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# (Un)stable vertical collusive agreements\*

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## Abstract

In this paper, we extend the concept of stability to vertical collusive agreements, involving downstream and upstream firms, using a setup of successive Cournot oligopolies. We show that a stable vertical agreement always exists: the unanimous vertical agreement involving all downstream and upstream firms. Thus, stable vertical collusive agreements exist even for market structures in which horizontal cartels would be unstable. We also show that there are economies for which the unanimous agreement is not the only stable one.

**Keywords:** collusion, stability, vertical agreement.

**JEL codes:** D43; L13

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# 1 Introduction

In the present paper, we study the existence of stable *vertical collusive agreements* in the context of successive oligopolies. Such collusive agreements simultaneously embody downstream and upstream firms. Collusion is represented as an agreement through which the insiders act in unison, reducing thereby the total number of decision units operating in the downstream and upstream markets and, thus, the corresponding number of oligopolists in each of them. Collusive outcomes are the Cournot equilibria corresponding to these reduced numbers of oligopolists, which are then compared with those arising when downstream and upstream firms act independently from each other in their respective markets.

More than half a century ago, Stigler (1950) has stressed the main difficulty encountered by a cartel promoter: "the major difficulty in forming a merger is that it is more profitable to be outside a merger than to be a participant. The outsider sells at the same price but at the much larger output at which marginal cost equals price. Hence, the promoter of a merger is likely to receive much encouragement from each firm, almost every encouragement, in fact, except participation". This sentence clearly illustrates the need for analyzing carefully under which conditions a cartel is expected to resist to the forces acting against its stability.

A definition of cartel stability, relying on two natural requirements, namely, external stability and internal stability, has been proposed by d'Aspremont *et al.*, 1983, in the context of horizontal mergers. A cartel with  $n$  firms in an industry embodying  $K$  firms ( $n \geq K$ ) is said *internally stable* when the profits realized by each firm member of the cartel exceeds the profits obtained when being outside of it, *taking into account the change in the profits of an outsider resulting from this exit*. Similarly, a cartel with  $K$  members is *externally stable* when the profits of a firm member of a cartel of size  $K + 1$  are smaller than the profits realized by an outsider when the cartel is of size  $K$ . A cartel of size  $K$  is stable when it is both internally and externally stable. Formally, assuming that all firms are identical, and defining  $\Pi^f(K)$  (resp.  $\Pi^c(K)$ ) the payoff received by each outsider (resp. by each cartel member), a cartel of size  $K$  is stable if both the inequalities

$$\Pi^c(K) \geq \Pi^f(K - 1)$$

(internal stability) and

$$\Pi^f(K + 1) \leq \Pi^c(K)$$

(external stability) hold simultaneously.

This definition of stability is rather abstract since it does not state how the profits of an insider or an outsider are defined or, equivalently, to which market structure it corresponds. As a consequence, the above abstract definition can be applied to a wide variety of market situations and corresponding payoffs' structures. Nevertheless, the definition of stability assumes that each member of the agreement receives the same share of profits,  $\Pi^c(K)$ , and, similarly, that each outsider obtains an equal amount of profits,  $\Pi^f(K)$ . One way to rationalize this assumption consists in supposing that all firms in the industry are identical, with the sole exception that either they participate to the collusive agreement, or decide to remain outsiders. This assumption then allows to share equally the profits inside the entity among its members through an argument of equal treatment. Furthermore, supposing an identical strategic behavior for the outsiders allows to state by an argument of symmetry that each one of them also should obtain an equal amount of profits at the solution. With these assumptions, given an entity of size  $K$ , profit sharing among firms is fully described by two numbers: the share of profits received by each participant,  $\Pi^c(K)$ , and the profits received by each outsider,  $\Pi^f(K)$ .

A first example of market structure and associated payoffs for which stability has been analyzed can be found in the paper referred to above (d'Aspremont *et al.*, 1983). The solution at which the profits are evaluated are those corresponding to the *price leadership* model introduced by Markham (1951). In this version, the

cartel (collusive agreement) is assumed to be the price-leader (*the dominant firm*) maximizing its profits on the "residual demand function", while the outsiders are behaving competitively, taking the price set by the cartel as given. It is easily seen that, for this specific market situation and payoff's structure, any cartel is always internally unstable simply because, according to the argument put forward by Stigler (1950), the per-firm profits of a cartel member are smaller than the profits obtained by each outsider. But this remark does not prevent the existence of at least one stable cartel, as demonstrated in d'Aspremont *et al.* (1983).

A second example of market structure in which a collusive agreement is considered corresponds to the version proposed by Salant, Schwitzer and Reynolds (1983) of the Cournot model. Compared with the price-leadership model, this approach consists in assuming that a collusive agreement among  $K$  firms takes place, according to which these firms maximize per firm profits against the output choice of the outsiders. This situation represents the Cournot equilibrium of the game consisting of the entity and  $n - K$  outsiders. In this setup, stability has been analyzed by Shaffer (1995) and Belleflamme and Peitz (2010). Assuming linear inverse demand and constant marginal cost, Shaffer (1995) shows that, whenever  $n \geq 3$ , there exists no stable horizontal cartel in the Cournot game. In a recent paper, Zu *et al.* (2012), borrowing from Konishi and Lin (1999), study the size of horizontal stable cartels using general specifications for the output demand and cost function. These authors confirm that, even with a more general specification, the size of stable horizontal cartels remains quite reduced. The main reason why cartel stability fails in Cournot competition refers to the Stigler statement reminded in the beginning of this paper: outsiders are always better off than insiders, destroying thereby internal stability in the Cournot game. Accordingly, contrary to the price leadership model in which there always exists a stable cartel, the Cournot game with linear output demand and constant marginal cost has never a stable cartel when the number of firms exceeds three.

The above studies all refer to collusive agreements embodying downstream firms only, excluding thereby more general forms of collusive agreements, like those arising when some downstream and upstream firms are allowed to combine together. Extending stability analysis to such collusive agreements is undoubtedly interesting and important. Real-life collusive entities often share the property that, among the participating firms, some of them operate in the upstream market(s) and produce the input(s) used by the downstream firms in the production of the final good. Such agreements have been studied by Salinger (1988), Gaudet and van Long (1996) and, more recently, by Gabszewicz and Zanaj (2011). However, to the best of our knowledge, the analysis of collusive stability in this context has not yet been pursued. Probably this is so because some assumptions used in the traditional approach seem inappropriate in this new set-up. Among these assumptions is the one stating that each member of the agreement must receive the same share of the entity's profits. From the very nature of the problem, the firms participating the entity belong to different types, some producing the final good (downstream firms) while the others produce the input (upstream firms). When downstream and upstream firms are allowed to combine together, with both upstream and downstream firms in the collusive agreement, there is no longer any reason to assume that both types of firms should get the same share of the entity's profits. Thus a conceptual problem arises: how profits should be shared among the members of the collusive entity, knowing that these members are not all identical, but belong to two different types? We meet this difficulty in the present paper by requiring that the redistribution of total profits of the entity among its members guarantees to each of them an amount of profits at least as high as the profits it would obtain when leaving the entity, taking into account the change in profits resulting from its move .

This definition of stability is a direct extension of the definition of stability in d'Aspremont *et al.* (1983). It requires that no firm in the entity would get more when leaving the entity than when staying inside (internal stability), taking into account the change in profits resulting from its move. Furthermore it requires that no firm outside the identity would obtain more when entering the entity than staying outside, again taking into

account the change in profits resulting from its move (external stability).

A *vertical* collusive agreement has three effects: (i) it softens double marginalization for the entity boosting its profit; (ii) it reduces the number of active upstream firms in the upstream market leading to a shift upwards of the input supply and it reduces the number of downstream firms that buy the input in the input market causing a shift downwards of the input demand schedule. The balance of these two shifts can lead to an increase of a decrease of the equilibrium input price; and, finally, (iii) it creates asymmetries in the production costs of the downstream firms: the entity produces at the marginal cost while the downstream firms do it at the market price. The effect on the equilibrium output price is ambiguous. The equilibrium output price increases if the output quantity increases due to the presence of the entity, or it may decrease, otherwise. The stability of the vertical entity depends on the balance of these three effects. It is clear that the combination of these effects leads to extremely complex consequences difficult to disentangle. However we are able to show that, in the class of linear economies (linear demand function in the downstream market and constant returns to scale in production), the vertical agreement involving *all* firms, downstream and upstream, always constitutes a stable vertical agreement. We also provide examples of some economies, in which this agreement is not the only one to be stable: we exhibit an example in which both the unanimous agreement and another one, involving a strictly smaller number of firms, are simultaneously stable.

To conclude, while the literature on the stability of vertical collusive agreement is not developed yet, there is a related literature to our paper on the sustainability of a cartel in vertically related industries. Two important contributions in this literature are Nocke and White (2007) and Piccolo and Miklos-Thal (2012). Nocke and White (2007) analyse the collusive agreement among upstream firms, some of which might be vertically integrated with downstream firms. In their paper, the vertical mergers impacts positively the upstream collusion. Piccolo and Miklos-Thal (2012) study collusive agreements among downstream firms who collude on their input supply contracts. They show that an implicit agreement on input supply contracts with above cost wholesale prices and slotting fees facilitates collusion in the downstream market.

## 2 Stability of vertical agreements and successive oligopolies

### 2.1 The definition of stability

In the following we call *vertical* agreement any collusive entity involving simultaneously both downstream and upstream firm(s). In order to examine the question of stability in the case of vertical agreements, we need a framework in which these are analyzed and firms' payoffs defined. This framework is provided in Salinger (1988) and, more recently, by the authors in Gabszewicz and Zanaj (2011), in which they propose a definition of *successive oligopolies* allowing a precise concept of vertical agreement. The nature of the agreement concerns (i) the payoff division among downstream and upstream participants and the behavior of the insider upstream firms with respect to the input market. In the present paper, we assume that upstream participants in the vertical agreement do not participate directly in the upstream market and receive at least the profit of an upstream firm outside the entity. More specifically, we assume that each upstream firm receives the profit of an outsider upstream firm plus a profit "bonus" that should in principle guarantee the interest of the upstream firms to participate in the agreement. Of course, this profit bonus should leave the downstream firms still interested to participate in the agreement. We leave open the door to any agreement between the firms on the exact amount of such a bonus.

To define precisely the notion of stability of vertical agreements, consider two successive markets embodying  $n$  identical downstream firms and  $m$  identical upstream firms,  $m \geq 2$ . Assume that  $K, K \leq n$ , downstream

firms and  $H, H \leq m$ , upstream firms decide to collude. For further use, we define the specific vertical agreement in which  $K = n$  and  $H = m$ , the *unanimous vertical agreement*: this vertical agreement involves all firms operating in either of the successive markets. Notice this entity now involves *two* types of agents, all identical in each type. An entity of size  $K + H$  is stable when it is both internally and externally stable, for each type of agents. Define formally  $\Pi^f(K, H)$  (resp.  $\Pi^c(K, H)$ ) the payoff received by each downstream outsider (resp. by each participant), and  $\Gamma^f(K, H)$  (resp.  $\Gamma^c(K, H)$ ) the payoff received by each upstream outsider (resp. by participant). Then,

**Definition** *A vertical entity of size  $K + H$  is stable if both the sets of inequalities*

$$\Pi^c(K, H) \geq \Pi^f(K - 1, H) \text{ and } \Gamma^c(K, H) \geq \Gamma^f(K, H - 1) \quad (1)$$

*(internal stability) and*

$$\Pi^f(K + 1, H) \leq \Pi^c(K, H) \text{ and } \Gamma^f(K, H + 1) \leq \Gamma^c(K, H) \quad (2)$$

*(external stability) hold simultaneously.*

This definition directly extends the definition of stability provided by d'Aspremont *et al* (1983) to agreements that include two types of firms. More precisely, a vertical collusive agreement embodying  $K + H$  is said *internally stable* when the profits realized by each type of firm, downstream and upstream, member of the entity, exceeds the profits obtained when being outside of it, taking into account the change in the profits of an outsider, resulting from this exit. Similarly, a vertical entity with  $K + H$  members is *externally stable* when the profits of a firm, upstream or downstream, member of an entity of size  $K + 1$  or/and  $H + 1$  are smaller than the profits realized by an outsider, downstream and upstream, when the entity is of size  $K + H$ . This definition of stability translates into requiring four conditions to be satisfied.

## 2.2 The linear model

Now let us apply this definition of stability to the well-known case of linear output demand and constant returns to scale in successive Cournot oligopolies. Let the demand function for some output in the downstream market be given by  $p(Q) = 1 - Q$ , where  $Q$  denotes aggregate supply. Consider  $n$  *downstream* firms producing the output *via* a constant returns technology  $f(z) = \alpha z, \alpha > 0$ , as well as  $m$  *upstream firms* initially supplying the market for the input  $z$  at a constant marginal cost equal to  $\beta, \beta > 0$ . Now assume that  $H$  upstream firms,  $h = 1, 2, \dots, H$ , form a vertical collusive agreement with  $K$  downstream firms  $k = 1, 2, \dots, K$ , and maximize joint profits together. Also assume that  $K < n$  and  $H < m$ .<sup>1</sup> After this agreement, the downstream and upstream markets move from an initial situation with  $n$  active downstream firms and  $m$  active upstream firms, to a market structure with  $n - K + 1$  active firms in the downstream market and  $m - H$  in the upstream one.<sup>2</sup>

Consider first how the profit functions write in the downstream market after the collusive agreement. To this end denote by  $I$  the entity resulting from the agreement. The profits  $\Pi_I$  of the entity  $I$  to which  $K$  downstream firms participate is given by

$$\Pi_I(q_I, q_{-I}) = (1 - q_I - \sum_{i \neq I} q_i)q_I - \beta \frac{q_I}{\alpha} \quad (3)$$

<sup>1</sup>This assumption guarantees that there always exists at least one firm on each side of the upstream market so that the cartel cannot exclude the outsider downstream firms to have access to the input. A similar assumption in another approach to collusion has been used by Gabszewicz and Hansen (1971).

<sup>2</sup>An example of this type of vertical collusive agreement is the agreement taking place in a market among one or several wholesalers with one or several retailers that fixes the price at which the market product is sold to the final consumers. For example, the French Competition Council in 2008 sanctioned five toys manufacturers and three distributors on grounds of collusion during the Christmas period between 2001 and 2004. ( La Revue, 2008)

where  $q_I$  (resp.  $\sum_{i \neq I} q_i$ ) denotes the supply of the entity (resp. firms not in the agreement) in the downstream market and  $\frac{\beta}{\alpha}$  is its unit production cost. As for the downstream firms that do not participate in the agreement, each of them obtains a payoff  $\Pi_i$  defined by

$$\Pi_i(q_i, q_k) = (1 - q_I - q_i - \sum_{k \neq i} q_k)q_i - \omega\left(\frac{q_i}{\alpha}\right), \quad (4)$$

with  $i, i \neq I$ , and  $\omega$  denoting the unit price in the input market.<sup>3</sup> Notice that from the comparison between (3) and (4), it appears immediately that, while the collusive members in the downstream market pay their input at marginal cost  $\beta$ , the rivals pay the input price  $\omega$ . Since  $\Pi_I$  is concave in  $q_I$ , we may use the first order condition to get the best reply function  $q_I$  of the entity in the downstream market game as

$$q_I(q_{i \neq I}) = \frac{1 - \frac{\beta}{\alpha} - \sum_{i \neq I} q_i}{2}.$$

As for an outsider downstream firm  $i$ , its best reply  $q_i$  in the downstream market is conditional on the input price  $\omega$  realized in the upstream market, namely

$$q_i(q_I, q_k, \omega) = \frac{1 - \frac{\omega}{\alpha} - \left(q_I + \sum_{k \neq i, k \neq I} q_k\right)}{2}.$$

Assuming a symmetric equilibrium *among the colluded downstream firms*, we get the resulting Cournot equilibrium in the downstream market, namely, the optimal supply coming from the entity  $q_I^*$  and from each of the rivals  $q_i^*$  which do not belong to the cartel, namely

$$q_I^*(K, \omega) = \frac{\alpha - \beta + (n - K)(\omega - \beta)}{\alpha(n - K + 2)}, \quad (5)$$

and

$$q_i^*(K, \omega) = \frac{\alpha + \beta - 2\omega}{\alpha(n - K + 2)}. \quad (6)$$

It is worth noting that the equilibrium in the downstream market depends on the input price obtained in the upstream market as an immediate consequence of supply and demand for the input. Taking into account (6) and the fact that  $q = f(z) = \alpha z$ , it is easy to derive the input demand resulting from the  $n - K$  outsider firms in the downstream market, i.e.  $\sum_{i \neq I} z_i(\omega) = (n - K)\left(\frac{\beta + \alpha - 2\omega}{\alpha^2(n - K + 2)}\right)$ . As for the input supply, it comes from the strategies  $s_j$ ,  $j \neq I$ , selected by the outsider upstream firms in the input market. Consider the  $j$ th upstream firm not participating in the entity. Its profits  $\Gamma_j$  at the vector of strategies  $(s_j, s_{-j})$  write as

$$\Gamma_j(s_j, S_{-j}) = \omega(s_j, S_{-j})s_j - \beta s_j, \quad (7)$$

with  $S_{-j} = \sum_{-j \neq I} s_{-j}$ . Taking into account that  $\omega(s_j, s_{-j})$  has to make demand equal to supply in the upstream market, namely,  $\sum_{j \neq I} s_j = \sum_{i \neq I} z_i(\omega)$ , we obtain

$$\omega(s_j, s_{-j}) = \frac{(\alpha + \beta)(n - K) - \alpha^2(n - K + 2) \sum_{j \neq I} s_j}{2(n - K)} \quad (8)$$

where  $\sum_{j \neq I} s_j = S_{-j} + s_j$ . Accordingly, the payoff of the  $j$ -th upstream firm writes as

$$\Gamma_j(s_j, s_{-j}) = \left( \frac{(\alpha + \beta)(n - K) - \alpha^2(n - K + 2) \sum_{j \neq I} s_j}{2(n - K)} \right) s_j - \beta s_j.$$

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<sup>3</sup>Notice that the set  $\{k : k \neq i\}$  includes the index  $I$ .

It is immediate to derive from the above the best reply function  $s_j = s_j(S_{-j})$ . Using the symmetry condition  $S_{-j} = (m - H - 1)s_{-j}$ , we derive the optimal input supply  $s_j^*$  coming from the  $j$ -th outsider firm, namely

$$s_j^*(K, H) = \frac{(\alpha - \beta)(n - K)}{\alpha^2(n - K + 2)(m - H + 1)}.$$

Substituting the expression of  $s_j^*$  in (8) we get the equilibrium input price

$$\omega^*(H) = \frac{\alpha + \beta + 2\beta(m - H)}{2(m - H + 1)}. \quad (9)$$

Substituting (9) in (6) and (5) we get the output supply of each outsider downstream firm  $q_i^*$  and that of the cartel  $q_I^*$ , respectively,

$$q_i^*(K, H) = \frac{(m - H)(\alpha - \beta)}{\alpha(n - K + 2)(m - H + 1)}, \quad (10)$$

and

$$q_I^*(K, H) = \frac{(\alpha - \beta)(n - K + 2(m - H + 1))}{2\alpha(n - K + 2)(m - H + 1)}.$$

It follows immediately that profits at equilibrium of the entity  $\Pi_I^*$ , and of the outsider firms  $\Pi_i^*(K, H)$  in the downstream market, write as

$$\Pi_I^*(K, H) = \frac{(\alpha - \beta)^2}{4\alpha^2} \frac{(2(m - H) + n - K + 2)^2}{(n - K + 2)^2(m - H + 1)^2}, \quad (11)$$

and

$$\Pi_i^*(K, H) = \frac{(\alpha - \beta)^2}{\alpha^2} \frac{(m - H)^2}{(n - K + 2)^2(m - H + 1)^2}, \quad (12)$$

respectively. The profit of an outsider upstream firm is

$$\Gamma_j^*(K, H) = \frac{(\alpha - \beta)^2}{2\alpha^2} \frac{(n - K)}{(n - K + 2)(m - H + 1)^2} \quad (13)$$

Notice for later use that the profits of an upstream and a downstream firm when the entity  $I$  is the empty set (namely,  $H = K = 0$ ) are given by (for details, see Appendix):

$$\Gamma^* = \frac{(\alpha - \beta)^2}{\alpha^2} \frac{n}{(m + 1)^2(n + 1)} \quad \text{and} \quad \Pi^* = \frac{(\alpha - \beta)^2}{\alpha^2} \frac{m^2}{(m + 1)^2(n + 1)^2}. \quad (14)$$

### 2.3 Stability in the linear model

Now we are in a position to examine stability properties for vertical agreements in the model we have just reminded. In the next proposition, we show that the unanimous vertical agreement plays a crucial role in the analysis.

Remind that the profit sharing rule consists of any rule guaranteeing to each member of the entity at least as much as it would receive when being outside of it, taking into account the resulting change in the profits. Given the total profits of the entity  $\Pi_I^*(K, H)$ , such a rule is equivalent to distributing at least an amount of profits  $K * \Pi_i^*(K - 1, H)$  to the insiders downstream firms and an amount of profits at least equal to  $H * \Gamma_j^*(K, H - 1)$  to insiders upstream firms. Translated into the linear model described above, the condition for internal stability thus rewrites as:

$$\Pi_i^*(K-1, H) + \frac{\Pi_I(K, H) - [K * \Pi_i^*(K-1, H) + H * \Gamma_j^*(K, H-1)]}{K+H} \geq \Pi_i^*(K-1, H), \quad (15)$$

$$\Gamma_j^*(K, H-1) + \frac{\Pi_I(K, H) - [K * \Pi_i^*(K-1, H) + H * \Gamma_j^*(K, H-1)]}{K+H} \geq \Gamma_j^*(K, H-1). \quad (16)$$

As for external stability, the conditions are given by

$$\Pi_i^*(K, H) + \frac{\Pi_I(K+1, H) - [(K+1) * \Pi_i^*(K, H) + H * \Gamma_j^*(K+1, H-1)]}{K+1+H} \leq \Pi_i^*(K, H), \quad (17)$$

$$\Gamma_j^*(K, H) + \frac{\Pi_I(K, H+1) - [K * \Pi_i^*(K-1, H+1) + (H+1) * \Gamma_j^*(K, H)]}{K+H+1} \leq \Gamma_j^*(K, H). \quad (18)$$

The natural question, one may ask at this point of the analysis is whether there always exists some stable vertical agreement. This question is particularly interesting to solve since we know that no stable agreement exists in the case of horizontal mergers (see Shaffer, 1995). Using the definition of stability, the payoffs in (11), (12), (13), (14) and the profit sharing rule, we can state the following

**Proposition 1** *For all  $m$  and  $n$ , the corresponding unanimous vertical agreement is stable.*

**Proof.** Notice that the unanimous vertical agreement is always externally stable. Indeed, it requires the inequalities (17) and (18) to be satisfied at  $K+1$  and  $H+1$ . However,  $K=n$  and  $H=m$  at the unanimous vertical agreement. Therefore these two inequalities are redundant and accordingly should not be taken into account when checking for stability of the unanimous vertical agreement. Accordingly, the check for stability is complete when the conditions for internal stability are satisfied. These conditions are immediately obtained from (15) and (16) by letting  $K=n$  and  $H=m$  in these expressions. It is easily seen that both these conditions are satisfied if and only if

$$\frac{\Pi_I(K, H) - [K * \Pi_i^*(K-1, H) + H * \Gamma_j^*(K, H-1)]}{K+H} \geq 0.$$

For  $K=n$  and  $H=m$ , we have

$$\frac{\Pi_I(K, H) - [K * \Pi_i^*(K-1, H) + H * \Gamma_j^*(K, H-1)]}{K+H} = \frac{1}{4(m+n)} > 0.$$

Q.E.D. ■

This proposition should be contrasted with the result provided by Shaffer (1995) for the case of pure horizontal agreements. In Shaffer's framework, the corresponding unanimous agreement is never stable for  $n$  exceeding 3. Thus, allowing the upstream firms to also participate in the collusive agreement considerably enhances the interest of the participants to agglomerate all firms in a single entity.

Notice that the main difficulty of proving stability for vertical agreements different from the unanimous one comes from establishing the inequality (18). Indeed, when the vertical agreement comprises  $H < m$  and  $K < n$  firms, then the outsider upstream firms are always willing to enter into the entity because the inequality (18) is generally not satisfied (for instance, it is always the case for the entity composed of one downstream and one upstream firm, as in Gaudet and Van Long (1996)). Thus, it appears that the entity exerts a strong attraction force on the outsiders upstream firms. In fact, proposition 1 only holds because there remains no outsider upstream firm available when all of them are already members of the entity!

It would be interesting to know whether the unanimous vertical agreement is the only stable one. The proof of this uniqueness property should require that the two following inequalities are satisfied:

$$\frac{\Pi_I(K,H) - [K*\Pi_i^*(K-1,H) + H*\Gamma_j^*(K,H-1)]}{K+H} > 0,$$

$$\frac{\Pi_I(K,H+1) - [K*\Pi_i^*(K-1,H+1) + (H+1)*\Gamma_j^*(K,H)]}{K+H+1} > \frac{\Pi_I(K,H) - [K*\Pi_i^*(K-1,H) + H*\Gamma_j^*(K,H-1)]}{K+H}.$$

The first inequality guarantees that the entity involving  $K$  downstream firms and  $H$  downstream firms is internally stable. Whereas, the second inequality implies that, whenever an entity involving  $K$  downstream firms and  $H$  downstream firms is internally stable, then the same entity is never externally stable for the upstream firms. These two conditions clearly imply that the entity involving  $K$  downstream firms and  $H$  downstream firms is unstable. The full-fledged analysis of these two inequalities turns out to be rather algebraically intricate due to the complexity of the functions and the number of variables. However, we were able to show that

**Proposition 2** *There are values for  $K, H, m$  and  $n$  for which there exists a stable agreement which differs from the unanimous one.*

**Proof.** Internal stability of an entity including  $K = n - 1$  and  $H = m - 2$  requires  $m < (518 - 72n) / 27$ .

External stability of a downstream firm requires  $n > 5$ . External stability for an upstream firm in turn is satisfied if and only if  $m > (110 - 27n) / 8$ . It turns out that all these inequalities are simultaneously compatible: for instance they are all simultaneously satisfied for  $K = 5, H = 1, m = 3, n = 6$ . Q.E.D. ■

Hence, in an economy with three firms in the input market and six firms in the output market, not only the unanimous agreement is stable. A vertical agreement involving one upstream firm and five downstream firms is also stable. It can easily be checked numerically that these two vertical agreements are the only two stable agreements in this economy. Why is this case? Why only that "interior" vertical agreements is stable? The intuition of the existence of the unanimous stable agreement is, as we explained above, that the upstream external stability is satisfied when no other firm is active in the market but all firms are inside the entity. While the stability of the interior agreement is obtained by the balance of the three effects of a collusive agreement: the balance among the profit generated by the entity shared per capita to the downstream and upstream firms in the agreement, the profit of the downstream firm as an outsider and the profit of an upstream as an outsider. This balance is not a monotonic function of neither  $n, m, H$  nor  $K$ . Consequently, a general rule on  $n, m, H$  and  $K$  that satisfy conditions (15), (16), (17) and (18) for the linear model is not feasible.

Another interesting issue is the effect on stability of the simultaneous entry of a new firm in the upstream and the downstream markets. This question can easily be answered.

**Proposition 3** *The entry of a new pair of downstream and upstream firms destroys the stability of the unanimous agreement with  $n$  downstream and  $m$  upstream firms*

**Proof.** The condition for internal stability of a vertical agreement including  $n - 1$  downstream and  $m - 1$  upstream firms requires that  $m < (359 - 27n) / 32$ . Similarly, the condition for downstream firms for external stability of a vertical agreement including  $n - 1$  downstream and  $m - 1$  upstream firms requires that  $n > 9$ ; whereas the condition for upstream firms for external stability requires  $m > 6$ . Consequently, there exists no value of  $n$  and  $m$  for which these conditions can be simultaneously satisfied, implying that a vertical agreement with  $n - 1$  downstream and  $m - 1$  upstream firms can never be stable. Q.E.D. ■

Of course, after the entry of a new pair of firms, the stability is restored for the unanimous agreement involving now  $n + 1$  downstream and  $m + 1$  upstream firms.

### 3 Conclusion

In this paper, we tackle the stability problem of collusive agreements not only involving some downstream firms, but also embodying some upstream firms, providing the final market with a specific input. In other words, we extend the stability analysis from pure horizontal collusive agreements to entities involving some degree of vertical agreements. This endeavour is made possible due to the flexibility of the stability concept introduced above, but also to the framework of *successive oligopolies* introduced elsewhere by the authors (see Gabszewicz and Zana (2011)). This extension is important because many real-life collusive agreements embody both upstream and downstream firms, influencing thereby the outcomes obtained both in the upstream and downstream market.

Our analysis reveals that, in the linear model, the unanimous vertical agreement is always stable: this proposition holds in full generality and should be contrasted with the proposition by Shaffer (1995) according to which no stable cartel exists in the case of horizontal cartel agreements when  $n > 3$ : thus, stable vertical entities exist for market structures in which pure horizontal cartels would all be unstable. It also reveals that the introduction of vertical agreements weakens the Stigler statement according to which "the major difficulty in forming a cartel is that it is more profitable to be outside a cartel than to be a participant". In the framework of successive oligopolies, the marginal cost of downstream firms is no longer exogenous, as in the case of horizontal agreements. When the entity also comprises upstream firms, it reduces the number of input suppliers in the upstream market, restricting thereby competition in this market, leading in turn to an increase in the input price. Furthermore, the presence of the upstream firms in the agreement increases the differences in production costs between the insider and outsider downstream firms. Therefore, the entity exerts a strong attraction force on the outsiders' upstream firms, differently from what is argued by Stigler (1950), according to whom the only firms that gain from a cartel are the outsiders. In the case of vertical agreements, often the outsiders are willing to become insiders. This attraction force explains why external stability is so difficult to obtain in the framework of vertical agreements.

Our paper has only scratched the surface of what looks like a promising area for further research. Many questions are still remaining open after our analysis. First, a natural question consists in evaluating the welfare effects of vertical agreements. With this respect, two forces operate in opposite directions. On the one hand, the entry of *upstream* firms should increase welfare to the extent that it dampens the negative effects of double marginalization. But, on the other hand, the higher the number of *downstream* firms entering the agreement, the more concentrated the power in the downstream market.

Second, how robust are the conclusions of the paper ? It is clear that, like most of the previous research in this field, its conclusions hold in the framework of the linear model. It would be interesting to examine more efficient types of pricing as the two part-tariffs. In that case, the first effect of the vertical collusive agreement, namely the elimination of the double marginalization, would disappear. The second effect would remain under the assumption that downstream firms do not buy from suppliers that are not part of the collusive agreement (for instance see Nocke and White (2007) on the "*outlet effect*"). Clearly, the profitability of the agreement in that setup would be different so different would be the type of stable equilibria that would emerge. Nevertheless, the main message of our paper would be invariant.

Finally, the institutional forms of collusive agreements observed in real life are by far more complex than those evoked in this paper where the agreement reduces simply to the acceptance to belong to the entity or not. In particular, merging existing firms often takes the form of acquisition of one firm by another. Such acquisitions reveal the existence of a market where firms are exchanged among firms, opening the door of a whole range of potential arrangements among firms. The various potential forms of vertical agreements raises

the interesting question of what could be the optimal structure of the market. Also this interesting issue has still to be explored in depth in the future.

## Appendix

In here, we briefly summarize the equilibrium market solution of successive oligopolies when no vertical agreement takes place. Reconsider the same economy as in Section (2.2) in absence of any agreement. Then, the profit  $\Pi_i(q_i, q_{-i})$  of a downstream firm is  $\Pi_i(q_i, q_{-i}) = ((1 - q_i - \sum_{j \neq i} q_j)q_i - \beta \frac{q_i}{\alpha})$ . Assuming a symmetric equilibrium in the downstream market, we obtain the optimal output quantity as a function of the input price  $\omega$ , namely  $nq_i = \frac{n(\alpha - \omega)}{(n+1)\alpha}$ . The market clearing condition in the input market,  $nq_i = \sum_{k=1}^m s_k$ , gives the inverse input demand function:  $\omega(\sum_{j=1}^m s_j) = \alpha - \alpha^2 \frac{n+1}{n} \sum_{k=1}^m s_k$ . Then, the profit function of the  $j$ -th upstream firm is given by  $\Gamma_j(s_j, s_{-j}) = (\omega(\sum_{j=1}^m s_j) - \beta)s_j$ . At the symmetric equilibrium, we obtain  $s^*(n, m) = \frac{n(\alpha - \beta)}{\alpha^2(n+1)(m+1)}$  and  $\omega = \frac{\alpha + m\beta}{m+1}$ ,  $q^* = \frac{m(\alpha - \beta)}{\alpha(n+1)(m+1)}$ . This equilibrium solution yields the level of profits in (14).

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