# MATHEMATICS SEMINAR
# of the
# UNIVERSITY OF LUXEMBOURG
# in cooperation with the
# LUXEMBOURG MATHEMATICAL SOCIETY

**March 2009**

**3 March 2009, at 5 pm** **Room 3.04 bs**

Robert Coquereaux
CNRS, Centre de Physique Théorique, Luminy

**Quantum subgroups of Lie groups and modular invariance**

Abstract

From quantum groups at roots of unity, or from affine Lie algebras at some level,ă one can construct a monoidal category of representations that admits, for special values of the chosen root (or of the level), module-categories, ie additive categories on which the previous one acts. In the case of quantum SU2, those "quantum subgroups" are classified by the usual ADE Dynkin diagrams.ă This classification is equivalent to another problem solved long ago in the case of SU2 by theoretical physicists, in the context of conformal field theories with boundaries, namely the classification of modular-invariant sesquilinear forms,ă for the Hurwitz - Verlinde representations of SL(2,Z). Each such quantum subgroup is associated with a weak Hopf algebra of a special kind (an Ocneanu quantum groupoid) that admits two, usually distinct, representations theories whose multiplicative structures can be encoded by graphs: the fusion graph and the graph of quantum symmetries. The purpose of the seminar is to provide a general introduction to the above ideas and to describe what happens when SU2 is replaced by more general Lie groups. This leads in particular to higher analogues of Coxeter-Dynkin diagrams (that will be presented for SU3 and SU4) and to higher graphs of quantum symmetries.

**17 March 2009, at 5 pm** **Room 3.04 bs**

Janusz Grabowski
Polish Academy of Sciences

**Geometry of quantum systems: density states and entanglement**

Abstract

Various problems concerning the geometry of the space of Hermitian operators on a Hilbert space $H$ are addressed. In particular, we study the canonical Poisson and Riemann-Jordan tensors and the corresponding foliations into Kähler submanifolds. It is also shown that the space $D(H)$ of density states on an $n$-dimensional Hilbert space $H$ is naturally a manifold stratified space with the stratification induced by the the rank of the state. This stratification is maximal in the sense that every smooth curve in $D(H)$, viewed as a subset of the dual $u^*(H)$ to the Lie algebra of the unitary group $U(H)$, at every point must be tangent to the strata it crosses. For a quantum composite system entangled states are defined in a geometrical way and an abstract criterion of entanglement is proved.

**24 March 2009, at 5 pm** **Room 3.04 bs**

Dmitri Alekseevsky
Edinburgh University and Maxwell Institute for Mathematical Sciences

**Para-CR structures and related structures**

Abstract

A para-CR structure is a para-complex analogue of a CR structure. It is defined as a distribution $H$ on a manifold $M$ together with a para-complex structure $K$ on $H$, i.e. a field of endomorphisms $K$ such that $K^2 = \mathrm{Id}$ and the eigendistributions $H^{\pm}$ of $K$ are involutive. Many notions and results of CR geometry remain valid in para-CR case. We present a survey of basic facts of para-CR geometry. A description of maximally homogeneous para-CR manifolds of semisimple type will be given. We consider also some structures subordinated to para-CR structure, for example, quaternionic para-CR structure, which is a para-analogue of 3-Sasakian structure, and pseudo-conformal quaternionic para-CR structure and describe their relations with pseudo-hyperKähler structure and pseudo-quaternionic Kähler structure. An interesting special case of para-CR structures consists of non degenerate codimension one para-CR structures. Such structure can be defined as a decomposition $H = H^+ + H^-$ of a contact distribution $H$ into direct sum of two integrable Lagrangian subdistributions. We discuss relations of such structures with second order ODE discovered by P. Nurowski and G.A.J. Sparling and to parabolic Monge-Ampere equations.

**31 March 2009, at 5pm** **room 3.04 bs**

Prof. Andreas Kollross (Universität Augsburg)

**Low cohomogeneity and polar actions on symmetric spaces**.

Abstract:

A Lie group action on a Riemannian manifold is called polar if there exists a section, i.e. a submanifold which meets all orbits orthogonally. A natural example is given by the action of a compact Lie group on itself by conjugation, where the maximal tori are sections.
Another class of examples is given by actions of cohomogeneity one. I will talk about classification results for polar actions on symmetric spaces.