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On the desirability of tax coordination when countries compete in taxes and infrastructures*

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Abstract

In our paper we show that when countries compete in taxes and infrastructures, coordination through a uniform tax rate or a minimum rate does not necessarily create the welfare effects observed under pure tax competition. The divergence is even worse when the competing jurisdictions differ in the quality of their institutions. If tax revenue is used to gauge the desirability of coordination, our model shows that imposing a uniform tax rate is Pareto-inferior to the non cooperative equilibrium when countries compete in taxes and infrastructures. This result is completely reversed with pure tax competition if countries are not too uneven in size. If a minimum tax rate lying between those resulting from the non-cooperative equilibrium is set, the low tax country will never be better off. Finally the paper shows that the potential social welfare gains from tax harmonization crucially depend on how heterogeneous the competing countries are.

Keywords: Tax competition, infrastructures, tax coordination, tax revenue, social welfare

JEL classification: H21; H87; H73; F21; C72

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1 Introduction

The debate about corporate tax coordination among international jurisdictions is still open. In particular it has been argued that the member states of the European Union should coordinate tax policy[1] to avoid a "race to the bottom" that would undermine their modern welfare states (Baldwin and Krugman, 2004). For this purpose, the Organization for Economic Cooperation and Development (OECD) launched in the 1990s a "harmful tax competition" initiative. In addition, the United Nations (UN) has called for the creation of an International Tax Organization, which would be specifically responsible for curtailing tax competition.

These concerns are in congruence with the large tax competition literature (for systematic reviews see Wilson, 1999; Wilson and Wildasin, 2004; Boadway and Tremblay, 2011). The main point is that independent governments engage in wasteful competition[2] for scarce capital through inefficient low tax rates and public expenditure levels. This has been formally modeled by Zodrow and Mieszkowski (1986) and Wilson (1986). Extensions have followed (see Wildasin and Wilson, 1996; Wilson, 1995; Hoyt, 1991; Bucovetsky, 1991).

In the literature two coordination devices aimed at correcting the inefficiencies[3] caused by tax competition have been highlighted: tax harmonization and the imposi-

[1]The Ruding Report (1992) made several far-reaching harmonization proposals for corporate taxation, including the imposition of an EU-wide minimum corporate tax rate (Haufler, 1999).

[2]Some other existing literature is related to welfare-improving tax competition. From the perspective of public choice, Brennan and Buchanan (1980) argue that tax competition reduces the excessive size of government and improves welfare. Rouscher (1998) and Edwards & Keen (1996) formalize the idea by tax competition in various 'Leviathan models'. Another branch is stimulated by Tiebout (1956), who was the first to investigate that competition between jurisdictions may promotes efficiency if households are able to sort themselves into jurisdictions composed of similar preferences, and receive public goods that are tailoring to their incomes and preferences. However, this is not our focus.

[3]The existing literature suggests other ways of coordination. Wildasin (1989) suggest that central governments can provide regions with a 'corrective subsidy', while Boadway and Flatters (1982) discuss the intergovernmental transfer when facing inefficiencies due to tax competition.

tion of a minimum tax rate. Tax harmonization is generally considered as a movement towards a common rate structure (Keen, 1987; Zissomos and Wooders, 2008)). More specifically, in the present paper we define tax harmonization as the equalization of tax rates which is consistent with the tax competition literature (see for example, Kanbur and Keen, 1993; Baldwin and Krugman, 2004; Zissimos and Wooders, 2008) and with general policy prescriptions[4]. The general conclusion of the classical literature is that appropriately chosen uniform tax rates improve efficiency compared to tax competition. The reason is that an upward harmonization of capital tax rates can produce a Pareto improvement (Baldwin and Krugman, 2004). This conclusion also holds true when the competing countries are asymmetric in size (Kanbur and Keen, 1993).[5] Another type of coordination is the adoption of a minimum tax rate which leaves some room to tax competition. An interesting result having been highlighted in the literature (see Keen and Konrad, 2012) is that the imposition of a minimum tax rate can be Pareto-improving for all the partner countries.

Many authors argue that jurisdictions not only compete in taxes but also in infrastructures (for example, Hindriks et al., 2008; Zissimos and Wooders, 2008 ; Justman et al, 2002, and Pieretti and Zanaj, 2011). However, the existing literature on the desirability of tax coordination is mainly based on the assumption that countries uniquely compete in tax rates. Perhaps one exception is Zissimos and Wooders (2008). These authors alternatively consider the setting of a minimum tax and tax harmonization as coordination devices. Governments are supposed to set simultaneously their tax level subject to the constraints imposed by policy coordination for given levels of public goods fixed at the non-cooperative equilibrium. But why should jurisdictions not adapt their infrastructures to the new environment caused by tax coordination when these expenditures are no more their best choices? Sticking to given infrastructure ex-

[4]In 2003 the EU Council adopted a voluntary Code of Conduct against harmful tax competition and more ambitious proposals for corporate tax harmonization have been put forward, including the introduction of a single EU corporate tax.(Conconi et al., 2008)

[5]Kanbur and Keen (1993) show that there exists a critical level above which harmonization results in tax revenue exceeding, for each jurisdiction, that of the non-cooperative equilibrium. However, a uniform level between the Nash equilibrium rates is certain to harm the small country.

penditures would not be rational. It would also drastically limit sovereign policy making since many infrastructure expenditures satisfy primarily internal policy goals and are incidentally attractive to foreign investments. Aware of this problem, we assume in our paper that tax coordination does not constrain infrastructure competition among sovereign jurisdictions. In other words, jurisdictions still compete in infrastructures though they may coordinate their taxes.

The purpose of this paper is to analyze the desirability of tax coordination when two heterogeneous jurisdictions compete for mobile entrepreneurs using taxes and infrastructures that improve firm productivity. These infrastructures may represent material and immaterial public goods such as law and regulations protecting intellectual property and specifying accurate dispute resolution rules. We thus model two dimensional strategic interactions within a game-theoretical approach. Furthermore, firms are supposed to be heterogeneous in their preference or their ability to relocate abroad. The model also accounts for two characteristics of the real world which are asymmetric both in country size[6] and in institutional quality. The importance of size asymmetry in tax competition has already been addressed by various authors (Bucovetsky, 1991; Wilson, 1991; Kanbur and Keen, 1993) but the role of institutional differences across jurisdictions has been neglected. It is also largely recognized that the institutional environment of a country impacts its economic performance. The reason is that the quality of the existing institutional framework in which activity takes place significantly shapes the business environment[7]. Indeed, nations endowed with higher qualitative institutions provide firms with better auditing and judicial systems and offer more efficient protection and enforcement of property rights[8]. In our model, we assume that the institutional environment of a country translates into territorial-specific productiv-

[6]Country size may be defined by population, area, or national income (Streeten, 1993). In this study, population, rather than area, is used to denote country size. More precisely, size is defined with respect to the number of capital owners who populate the country.

[7]La Porta et al. (2000) argue that countries have varying abilities to offer investors an attractive institutional environment. According to Acemoglu et al. (2001), institutions positively influence per capita GDP.

[8]Besley (1995) and Johnson et al. (2002) argue that strong property rights are attractive to investment.

ity conditions which are shared, independently of their country of origin, by all the firms located within a given jurisdiction.[9]

The main results may be summarized as follows. When tax revenue is used to gauge whether tax coordination dominates a non-cooperative equilibrium, the following results arise. If the jurisdictions decide to set uniform tax rates, coordination is Pareto-inferior to the non cooperative equilibrium when countries compete in tax and non-tax instruments. By contrast, if jurisdictions only compete in taxes, our model shows that tax harmonization can be Pareto-improving. Coordination consisting in the imposition of a lower bound on tax rates, increases only the revenue of the high tax country if jurisdictions compete in taxes and infrastructures. In other words, the low tax country will never be better off and the revenue loss increases with the weakness of its institution. If inter-jurisdictional tax redistribution is feasible, it is however conceivable that the country incurring a tax loss could be compensated if coordination increases joint revenue. We show however that for a range of minimum rate choices, there is no room for compensation. These results are however at odds with the classical outcome that imposing an appropriate minimum rate in pure tax competition improves the revenue of each jurisdiction (see Keen and Konrad, 2012). The results are also contrasted when we look at the potential welfare gains from tax coordination. When the jurisdictions decide to set uniform tax rates, the profitability of coordination crucially depends on how asymmetric countries are. In particular, if countries are symmetric in population size but have very unequally developed institutions, tax harmonization will be less efficient than tax and infrastructure competition. This result is however reversed if countries have equal institutional quality and are not too uneven in size. Finally, our model shows that minimum tax coordination always increases social welfare.

The rest of the paper is organized as follows. In section 2, we model tax and infrastructure competition between heterogeneous jurisdictions which try to attract im-

[9] Hindriks et al. (2008) also consider a model with uneven productivity levels across two regions which compete in taxes and public inputs to attract capital. They however don't consider tax coordination.

perfectly mobile firms. Section 3 analyses under which conditions tax harmonization is more desirable than tax and infrastructure competition. Section 4 analyses minimum tax coordination versus tax competition. Section 5 concludes.

2 The model

Consider two jurisdictions denoted by h and f. The countries' population is evenly spread with a unit density on a segment $[0, 1]$. Country h is assumed to be small in terms of total population and its size is given by S with $0 < S < \frac{1}{2}$. It follows that the size of country f equals $\frac{1}{2} < 1 - S < 1$. Similar to Pieretti and Zanaj (2011), we assume that each individual owns one unit of capital and is at the same time an entrepreneur and a worker. In other words, to each member of the population corresponds a one-person company[10]. The entrepreneurs can move their activity abroad, but we assume (see Ogura, 2006) that they are heterogeneous in their preference to leave their current location. The entrepreneurs are thus ranked according to their willingness to relocate abroad [11]. The closer an individual is to the border separating countries h and f, the easier she is able to relocate abroad. In other words, an entrepreneur of type $\alpha \in [0, 1]$ who moves abroad incurs a mobility cost equal to $|\alpha - S|$ which is the "distance" between the border S and the entrepreneur of type α.

Firms

Using one unit of capital, each individual living in country j $(j = h, f)$ is able to produce $y_j = q_j + \overline{\theta}_j$ units of one final good[12]. The parameter q_j $(j = h, f)$ represents

[10]It follows that the world population coincides with the population of firms. We could assume that each firm is run by more than one persons but this would unnecessarily complicate the model without further insights.

[11]As in Ogura (2006), we assume that this population of entrepreneurs is heterogeneous in the degree of their attachment to the home country. The sources of this home bias can be different. For example, transferring activities abroad requires a lot of information which can be different for each entrepreneur. Another cause can be linked to material relocation costs which can be specific to each firm.

[12]This production function is of the AK type, where $K = 1$ and $A = q + \theta$.

firm specific productivity, whereas $\overline{\theta}_j$ is the output fraction which is country-specific. More precisely, we write $\overline{\theta}_j = \theta_j^0 + \theta_j$, where θ_j^0 is a state parameter describing the institutional environment in country j and θ_j is the level of infrastructure spending planned by the policy-makers in country j. In other words, the quality of institutions and infrastructures results from history and current decisions.

The focus of the paper is on how uneven institutional quality and infrastructure expenditures impact the welfare effects of tax competition. So we assume that the firms-specific productivity is uniform across firms which means that $q_j = q$ $(j = h, f)$. For simplification purposes, we normalize $\theta_h^0 = 1$ and consider $\theta_f^0 = a\theta_h^0 = a \geq 0$, where the ratio a reflects the difference in institutional quality between the two economies. The ratio can be equal to one (the two economies are equal in quality), larger than one (the small economy has poorer institutional quality) or smaller than one (the large economy has poorer institutional quality). Finally, we assume that the final goods are sold in a competitive market with a price normalized to one. The unit cost of production is assumed to be constant and normalized to zero.

A firm of type $x \in [0, S]$ living in country h is indifferent between producing at home and in the foreign country f if

$$q_h + \overline{\theta}_h - t_h = q_f + \overline{\theta}_f - t_f - (S - x)\,, \tag{1}$$

where t_h and t_f are source-based tax rates levied on capital in countries h and f, respectively.

Similarly, a firm of type $x \in [S, 1]$ located in country f is indifferent between investing at home and investing abroad if

$$q_f + \overline{\theta}_f - t_f = q_h + \overline{\theta}_h - t_h - (x - S). \tag{2}$$

The above two conditions yield

$$x = (1 - a) + (\theta_h - \theta_f) + (t_f - t_h) + S. \tag{3}$$

Note that if $x > S$, firms move from the large to the small country, while if $x < S$, firms move from the small country to its large rival.

Governments

We now consider that countries try to attract companies by competing in taxes and public infrastructures which enhance private productivity. Jurisdictions h and f are thus able to influence the productivity parameter θ_j $(j = h, f)$ of the firms located within their respective boundaries. As in Hindriks et al. (2008) and Pieretti and Zanaj (2011), we assume that one additional unit of public good produces one additional unit of private good. It follows that θ_j also represents the amount of public good supplied by jurisdiction j $(j = h, f)$. The cost of providing this public good in each country j is given by the quadratic cost function $C(\theta_j) = \frac{1}{2}\theta_j^2$. Each jurisdiction j $(j = h, f)$ is supposed to maximize its total tax revenue[13], net of public expenditures, by choosing the appropriate tax rate t_j and infrastructure level θ_j. The governments' objective functions are given by

$$B_h = t_h x - \frac{1}{2}\theta_h^2, \;\; B_f = t_f(1-x) - \frac{1}{2}\theta_f^2. \tag{4}$$

We now assume that the two jurisdiction wish to attract productive capital by competing in taxes and infrastructures. Toward this end we consider a two-stage game[14]. First, the governments choose the level of infrastructures non-cooperatively. Then, they set the level of tax rates. Finally, firms decide where to locate their production process. We solve the game by backward induction.

Starting from the second stage, each government maximizes its objective with respect to its tax rate assuming its rival's rate as given. The first order conditions[15] yield

[13]For a similar assumption, see Kanbur and Keen (1993), Zissimos and Wooders (2008) or Pieretti and Zanaj (2011). In doing so we don't consider that jurisdictions are self-interested governments. We simply assume that collected taxes are used to finance public goods in the interest of their populations.

[14]The choice of sequentiality follows from the rule that the most irreversible decision must be made first.

[15]The second order conditions can easily be verified.

the following unique equilibrium tax rates

$$\begin{aligned} t_h &= \frac{1-a+(1+S)-\theta_f+\theta_h}{3}, \\ t_f &= \frac{a-1+(2-S)+\theta_f-\theta_h}{3}. \end{aligned} \tag{5}$$

It follows that the number of companies respectively located in countries h and f , are x and $1-x$, with

$$x = \frac{1-a+(1+S)+\theta_h-\theta_f}{3}.$$

After substituting the above tax rates into the jurisdictions' objective functions, we can solve for stage 1 of the game, where the two governments compete in public infrastructures θ_h and θ_f. It is easy to check that the objective function B_j $(j = h, f)$ is strictly concave in θ_j $(j = h, f)$. The first order conditions lead thus to the unique equilibrium expenditures

$$\theta_h^* = \frac{2}{15}(4-3a+3S), \;\; \theta_f^* = \frac{2}{15}(1+3a-3S). \tag{6}$$

Introducing (6) into (5) yields the equilibrium values

$$t_h^* = \frac{3}{2}\theta_h \;, \;\; t_f^* = \frac{3}{2}\theta_f^*. \tag{7}$$

Equation (7) shows that the country which taxes more than its rival also provides more public infrastructures.

The number of firms located at equilibrium in country h is given by

$$x^* = \frac{1}{5}\,(4-3a+3S). \tag{8}$$

It is straightforward to show that $x^* \in [0,1]$ and $\theta_j^* \geq 0 (j = h, f)$ if $a \in \left(S - \frac{1}{3}, S + \frac{4}{3}\right)$.

The tax differential between the large and the small countries equals

$$t_f^* - t_h^* = \frac{3}{2}\left(\theta_f^* - \theta_h^*\right) = \frac{3}{5}\,(a - \overline{a}) \text{ where } \overline{a} = \frac{1}{2} + S. \tag{9}$$

According to (9), it follows that $t_f^* > t_h^*$ if $a \in \left(\overline{a}, S + \frac{4}{3}\right)$, and $t_h^* > t_f^*$ if $a \in (0, \overline{a})$. The intuition underlying equation (9) is best understood if we suppose that countries would share the same institutional environment ($a = 1$). In this case, we get the standard result that the smallest country sets the smallest tax rate. If we assume now that the small country has the best institutional environment, it will be able to increase its tax rate. Consequently, competition will equalize the tax rates across jurisdictions if the small country's institutional quality is high enough. This case precisely happens when $a = \overline{a}$. In other words, $\overline{a}$ is the level of institutional disparity which exactly compensates the effect of size asymmetry on inter-jurisdictional tax differences. If countries are even in size ($S = \frac{1}{2}$), the level of $\overline{a}$ equals 1. But if country h becomes smaller relative to f (S decreases), the compensation level of institutional quality of country h has to increase relative to f. So $\overline{a}$ decreases with S. Finally we can say that the higher the gap $|a - \overline{a}|$, the more the competing jurisdiction differentiate themselves.

The equilibrium tax revenue of both countries are

$$B_h^* = \frac{7}{225}\left(4 - 3a + 3S\right)^2 \text{ and } B_f^* = \frac{7}{225}\left(1 + 3a - 3S\right)^2. \tag{10}$$

Hence, the joint tax revenue is $B^* = B_h^* + B_f^*$. As in Zissimos and Wooders (2008), we define efficiency as the maximum level of surplus available to all individuals in the two economies

$$W(x) = (\pi_h + \pi_f) + (B_h + B_f) - \int_0^{\left|x^T - S\right|} y dy. \tag{11}$$

The two terms in the brackets include, respectively, the total firms' profits[16] and the total tax revenues. The last term is the relocation cost of the moving companies. After simplification, the (joint) social welfare W^* resulting from inter-jurisdictional competition equals

[16]The profit in country j ($j = h, f$) is $\pi_j = (q + \overline{\theta}_j - t_j)x_j$.

$$W^* = \left[q + (1+\theta_h^*)\, x^* + \left(a+\theta_f^*\right)(1-x^*) - \frac{\left(\theta_h^*\right)^2}{2} - \frac{\left(\theta_f^*\right)^2}{2}\right] - \int_0^{|x^*-S|} y dy. \tag{12}$$

The sum of the terms included in the bracket is the global output and the second term is the total mobility cost of firms.

Plugging the equilibrium values of θ_h^*, θ_f^* and x^* into (12) leads to

$$W^* = q + \frac{1}{450}[333a^2 - 18a(37S+6) + 18S(6S+31) + 352]. \tag{13}$$

It is convenient to show that $W^* > 0$ for all $a \in \left(S - \frac{1}{3}, S + \frac{4}{3}\right)$ with $S \in \left(0, \frac{1}{2}\right)$.

3 Harmonization versus tax competition

We now consider that the two countries decide cooperatively to set uniform taxes, for given infrastructure expenditures. So they only compete in infrastructures. We further assume that the uniform tax rate is designed to maximize either global tax revenue or global social welfare. Both cases will be considered successively. Then we analyze under which conditions harmonization is desirable, adopting respectively the perspective of tax revenue and social welfare.

3.1 Tax harmonization

We define the uniform tax rate as follows:

$$t_h = t_f = t,\ t \geq 0.$$

Hence, the number of firms which located in the small country is given by

$$x = (1-a) + (\theta_h - \theta_f) + S.$$

We first solve the infrastructure game. Each jurisdiction chooses the level of public infrastructure θ_j by maximizing its revenue for a given tax rate t.

At equilibrium, we get

$$\theta_h^u = \theta_f^u = t.$$

It follows that

$$x^u = (1-a) + S.$$

If the institutional quality is higher in the small country ($1 > a$) it attracts $x^u - S$ firms from its large rival. Otherwise ($1 < a$), $S - x^u$ firms leave the small country. Since $x^u \in [0,1]$, we impose $a \in [S, 1+S]$.

The tax revenue of countries h and f resulting from infrastructure competition for given uniform tax rate is given as follows

$$B_h^u = t\left((1-a)+S\right) - \frac{1}{2}t^2 \text{ and } B_f^u = t(a-S) - \frac{1}{2}t^2. \tag{14}$$

The joint tax revenue becomes

$$B^u(t) = B_h^u + B_f^u = t\left(1-t\right), \tag{15}$$

where $B^u(t)$ is positive if $t \in (0,1)$.

The aggregate social welfare resulting from infrastructure competition with uniform tax rates is

$$W^u = q + \left[a^2 - (1+S)\,a + (1+S+t\,(1-t))\right] - \frac{1}{2}\,(1-a)^2. \tag{16}$$

Given that $t \in (0,1)$ and $S \in \left(0, \frac{1}{2}\right)$, W^u is always positive.

Now we are able to calculate the harmonized tax rate. Consider first the case where the jurisdictions agree upon a uniform rate which maximizes joint tax revenue. It is easy to see that $\overline{t} = \arg\max B^u(t) = \frac{1}{2}$. It follows that $\overline{B}^u = B^u(\overline{t}) = \frac{1}{4}$, $B_h^u(\overline{t}) = \frac{1}{8}\,(4S-4a+3)$ and $B_f^u(\overline{t}) = \frac{1}{8}\,(4a-4S-1)$. If tax harmonization is intended to maximize global social welfare we show that $t^s = \arg\max W(t) = \frac{1}{2}$. The resulting maximum social welfare equals $W(t^s) = \frac{1}{4}\left[4S\,(1-a) + 2a^2 + 3\right]$.

3.2 Comparing tax revenue

In this section we analyze the desirability of tax harmonization measured in terms of tax income. Comparing tax income resulting from tax and infrastructure competition with the maximum revenue resulting from tax harmonization shows that $B_h^* > B_h^u(\bar{t})$ and $B_f^* > B_f^u(\bar{t})$ for all $a \in [S, 1+S]$ and all $S \in (0, \frac{1}{2})$. In other words, if the common rate equals $\bar{t}$, tax harmonization does not make each country better off. The intuition underlying this result can be explained as follows. As in (Hindriks et al., 2008), our model implies that the more jurisdictions improve their attractiveness by currently investing in infrastructures, the fiercer tax competition will be in the second stage. The competing jurisdictions anticipate this effect in the first stage and thus underinvest in infrastructures compared to the tax harmonization scenario. This last case boils down to a one stage game without strategic interaction between taxes and public investments. As a result, tax revenue net of infrastructure expenditures is lower with the tax harmonization option.

The above finding is at odds with classical results according to which tax harmonization dominates pure tax competition if the uniform tax rate is high enough (see for example, Kanbur and Keen, 1993; Baldwin and Krugman, 2004 and Boadway and Tremblay, 2011). Our model leads to a similar conclusion (see Appendix A) if we restrict ourselves to pure tax competition with symmetric institutional quality by setting $a = 1$and $\theta_h = \theta_f = 0$. Indeed, in that case, tax harmonization generates more revenue than tax competition for both jurisdictions if size asymmetry is not too high. However, if revenue transfers are feasible, both countries are always better off with an appropriate common rate.

We can now state the following proposition

Proposition 1 *Moving from tax and infrastructure competition to tax harmonization decreases the tax revenue of all the competing countries. However if countries compete in taxes only, harmonization can be Pareto-improving in tax revenue.*

3.3 Comparing social welfare

Now we use social welfare to gauge the desirability of tax harmonization. Towards this end consider the difference

$$W^* - W(t^s) = Aa^2 + Ba + C,$$

where $A = \frac{6}{25}$, $B = -\frac{6}{25}(1+2S)$ and $C = \frac{1}{900}(216S + 216S^2 + 29)$. It is straightforward to show that $W^* < W(t^s)$ if $a \in (a_1, a_2)$ where[17] $a_1 = \overline{a} - \frac{5}{36}\sqrt{6}$ and $a_2 = \overline{a} + \frac{5}{36}\sqrt{6}$ and $W^* > W(t^s)$ if $a \in (S, a_1)$ or $a \in (a_2, 1+S)$. It follows that harmonization dominates tax competition as long as the disparity of institutional quality across countries is not too distant from $\overline{a}$. Recall that the larger the gap between a and $\overline{a}$ the more the competing jurisdiction are differentiated from a tax point of view. In the same vein, countries will be considered similar from a tax point of view if $a = \overline{a}$.

To explain in detail what happens, we can decompose the welfare difference $W^* - W(t^s)$ in the following way

$$W^* - W(t^s) = \Delta B + \Delta\pi, \tag{17}$$

where $\Delta B = (B_h^* + B_f^*) - [B_h(t^s) + B_f(t^s)]$ and $\Delta\pi = (\pi_h^* + \pi_f^*) - [\pi_h(t^s) + \pi_f(t^s)]$. From the previous section we know that the movement from interjurisdictional competition to harmonization increases the net joint tax revenue ($\Delta B > 0$ for all a). On the other hand, it can readily be shown that the same change of regime decreases joint profit (net of moving cost) ($\Delta\pi < 0$ for all a). The opposite signs of ΔB and $\Delta\pi$ have however a common cause. Indeed, inter-state competition generates more tax revenue than harmonization but less infrastructure expenditures. This benefits the governments and, by the same token, it hurts the private economy. Which of the two effects will dominate depends[18] on the value of a. Indeed, it is convenient to show that $\Delta B + \Delta\pi < 0$

[17] It can be shown that a_1 and a_2 respectively satisfy the conditions $S < a_1$ and $a_2 < 1 + S$.

[18] More exactly, we have $\Delta B = \frac{14}{25}a^2 + -\frac{14}{25}(1+2S)a + \frac{1}{900}\left(504S + 504S^2 + 251\right)$ and $\Delta\pi = -\frac{8}{25}a^2 + \frac{8}{25}a(1+2S) - \frac{1}{150}\left(48S + 48S^2 + 37\right)$.

if $a \in (a_1, a_2)$ and $\Delta B + \Delta\pi > 0$ if $a \notin (a_1, a_2)$. In order to explain the intuition underlying this result, note that ΔB increases faster than $-\Delta\pi$ when a moves away from $\overline{a}$ (see Figure 1). When a deviates increasingly from $\overline{a}$, the competing jurisdictions differentiate more and more from each other and tax competition becomes less intense. Consequently, joint tax revenue increases at a faster pace than infrastructure expenditures. Less intense inter-jurisdictional competition makes tax-payers more captive but decreases the importance of infrastructure attractiveness. At the governmental level, tax receipts augment faster than infrastructure expenditures and from the firms' perspective, productivity induced by public expenditures grows more slowly than tax payments. Two cases can then be considered.

a) When $a \in (a_1, a_2)$, the competing jurisdictions are not too different and tax harmonization is the most preferable option from the social point of view. Tax payers are then moderately captive compared to the importance of infrastructure attractiveness. As a result (see Figure 1), the relative gain induced by tax and infrastructure competition is not high enough to compensate the benefit of tax harmonization ($\Delta B < -\Delta\pi$).

b) When the institutional gap between the competing countries is large enough, i.e., $a \in (S, a_1) \cup (a_2, 1 + S)$, tax harmonization is no more the most efficient option. Tax competition has become less intense and taxing captive firms is relatively more beneficial than providing infrastructures. Accordingly, the relative benefit of tax and infrastructure competition (ΔB) increases to such an extent that it exceeds (see Figure 1) the benefit of harmonization ($-\Delta\pi$).

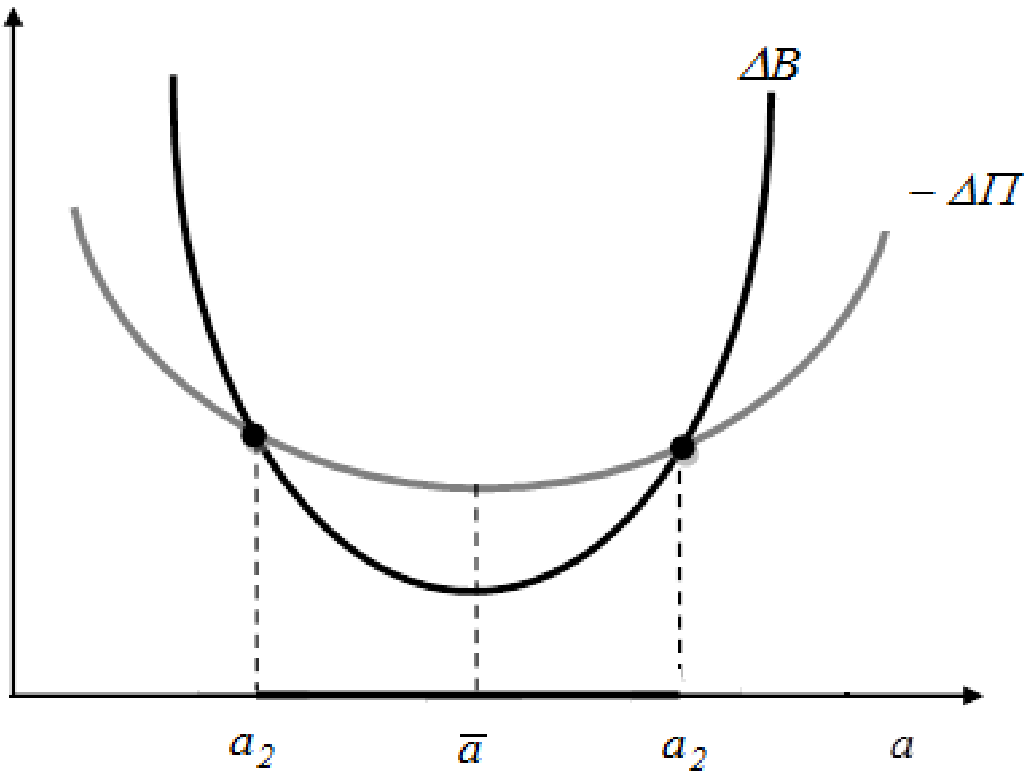

Figure 1

As a corollary of the above analysis, if countries are symmetric in size and have equally developed institutions[19], tax harmonization always dominates tax and infrastructure competition. However, if we consider asymmetric size while still supposing uniform institutional development, the result can be reversed. Indeed, tax and infrastructure competition is more efficient than tax harmonization ($W^* > W(t^s)$) if the size S of the small country is smaller than $\widehat{S} = \frac{1}{2} - \frac{5}{36}\sqrt{6}$. This last finding does not appear if we restrict ourselves to pure tax competition. Indeed, our model shows (see Appendix A) that moving from tax competition to tax harmonization always improves social welfare if $\theta_h = \theta_f = 0$ and $a = 1$.

The following proposition concludes

[19]This assumption is generally taken for granted in the tax competition literature.

Proposition 2

(a) If countries have equally developed institutions and are symmetric in size, tax harmonization is more efficient than tax and infrastructure competition. This result is however reversed if country size asymmetry is high enough.

(b) If countries have unequal institutional quality, harmonization can be less efficient than tax and infrastructure competition. This result arises if the competing jurisdictions are differentiated enough.

4 Minimum tax versus tax competition

Now we assume that the jurisdictions agree upon a minimum tax rate τ which is in between the tax rates resulting from tax and infrastructure competition. This option has been analyzed by some authors (see for example, Kanbur and Keen, 1993). We showed above that $a > \overline{a}$ implies $t_h^* < t_f^*$. Hence the minimum tax rate τ will be $\tau > t_h^*$. However, when $a < \overline{a}$ we have $t_h^* > t_f^*$ and thus $\tau > t_f^*$. In the following, we only analyze in detail the case where $t_h^* < t_f^*$ since the conclusions we derive still hold true for the alternative case.

4.1 Competition with a minimum tax

We now assume that the jurisdictions compete first in providing infrastructure expenditures and then in tax rates which are bounded from below. Since we assume that $a > \overline{a}$, the non-cooperative tax rate of the small country will be the lower bound ($\tau > t_h^*$). We know that the objective function $B_h(t_h)$ is concave in t_h. Consequently, the small country chooses its best tax rate which is $t_h^o(\tau) = \tau$. If the common lower bound τ is higher than t_f^*, the large country will also set $t_f^o(\tau) = \tau$, and we recover the case of harmonization. Hence we assume that $t_h^* < \tau < t_f^*$. The large country chooses then the tax rate $t_f^o[t_h^o(\tau)]$ which is its best reply to $t_h^o(\tau)$. Solving the game backwardly, we first analyze tax competition for given infrastructure expenditures and then we consider infrastructure competition. The solution of the game yields the following subgame perfect equilibrium values

$$\theta_h^o = \frac{\tau}{2},\ \ \theta_f^o = a - S + \frac{\tau}{2}, \tag{18}$$

$$t_h^o = \tau,\ \ t_f^o = a - S + \frac{\tau}{2}. \tag{19}$$

The share of firms that locate in the small country is $x^o = S - a - \frac{1}{2}\tau + 1$. Since $x \in [0, 1]$, we impose $\tau < \tau_m = 2(1 - a + S)$, which requires $a < 1 + S$. Furthermore, to guarantee that $\tau_m > t_h^*$, we impose that $a < \frac{6}{7} + S$. So, in the sequel we assume that $\tau \in [t_h^*, \min\{t_f^*, \tau_m\}]$ and $a \in \left(\frac{1}{2} + S, \frac{6}{7} + S\right)$.

The tax revenue of the small and the large countries are respectively $B_h^o = (1 - a + S)\tau - \frac{5}{8}\tau^2$, and $B_f^o = \frac{1}{8}(2a - 2S + \tau)^2$.

The joint tax revenue becomes

$$\begin{aligned} B^o &= B_h^o + B_f^o \\ &= \frac{1}{2}[(a - S)^2 + (2 - a + S)\tau - \tau^2]. \end{aligned} \tag{20}$$

The equilibrium social welfare resulting from the above equilibrium is

$$W^o = q + a^2 - 2aS + \frac{1}{8}[4S(2 + S) + (4 - 3\tau)\tau + 4]. \tag{21}$$

4.2 Comparing tax revenue and social welfare

Tax revenue

We first analyze if tax coordination by imposing a minimum tax rate increases the tax revenue of the competing countries. To this end, we compare for each country j ($j = h, f$) the difference $B_j^* - B_j^o$. In the Appendix B (Claims 1 and 2) we show that for $a > \overline{a}$ we get $B_h^* > B_h^o$ and $B_f^o > B_f^*$. In other words, imposing a lower bound on tax rates does not unanimously improve the revenue of the coordinating countries.

Indeed, it appears that the lower tax country[20] loses tax revenue by moving from a non-cooperative equilibrium to minimum tax coordination. Consequently, taking account of the fact that countries can, in addition to tax competition, also compete in infrastructures qualifies the classical result (see Kanbur and Keen, 1993) according to which imposing a minimum tax rate Pareto-improves countries' tax revenue (see Appendix A).

However, if coordination improves joint revenue the winner could possibly compensate the loser and each country could thus be made better off. So let us analyze if joint revenue improvement ($B^o > B^*$) is possible. In Appendix B (Claim 3) we show that $B^* > B^o$ if and only if $\tau \in \left(t_h^*, \min\{\underline{\tau}, t_f^*\}\right)$ which is only possible for $a \in (\overline{a}, a_m)$ where $a_m = \frac{5\sqrt{3}-3}{9} + S$. In other words, for certain minimum rate choices, there is no room for compensation if the institutional quality of the high tax country is not high enough. The following figure illustrates the just described conditions. If the minimum tax rate is included in the grey area, we have $B^* > B^o$ and $B^o > B^*$ in the yellow area. Note that the figure merges the case where $a > \overline{a}$ with the one where $a < \overline{a}$ for which we did not provide calculations since both cases are symmetric.

[20]Note that the lower tax country is the small one if $a > \overline{a}$, but it will be the large country if $a < \overline{a}$.

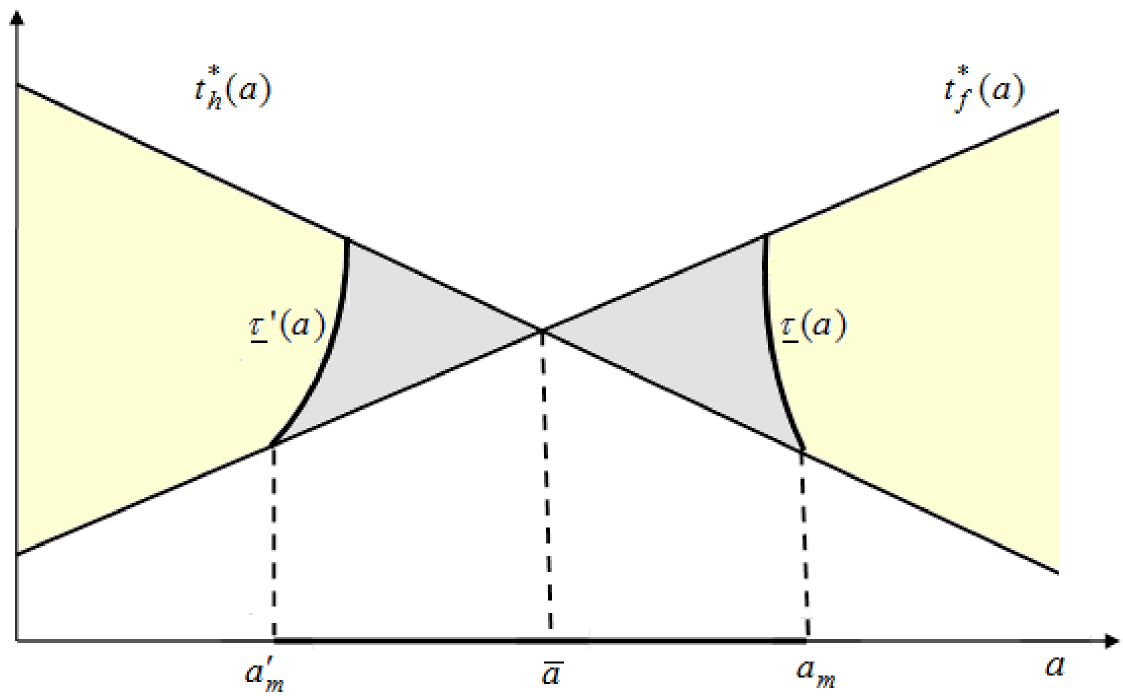

Figure 2

We just showed that, for a given level of institutional quality, moving from a non-cooperative situation to the imposition of a minimum tax rate reduces the revenue of the low tax country. Does this loss increase with lower institutional quality? In other words when the losing country is h, does the difference $B_h^* - B_h^o$ increase with a? It is straightforward to show[21] that $\frac{\partial}{\partial a}\left(B_h^* - B_h^o\right) > 0$. So the more the low tax country is institutionally underdeveloped the more it will lose by moving from competition to minimum tax coordination. If countries are equal in size $(S = \frac{1}{2})$ we see that the country which has the less developed institutions[22] will lose by the imposition of a minimum tax rate.

Social welfare

[21]Indeed we show that $\frac{\partial}{\partial a}\left(B_h^* - B_h^o\right) = \frac{14}{25}[(a-S) - \frac{4}{3}] + \tau > \frac{14}{25}\left(x - \frac{4}{3}\right) + t_h^*,$ with $x = a - S$ and $x \in \left(\frac{1}{2}, \frac{6}{7}\right)$. Since $t_h^* = \frac{4-3x}{5}$, we can write $\frac{\partial}{\partial a}\left(B_h^* - B_h^o\right) = \frac{4}{75} - \frac{1}{25}x$. This last expression is decreasing in x. So $\frac{\partial}{\partial a}\left(B_h^* - B_h^o\right) > 0$ for all $x \in \left(\frac{1}{2}, \frac{6}{7}\right)$ since it is positive for the highest value of x.

[22]Note that low institutional development is generally linked to economic underdevelopment.

In Appendix B (Claim 4) we show that moving from a non-cooperative equilibrium to minimum tax coordination always increases social welfare.

The following proposition can now be stated

Proposition 3

(a) Moving from tax and infrastructure competition to minimum tax coordination has an opposite effect on the jurisdictions' tax revenue. The high tax country's revenue is improved while the low tax country is made worse off.

(b) If the institutional quality of the high tax country is not high enough, there is no scope for compensating the loser, even if a compensation mechanism does exist.

(c) Moving from tax and infrastructure competition to minimum tax coordination always increases social welfare.

5 Conclusion

The purpose of the paper is to investigate whether tax coordination is desirable when countries compete in taxes and infrastructures. To address this question, we develop a model where governments strategically choose tax rates and the level of public expenditures to maximize net tax revenue. In addition to size asymmetry, the model incorporates asymmetry in institutional quality, which has been largely ignored in the tax competition literature. The desirability of tax coordination is then analyzed through its impact on tax revenue and social welfare respectively.

Our results are in stark contrast to the findings of the pure tax competition literature. This is particularly relevant for policy issues since the belief that tax competition generally causes the "erosion of national tax bases" may prove erroneous if countries compete in tax and non-tax instruments. Indeed, in our two-country model we show that a uniform tax causes a tax loss to each country and imposing a minimum tax rate only hurts the low tax jurisdiction. These results are however strongly contrasted if jurisdictions only compete in taxes.

It is also worth noting that asymmetries in size and institutional quality play an important role in gauging the desirability of tax coordination. For example, we show that tax harmonization is less efficient than tax and infrastructure competition if the competing countries are equal in size but have very different levels of institutional development.

If we focus on international institutional disparities which often reflects uneven economic development, our model shows that tax coordination is harmful to the less developed jurisdiction which is also the low tax one.[23] This begs the following question. Can tax and infrastructure competition be a way for lagging countries to catch-up with economic development? Future research could address this question by using a dynamic version of our model. This would allow to investigate under which conditions tax and infrastructure competition could, in the long run, promote convergence across unequally developed countries.

[23] Assuming that countries are equal-sized.

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A Pure tax competition

In the case of pure tax competition with symmetric institutional quality ($a = 1$), we have $\theta_h = \theta_f = 0$. Solving the tax game yields the equilibrium rates of countries h

and f , which are respectively $t_h^T = \frac{1}{3}(1+S)$ and $t_f^T = \frac{1}{3}(2-S)$. The corresponding countries' tax revenue are $B_h^T = \frac{1}{9}(S+1)^2 = \left(x^T\right)^2$ and $B_f^T = \frac{1}{9}(2-S)^2 = \left(1-x^T\right)^2$. The joint tax income is thus $B^T = \frac{1}{9}\left(2S^2 - 2S + 5\right)$.

A.1 Tax harmonization

The impact on tax revenue

If both countries opt for tax harmonization, the uniform tax rate can equal any value $t^u \in [0,1]$. As a result, $x^u = S$ companies will be located in the small country and $1 - x^u$ in the large economy. The tax revenue of the two countries are then respectively $B_h^u = t^u S$ and $B_f^u = t^u(1-S)$. The joint maximal revenue is $B^u = B_h^u + B_f^u = t^u$. It is now convenient to show that $B^u > B^T$, if $t \in [\frac{5}{9}, 1]$ for all $S \in (0, \frac{1}{2})$. It implies that if the unified tax rate is higher than $\frac{5}{9}$, tax harmonization generates higher total tax revenue than pure tax competition. This is consistent with the tax competition literature (see for example, Kanbur and Keen, 1993; Baldwin and Krugman, 2004 and Boadway and Tremblay, 2011).

We now consider each country individually. For the large country we can easily show that $B_f^u > B_f^T$ for $S \in \left(0, \frac{1}{2}\right)$ and $t \in [\frac{(2-S)^2}{9(1-S)}, 1]$. In the same way we can show for the small country that $B_h^u > B_h^T$ for all $S \in \left(\frac{7}{2} - \frac{3}{2}\sqrt{5}, \frac{1}{2}\right)$ and $t \in [\frac{(1+S)^2}{9S}, 1]$. In other words, if the competing economies are not too uneven in size, the presence of a uniform tax rate, which is high enough, leads to a Pareto-improvement in tax revenue. However, each country can be made better off for any $S \in \left(0, \frac{1}{2}\right)$, by imposing a uniform tax rate $t \in [\frac{5}{9}, 1]$, if inter-jurisdictional revenue redistribution is feasible.

The impact on social welfare

If tax rates are the same across jurisdictions, the social welfare equals $W^u = q$. The aggregate welfare resulting from pure tax competition is $W^T = q - \frac{1}{18}(2S-1)^2$. Consequently we get $W^T - W^* = -\frac{1}{2}(\ t_f^T - t_h^T)^2 < 0$. Moving from tax competition to tax harmonization is thus welfare improving.

A.2 Minimum tax

The impact on tax revenue

We assume that the tax rates set by the jurisdictions are now bounded from below by τ such that $\tau \in \left(t_h^T, t_f^T\right)$. In that we follow Kanbur and Keen (1993). The small country will set $\widetilde{t}_h = \tau$ since it is its best choice. The large country chooses its best reply $\widetilde{t}_f = \frac{\tau}{2} + \frac{1-S}{2}$. It follows $\widetilde{x} = \frac{1}{2}(1 + S - \tau)$. The tax income for each country is respectively $\widetilde{B}_h = \frac{1}{2}\tau(1 + S - \tau)$ and $\widetilde{B}_f = \frac{1}{4}(\tau - S + 1)^2$. The aggregate tax income is then $\widetilde{B} = \frac{1}{4}\left(S^2 - 2S + 4\tau - \tau^2 + 1\right)$.

It is then easy to check that for $\tau > t_h^T$ we have $\widetilde{B}_h > B_h^T$ and $\widetilde{B}_f > B_f^T$. It follows that imposing a minimum tax rate to the competing jurisdictions is a Pareto-improvement in tax revenue. This result is reminiscent of Kanbur and Keen (1993).

The impact on social welfare

The social welfare resulting from a minimum tax bound $\tau \in \left(t_h^T, t_f^T\right)$ equals $\widetilde{W} = q - \frac{1}{2}(\frac{1}{2} - \frac{1}{2}\tau - \frac{1}{2}S)^2$. Hence, $\widetilde{W} - W^T = \frac{1}{72}\left(S - 3\tau + 1\right)\left(7S + 3\tau - 5\right)$. Since $\tau \in \left(t_h^T, t_f^T\right)$ it is straightforward to show that $W(\tau) > W^T$.Consequently, a minimum tax which lying between the non-cooperative equilibrium tax rates is welfare improving. This result is in line with Kanbur and Keen (1993).

B Claims and their proofs

Claim 1. *With $\tau \in [t_h^*, t_f^*]$, we always have $B_h^* > B_h^o = B_h(\tau)$.*

Proof. We recall that the tax revenue of country h resulting from tax and infrastructure is $B_h^* = \frac{7}{225}\left(3S - 3a + 4\right)^2$ with $S + \frac{1}{2} < a < 1 + S < S + \frac{4}{3}$. The non-cooperative equilibrium tax rates are $t_h^* = \frac{4-3a+3S}{5}$ and $t_f^* = \frac{1+3a-3S}{5}$. In addition, $B_h(\tau) = (1-a+S)\tau - \frac{5}{8}\tau^2$ is positive only if $0 < \tau < \frac{8(1-a+S)}{5}$. It is easy to check that $B_h(\tau)$ reaches its maximum at $\widehat{\tau} = \frac{4(1-a+S)}{5}$. Furthermore, $B_h(\tau)$ is decreasing in τ for $\tau \in [\widehat{\tau}, t_f^*]$. Since $\widehat{\tau} - t_h^* = -\frac{a-S}{5} < 0$, it follows that $B_h(\tau)$ decreases in τ for $\tau \in [t_h^*, t_f^*]$ and reaches its maximum at t_h^*. Therefore, to prove the claim, we only need to show that $B_h^* > B_h(\tau = t_h^*)$. It is straightforward to show that $B_h(\tau = t_h^*) = t_h^* \frac{4-5a+5S}{8}$. Thus, $B_h^* - B_h(\tau = t_h^*) = \frac{t_h^*}{360}(44 + 57a - 57S) > 0$. That finishes the proof.

Claim 2. *There is $B_f^* < B_f^o = B_f(\tau)$ for $\tau \in [t_h^*, t_f^*]$.*

Proof. We know that $B_f(\tau) = \frac{1}{8}(2a - 2S + \tau)^2$ and $B_f^* = \frac{7}{225}\left(3S - 3a - 1\right)^2$. Given that $B_f(\tau)$ is increasing in τ for $\tau \in [t_h^*, t_f^*]$, the claim is proved if the inequality $B_f(\tau) > B_f^*$ holds true for the minimum value of $B_f(\tau)$ which equals $B_f(\tau = t_h^*) = \frac{1}{8}\left(\frac{4+7a-7S}{5}\right)^2$. After straightforward calculations, we get $B_f(t_h^*) - B_f^* = \frac{1}{25 \cdot 72}\left[88 + 56 \times 3(a-S) - 63(a-S)^2\right]$ with $S + \frac{1}{2} < a < 1 + S$, which implies, $\frac{1}{2} < a - S < 1$.

If we set $x = a - S$, we can write $B_f(t_h^*) - B_f^*$ as a second order polynomial in x, which equals $f(x) = -63x^2 + 56 \times 3x + 88$ with $\frac{1}{2} < x < 1$. The function $f(x)$ is concave and it is easy to check that it is positively signed for $x \in \left(\frac{1}{2}, 1\right)$. Consequently, $B_f(t_h^*) - B_f^* > 0$ for all $a \in \left(\frac{1}{2} + S, 1 + S\right)$ and $B_f^* < B_f(\tau)$ for $\tau \in [t_h^*, t_f^*]$.

Claim 3. *If $a \in \left(\frac{1}{2} + S, \frac{5\sqrt{3}-3}{9} + S\right)$ and $\tau \in (t_h^*, \underline{\tau})$ it follows that $B^* > B(\tau) = B^o$.*

Proof. Set $x = a - S$ and let $\underline{\tau} = 1 - \frac{1}{2}x - \sqrt{\frac{13x^2}{100} + \frac{3x}{25} - \frac{13}{225}}$ and $\overline{\tau} = 1 - \frac{1}{2}x + \sqrt{\frac{13x^2}{100} + \frac{3x}{25} - \frac{13}{225}}$ be the solutions of $\Psi(\tau) = B^* - B(\tau) = 0$. The function $\Psi(\tau)$ is negative for $\tau \in (\underline{\tau}, \overline{\tau})$, since it is convex in τ. It can further be checked that $\underline{\tau} > t_h^*$ if $\frac{1}{2} < x < \frac{5\sqrt{3}-3}{9}\left(< \frac{6}{7}\right)$ and that $t_f^* < \overline{\tau}$ if $\frac{1}{2} < x < \frac{6}{7}$. It follows that $\Psi(\tau) > 0$ for $\tau \in (t_h^*, \underline{\tau})$ which is only possible for $x \in \left(\frac{1}{2}, \frac{5\sqrt{3}-3}{9}\right)$, or for $a \in \left(\frac{1}{2} + S, \frac{5\sqrt{3}-3}{9} + S\right)$.

Claim 4. *$W(\tau) > W^*$ for $\tau \in [t_h^*, \min\{\tau_m, t_f^*\}]$ with $\tau_m = 2(1 - a + S)$ and $\frac{1}{2} + S < a < \frac{6}{7} + S$.*

Proof. It is convenient to show that $W(\tau)$ is strictly concave in τ and reaches its maximum at $\tau^+ = \frac{2}{3}$. Thus, the minimum of $W(\tau)$ can only be attained at one of the two boundaries, t_h^* or $\min\{t_f^*, \tau_m\}$. Now we determine the minimum value of $W(\tau)$. First, it is easy to show that $W(t_h^*) - W(t_f^*) = -\frac{t_f^* - t_h^*}{8} < 0$ and so $W(t_f^*) > W(t_h^*)$. It follows that $W(t_h^*) = \inf W(\tau)$ if $t_f^* < \tau_m$. If $\tau_m < t_f^*$, we consider the cases $\tau_m < \tau^+$ and $\tau^+ < \tau_m < t_f^*$. Because $W(\tau)$ is strictly concave and $\tau_m > t_h^*$, we must have $W(\tau_m) > W(t_h^*)$ if $\tau_m < \tau^+$ and since $W(t_f^*) > W(t_h^*)$, we must have $W(t_h^*) < W(t_f^*) < W(\tau_m)$, if $\tau^+ < \tau_m < t_f^*$. In any case, $W(\tau)$ reaches its minimum at t_h^*.

We now prove that the minimum value of $W(\tau)$ is above W^*. For that purpose it is sufficient to compare $W(t_h^*)$ with W^*. Direct calculation leads to $W(t_h^*) - W^* = \frac{1}{2}\left(\frac{x^2}{4} + \frac{3x}{5} - \frac{11}{45}\right), \forall x \in \left[\frac{1}{2}, \frac{6}{7}\right]$ with $x = a - S$. It is easy to see that the polynomial $f(x) = \frac{x^2}{4} + \frac{3x}{5} - \frac{11}{45}$ is convex in x and reaches its minimum at $\overline{x} = -\frac{6}{5}$.The function $f(x)$ is thus increasing in $(-\frac{6}{5}, \infty)$. Noticing that $f(0) = -\frac{11}{45}$ and $f(\frac{1}{2}) = \frac{1}{8} > 0$ it follows that $f(x) > 0$ for $x \in \left[\frac{1}{2}, \frac{6}{7}\right]$. In other words $W(t_h^*) > W^*$, and thus we prove that $W(\tau) > W^*$ for all $\tau \in [t_h^*, \min\{t_f^*, \tau_m\}]$ and $\frac{1}{2} + S < a < \frac{6}{7} + S$. We finish the proof.