\mathcal{R} = \mathcal{R}^o = \bar{\mathcal{R}}$) and sets the design of zones $\{\mathcal{T}_r\}$ such that $\bar{\beta}_r Z_r = B_r/\gamma, \quad \forall r \in \bar{\mathcal{R}}$.*

Proof. See Appendix. ■

Under Proposition 2 the market designer is able to implement its optimal allocation of flights by an appropriate design of zones only if he/she does not put too large a weight on the airport economic activity. In this case, he/she prefers a low aircraft activity and he/she can raise either the number of non-critical zones or the size of the critical zones so that the price of noise licenses increases. The number of zones

and the size of the critical zones are therefore substitutable instruments, which gives the planner some freedom in the market design. By contrast, the planner is not able to implement its optimal solution with a market for noise licenses when he/she puts too high a weight on profits or economic activities ($\gamma > 1$). In this case, he/she desires a flight activity that conflicts too strongly with the residents' interest. Residents indeed set prices of noise licenses that are too high and the aircraft activity remains too small compared to the optimal level even if the planner has reduced the price of flying over a route to its minimal value (by setting $Z_r = 1$).

It is interesting to discuss the situation where noise pollution is uniformly distributed over each route. For instance, this may happen when aircrafts keep the same altitude on neighborhoods. In this case, the location-noise disutility parameter is constant on each route: $\delta_r(t) = \delta_r$. So, condition (7) writes as

$$\frac{\int_{t_{r,\bar{z}-1}}^{t_{r,\bar{z}}} n_r(t) dt}{\int_0^T n_r(t) dt} = \frac{1}{Z_r \gamma} \quad \forall r$$

where \bar{z} is the index of a critical zone. The market design requires only information on population densities. The left hand side of this equation represents the proportion of the population of route r in the critical zone. Once this proportion is chosen, the optimal number of zones can be set. For example, suppose that $\gamma = 1/2$ and that the market design includes a critical zone covering 40 per cent of the population on the route. Then the optimal number of zones is $Z_r = 5$.

It also is interesting to discuss the dual situation where the population density is uniform over each route. This case approximates the situation where one route passes over a highly populated city and another route passes over a less dense sprawl. In this case, the population density is constant on each route: $n_r(t) = n_r$. So, condition (7) becomes

$$\frac{\int_{t_{r,\bar{z}-1}}^{t_{r,\bar{z}}} \delta_r(t) dt}{\int_0^T \delta_r(t) dt} = \frac{1}{Z_r \gamma} \quad \forall r$$

Here, the market design requires only information on noise profiles and *individual*

disutility of noise exposure. Assuming that noise levels are proportionally reflected in the individual disutility of noise exposure, the market designer can base his/her design of zones on the noise profiles.

In Proposition 2 the number of zones and the boundaries of critical zones are two substitutable instruments. Imposing a restriction on one of these instruments does not necessarily prevent the market designer from implementing the social optimum. For instance, if the number of zones Z_r cannot be freely chosen, the market designer is still able to choose the boundaries of the critical zone (*i.e.* choose $\bar{\beta}_r$). Conversely, if the critical zones cannot be freely chosen, then he/she is still able to choose the number of zones such that condition (7) holds. However, one may wonder about the feasibility of the implementation of the social optimum allocation when the market designer faces some constraints on both instruments. Of course, when the zones are exogenously defined, for instance by administrative boundaries, the market designer has no degree of freedom in his/her design and the social optimum cannot be implemented. In the following sections, we consider an intermediate situation in which the market design is still possible but for a smaller set of parameters.

4.2 An example of restricted design

Let us consider for expositional purposes the case of an additional constraint that imposes that routes must have equal lengths, $T_r \equiv T$, and equal number of zones, $Z_r \equiv Z$. As a result, routes are divided into equal intervals, $t_{z-1} - t_z = T/Z$. Because this restriction simultaneously fixes the number and the boundaries of zones, it reduces the market designer's degree of freedom. By equation (7) we get that,

$$\bar{\beta}_r Z = \frac{B_r}{\gamma}$$

which must be compatible with $Z \geq 1$. We remind that the function $\bar{\beta}_r Z$ is bounded below by B_r ($Z = 1$). This function also is bounded above by $T\beta_r^{\max}$ where $\beta_r^{\max} \equiv \max_{t \in [0, T]} \beta_r(t)$. Indeed, zone lengths are equal to T/Z and, when Z is large enough

the disutility parameter of the critical zone, $\bar{\beta}_r$, can be approximated by $\beta_r^{\max}*(T/Z)$. As a result, $\bar{\beta}_r Z$ tends to $T\beta_r^{\max}$ when $Z \rightarrow \infty$. The market designer is then able to implement the optimal number and allocation of flights with a market for noise licenses if B_r/γ takes any value between these two bounds. This argument yields the following proposition.

Proposition 3 *Suppose the market designer is constrained to design zones and routes with equal lengths. Then, the socially optimal allocation of flights can be implemented by a market for noise licenses if and only if $B_r/(T\beta_r^{\max}) < \gamma \leq 1$.*

Hence, the constraint of zones of equal length does not eliminate the possibility of implementation of the social optimum *via* a market for noise licenses, but it reduces the set of parameters γ for which this is possible. The market designer is not able to implement the optimal allocation of flights when γ is too low because he/she is unable to design a set of zones that induces a high enough price of noise licenses and thus a low enough aircraft activity. However, the restriction on equal lengths of zones obliges him/her to reduce at the same time the size and the total disutility of critical zones, which has the effect of decreasing the price of noise licenses.

In this section we have analyzed the design of markets for noise licenses and we have established the conditions under which a market designer can implement the socially optimal allocation of flight under the three following conditions: all zones belong to the same jurisdiction, all zone representatives are price takers and the spatial distribution of residents is given. We relax each of these assumptions in the following sections.

5 Flights over independent jurisdictions

In some situations airport noise pollution may cover different jurisdictions with conflicting interests. The airport may for instance bring local benefits to only one

jurisdiction (employment, accessibility, ...) whereas flights pass over zones that belong to some other jurisdictions. One example is the case of the Brussels Airport (Belgium) that offers large job opportunities to the Flemish region and that dispatches flights over both the Flemish and Brussels regions. In many respects, the two regions' governments are independent institutions. Such a case can be handled in our setting. In this section, we establish the conditions under which a market for noise licenses can be an appropriate economic instrument for the jurisdiction that hosts the airport, receives the major part of its benefits and designs the market.

We assume two jurisdictions A and B , with jurisdiction A hosting the airport. Hence, each route r includes at least one zone in jurisdiction A and maybe some zones in jurisdiction B . Let $\mathcal{T}_r^A \equiv \{\dots, (t_{r,z-1}^A, t_{r,z}^A], \dots\}$ be the set of all zones under route r in jurisdiction A . Aggregate noise pollution measure for this jurisdiction is equal to $B_r^A \equiv \int_{\mathcal{T}_r^A} \beta_r(t) dt \leq B_r$.

We also assume that jurisdiction A is not altruistic with respect to jurisdiction B 's residents and that it has a share of economic returns: $\gamma^A \in [0, \gamma]$ ($\gamma^A + \gamma^B = \gamma$). When jurisdiction A plans its optimal airport activity, it maximizes

$$W^A = \gamma^A \int_0^y (\bar{\pi} - x) dx - \sum_{r \in \mathcal{R}} \int_0^{T_r^A} d_r(t, y_r) dt \quad \text{s.t.} \quad y = \sum_{r \in \mathcal{R}} y_r$$

which gives the same solution as in Section 2.4 except that γ and B_r must now be replaced by γ^A and B_r^A :

$$y^A = \frac{\bar{\pi}}{1 + [\gamma^A \sum_{r \in \mathcal{R}^A} (B_r^A)^{-1}]^{-1}} \quad \text{and} \quad \frac{y_r^A}{y^A} = \frac{(B_r^A)^{-1}}{\sum_{s \in \mathcal{R}^A} (B_s^A)^{-1}}$$

For the same reason as in Section 2, jurisdiction A chooses to open all routes: $\mathcal{R}^A = \bar{\mathcal{R}}$.

The first expression shows that if jurisdiction A has the full economic benefit of the airport activity ($\gamma^A = \gamma$) then it plans a higher flight activity than the multi-jurisdictional planner discussed in Section 2. Indeed, jurisdiction A promotes higher

flight activity because it accounts for a smaller number of disturbed individuals. Yet, this statement is not always true. One can indeed check that the necessary and sufficient condition for a higher optimal number of flights, $y^A \geq y^o$, is given by the condition: $\gamma^A/\gamma \geq [\sum_r(B_r^{-1})]/[\sum_r(B_r^A)^{-1}]$. Hence, if jurisdiction A gets a small enough share of economic benefits it will plan a flight activity smaller than the multi-jurisdictional planner's.

The second expression shows that flight allocation on a route decreases as noise pollution over jurisdiction A 's territory increases. In particular, jurisdiction A will allocate a large proportion of flights on the routes that pass over few residents of its jurisdiction (small B_r^A).

We now ask whether jurisdiction A is able to implement its own social optimal allocation of flights while not making jurisdiction B worse off. Because a market for noise licenses cannot make residents in any zone worse off, residents of jurisdiction B surely have an interest to accept the implementation of such a market. Therefore, the question simply becomes whether a market designer is able to choose the design of routes and zones that implements jurisdiction A 's social optimal allocation of flights. For a given design of routes and zones, the market satisfies the same conditions as in Proposition 1. As in Section 4.1, the airport activity depends on the number and the design of zones and routes, which can be chosen to satisfy the optimal flight allocation for jurisdiction A 's planner. Comparing the above expression to Proposition 1, jurisdiction A must choose a design such that

$$Z_r \bar{\beta}_r = \frac{1}{\gamma^A} B_r^A \quad \forall r \quad (8)$$

The argument is the same as in Section 4.1 except that there are two types of routes to consider. First, consider the routes that do not pass over jurisdiction B . In that case we get that $B_r^A = B_r$ and that expression (8) is identical to the one in Proposition 1. Therefore, we need the condition $\gamma^A \leq 1$. Second, consider the routes that pass over jurisdiction B . Note firstly that there exist two jurisdictions

with at least one zone in each. So, the number of zones must be larger than or equal to 2. Secondly, observe that the largest value of the coefficient $\bar{\beta}_r$ of critical zone is given by either B_r^A or $B_r - B_r^A$, depending on whether the critical zone is in jurisdiction A or B . Therefore the smallest value of $Z_r \bar{\beta}_r$ is obtained when the market designer sets the smallest number of zones. That is, when $Z_r = 2$ we get that $Z_r \bar{\beta}_r = 2 \max\{B_r^A, B_r - B_r^A\}$. Finally, it is possible to set zones so as to increase the value of $Z_r \bar{\beta}_r$ and find a market design that satisfies expression (8). As a result, a market design is feasible if and only if $2 \max\{B_r^A, B_r - B_r^A\} \leq \frac{1}{\gamma^A} B_r^A$. This argument is summarized in the following proposition.

Proposition 4 *Suppose the market designer of jurisdiction A is able to choose the routes, the number of zones and their boundaries. Then, the jurisdiction A 's optimal allocation of flights can be implemented by a market for noise licenses if and only if $\gamma^A \leq \min_{r \in \mathcal{R}^B} \{1/2, B_r^A / [2(B_r - B_r^A)]\}$ where \mathcal{R}^B is the set of routes that pass over jurisdiction B .*

The model with two jurisdictions imposes two restrictions on the parameter γ^A in order to allow the market designer to implement its preferred allocation with a market of noise licenses. When those restrictions are binding, the market yields a high price and a low flight activity although jurisdiction A puts a high weight on its own economic benefit and promotes a higher flight activity.

The first restriction ($\gamma^A \leq 1/2$) stems from the existence of a second jurisdiction that obliges jurisdiction A to design more than two zones. Since the price of the noise licenses increases with the numbers of zones, jurisdiction A cannot obtain a price as low as the one obtained by the multi-jurisdiction planner who is able to design a market with a unique zone.

The second restriction ($\gamma^A \leq B_r^A / [2(B_r - B_r^A)]$) stems from jurisdiction A ' opportunity to shift noise damage to the neighboring jurisdiction B . Jurisdiction A may indeed be willing to concentrate the flight activity on the routes that host only

a small share of its residents (small B_r^A) but that host a large share of the residents located in jurisdiction B (large $B_r - B_r^A$). Yet, this strategy cannot be achieved by a market for noise licenses because jurisdiction B 's residents do not allow the noise license price to fall. This line of argument gives some insight into how jurisdiction A should design the residential zones to reach its optimal outcome. If the optimal number of zones Z_r is large (low γ^A), jurisdiction A may ask for bids from small associations of residents (*e.g.* at district level) in both jurisdictions. If the optimal number of zones Z_r is small (high γ^A), it may organize one zone per route in its territory and ask the other jurisdiction to bid on behalf of its residents. In any case, the intervention of a federal or multi-jurisdictional government may not be required in such a context.

Note finally that *jurisdiction A should keep the control of the market design over the design of routes and zones passing over jurisdiction B to be able to implement its optimal number and spatial distribution of flights*. Indeed, suppose this is not the case. That is, jurisdiction A aims at inducing its optimal aircraft activity $\{y_r^A\}$ by setting its market design $\{\mathcal{T}_r^A\}$ that satisfies condition (8) while jurisdiction B is able to choose its design $\{\mathcal{T}_r^B\}$. Because the aircraft activity is fixed to $\{y_r^A\}$, the noise disutility is fixed everywhere. For the same reason, the demand is fixed to y^A and the price of routes is given by a same value P . As a result, the profits of airline companies are also fixed. Therefore, two terms of jurisdiction B 's objectives, noise disutility and profits, are fixed. Jurisdiction B is nevertheless able to alter the last term in its objective related to the proceeds from noise licenses on each route: $P y_r^A Z_r^B / (Z_r^A + Z_r^B)$, $\forall r \in \mathcal{R}^B$. As a result, jurisdiction B 's best strategy is to augment those proceeds by infinitely dividing its routes, so that $Z_r^B \rightarrow \infty$. Jurisdiction A is obviously unable to respond to this strategy because it cannot diminish both Z_r^A and $\bar{\beta}_r$ to zero in order to keep the condition (8) binding. As a result, a jurisdiction is able to implement its optimal solution only if it receives the full authority on the market design. Even though the rights for a quiet environment are spatially

distributed, the authority on the market design must be given to a single agency.

6 Non competitive markets

In the above market design, zone representatives and airlines companies were assumed to be price takers in the market for noise licenses. This assumption may be questioned. Indeed, some airlines companies may be price makers as they demand a large share of noise licenses. At the same time, some critical zones are likely to be price makers because the number of technically feasible routes is not expected to be that large, and because non-critical zones have no impact on the number of flights over routes. Because our paper focuses on the issue of granting noise licenses to residents, it is natural to concentrate our discussion on residents' market power. Further, it is many times feared that residents would use their power to put a veto on airport activity. We therefore elaborate a game theoretic foundation for the market for noise licenses where the zone representatives can be strategic.

We propose a market design in which each zone representative simultaneously decides on a finite number of licenses to supply. The 'neutral auctioneer' allocates flights according to the minimum number of licenses on each route. This corresponds to a Cournot Nash equilibrium where zone representatives fix the number of flights. In this section we show the conditions under which there exists a design of zones $\{\mathcal{T}_r\}$ such that the social optimal allocation of flights can be implemented by a market for noise licenses under imperfect competition.

Let the utility of the zone representative be defined as $U_{rz}(y_{rz}) = p_{rz}y_{rz} - \beta_{rz}y_{rz}^2/2$ as before. Each representative sets a number of licenses y_{rz} for his/her zone. The market auctioneer sets the number of licenses to its minimum over each route, $y_r = \min_z\{y_{rz}\}$; as before, he/she allocates the same price $P = P_r$ for all routes and all zones $p_{rz} = P/Z_r$; and he/she balances supply $\sum_r y_r$ with demand $y \equiv \bar{\pi} - P$. Because the price on routes P depends on other zones, the utility in a zone will

depend on other zones. Let y_{-rz} be the set of supplies by all zones different from rz , *i.e.* $y_{-rz} \equiv \{y_{ij}\}_{ij \neq rz}$. The utility of zone z 's representative on route r is given by

$$U_{rz}(y_{rz}, y_{-rz}) = \frac{1}{Z_r} P(y_{rz}, y_{-rz}) \min_j \{y_{rj}\} - \beta_{rz} \left(\min_j \{y_{rj}\} \right)^2 / 2$$

where the market price is equal to

$$P(y_{rz}, y_{-rz}) \equiv \bar{\pi} - \sum_s \min_j \{y_{sj}\}$$

This utility depends on the number of flights supplied by the critical zone on route r , $\min_j \{y_{rj}\}$, and it depends on the revenues from the sales of licenses, which decrease with the number of zones Z_r and with the supplies of critical zones on all routes. A Cournot Nash equilibrium is defined as the number of flights y_{rz}^c such that

$$y_{rz}^c \in \arg \max_{y_{rz}} U_r(y_{rz}, y_{-rz}^c) \quad \forall r, \forall z$$

We formally derive the best response correspondences and the equilibrium in the Appendix. An informal proof is provided hereafter. The main idea is that non-critical zones are never enticed to supply less than critical zones' supply. So, the supply of critical zones binds in equilibrium. As a result, the supply of licenses on a route is given by the supply of its critical zone $y_r \equiv \min_z \{y_{rz}\}$. The utility of the critical zone can be re-written as function $U_r(y_r, y_{-r})$ that depends on the supply of the critical zone and of other critical zones, $y_{-r} \equiv \{y_i : i \neq r\}$. The best responses then can be written as

$$y_r^{BR}(y_{-r}) = \arg \max_{y_r} U_r(y_r, y_{-r}) = \frac{\bar{\pi} - \sum_{s \neq r} y_s}{2 + \beta_{rz} Z_r}$$

In equilibrium we must have that $y_r^c = y_r^{BR}(y_{-r}^c)$ for all r . Solving this equality for all routes r , we get

$$y^c = \frac{\bar{\pi}}{1 + \left[\sum_s (1 + \bar{\beta}_s Z_s)^{-1} \right]^{-1}} \quad \text{and} \quad \frac{y_r^c}{y^c} = \frac{(1 + \bar{\beta}_r Z_r)^{-1}}{\sum_s (1 + \bar{\beta}_s Z_s)^{-1}} \quad (9)$$

One can check that the number of flights falls with the number of zones Z_r on a route r and with noise pollution in the critical zones, $\bar{\beta}_r$. Also, the routes that are allocated fewer flights are those with higher aggregate noise damage in critical zones, $y_r^c \leq y_s^c \iff \bar{\beta}_r \geq \bar{\beta}_s$.

We may compare the Cournot equilibrium with the competitive equilibrium holding the design of routes and zones fixed. First, it is easy to check that the number of licenses is smaller in the Cournot situation ($y^c < y^*$). Market power naturally decreases the number of licenses in equilibrium. Yet, as the number of routes (not zones) increases the Cournot number of licenses converges to the competitive number, because $y^c \rightarrow \bar{\pi}$ and $y^* \rightarrow \bar{\pi}$. Note that a planner can restore the equilibrium number of flights by reducing the number of zones Z_r . Second, comparing (4) and (9), one can show that

$$\frac{y_r^c}{y^c} \geq \frac{y_r^*}{y^*} \iff \sum_{s \in \mathcal{R}} \frac{(\bar{\beta}_s Z_s)^{-1} - (\bar{\beta}_r Z_r)^{-1}}{(1 + \bar{\beta}_s Z_s)} \geq 0$$

which is true for $r = \arg \max \bar{\beta}_s Z_s$ and false for $r = \arg \min \bar{\beta}_s Z_s$. Hence there exists a subset of routes $\mathcal{R}_c \in \mathcal{R}$ that accepts a larger proportion of flights under imperfect competition than under perfect competition, and a complementary subset $\mathcal{R} \setminus \mathcal{R}_c$ that accepts a smaller proportion of flights. Routes that accept a proportionally larger number of flights under imperfect competition bear higher noise pollution, $\bar{\beta}_r Z_r$. It is instructive to study the case where the number of zones is equal, $Z_r = Z$. It naturally comes that *a route hosting a critical zone with higher noise pollution accepts a higher proportion of flights under the non-competitive equilibrium than under the competitive one*. In other words, when shifting from price taking to price making behavior, such a route reduces proportionally less its supply of licenses. This is consistent with the fact that the critical zone on this route displays a higher marginal noise disutility and, as a result, has a lower supply elasticity of noise licenses. Similarly, one can study the case where critical zones have the same noise pollution parameters, $\bar{\beta}_r = \bar{\beta}$. Then, a critical zone on a route with a larger number

of zones will proportionally reduce less its offer of flights when it is able to exert its market power. Indeed, the revenue increase caused by the contraction of supply must be shared amongst more zones, so that incentives to exert market power are lower.

Is it possible to replicate the optimal number and allocation of flights under this non competitive equilibrium? In particular, one may expect that representatives of critical zones use their market power to reduce the flight activity below its socially optimal level. Comparing the non competitive equilibrium and the first best allocation yields the following proposition.

Proposition 5 *The social planner's optimal allocation of flights can be implemented by a non competitive market for noise licenses if and only if $\gamma \leq \min_r \{B_r/(B_r + 1)\} < 1$. In this case, the market designer opens all feasible routes ($\mathcal{R}^c = \mathcal{R}^o = \overline{\mathcal{R}}$) and sets the design of zones $\{\mathcal{T}_r\}$ such that $\overline{\beta}_r Z_r = B_r/\gamma - 1, \forall r \in \overline{\mathcal{R}}$.*

Proof. See Appendix. ■

As in the case of perfectly competitive markets for noise licenses, the optimal allocation of flights can be replicated by the market. Yet, the higher market power of zone representatives reduces the set of parameters γ for which the market for noise licenses implements the first best. At a given design, critical zones offer less licenses in the non competitive markets, so that equilibrium prices are higher and flight activity smaller. Therefore, the same design under a non competitive market implies a smaller flight activity than under a competitive market.

7 Residential mobility and city structure

We have shown in the previous sections that a market for noise licenses allows the implementation of a central planner's solution. The main advantage of noise licenses is their ability to automatically adjust for short term perturbations (*e.g.*, a demand

increase) and to allow the revelation of residents' noise disutility. Thus, from a short term viewpoint, the flexibility and effectiveness of this instrument rank high. The question of how such a market may impact on the economy in the medium and long terms however deserves some attention. To be more precise, our objective in this section is to analyze the effects of implementing a market for noise licenses on the land market and, therefore, on the structure of residential areas. The rationale is that, through the proceeds received by residents from the selling of noise licenses, the housing and land markets may be affected.

The key feature in the following analysis is the time horizon in which agents make decisions and in which markets clear. One may reasonably assume that noise licenses are traded frequently, say on a quarterly basis in order to react to business cycles. This frequency contrasts with the lower frequency of households' decisions to relocate in other areas, and with the even lower frequency of the land owners' decisions about the housing structure.

For this analysis, let us introduce a new kind of agent, landlords. We model the relationship between airport, residents and landlords as a sequential game that goes as follows. First, landlords choose the lot size which hosts residents. Second, residents choose a place to locate and land prices adjust. Finally, residents organize themselves and participate in the market for noise licenses. This sequential game reflects the above mentioned idea that noise licenses are traded more frequently than land or houses. As in Section 3 we assume that residents and their representatives are price takers.¹⁸

In this section, we want to analyze how the market for noise licenses shapes the population distribution across zones. For that reason we neglect any heterogeneity within each zone. As a consequence, we now assume that zones are populated by residents with homogenous noise disutility, that is, $\delta_r(t) = \delta_{rz}$ and $n_r(t) = n_{rz}$, $t \in [t_{r,z-1}, t_{r,z}]$, where n_{rz} is the population size within zone z on route r and

¹⁸A similar analysis holds in the case of non competitive markets for noise licenses.

where δ_{rz} is the location noise disutility parameter in that zone. Therefore the noise disutility parameter $\beta(t)$ is constant over each zone and equal to $n_{rz}\delta_{rz} = \beta_{rz}$. Under this assumption, the objectives of residents and zones' representatives are perfectly congruent and the redistribution of the proceeds of noise licenses is not an issue.

Each resident's preference for his/her residence lot size s is described by a concave, increasing utility function $v(s)$ where $v(0) = 0$ and $\infty > v' > 0 > v''$ for all $s \geq 0$. Note that the population density is given by $1/s$. So, resident i 's utility is given by the revenues of noise licenses, the disutility of noise, the utility and the rent for residential space,

$$V_{rz}^i = \frac{p_{rz}y_r}{n_{rz}} - \delta_{rz}\frac{y_r^2}{2} + v(s_{rz}) - s_{rz}R_{rz}$$

where R_{rz} is the land rent (per acre). Because residents are homogenous they have the same use of space s_{rz} . The zone's representative has an aggregate utility given by

$$V_{rz} = p_{rz}y_r - \beta_{rz}\frac{y_r^2}{2} + n_{rz}v(s_{rz}) - n_{rz}s_{rz}R_{rz}$$

Finally, let us assume that the residential area of each disturbed zone consists of a land strip with unit width and with a length of $T_{rz} = t_{r,z-1} - t_{r,z}$. So, T_{rz} is both the length and the residential area of zone z on route r . In the long run, lot sizes cover the residential area so that $T_{rz} = n_{rz}s_{rz}$.

We solve this game backwards, starting with the equilibrium in the market for noise licenses.

7.1 Short term

In the last period, the land rent R_{rz} and the lot size s_{rz} of each resident in zone z on route r are given. The representative asks for a financial compensation for the aircraft activity that maximizes his/her utility and the market for noise licenses clears. The first order condition is the same as in Section 3 and yields the same

supply of noise licenses, $y_{rz}^S = p_{rz}/\beta_{rz}$. Hence, the market for noise licenses yields the same outcome; that is, the equilibrium price P^* and the flight allocations y_r^* are given by Proposition 1. The identity of critical zones is still given by $z = \arg \min_z \beta_{rz}$. The property rights on quietness give the residents a benefit that we call *windfall gain*. This corresponds to the additional utility each resident obtains from the sales of the licenses. Indeed, using (2) the total utility of the zone z 's representative can be computed as

$$U_{rz} = p_{rz}^2/(2\beta_{rz}) > 0 \quad (10)$$

so that residents get a positive rent from the sale of noise licenses. Therefore, in equilibrium, resident i 's utility is given by

$$U_{rz}^i = \frac{U_{rz}}{n_{rz}} = \frac{p_{rz}^{*2}}{2n_{rz}\beta_{rz}} = \frac{1}{2\delta_{rz}} \left(\frac{P^*}{Z_r} \frac{s_{rz}}{T_{rz}} \right)^2 \quad (11)$$

which is positive, increases with the equilibrium price of noise licenses P^* and falls as lot size s_{rz} shrinks (and therefore population n_{rz} rises).

The existence of windfall gains is well-known in the literature on environmental economics. It stems from the fact that a property right is given for free on a productive input which acquires a price when the market becomes operational. As the compensation offered by licenses is larger than the noise-induced loss of utility, residents earn a windfall gain. This is the very existence of windfall gains that shape medium and long term equilibria.

7.2 Mid term

In the second period, residents move across locations while the land prices R_{rz} adjust. The lot sizes and critical zones are still fixed. Let V_o^i be residents' utility outside the disturbed zones. Reasonably, the city is supposed to be large enough so that residents' utility outside the disturbed zones, V_o^i , is independent of the land and the market for noise licenses in the disturbed zones. Empirical evidence supports

this assumption in many cases.¹⁹ As soon as residents are mobile, they locate in disturbed zones only if, when doing so, they do not get less utility than outside, that is, only if $V^i \geq V_o^i$. At the mid-term equilibrium, this inequality binds and land rents absorb any utility difference. In equilibrium, this leads to a land rent defined as

$$R_{rz}^* = \frac{v(s_{rz}) - V_o^i + U_{rz}^i}{s_{rz}} \quad (12)$$

Hence, windfall gains are transferred to landlords in the mid term through higher land rents. Land rents are higher in zones where residents earned a larger windfall gain. Resident utility is the same as in other parts of the city. For tractability we will assume in the sequel that the windfall gains do not outweigh residents' utility in non-disturbed areas ($V_o^i > U_{rz}^i$).

7.3 Long term

In the first period, landlords decide on the lot size s_{rz} . We assume a competitive land market where landlords are numerous and price takers. They do not consider others' behavior in their decision process. In particular, they do not anticipate the aggregate migration of residents that may result from their lot size choices. Hence, each landlord finds the optimal lot size s_{rz} that maximizes his/her land rent R_{rz}^* given by (12) and taking the zone's population n_{rz} as given. So, his/her optimal rent solves the following first order condition

$$v(s_{rz}) - s_{rz}v'(s_{rz}) = V_o^i - U_{rz}^i. \quad (13)$$

By our assumption on v , the left hand side of this equality strictly increases from and above zero as the lot size s_{rz} increases from zero. Therefore, landlords further

¹⁹By using noise exposure maps for 35 major US airports, Morrison *et al.* (1999) conclude that noise damages typically affect less than 2 percent of the total number of housing units in each considered metropolitan area.

reduce lot sizes in zones earning larger windfall gains U_{rz}^i . Because the proceeds of noise licenses are redistributed to residents, each landlord has an incentive to divide his/her lots and to offer a place to more residents, recouping a larger share of the license proceeds. At a given price of noise licenses, each resident's windfall gain U_{rz}^i decreases as lot sizes shrink.

On the long run, as landlords reduce lot sizes, the number of residents in disturbed zones rises so that the price licenses (weakly) increases but so that the proceeds of each individual resident decrease. Windfall gains then decrease and an equilibrium can be reached with smaller lot sizes. In the Appendix we indeed show that the conditions (13) and $T_{rz} = n_{rz}s_{rz}$ yield a unique solution with non-zero lot sizes s_{rz}^* . Therefore the population in noisy areas T_{rz}/s_{rz}^* never explodes.

We can compare this value either to the lot size in the absence of both noise licenses and aircraft activity, or to the lot size in the absence of noise licenses but in the presence of aircraft activity. First, in the absence of both noise licenses and aircraft activity, residents get no windfall gain ($p^* = U_o^i = 0$) and the long-term lot space is equal to s^o which solves $v(s) - sv'(s) = V_o^i$. This is the lot size of the undisturbed areas. Comparing this to expression (13), we can readily infer that the long term lot space with noise licenses s_{rz}^* is smaller than s_{rz}^o . Therefore, the introduction of the market for noise licenses and aircraft activities decreases lot sizes and thus increases the population in each zone ($s_{rz}^* < s^o$ and $n_{rz}^* \equiv T_{rz}/s_{rz}^* > T_{rz}/s^o \equiv n_{rz}^o$).

Second, in the absence of noise licenses but in the presence of aircraft activity, residents get the noise damage without any compensation. The noise disutility must be compensated by higher lot size. This implies a fall in population size in each zone. Indeed, in the long run equilibrium each resident gets a utility equal to $V_{rz}^i = -\delta_{rz}y_r^2/2 + v(s_{rz}) - s_{rz}v'(s_{rz})$, which must be equal to the utility level in non disturbed zones, V_o^i . Therefore, the long run equilibrium condition is equal to $v(s_{rz}) - s_{rz}v'(s_{rz}) = V_o^i + \delta_{rz}y_r^2/2$, which gives the solution s_{rz}^{oo} . It is easy to check

that the lot size is even larger and that population is even smaller ($s_{rz}^{oo} > s_{rz}^o > s_{rz}^*$ and $n_{rz}^{oo} \equiv T_{rz}/s_{rz}^{oo} < n_{rz}^o < n_{rz}^*$). We summarize this discussion in the following proposition.

Proposition 6 *There exists a unique long term equilibrium with a market for noise licenses. In this equilibrium, land rents fully capture the windfall gain that residents obtain in the market for noise licenses. The introduction of the market for noise licenses thus decreases lot sizes and increases the population level in every zone ($s_{rz}^* < s^o < s_{rz}^{oo}$ and $n_{rz}^* < n_{rz}^o < n_{rz}^{oo}$).*

Proof. See appendix. ■

In the long run landlords entice the residents located outside the disturbed zones to come in the disturbed areas and to share the windfall gains offered to residents. Hence, the population size increases in these zones, raising in turn the price of noise licenses and reducing the flight activity in equilibrium.

Whereas the previous proposition compares lot and population sizes with their level without market for noise licenses, we now compare lot and population sizes between zones of different routes. Using expression (13) we have already established that landlords reduce lot sizes in zones earning larger windfall gains U_{rz}^i . Therefore, zones with higher population density will be associated to larger (individual) windfall gains. Yet, population densities are determined by the long run equilibrium. The following proposition determines the relationship between those population densities and the characteristics of zones, namely their exposure to noise and their land supply.

Proposition 7 *In the long term equilibrium, each resident benefits from a larger windfall gain in zones with higher population density. The population density is higher in zones with lower individual noise exposure (δ_{rz}) and with smaller land supply (T_{rz}). That is, $U_{rz}^i > U_{rz'}^i \iff 1/s_{rz}^* > 1/s_{rz'}^* \iff \delta_{rz}T_{rz}^2 < \delta_{rz'}T_{rz'}^2$, $\forall z \neq z' \forall r$.*

Proof. See appendix. ■

The intuition goes as it follows. First, landlords divide their lots in smaller parcels when residents benefit from high individual windfall gains. The population density is therefore higher in zones with larger windfall gains. Second, consider two zones of equal size. The one with the lower noise exposure will have the higher population density. This is because lower noise exposure increases windfall gains and entices landlords to attract more residents.

Finally, consider two zones with equal individual noise exposure. Then, the zone with the smaller area will have the higher population density in the long run. Indeed, if the smaller area had the same population density, then it would host a smaller population and individual windfall gains would be larger there. This would entice landlords to attract additional residents, which would increase the population density in that zone. As a result, the population density is necessarily higher in the zone with the smaller area.

The above Proposition also gives us an additional message: the identity of critical zones may eventually be different in the short and in the long run because population densities are different.²⁰ This result suggests that, in order to preserve the optimality of the market design, *the implementation of the market for noise licenses must be accompanied with a land use policy that restricts changes in lot sizes.*

²⁰To make things simple, let us concentrate on an example in which residents have initially no noise pollution and have same lot sizes (*i.e.* $s_{rz}^o = s^o$). Furthermore, suppose that the planner is able to set zones such that each zone offers the same windfall gain in the short run. This means that all zones are critical and have the same disutility parameter $\beta_{rz}^o = \beta_{rz'}^o$, which implies that $\delta_{rz}T_{rz} = \delta_{rz'}T_{rz'}$ since zones initially have the same short term population density ($1/s_{rz}^o = 1/s^o$). In the long run, we then get $\beta_{rz}^* > \beta_{rz'}^*$ which is equivalent to $\delta_{rz}T_{rz}/s_{rz}^* > \delta_{rz'}T_{rz'}/s_{rz'}^*$, or $s_{rz}^* < s_{rz'}^*$, or, by virtue of Proposition 6, is true iff $T_{rz} < T_{rz'}$ or equivalently iff $\delta_{rz} > \delta_{rz'}$. That is, population density ($1/s_{rz}$) is larger in zones with higher individual noise disutility. In this case, only the zone with the highest individual noise disutility becomes critical in the long term although all zones are critical in the short run. As a result, in the long term changes in lot size and population density may alter the identity of critical zones.

8 Conclusion

The main strand of literature on noise pollution around airports has neglected the issue of policy design to make airport facilities internalize the negative externality to surrounding residents. In this paper we focus on Coase's (1960) idea to achieve this goal with a market for noise licenses. We suggest to organize residents in zones and to allow them to offer noise licenses that must be bought by airline companies to fly over their zones. In such a design the noise externality is internalized and residents can never be worse off with the airport traffic. Local governments need no longer be involved in cumbersome information collection/studies on noise damages and in political arbitrage between supporters of environmental quality and airport economic activity.

We show that the market for noise licenses allows to achieve the planner's optimal allocation of flights provided that she/he does not put too much weight on the benefits of the economic activity compared to the disutility of noise pollution. The possibility that some zones may be strategic players does not fundamentally alter this finding. In the long run, because the market auctioneer is not allowed to perfectly discriminate, noise licenses offer a windfall gain to residents located on the routes. This entices landlords to increase their land/house rents and to set smaller houses in the long run.

This article proposes an original solution to the regulation of noise pollution around airports. Further research must be undertaken on additional issues like, for example, the possible cooperative behaviors of zones, the dominance of some airline companies in the market for licenses, a finer organization of the market place (auctioneer's task/algorithm), weather constraints, the heterogeneity of aircraft noise levels and the heterogeneity of residents.

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9 Appendices

Proof of Proposition 1 Demands for noise licenses are non-increasing and supplies are non-decreasing functions of prices P_r . There thus exists a unique equilibrium. Because on every route r , supply is smaller than demand at $P_r = 0$, the equilibrium price is interior ($P_r > 0$). Therefore, in equilibrium, routes have same price: $P_r = P^*$. Hence, $\sum_r y_r^S(P^*/Z_r) = \sum_r y_r^D(P^*, \dots, P^*)$ which gives $\sum_r P^* \bar{\beta}_r^{-1} Z_r^{-1} = \bar{\pi} - P^*$. This yields equations (3) and (4). \square

Proof of Proposition 2 Let $F(\mathcal{T}_r)$ be the function that returns the value of $\bar{\beta}_r Z_r$ for any design of zones \mathcal{T}_r on route r . We just need to prove that F is defined over $[B_r, \infty)$. We first prove that there exists a design of zones, \mathcal{T}_r , such that $\bar{\beta}_r Z_r$ can be made equal to any real number above B_r . That is, $\forall x \geq B_r, \exists \mathcal{T}_r$ such that $F(\mathcal{T}_r) = x$. This is true for the following possible design \mathcal{T}_r . It is such that the

number of zones Z_r is set to the integer strictly above $[x/B_r]$, the boundaries of the critical zone \bar{z} are set so that $\bar{\beta}_r = x/Z_r$ and the boundaries of other zones z are set so that $\bar{\beta}_r > \beta_{rz}$. Second, we prove that $F(\mathcal{I}_r)$ is not bounded from above. Indeed, choose for instance a critical zone with small length εT_r ($\varepsilon > 0$) that includes the location $t_r^{\max} \equiv \max_{t \in [0, T_r]} \beta_r(t)$ and, choose non-critical zones with equal length $\varepsilon T_r/M$ ($M \geq 1$) which is smaller than the length of the critical zone. Then, the number of critical zones is equal to $Z_r = 1 + M(1 - \varepsilon)/\varepsilon$. For small enough ε , Z_r can be approximated by M/ε and $\bar{\beta}_r Z_r$ by $\beta_r(t_r^{\max})\varepsilon T_r Z = \beta_r(t_r^{\max})T_r M$. It results from this design that $\bar{\beta}_r Z_r$ can be set as large as wanted if M is set to a large enough value. \square

Proof of Expression (9) Let y_{-rz} be the set of all supplies of zones different from rz : *i.e.* $y_{-rz} \equiv \{y_{r'z'}\}_{r'z' \neq rz}$. Utility of zone z on route r is given by $U_{rz}(y_{rz}, y_{-rz}) = \frac{1}{Z_r} [\bar{\pi} - \sum_{r'} \min_{z'} \{y_{r'z'}\}] \min_{z'} \{y_{rz}\} - \beta_{rz} (\min_{z'} \{y_{rz}\})^2 / 2$. A Cournot Nash equilibrium is defined as the number of flights y_{rz}^c such that $y_{rz}^c \in \arg \max_{y_{rz}} U_r(y_{rz}, y_{-rz}^c)$ for all r, z . The equilibrium is easily characterized. Suppose first that zone rz is the critical zone on route r . Then, its best response to other zones' supplies is given by

$$\begin{aligned} \hat{y}_{rz}(y_{-rz}) &= \arg \max_{y_{rz}} U_r(y_{rz}, y_{-rz}) \\ &= \arg \max_{y_{rz}} \frac{1}{Z_r} \left[\bar{\pi} - y_{rz} - \sum_{r' \neq r} \min_{z'} \{y_{r'z'}\} \right] y_{rz} - \beta_{rz} (y_{rz})^2 / 2 \\ &= \frac{\bar{\pi} - \sum_{r' \neq r} \min_{z'} \{y_{r'z'}\}}{2 + Z_r \beta_{rz}} \end{aligned} \quad (14)$$

This best response decreases with the aggregate noise pollution β_{rz} . Second, suppose that zone zr is not the critical zone on route r but that zone rz' ($rz' \neq rz$) is critical. Therefore, we must have that $\beta_{rz} < \beta_{rz'}$ and $\hat{y}_{rz}(y_{-rz}) > \hat{y}_{rz'}(y_{-rz'})$. In this case, zone rz is indifferent to any offer higher than $\hat{y}_{rz'}(y_{-rz'})$ since such offers will not change the number of flights on route r . Also, zone rz will not offer any number

of flights below $\widehat{y}_{rz'}(y_{-rz'})$. Indeed, this would reduce the number of flights further below its preferred level as a critical zone. Hence, the best reply of zone rz is given by the following correspondence:

$$y_{rz}^{BR}(y_{-rz}) = \begin{cases} \widehat{y}_{rz}(y_{-rz}) & \text{if } \widehat{y}_{rz}(y_{-rz}) \leq \min_{z' \neq z} y_{rz'} \\ [\min_{z' \neq z} y_{rz'}, \infty) & \text{otherwise} \end{cases} \quad (15)$$

In equilibrium we must have that $y_{rz}^c = y_{rz}^{BR}(y_{-rz}^c)$ for all rz . Then, we successively get that

$$\begin{aligned} y_r^c &= \min_z y_{rz}^{BR}(y_{-rz}^c) \\ &= \min_z \widehat{y}_{rz}(y_{-rz}^c) \\ &= \min_z \frac{\bar{\pi} - \sum_{r' \neq r} \min_{z'} \{y_{r'z'}^c\}}{2 + \beta_{rz} Z_r} \\ &= \frac{\bar{\pi} - \sum_{r' \neq r} \min_{z'} \{y_{r'z'}^c\}}{2 + \bar{\beta}_r Z_r} \\ &= \frac{\bar{\pi} - \sum_{r' \neq r} y_{r'}^c}{2 + \bar{\beta}_r Z_r} \end{aligned}$$

where the first and last equalities stem from the fact that the auctioneer takes the minimum offer of licenses on each route, where the second and third equalities follow from (15) and (14), and where the fourth equality uses $\bar{\beta}_r = \max_z \beta_{rz}$. Solving this equality for all routes r , we get expressions (9). \square

Proof of Proposition 5 By comparing the Cournot equilibrium (9) with the first best (1), we obtain

$$\begin{aligned} y^c = y^o &\iff \gamma \sum_s B_s^{-1} = \sum_s (1 + \bar{\beta}_s Z_s) \\ \frac{y_r^c}{y^c} = \frac{y_r^o}{y^o} &\iff \frac{1 + \bar{\beta}_r Z_r}{B_r} = \frac{\sum_s B_s^{-1}}{\sum_s (1 + \bar{\beta}_s Z_s)^{-1}} \quad \forall r \end{aligned}$$

Combining both equalities we get $\frac{1 + Z_r \bar{\beta}_r}{B_r} = \frac{1}{\gamma} \quad \forall r$ or equivalently,

$$Z_r \bar{\beta}_r = \frac{1}{\gamma} B_r - 1 \quad \forall r.$$

□

Proof of Proposition 6 We first prove the property $\delta_{rz}T_{rz}^2 > \delta_{rz'}T_{rz'}^2 \iff s_{rz}^* > s_{rz'}^*, \forall z \neq z' \forall r$. Indeed, suppose the contrary statement that, for a given price p^* , the inequalities $\delta_{rz}T_{rz}^2 \geq^*$ hold together. Then we get $\frac{1}{2\delta_{rz}} \left(\frac{p^*s_{rz}}{T_{rz}} \right)^2 < \frac{1}{2\delta_{rz'}} \left(\frac{p^*s_{rz'}}{T_{rz'}} \right)^2$ which is equivalent to $v(s_{rz}) - s_{rz}v'(s_{rz}) > v(s_{rz'}) - s_{rz'}v'(s_{rz'})$. By (13), this implies that $s_{rz} > s_{rz'}$, a contradiction. We then prove the existence and uniqueness of the equilibrium. Using $\bar{\beta}_r^{-1} = (\max_z \beta_{rz})^{-1} = (\max_z n_{rz}\delta_{rz})^{-1} = (\max_z s_{rz}^{-1}T_{rz}\delta_{rz})^{-1} = \min_z (s_{rz}T_{rz}^{-1}\delta_{rz}^{-1})$, one can show that

$$\begin{aligned} P^*s_{rz} &= \frac{\bar{\pi}s_{rz}}{1 + \sum_s \bar{\beta}_s^{-1}Z_s^{-1}} \\ &= \frac{\bar{\pi}s_{rz}}{1 + \sum_s \min_z (s_{sz}T_{sz}^{-1}\delta_{sz}^{-1})Z_s^{-1}} \\ &= \frac{\bar{\pi}s_{rz}}{1 + \min_z (s_{rz}T_{rz}^{-1}\delta_{rz}^{-1})Z_r^{-1} + \sum_{s \neq r} \min_z (s_{sz}T_{sz}^{-1}\delta_{sz}^{-1})Z_s^{-1}} \end{aligned}$$

This is a strictly increasing and continuous function of s_{rz} that has a zero at $s_{rz} \rightarrow 0$ and that is linear for sufficiently high s_{rz} . Therefore, using expression (11) it is easy to check that the right hand side of equation (13) decreases from and below V_o^i and falls to $-\infty$ as the lot size s_{rz} rises from 0 to ∞ . As a result, equation (13) has a unique a solution. So, the long-term equilibrium lot size exists and is unique. □

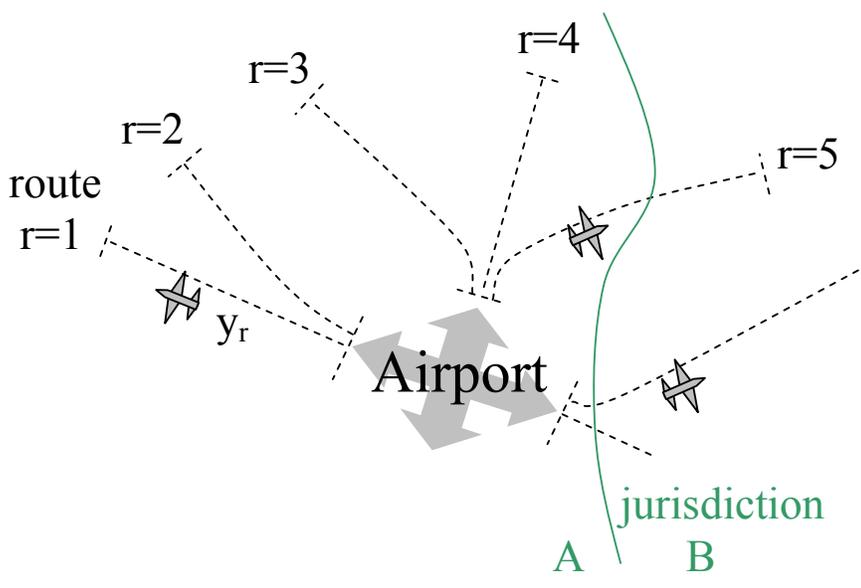


Figure 1 : Feasible routes from an airport.

route r

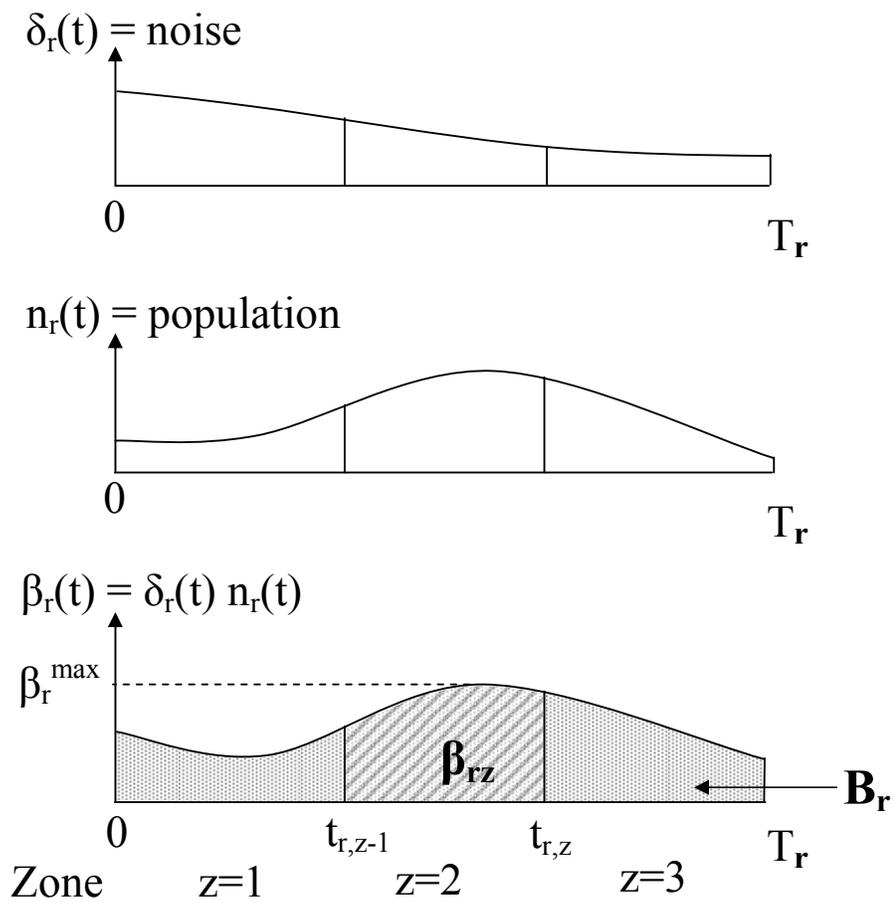


Figure 2 : Distribution of noise disutility parameter on route r and possible design of zones.

