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## **Urban Spatial Structure, Employment and Social Ties**

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Pierre Picard, CREA, Université du Luxembourg  
Yves Zenou, Stockholm University, IFN and GAINS

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For editorial correspondence, please contact: [crea@uni.lu](mailto:crea@uni.lu)  
University of Luxembourg  
Faculty of Law, Economics and Finance  
162A, avenue de la Faiencerie  
L-1511 Luxembourg

# Urban Spatial Structure, Employment and Social Ties

Pierre M. Picard\*      Yves Zenou†

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## Abstract

We develop a model where workers both choose their residential location (geographical space) and social interactions (social space). In equilibrium, we show under which condition the majority group resides close to the job center while the minority group lives far away from it. Even though the two populations are ex ante totally identical, we find that the majority group experiences a lower unemployment rate than the minority group and tends to socially interact more with other workers of its own group. Within each group, we demonstrate that workers residing farther away from the job center tend to search less for a job and are less likely to be employed. This model is thus able to explain why ethnic minorities are segregated in the urban and social space and why this leads to adverse labor-market outcomes in the absence of any discrimination against the minority group.

**Key words:** Social interactions, segregation, labor market, spatial mismatch.

**JEL Classification:** A14, J15, R14, Z13.

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\*CREA, University of Luxemburg and CORE, Université Catholique de Louvain. Email: pierre.picard@uni.lu.

†Stockholm University, Research Institute of Industrial Economics (IFN) and GAINS. E-mail: yves.zenou@ne.su.se.

# 1 Introduction

Economists have long been interested in how the socio-economic outcomes of individuals are shaped by their interactions with those around them. This question is especially important in urban areas where cities provide the homes, workplaces, and social environments for most individuals and families and present a substantial stratification across ethnic groups.

The aim of this paper is to analyze the relationship between workers' social interactions and their labor-market outcomes in an urban spatial framework and analyze how minority and majority workers are differently affected.

To be more precise, we develop a simple model where there are frictions in the labor market and where, in order to find a job, workers need to interact with each other. They have to decide on how much time they want to spend with other workers. For each social interaction, the worker needs to commute to the location of the other worker. There is therefore a trade off since the more time they spend with other workers, the higher is their chance of finding a job but the more costly it is. We consider a closed and linear city where all jobs are located in the job center -or Central Business District (CBD). In the homogenous population case, we show that workers residing farther away from the job center end up searching less for a job and are less likely to be employed because they tend to interact less with other workers. This is because it is more costly to socially interact with other workers the farther away a worker lives from the CBD.

We then consider the case of two populations, say the majority and the minority group. We analyze an equilibrium where the majority group endogenously chooses to live close to the job center while the minority group prefers to reside far away from it. We show that the majority group experiences a lower unemployment rate than the minority group and tends to socially interact more with other workers of the same group. The workers from the majority group pay, however, a much higher housing price than workers from the minority group.

We then extend our model in two different directions. We first allow workers from

one group to socially interact with workers from the other group. We show under which conditions there exist spatial equilibria for which the two groups choose not to interact with each other. We also allow workers to direct their search so that they can decide with whom they want to socially interact more. We show that, under some conditions, the majority group still experiences a lower unemployment rate. Our model is thus able to explain why ethnic minorities tend to experience a higher unemployment rate than majority workers. Indeed, even though if both groups are *ex ante* identical, we can demonstrate under which conditions ethnic minorities choose to locate further away from the employment center, socially interact only with people from their own group and social interact less with them compared to the majority group. All these factors lead to adverse labor-market outcomes for the ethnic minorities.

The paper unfolds as follows. The next section highlights our contribution with respect to the literature. Section 3 presents the benchmark model where we determine the employment rate, workers' search activities and location decisions. Section 4 discusses the urban equilibrium for an homogenous population. Section 5 analyzes the urban equilibria with two populations. Section 6 extends the analysis to the case when workers choose the intensity of ties to each member of their own population (directed search). Finally, Section 7 concludes. All proofs of the propositions can be found in the Appendix at the end of the paper.

## 2 Related literature

Our paper contributes to the literature on “social interactions and cities”, which is a small but growing field.

**Urban economics and economics of agglomeration** There is an important literature in urban economics looking at how interactions between agents create agglomeration

and city centers.<sup>1</sup> It is usually assumed that the level of the externality that is available to a particular firm or worker depends on its location relative to the source of the external effect – the spillover is assumed to attenuate with distance – and on the spatial arrangement of economic activity. This literature (whose keystones include Beckmann, 1976; Fujita and Ogawa, 1980; and Lucas and Rossi-Hansberg, 2002; Behrens et al., 2014; Helsley and Strange, 2014) examines how such spatial externalities influence the location of firms and households, urban density patterns, and productivity. For example, Glaeser (1999) develops a model in which random contacts influence skill acquisition, while Helsley and Strange (2004) consider a model in which randomly matched agents choose whether and how to exchange knowledge. Similarly, Berliant et al. (2002) show the emergence of a unique centre in the case of production externalities while Berliant and Wang (2008) demonstrate that asymmetric urban structures with centres and subcenters of different sizes can emerge in equilibrium. More recently, Mossay and Picard (2011, 2013) propose interesting models in which each agent visits other agents so as to benefit from face-to-face communication (social interactions) and each trip involves a cost which is proportional to distance. These models provide an interesting discussion of spatial issues in terms of use of residential space and formation of neighborhoods and show under which condition different types of city structure emerge. All these models are different from ours since the labor market is not explicitly modeled and therefore the impact of social interactions on the labor-market outcomes is not analyzed.

**Peer effects, social networks and urbanization** There is a growing interest in theoretical models of peer effects and social networks (see e.g. Akerlof, 1997; Glaeser et al., 1996; Ballester et al., 2006; Calvó-Armengol et al., 2009; Jackson, 2008; Jackson and Zenou, 2014). However, there are very few papers that explicitly consider the interaction between the social and the geographical space.<sup>2</sup> Brueckner et al. (2002), Helsley and

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<sup>1</sup>See Fujita and Thisse (2013) for a literature review.

<sup>2</sup>Recent empirical researches have shown that the link between these two spaces is quite strong, especially within community groups (see e.g. Topa, 2001; Bayer et al., 2008; Ioannides and Topa, 2010;

Strange (2007), Brueckner and Largey (2008), Zenou (2013) and Helsley and Zenou (2014) are exceptions but, in all these models either the labor market is not included or social interactions are exogenous. Sato and Zenou (2014) is the only paper that has both aspects but the focus is totally different since it mainly analyzes on the role of weak and strong ties in the labor market and explains why, in denser areas, individuals choose to interact with more people and meet more random encounters (weak ties) than in sparsely populated areas. Finally, Schelling (1971) is clearly a seminal reference when discussing social preferences and location. Shelling’s model shows that, even a mild preference for interacting with people from the same community can lead to large differences in terms of location decision. Indeed, his results suggest that total segregation persists even if most of the population is tolerant about heterogeneous neighborhood composition.<sup>3</sup> Our model is very different from models a la Schelling since we focus on the interaction between the labor market and social interactions.

To the best of our knowledge, our paper is the first one to provide a model that shows how the urban spatial structure of a city affects both social interactions and the labor-market outcomes of workers.

**Spatial mismatch** There is ample evidence showing that distance to jobs is harmful to workers, in particular, ethnic minorities. This is known as the “spatial mismatch hypothesis”. Indeed, first formulated by Kain (1968), the spatial mismatch hypothesis states that, residing in urban segregated areas distant from and poorly connected to major centres of employment growth, black workers face strong geographic barriers to finding and keeping well-paid jobs. There are, however, very few theoretical models explaining these stylized facts (for a survey see Gobillon et al. 2007, and Zenou, 2009). The standard

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Hellerstein et al., 2011; Patacchini and Zenou, 2012; Del Bello et al., 2014). See also Ioannides (2012, Chap. 5), Ross (2012) and Topa and Zenou (2015) who review the literature on social interactions and urban economics.

<sup>3</sup>This framework has been modified and extended in different directions, exploring, in particular, the stability and robustness of this extreme outcome (see, for example, Zhang, 2004 or Grauwin et al., 2012).

approach is to use a search model to show that distant workers tend to search less (due to lack of information about jobs or less opportunities to find a job) and thus stay longer unemployed (Coulson et al., 2001; Wasmer and Zenou, 2002).<sup>4</sup> The only paper that explains the spatial mismatch of the minority workers uses a social-interaction approach is that of Zenou (2013). He shows that if workers only find jobs through weak and strong ties (social networks), then minority workers may experience adverse labor outcomes because, by living far away from jobs, they will mainly interact with other minority workers who are themselves more likely to be unemployed. In this literature, all models have to assume some discrimination against minority workers (usually in the housing market) to obtain the different outcomes for minority and majority workers.

Our main contribution to this literature is twofold. First, we propose a model where, without any form of discrimination in the labor and housing markets, segregation in the urban and social space arises endogenously in equilibrium. Second, we are able to explain why ex ante identical workers can end up with very different labor-market outcomes. This is because ethnic minorities choose to locate further away from the job center, socially interact only with people from their own group and social interact less with them compared to the majority group. As a result, the *separation in both the urban and the social space* make minority workers more vulnerable and therefore more likely to experience higher unemployment rates than majority workers.

## 3 The benchmark model

### 3.1 Employment

We assume a linear city with unit width, two working populations of exogenous size  $P_i$  each, and a geographical support (set)  $D_i \subset \mathbb{R}$ ,  $i = 1, 2$ , for these two populations. All

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<sup>4</sup>See also Brueckner and Zenou (2003) for a model of spatial mismatch but without an explicit search model. In an efficiency wage model where, in equilibrium, no worker shirks, they show that housing discrimination can lead to adverse labor-market outcomes for black workers.

workers (belonging to population 1 or 2) work in the Central Business District (CBD), located at  $x = 0$ . The workers of these two populations have exactly the same characteristics: they have the same productivity, the same wage  $w$ ,<sup>5</sup> the same unit use of residential space and the same linear commuting costs  $t$  (per unit of distance) to commute to the CBD.

Each individual of type  $i$  (i.e. belonging to population  $i = 1, 2$ ) located at a distance  $x$  from the CBD can have social interactions with the members of her own population and decides with how many of them she wants to interact with. Each social interaction implies a travel cost  $\tau$  (per unit of distance) and allows the individual to acquire a piece of job information.<sup>6</sup> Individuals only interact with individuals from the same population because of cultural differences and/or language barriers. There is also strong evidence that ethnic minorities use extensively their social networks in finding a job (Battu et al., 2011) and that the majority and minority groups, for example blacks and whites in the United States, do not interact much with each other (Sigelman et al., 1996; Topa, 2001). Another way to justify this assumption is that, even at the same skill level, blacks and whites (in the US) typically do not compete for the same jobs, so that their labor markets tend to be separated (or segmented). Indeed, evidence suggests that blacks are much likely to be employed at some types of firms than at others (Holzer and Reaser, 2000).<sup>7</sup>

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<sup>5</sup>In the presence of an unemployment benefit  $B$ , the wage  $w$  should be replaced by  $w - B$ , i.e. the gain over the unemployment benefit. For simplicity, we normalize  $B$  so that  $B = 0$ .

<sup>6</sup>There is strong evidence that many jobs are found through social interactions and networks. See, in particular, Calvó-Armengol and Jackson (2004), Ioannides and Loury (2004), Galenianos (2014) and Zenou (2015).

<sup>7</sup>For instance, federal contractors are more likely to employ blacks than are non-contractors (Leonard, 1990); larger firms are more likely to employ blacks than small firms (Holzer, 1998); and firms having more black customers are more likely to employ blacks than others (Holzer and Ihlanfeldt, 1998). Also, the employment of blacks in manufacturing has declined dramatically in the recent years and recent evidence suggests that most low-educated blacks work in services, like e.g. business and consumer services (Bound and Holzer, 1993). Another way to justify the fact that blacks and whites do not compete for the same jobs is that unskilled jobs are usually performed in teams. Thus, employers prefer to have teams composed of either blacks or whites but not mixed. Finally, it has also been argued that blacks and whites do not

In Section 5.4 below, we will relax this assumption and discuss realistic conditions under which individuals optimally choose not to interact with people from the other group.

In this paper, we assume that social interactions are the main channel for finding employment.<sup>8</sup> As in Zenou (2006, 2009), we also assume perfect capital markets with a zero interest rate.<sup>9</sup> As a result, workers engage in income smoothing as they cycle in and out of unemployment. Thus, workers save while employed and draw down their savings when out of work, with their consumption expenditure reflecting average income. This means that all workers have identical disposable incomes, equal to the average income over the job cycle. As a result, individuals choose their residence given their expected income and utility. This fits the recent US labor market with low long term unemployment. It also fits the case where moving costs are important so that workers are unlikely to change location during their unemployment spells. In this context, the expected utility of an individual of type  $i$  (i.e. belonging to population  $i = 1, 2$ ) residing at location  $x$  is given by:

$$U_i(x) = e_i(x) (w - t|x|) - C_i(x) - R(x) \quad (1)$$

where  $e_i(x)$  is the individual's employment probability,  $C_i(x)$  is the total travel cost at a distance  $x$  due to social interactions (which will be determined below) and  $R(x)$  is the land rent at a distance  $x$  from the CBD. In this expression, all workers from the same group, employed and unemployed, socially interact with each other. Given the unit city width and the individuals' unit use of residential space, the total number of workers for each population  $i$  is given by  $P_i = \int_{D_i} \lambda_i(y) dy$  where  $\lambda_i(y)$  denotes the number of individuals

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specialise in the same type of jobs because of cultural differences (Wilson, 1996).

<sup>8</sup>There is strong evidence that firms mainly rely on referral recruitment (Bartram et al. 1995; Barber et al., 1999; Mencken and Winfield, 1998; Pellizzari, 2010) and it is even common and encouraged strategy for firms to pay bonuses to employees who refer candidates who are successfully recruited to the firm (Berthiaume and Parsons, 2006). It is also well documented that workers use a lot their social networks to find a job (Holzer, 1987, 1988; Ioannides and Loury, 2004).

<sup>9</sup>When there is a zero interest rate, workers have no intrinsic preference for the present so that they only care about the fraction of time they spend employed and unemployed. Therefore, the expected utilities are not state dependent.

at location  $y$ . The employment for population  $i$  is equal to:

$$E_i = \int_{D_i} \lambda_i(y) e_i(y) dy$$

while the number of unemployed workers is simply:

$$\int_{D_i} \lambda_i(y) [1 - e_i(y)] dy = P_i - E_i$$

Workers are either employed or unemployed. When working, they may lose their job with an exogenous probability  $\beta$  (firm bankruptcy, restructuring, etc.). When they are unemployed, workers residing at location  $x$  search for a job with a success probability of  $\pi_i(x)$ . In a steady-state equilibrium, flows in and out unemployment must be equal so that  $\beta e_i(x) = \pi_i(x) [1 - e_i(x)]$ . This yields the following employment rate:

$$e_i(x) = \frac{\pi_i(x)}{\pi_i(x) + \beta} \tag{2}$$

In this paper, we focus on the relationship between social interactions and employment. The benefits of social interactions are through the information flows workers obtain about employment opportunities. We assume that each social interaction with an employed individual is associated to a probability of finding a job in the CBD.

We initially assume that individuals choose the number of interactions entertained with their own population mates whom they *randomly* meet (*random search*). Specifically, each individual of type  $i$  residing at  $x$  chooses to meet  $n_i(x)$  persons from her own population to socially interact with them. This set-up has both deterministic and probabilistic interpretations about the individuals' social networks. Firstly, we can consider that each individual meets  $n_i(x)$  times all her population mates in a deterministic way during the period considered in the model. In this case, the model discusses the social interactions during the individual's life time in the city. Secondly, we can consider that each individual chooses her residence location and then build up a permanent social network of random ties after her arrival in the city. Finally, we can interpret this set-up as a repetition of time periods where each individual meet  $n_i(x)$  different individuals whose

identities are randomly drawn within her population in the city. In that case,  $n_i(x)$  is the expected number of people individuals of type  $i$  meet over their lifetime. In all these interpretations,  $n_i(x)$  corresponds to the concept of *weak ties* introduced by Granovetter (1973) in which weak ties are generated through professional meetings, casual acquaintances, encounters in sport events, etc. The important part of the assumption of random search is that individuals do not choose their frequency of interaction according to the residential location of their interaction partners. This assumption is made for analytical tractability and is relaxed in Section 6.

Given the employment rate for workers of type  $i$ ,  $E_i/P_i$ , the individual's probability of finding a job for a worker of type  $i$  residing at  $x$  is equal to:

$$\pi_i(x) = \alpha n_i(x) \frac{E_i}{P_i} \quad (3)$$

where  $\alpha$  is a positive constant. This is the key equation that captures the fact that each individual  $i$  located at  $x$  chooses to meet a number  $n_i(x)$  of workers from her own population but only those who are employed provide some information about jobs. This equation highlights the random search process since the probability of employment of each person met by worker  $i$  is just  $E_i/P_i$  (the aggregate employment rate for workers of type  $i$ ) and is not specific to the person met. Quite naturally, the individual's probability of finding a job increases with the number of social interactions  $n_i(x)$  and with higher employment rate from own population. From (2), we can see that  $e_i(x) = \pi_i(x) / [\pi_i(x) + \beta]$  or equivalently  $\pi_i(x) = \beta e_i(x) / [1 - e_i(x)]$ . Plugging this value of  $\pi_i(x)$  into (3), we obtain:

$$e_i(x) = \frac{\alpha n_i(x) E_i/P_i}{\beta + \alpha n_i(x) E_i/P_i} \quad (4)$$

or equivalently

$$e_i(x) = f(n_i(x) E_i/P_i) \quad (5)$$

where

$$f(z_i) \equiv \frac{\alpha z_i}{\beta + \alpha z_i} \quad (6)$$

with  $f'(z_i) > 0 > f''(z_i)$ ,  $f(0) = 0$  and  $\lim_{z_i \rightarrow +\infty} f(z_i) = 1$  and where  $z_i \equiv n_i(x) E_i/P_i$ .

Indeed, for a given location  $x$ , higher social contacts and/or higher employment rate in own population raises own probability of finding a job. In this case, the steady-state aggregate employment rate in population  $i$  is given by

$$E_i = \int_{D_i} e_i(x) \lambda_i(y) dx = \int_{D_i} f(n_i(x) E_i/P_i) \lambda_i(y) dx$$

### 3.2 Search and social interactions

Since social interactions occurs at the residence place of the potential information holder, the cost of those social interactions for a worker of type  $i$  residing at  $x$  is equal to  $C_i(x) = n_i(x) c_i(x)$  where  $n_i(x)$  is her chosen number of interactions and

$$c_i(x) = \frac{1}{P_i} \int_{D_i} \tau |x - y| \lambda_i(y) dy \quad (7)$$

measures the average cost of a single social interaction where  $\lambda_i(y)$  denotes the number of individuals at location  $y$ .<sup>10</sup> As a result, each worker  $i$  residing at  $x$  socially interacts with all members of her own population and each of these interactions implies a commuting cost of  $\tau$  per unit of distance. Observe that the location  $x$  of a worker  $i$  is crucial to determine  $c_i(x)$ . If, for example, a worker  $i$  lives close to the CBD, then her cost  $c_i(x)$  will be relatively low since this worker will be at the same distance from the left and the right of  $x$ . But, if this worker is located at one end of the city, then  $c_i(x)$  will be higher because she must travel longer distances to meet her peers.

In the land market, as it is usually assumed (Zenou, 2009; Fujita and Thisse, 2013), land is offered to the highest bidders. Let  $u_i$  be the equilibrium (expected) utility obtained by an individual of type  $i$ . It should be clear that, in equilibrium, all individuals of type  $i$  should have the same expected utility  $u_i$ . From (1), it is easily verified that the bid rent of a worker  $i$  located at  $x$  is given by:

$$\psi_i(x, u_i) = e_i(x) (w - t|x|) - n_i(x) c_i(x) - u_i \quad (8)$$

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<sup>10</sup>It measures the expected cost of a single interaction under the probabilistic interpretation of the model.

where  $e_i(x)$  and  $c_i(x)$  are given by (5) and (7). We assume that  $w - t|x| > 0, \forall x \in [0, 1]$  so that workers always have incentives to search for a job.

The number of social interactions  $n_i(x)$  is a choice variable. Thus, a worker  $i$  located at  $x$  chooses  $n_i(x)$  that maximizes her expected utility (1) or equivalently her bid rent (8), i.e.

$$\Psi_i(x, u_i) = \max_{n_i(x)} [e_i(x)(w - t|x|) - n_i(x)c_i(x)] - u_i \quad (9)$$

where  $e_i(x)$  is given by (5). The first-order condition is equal to:<sup>11</sup>

$$(E_i/P_i) f'(n_i^*(x) E_i/P_i) = \frac{c_i(x)}{w - t|x|} \quad (10)$$

which solves for  $n_i^*(x)$ . When deciding the optimal level of social interactions, an individual  $i$  located at  $x$  trades off the benefits of an increase in  $n_i(x)$ , which raises her chance of obtaining a job (i.e.  $\frac{\partial e_i(x)}{\partial n_i(x)} > 0$ ), with its costs since more social interactions imply more travelling and thus higher  $c_i(x)$ . Furthermore, since  $f'(\cdot)$  is a decreasing function,  $n_i^*(x)$ , the optimal number of interactions in population  $i$ , increases with the benefits of being employed, i.e.  $w - t|x|$ . Finally, observe that  $n_i^*(x)$  decreases with  $x$ , the distance to the CBD, if and only if the right-hand side (RHS) of (10), i.e.  $\frac{c_i(x)}{w-t|x|}$ , increases in  $x$ . By (5) the employment probability will then also falls with  $x$ .

To be more specific, we can use the definition of  $f(\cdot)$  given in (6), to determine (10). We obtain:

$$[\beta + \alpha n_i^*(x) (E_i/P_i)]^2 = \frac{\alpha \beta (w - t|x|) (E_i/P_i)}{c_i(x)} \quad (11)$$

which using (4) can be written as:<sup>12</sup>

$$[1 - e_i^*(x)]^2 = \frac{\beta c_i(x)}{\alpha (w - t|x|) (E_i/P_i)} \quad (12)$$

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<sup>11</sup>Given the concavity of  $f(\cdot)$ , there is a unique maximum given by  $n_i^*(x)$ .

<sup>12</sup>Observing that (4) implies that

$$\alpha n_i(x) \frac{E_i}{P_i} = \frac{e_i(x)\beta}{[1 - e_i(x)]}$$

Equations (11) or (12) are well-defined if the right-hand side of (12) is lower than one. Otherwise, we have a corner solution:  $e_i^*(x) = n_i^*(x) = 0$ . In the sequel, we focus on the situation where  $e_i^*(x) > 0$  and  $n_i^*(x) > 0$  for all locations  $x$  in the city. For that, we impose that the right-hand side of (12) is less than one, which is equivalent to:

$$\frac{\alpha E_i}{\beta P_i} > \max_x \left[ \frac{c_i(x)}{w - t|x|} \right] \quad (13)$$

We can discuss the basic properties of the employment probability  $e^*(x)$  and number of social interactions  $n_i^*(x)$ . First, when  $c_i(x)/(w - t|x|)$  increases in  $x$ , both the employment probability  $e_i^*(x)$  and the number of interactions  $n_i^*(x)$  fall with the distance from the city center. This occurs for two reasons. On the one hand, as in Zenou and Wasmer (2002), the workers who live further away from the job center have a lower income net of commuting cost,  $w - t|x|$ , which reduces their incentives to search for a job. On the other hand, when  $c_i(x)$  rises, workers reside further away from their social networks that are a source of job information. In this case, their job search efforts become more costly and workers have smaller incentives to search for a job. One of the contributions of the present paper is to highlight the consequences of this new effect that has not been discussed in the literature.

Also, from (12), it can be shown that the employment probability  $e_i^*(x)$  increases with higher aggregate employment rate  $E_i/P_i$ . As workers have higher chance of obtaining information about job opportunities when the individuals in their own social networks are employed, they have higher incentives to search for a job and ultimately are less likely to stay unemployed. However, the impact of the aggregate employment rate  $E_i/P_i$  on the number of interactions  $n_i^*(x)$  is ambiguous and depends on the shape of the function  $f(\cdot)$ . Indeed, one can show from (11) that the number of interactions  $n_i(x)$  decreases with  $E_i/P_i$  if and only if

$$\frac{-z f''(z_i)}{f'(z_i)} > 1$$

evaluated at  $z_i = n_i(x) E_i/P_i$ . This reflects a *substitution effect* between social interactions and employment level in the population (see below). In particular, the impact of

the employment rate on the number of interactions is not monotonic. It is easily checked that  $n_i^*(x)$  falls with  $E_i/P_i$  if and only if

$$\frac{E_i}{P_i} > 4\frac{\beta}{\alpha} \left[ \frac{c_i(x)}{w - t|x|} \right] \Leftrightarrow e_i^*(x) > \frac{1}{2} \quad (14)$$

Hence, when the aggregate employment rate is not too low, workers react to an increase in aggregate employment rate by reducing their job searches amongst their social ties. Workers have indeed better chance to find a job and reduce their efforts in entertaining social interactions. This substitution effect is more important for workers who bear low search costs and reside closer to the city center (low  $c_i(x)/(w - t|x|)$ ).

Applying the envelop theorem, we finally obtain the following land gradient for  $x > 0$ :<sup>13</sup>

$$\Psi'_i(x, u_i) = -e_i^*(x) t [\text{sign}(x)] - n_i^*(x) c'_i(x) \quad (15)$$

So far, we have analyzed the properties of the model for any possible urban configuration. We would like now to study the possible urban configurations under such model. We first study the case of a unique and homogenous population.

## 4 Urban equilibrium with an homogenous population

Assume a single homogenous population residing on the city support  $D = [-b, b]$  where  $b$  is the city border and  $x = 0$  is the CBD. We can drop the subscript  $i$ . Let the city border be  $b = P/2$  where  $P$  is the population size. Remember that we assume that there is a uniform distribution of workers in the city and that each worker consumes one unit of land. Therefore, in the case of a uniform distribution of an homogenous population  $P$  on the interval  $[-b, b]$ , we have  $\lambda_i(y) = \lambda(y) = 1$ . In that case, the total social-interaction

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<sup>13</sup>We adopt the following notation:

$$\Psi'_i(x, u_i) \equiv \frac{\partial \Psi_i(x, u_i)}{\partial x}$$

cost (7) of an individual residing at  $x$  is given by:<sup>14</sup>

$$c(x) = \frac{\tau}{P} \left( \frac{P^2}{4} + x^2 \right) \quad (16)$$

Therefore, the ratio

$$\frac{c(x)}{w - t|x|} = \frac{\tau}{P(w - t|x|)} \left( \frac{P^2}{4} + x^2 \right)$$

increases as one moves from the city center to the border  $b$ . By (10) and (11), we can conclude that the optimal number of social interactions  $n_i^*(x)$  and the individual employment probability  $\pi^*(x)$  and  $e^*(x)$  falls with distance  $x$  from the center.

**Proposition 1** *Consider a homogenous population where workers chose their intensity of social interactions. Then, in any equilibrium, the employment probability  $e(x)$  and the optimal number of social interactions  $n(x)$  fall with distance from the city center.*

Let us now determine the urban configuration. Observe that we consider a closed city model so that the equilibrium utility  $u$  is endogenous while the total population  $P$  is exogenous and equal to  $P = 2b$ .

**Definition 2** *Given that  $c(x)$  is determined by (16), a closed-city competitive spatial equilibrium with an homogenous population is defined by a 5-tuple  $(R^*(x), e^*(x), E^*, n^*(x), u^*)$  satisfying the following conditions:*

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<sup>14</sup>Indeed,

$$\begin{aligned} c(x) &= \frac{1}{P} \int_{-b}^b \tau |x - y| dy \\ &= \frac{\tau}{P} \left( \int_{-b}^0 (x + y) dy + \int_0^x (x - y) dy + \int_x^b (y - x) dy \right) \\ &= \frac{\tau}{P} \left( \int_0^b (x + y) dy + \int_0^x (x - y) dy + \int_x^b (y - x) dy \right) \\ &= \frac{\tau}{P} (b^2 + x^2) \\ &= \frac{\tau}{P} \left( \frac{P^2}{4} + x^2 \right) \end{aligned}$$

(i) *land rent (land-market condition):*

$$R^*(x) = \begin{cases} \max \{ \Psi(x, u^*), 0 \} & \text{for } -b < x < b \\ \Psi(x, u^*) = 0 & \text{for } x = -b \text{ and } x = b \\ 0 & \text{for } x > |b| \end{cases} \quad (17)$$

where  $\Psi(x, u^*)$  is given by (9) without subscript  $i$ .

(ii) *spatial distribution of employment:*

$$e^*(x) = \frac{\alpha n^*(x) E^*/P}{\beta + \alpha n^*(x) E^*/P} \quad (18)$$

(iii) *aggregate employment (labor-market condition):*

$$\frac{E^*}{P} = \frac{1}{2b} \int_{-b}^b e^*(x) dx \quad (19)$$

(iv) *spatial distribution of social interactions:*

$$[\beta + \alpha n^*(x) (E^*/P)]^2 = \frac{\alpha \beta (w - t|x|) (E^*/P)}{c(x)} \quad (20)$$

Because of perfect competition in the land market and continuous land rent, equation (17) says that the land has to be allocated to the highest bidders and that, at the city fringe  $x = b$  or  $x = -b$ , it has to be equal to the price of land outside the city, which we normalize to zero. As explained above, the spatial distribution of employment is determined by a steady-state condition, which is equal to (18). In equilibrium, the aggregate employment rate has to be consistent with the individuals' employment probabilities across the city, so that the total employment is given by (19). Finally, the equilibrium level of social interactions is the result of individuals' maximization problem as expressed by (20).

Let us now determine the equilibrium value of all endogenous variables. By (12), we have

$$e^*(x) = 1 - \sqrt{\frac{\beta c(x)}{\alpha (w - t|x|) (E^*/P)}}$$

and thus (19) can be written as (noticing that  $P = 2b$ ):

$$\frac{E^*}{P} = 1 - \frac{1}{P} \sqrt{\frac{\beta}{\alpha (E^*/P)}} \int_{-P/2}^{P/2} \sqrt{\frac{c(x)}{(w-t|x|)}} dx \quad (21)$$

This is the key equilibrium equation that determines  $E^*$  where  $c(x)$  is given by (16). Once we have calculated  $E^*$ , we obtain  $n^*(x)$  using (20),  $e^*(x)$  using (18), and finally the utility  $u^*$  and the land rent  $R(x)$  using (17).

As can be seen from (21), in the absence of commuting and search costs ( $t = \tau = c(x) = 0$ ), all workers find automatically a job and  $E^*/P = 1$ . The presence of commuting and search costs deter, however, workers to search and take a job. As a result the employment probability is lower. After some algebra, we get the following labor market condition:

$$\sqrt{\frac{\alpha}{\beta}} \left(1 - \frac{E^*}{P}\right) \sqrt{\frac{E^*}{P}} = \Gamma(P) \quad (22)$$

where

$$\Gamma(P) \equiv \frac{1}{P} \int_{-P/2}^{P/2} \sqrt{\frac{c(x)}{w-t|x|}} dx \quad (23)$$

Note that the LHS of (22) represents the benefits from job search (or social interactions). It is a bell-shape curve in  $E/P$  with a maximum at  $E/P = 1/3$ . In the RHS of (22), the function  $\Gamma(P)$  reflects the average share of commuting and search cost in the employment earnings. Higher commuting and search costs indeed increase  $\Gamma(P)$ .

Since the only endogenous variable is  $E^*$ , we can depict the equilibrium in Figure 1.

*[Insert Figure 1 here]*

We have the following result:

**Proposition 3** *Consider the equilibrium defined in Definition 2. If  $w$  is large enough and*

$$\Gamma(P) \leq 0.384\sqrt{\frac{\alpha}{\beta}} \quad (24)$$

*holds, then there exists a unique equilibrium for which  $1/3 < E^*/P < 1$ . In this equilibrium, the employment rate  $E^*$  decreases with the commuting cost  $t$ , the search cost  $\tau$  and the job-destruction rate  $\beta$  but increases with the wage  $w$  and the effectiveness of social interactions in finding a job  $\alpha$ .*

First, observe that condition (24) puts an upper bound on commuting and search costs. It also puts an upper bound  $\bar{P} \equiv \Gamma^{-1}\left(0.384\sqrt{\alpha/\beta}\right)$  on the city size (where  $\Gamma^{-1}$  is the inverse of the function  $\Gamma$ ). Too large city sizes imply too much dispersed searches so that workers have no incentive to search and take jobs. Second, we assume that  $w$  is large enough to avoid a corner solution for which  $E^*/P = e^*(x) = n^*(x) = 0$ . Third, if the commuting cost  $t$  and the search cost  $\tau$  are too high, then equilibrium employment  $E^*$  decreases because it is more costly to be employed (higher  $t$ ) and to search for a job (through social interactions  $\tau$ ). Since  $\alpha$  is the effectiveness of searching for a job via social interactions and  $\beta$  is the job destruction rate, the ratio  $\alpha/\beta$  can be viewed as an indicator of the efficiency of the labor market. When this ratio increases, it becomes easier to find a job and jobs last longer and so employment increases. Finally, when wages  $w$  are higher, the value of employment is higher and thus workers search more intensively for a job (by increasing  $n^*(x) > 0$ ) and therefore employment increases.

Let us now investigate the case of two populations.

## 5 Urban equilibrium with two populations

We now discuss the urban equilibrium when the city hosts two populations. We begin with the case where the two populations are spatially integrated. We then discuss the

case where the populations are spatially segregated. We finally discuss the role and the choice of intra-group interactions.

## 5.1 Spatial integration

We first consider an *integrated city* where the two populations  $i = 1, 2$  reside at every location. As exposed in Section 3, the two populations have exactly the same characteristics except for the fact that they do not mix in terms of social interactions and job searches. Each member of population  $i$  only meets the members of her own population. We want to show here that the absence of social interactions between populations has no impact on labor outcomes in a spatial equilibrium where the two populations are spatially integrated.

Let the total population with sizes  $P_1$  and  $P_2$  with  $P_1 + P_2 = P$  locate on the intervals  $[-b_1, b_1]$  and  $[-b_2, b_2]$ . With a uniform distribution, we have:  $\lambda_i(y) = P_i/P$ , which is the proportion of individuals  $i$  on each plot of land. We consider the *symmetric equilibria* where each population has a residential density proportional to its constant share  $P_i/P$  across the city. In this case, the city border is the same for all populations and equal to  $b_i = b = P/2$ ,  $i = 1, 2$ . The cost of search interactions is given by  $n_i(x) c_i(x)$  where

$$c_i(x) = \frac{1}{P_i} \int_{-b}^b \tau |x - y| \left( \frac{P_i}{P} \right) dy = \frac{1}{P} \int_{-b}^b \tau |x - y| dy$$

As a result,  $c_i(x)$  is equal to  $c(x)$  and given by (16). For a given population size and city border, the cost of each single interaction is the same whenever the city hosts one population or two integrated populations. “Random” searches imply that workers occur the same expected travel distance since the two populations are equally spread. This stems from the population symmetry in both terms of their characteristics and spatial distributions.

Because  $c_i(x) = c(x)$ , the number of interactions and employment probability of each worker  $(n_i^*(x), e_i^*(x))$  depends only on the aggregate employment  $E_i/P_i$  (see (11) and (12)). It is then clear that this spatial configuration is an equilibrium when  $E_1/P_1 =$

$E_2/P_2$ . In this case, the number of interactions and employment probability are identical across populations and have the same values as the ones found under homogenous population. As a result, the bid rents  $\Psi_1(x)$  and  $\Psi_2(x)$  are also equal for all  $x$  and equal to the bid rents in the homogenous population case. No population can offer a higher bid than the other for any piece of land. The equilibrium is defined similarly to Definition 2. The total employment is then given by

$$E_i = \int_{-b}^b f [n_i^*(x) E_i/P_i] \left(\frac{P_i}{P}\right) dy = \int_{-b}^b f [n^*(x) E/P] \left(\frac{P_i}{P}\right) dy$$

where  $(P_i/P)$  is again the proportion of individuals  $i$  on each plot of land. Of course, we consistently get that  $E_i/E = P_i/P$  where  $E = E_1 + E_2$  is the total employment under two spatially integrated populations. This total employment  $E$  is also equal to the employment level of a homogenous population. We summarize this result in the following proposition:

**Proposition 4** *Suppose two identical populations that socially interact only with their own group and are spatially integrated. Then, the equilibrium urban structure and employment rates are the same as in the case where there is one homogenous population.*

It must be noted that the spatially integrated configuration should be seen as a benchmark. Indeed, it is not immune to small perturbations of preferences and technologies. Indeed, this equilibrium would break if population 1 earned slightly higher salaries, needed slightly smaller land plots, had a slightly lower commuting or search cost, etc. It would also not be sustainable if the population size would increase the probability of finding a job (e.g. if  $\pi_i(x) = \alpha n_i(x) (E_i/P_i) \times (P_i)^\delta$ ,  $\delta > 0$ ). Those small perturbations would lead to segregated outcomes.

While the absence of intergroup interactions does not alter the equilibrium employment rates when the populations are spatially integrated, we will show that this is not the case when there is spatial segregation. In that case, different employment outcome may arise. This is what we study now.

## 5.2 Spatial segregation and spatial mismatch

Suppose that population 1 resides close to the city center, i.e. in the interval  $[-b_1, b_1]$ , while population 2 resides at the outskirts of the city, i.e. at  $[-b_2, -b_1] \cup (b_1, b_2]$ , where  $b_1 > 0$  and  $b_2 > b_1$  are the borders of populations 1 and 2. The population sizes are now given by  $P_1 = 2b_1$  and  $P_2 = 2(b_2 - b_1)$  while the total population size is still equal to  $P$ . In that case, with a uniform distribution, we have:  $\lambda_1(y) = 1$  and  $\lambda_2(y) = 0$  for  $y \in [-b_1, b_1]$  while  $\lambda_1(y) = 0$  and  $\lambda_2(y) = 1$  for  $y \in [-b_2, -b_1] \cup (b_1, b_2]$ .

We want to show under which conditions, this spatial configuration is an equilibrium. Notice that if population 1 corresponds to the “white” or “majority” population and population 2 to the “ethnic” or “minority” population, then this spatial equilibrium corresponds to a *spatial mismatch* equilibrium (see our discussion in Section 2 on the spatial-mismatch literature) where the minority workers reside far away from jobs.<sup>15</sup> In that case, the search costs are now given by:

$$c_1(x) = \begin{cases} \frac{\tau}{2b_1} (b_1^2 + x^2) & \text{if } |x| \leq b_1 \\ \tau |x| & \text{if } b_1 < |x| \leq b_2 \end{cases} \quad (25)$$

and

$$c_2(x) = \begin{cases} \frac{\tau}{2} (b_1 + b_2) & \text{if } |x| \leq b_1 \\ \frac{\tau}{2(b_2 - b_1)} (b_2^2 - 2b_1|x| + x^2) & \text{if } b_1 < |x| \leq b_2 \end{cases} \quad (26)$$

Figure 2 displays these two cost functions. It can be checked that the cost  $c_i(x)$  for each type of worker  $i = 1, 2$  is a symmetric and convex function of  $x$ . The search costs increase as workers locate away from the city center. Furthermore,  $c_1(x) < c_2(x)$  for all  $|x| < b_2$  and  $c_1(x) = c_2(x)$  at  $|x| = b_2 = (P_1 + P_2)/2$ . Also, the ratio of average travel costs  $c_2(x)/c_1(x)$  is a monotonically increasing function of  $x$ , for  $x > 0$ , and a decreasing function, for  $x < 0$ .

[Insert Figure 2 here]

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<sup>15</sup>The labels “majority” and “minority workers” do not necessary imply that the size of the population of the “majority” group is larger than that of the “minority” group.

As a result, we can readily conclude that the employment probability in each population decreases as workers reside further away from the city center and that population 2 (the minority group) has a disadvantage in terms of access to its own members and thus to find a job. This is mainly because workers of type 2 are spread around in the city while workers of type 1 are concentrated at the vicinity of the city-center and geographically closer from each other. We need to have a definition of the equilibrium similar to Definition 2.

**Definition 5** *Given that  $c_1(x)$  and  $c_2(x)$  are determined by (25) and (26), and  $P_1 = 2b_1$  and  $P_2 = 2(b_2 - b_1)$ , a closed-city competitive spatial equilibrium with two populations, where population 1 (majority group) resides close to the job center while population 2 (minority group) lives far away from the job center, is defined by a 9-tuple  $(R^*(x), e_1^*(x), e_2^*(x), E_1^*, E_2^*, n_1^*(x), n_2^*(x), u_1^*, u_2^*)$  satisfying the following conditions:*

(i) *land rent (land-rent condition):*

$$R^*(x) = \begin{cases} \max\{\Psi_1(x, u_1^*), \Psi_2(x, u_2^*), 0\} & \text{for } -b_2 < x < b_2 \\ \Psi_1(x, u_1^*) = \Psi_2(x, u_2^*) & \text{for } x = -b_1 \text{ and } x = b_1 \\ \Psi_2(x, u_2^*) = 0 & \text{for } x = -b_2 \text{ and } x = b_2 \\ 0 & \text{for } |x| > b_2 \end{cases} \quad (27)$$

(ii) *spatial distribution of employment for type  $i$  workers:*

$$e_i^*(x) = \frac{\alpha n_i^*(x) E_i^*/P_i}{\beta + \alpha n_i^*(x) E_i^*/P_i} \quad (28)$$

(iii) *aggregate employment (labor-market conditions):*

$$\frac{E_1^*}{P_1} = \frac{2}{P_1} \int_0^{b_1} e_1^*(x) dx \quad (29)$$

$$\frac{E_2^*}{P_2} = \frac{2}{P_2} \int_{b_1}^{b_2} e_2^*(x) dx \quad (30)$$

(iv) *spatial distribution of social interactions for type  $i$  workers:*

$$[\beta + \alpha n_i^*(x) (E_i^*/P_i)]^2 = \frac{\alpha\beta (w - t|x|) (E_i^*/P_i)}{c_i(x)} \quad (31)$$

The interpretation of the equations are similar to that of Definition 2. As above, we look at equilibria for which  $e_i^*(x) > 0$  and  $n_i^*(x) > 0$  for all locations in the city. To guarantee that this is always true, we impose that

$$(E_i/P_i)(\alpha/\beta) > \frac{c_i(b_i)}{w - tb_i}, \quad i = 1, 2 \quad (32)$$

To obtain the labor market conditions for each population  $i = 1, 2$ , using (12), we can write (29) and (30) as follows (noticing that  $b_1 = P_1/2$  and  $b_2 = (P_1 + P_2)/2$ ):

$$\sqrt{\frac{\alpha}{\beta}} \left(1 - \frac{E_i^*}{P_i}\right) \sqrt{\frac{E_i^*}{P_i}} = \Gamma_i(P_1, P_2) \quad (33)$$

where that the average share of commuting and search costs are given by

$$\Gamma_1(P_1, P_2) \equiv \frac{2}{P_1} \int_0^{P_1/2} \sqrt{\frac{c_1(x)}{w - t|x|}} dx \quad (34)$$

and

$$\Gamma_2(P_1, P_2) \equiv \frac{2}{P_2} \int_{P_1/2}^{(P_1+P_2)/2} \sqrt{\frac{c_2(x)}{w - t|x|}} dx \quad (35)$$

We have the following result.

**Proposition 6** *Consider the equilibrium defined in Definition 5. If the wage  $w$  is large enough,  $P_2$  small enough and the following labor-market condition*

$$\max\{\Gamma_1(P_1), \Gamma_2(P_1, P_2)\} < 0.384\sqrt{\frac{\alpha}{\beta}} \quad (36)$$

*holds, then there exists a unique equilibrium for which  $1/3 < E_1^*/P < 1$  and  $1/3 < E_2^*/P < 1$ .*

Condition (36) is similar to condition (24) for the homogenous-population case. The first constraint,  $\Gamma_1(P_1) < 0.384\sqrt{\alpha/\beta}$ , puts an upper bound on population 1,  $\bar{P}_1 \equiv \Gamma_1^{-1}(0.384\alpha/\beta)$ , which is the same as for an homogenous population. The second constraint,  $\Gamma_2(P_2) < 0.384\alpha/\beta$ , puts an upper bound for population 2,  $\bar{P}_2(P_1) \equiv \Gamma_2^{-1}(P_1, 0.384\alpha/\beta)$ .<sup>16</sup>

<sup>16</sup>Here,  $\Gamma_2^{-1}$  is the inverse of  $\Gamma_2(P_1, P_2)$  with respect to the second argument.

We also assume that  $w$  is large enough so that there is no corner solution for which  $e_i^*(x) = n_i(x) = 0$  and that  $P_2$  is small enough so that population 2 can outbid population 1 at the periphery of the city. This is a reasonable assumption since population 2 is the minority group. As a result, these conditions will hold if both populations  $P_1$  and  $P_2$  are not too large.

**Proposition 7** *Consider the equilibrium defined in Definition 5. Then, the employment rate of population 1 is always higher than that of population 2 whatever their relative sizes, i.e.  $E_1/P_1 > E_2/P_2$ . Moreover,  $E_2/P_2$  decreases with higher  $P_1$  and  $P_2$ . In addition, the worker's employment probability  $e_i(x)$  decreases with  $x$ , the distance to the city center and abruptly falls at the border  $|b_1|$  between the two populations. The number of social interactions  $n_i(x)$  also decreases with distance from the center but abruptly rises or falls at the border  $b$  depending on whether their employment probability is high or low.*

Figure 3 (upper panel) depicts the employment levels in this equilibrium where population 1 locates at the city center and population 2 at the periphery of the city. We see that population 1 always experiences a higher employment rate than population 2, the reason being that it has a better average access to its social network. As a result, individuals have more incentives to find a job. The employment level falls dramatically at the border between the two populations. Individuals from population 2 have a different social network from those of population 1 and a lower average access to their interaction partners. Figure 3 (lower panel) displays the equilibrium land rent for the two populations. Even though workers from population 1 experience a higher employment rate, they pay a higher land price to occupy locations close to the job center. As the periphery of the city, they bid less for land and thus workers from population 2 reside in this part of the city.

[Insert Figure 3 here]

This result is new and interesting because it highlights the feedback effect of space and segregation on labor-market outcomes. If we take two identical populations in all possible characteristics, then employment differences result from the existence of spatial segregation and the resulting spatial organization of workers' social networks. Workers obtain job information through their social contacts that belong to the same type but organize in a different way through the urban area. This mechanism contrasts with the analysis presented by the literature that assumes some exogenous discrimination by landlords (see e.g. Brueckner and Zenou, 2003 or Zenou, 2013) or by employers (Verdier and Zenou, 2004).

What is interesting here is that minority workers end up with adverse labor-market outcomes because they reside far away from jobs (*spatial mismatch*), far away from their social networks (*social mismatch*) and the *quality of their social network* is low (the employment rate of population 2 is lower than that of population 1). As in all these coordination models, one could have another equilibrium where population 1 resides at the periphery of the city while population 2 lives close to the city-center. In other words, the labor market can support equilibria with the largest or the smallest population in the city center.

One may interpret our model in terms of black (population 2) and white workers (population 1) in the United States. Then, there is strong evidence on the segregation of black workers, racial homogeneity and disconnection to jobs. Indeed, in 1980, after a century of suburbanization, 72% of metropolitan blacks lived in central cities, compared to 33% of metropolitan whites (Boustant, 2010). The fact that black families who tend to live in central cities are far away from jobs is well-documented (see, in particular, the literature surveys by Ihlanfeldt and Sjoquist, 1998, and Ihlanfeldt, 2006).<sup>17</sup> The racial homogeneity of neighborhoods is also a well documented phenomenon in US cities.

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<sup>17</sup>The fact that, in the real-world, black workers tend to live in central cities is not in contradiction with our model because what matters is the distance to jobs and the access to the social networks of workers from the same group. Indeed, in our model, black workers are far away from the job center (CBD) and could reside either in the center or the periphery of the city.

In 1979, for example, the average black lived in a neighborhood that was 63.6% black, even though blacks formed only 14.9% of the population (Borjas, 1998). In the 1990 census, the figures were similar (Cutler et al., 1999). Racial segregation by jurisdiction and neighborhoods has historical roots in two population flows: black migration from the rural South and white relocation from central cities to suburban rings. Both flows peaked during World War II and the subsequent decades (Boustant, 2010). As in our model, many studies find that blacks who live in segregated metropolitan areas have lower labor-market outcomes than their counterparts in more integrated areas (for an overview, see Boustant, 2012). This difference appears to reflect the causal effect of segregation on economic outcomes. This literature shows that the association between segregated environments and minority disadvantage is driven in part by physical isolation of black neighborhoods from employment opportunities and in part by harmful social interactions within black neighborhoods, especially due to concentrated poverty.

### 5.3 Numerical simulations

To illustrate better our model, let us perform some simple numerical simulations. Table 1 shows the value of each population's aggregate employment rates  $E_1^*/P_1$  and  $E_2^*/P_2$  with varying population sizes  $P_1$  and  $P_2$ . For instance, a city with  $P_1 = P_2 = 1$  has aggregate employment rates equal to  $E_1^*/P_1 = 0.94$  and  $E_2^*/P_2 = 0.90$ . The table confirms the results of Proposition 7: whatever its relative size, population 1 residing close to the job center (majority group) has the largest aggregate employment rate. We also see that,  $E_2^*/P_2$ , the aggregate employment rate of population 2 (which resides far away from jobs) decreases with its population size ( $P_2$ ) and with the size of population 1 ( $P_1$ ). Note that the table also shows that the city can support an equilibrium for which the peripheral population is larger than the central one (see for instance the configuration where  $P_1 = 0.1$  and  $P_2 = 30$ ).

$P_1$	$P_2$								
	0.1	0.5	1	5	10	15	20	30	40
0.1	(98,97)	(98,95)	(98,94)	(98,86)	(98,79)	(98,73)	(98,68)	(98,56)	-
0.5	(96,95)	(96,93)	(96,92)	(96,85)	(96,78)	(96,73)	(96,67)	(96,55)	-
1	(94,92)	(94,91)	(94,90)	(94,84)	(94,78)	(94,72)	(94,66)	(94,54)	-
5	(86,82)	(86,82)	(86,81)	(86,76)	(86,71)	(86,65)	(86,59)	(86,44)	-
10	(79,73)	(79,73)	(79,72)	(79,68)	(79,62)	(79,56)	(79,48)	-	-
15	(74,64)	(74,64)	(74,63)	(74,59)	(74,52)	(74,42)	-	-	-
20	(68,55)	(68,54)	(68,54)	(68,47)	-	-	-	-	-
30	-	-	-	-	-	-	-	-	-

Table 1: Equilibrium employment rates (percent)  $E_1^*/P_1$  and  $E_2^*/P_2$ .

Parameters:  $t = \tau = \alpha = \beta = 0.1$  and  $w = 10$ .

Table 1 also confirms Proposition 6 according to which populations cannot be too large to sustain an urban equilibrium (see condition (36)). The “–” signs indicate when the urban equilibrium does not exist because either (33) have no solution or the bid rent condition  $\Psi_2(x) > \Psi_1(x)$  is violated on the interval  $(b_1, b_2]$ .<sup>18</sup> Importantly, the table shows the existence of *multiple equilibria* for many population configurations. For instance, there exist both an equilibrium with population sizes  $(P_1, P_2) = (0.1, 1)$  and employment rates  $(E_1^*/P_1, E_2^*/P_2) = (0.98, 0.94)$  and an equilibrium with  $(P_1, P_2) = (1, 0.1)$  and  $(E_1^*/P_1, E_2^*/P_2) = (0.94, 0.92)$ . We can see that the total equilibrium employment  $E_1^* + E_2^*$  is higher in the former than in the latter when the center population has a bigger size. The multiplicity of equilibria also takes place in the configurations where both populations have identical sizes. For example, when  $(P_1, P_2) = (1, 1)$ , one population has a employment rate of either 0.94 or 0.9 depending whether it locates at the city center

<sup>18</sup>Those conditions have been verified for each cell. Note that in this example only conditions (33) bind.

or in the periphery. The multiplicity arises because of the convex travel costs incurred for social interactions, which makes bid rents non-linear (see Figure 3). As stated above, when we interpret our model in terms of black and white workers, the fact that we have an equilibrium where blacks reside far away from jobs (population 2) could be due to historical reasons such that the black migration from the rural South to Northern cities and the white relocation from central cities to suburban rings.

This numerical example also suggests that the multiplicity of equilibria occurs as long as populations are not too large since when  $P_1 \geq 20$  and  $P_2 \geq 20$  no equilibrium can be sustained. Finally, there exist population configurations that support only one equilibrium. For example, the population configuration  $(P_1, P_2) = (10, 20)$  is an equilibrium whereas  $(P_1, P_2) = (20, 10)$  is not. In that case, the larger population splits and locates at the periphery. Such configurations are found close to the limit where the city stops to be an equilibrium.

We can also highlight the impact of the social interaction (travel) costs  $\tau$  on economic outcomes. First, observe that, in our model, workers are never unemployed when the travel cost  $\tau$  is zero since, in that case, workers social interact with other workers at no cost and thus choose an infinite number of social interactions  $n_i(x)$ , which implies that the probability of finding a job is equal to 1. In Table 2, we vary the travel costs from  $\tau = 0.01$  to  $\tau = 0.20$  when the two populations have equal sizes,  $(P_1, P_2) = (1, 1)$ .

$\tau$	$Un_{\text{tot}}(\%)$	$Un_1(\%)$	$Un_2(\%)$	$\text{avr}(n_1^*)$	$\text{avr}(n_2^*)$	$u_1^*$	$u_2^*$	$\text{avr}(\Psi_1^*)$	$\text{avr}(\Psi_2^*)$	$W_{\text{tot}}$
0.01	2.4	1.8	2.9	55.2	17.1	9.4	9.3	0.24	0.04	18.92
0.05	5.4	4.2	6.7	24.4	7.5	8.7	8.5	0.44	0.07	17.73
0.1	7.8	5.9	9.6	17.1	5.2	8.3	7.9	0.58	0.09	16.84
0.15	9.6	7.3	11.9	13.9	4.2	7.9	7.5	0.68	0.11	16.15
0.2	11.2	8.5	14.	11.9	3.6	7.6	7.1	0.77	0.12	15.58

Table 2: Impact of social interaction costs for equal populations  $(P_1, P_2) = (1, 1)$ .

Parameters:  $t = \alpha = \beta = 0.1$  and  $w = 10$ .

This table shows that higher social-interaction costs raise unemployment rates in each population and therefore in the whole population (see  $Un_i \equiv (P_i - E_i)/P_i$  and  $Un_{\text{tot}} \equiv \sum_i (P_i - E_i) / \sum_i P_i$ ). As expected, a rise in travel costs decreases the average number of social interactions ( $\text{avr}(n_i) \equiv \int_{D_i} n_i^* dx / P_i$ ). It also decreases the equilibrium utility ( $u_i^*$ ), increases the average land rent ( $\text{avr}(\Psi_i^*) \equiv \int_{D_i} \Psi_i^* dx / P_i$ ) and reduce the total welfare ( $W_{\text{tot}} = \sum_i P_i [u_i^* + \text{avr}(\Psi_i^*)]$ ). Also, the peripheral population has higher unemployment, exerts fewer effort in searching for a job, obtains a lower utility and pays lower land rents.

What is the impact of the separation of social networks on those economic variables? To discuss this, consider a very large shift of the populations so that population 1 increases from 1 to 1.99 while population 2 decreases from 1 to 0.01. Table 3 displays the results of the simulations.

$\tau$	$Un_{\text{tot}}(\%)$	$Un_1(\%)$	$Un_2(\%)$	$\text{avr}(n_1^*)$	$\text{avr}(n_2^*)$	$u_1^*$	$u_2^*$	$\text{avr}(\Psi_1^*)$	$\text{avr}(\Psi_2^*)$	$W_{\text{tot}}$
0.01	2.6	2.6	3.2	38.9	0.2	9.3	9.3	0.17	0.0	18.88
0.05	5.9	5.9	7.4	17.1	0.1	8.5	8.5	0.3	0.0	17.61
0.1	8.5	8.5	10.6	11.9	0.0	7.9	7.9	0.4	0.0	16.67
0.15	10.5	10.5	13.2	9.6	0.0	7.5	7.5	0.48	0.0	15.94
0.2	12.3	12.3	15.4	8.3	0.0	7.1	7.1	0.54	0.0	15.32

Table 3: Impact of interaction cost for unequal populations  $(P_1, P_2) = (1.99, 0.01)$ .

Parameters:  $t = \alpha = \beta = 0.1$  and  $w = 10$ .

In this new configuration, the larger central population 1 experiences longer average trips to access to their social network while population 2 crosses the city to access half of its social network, which is now much smaller in size. We see that the average number of social interactions decreases for both populations because of the longer average trips of population 1 and smaller network size of population 2. As a result, job search is more difficult for both populations and unemployment rates are higher in each population and

therefore in aggregate. Interestingly, the utility and welfare levels do not differ significantly (compared to Table 3) because most of the employment disadvantages are balanced by lower search efforts and land rents.

To sum up, in the absence of inter-group interactions, we show that the population located far away from the employment center experience higher unemployment rates and lowers social interactions and job search activities.

## 5.4 Inter-group interactions

Up to now, we have imposed that workers only socially interact within their own population. This was justified by the existing barriers between social networks such as ethnic or language barriers. In this section, we discuss the possibility of inter-group social interactions and show under which conditions workers choose to socially interact exclusively within their own population.

Workers from population  $i$  choose their numbers of interactions both with their own population  $i$  (denoted by  $n_{ii}(x)$ ) and with the other population  $j$  (denoted by  $n_{ij}(x)$ ). As before, the individual's probability of finding a job depends on the number of social interactions and the aggregate employment rate of the visited population. In addition, language and/or ethnic differences create communication and/or trust issues that may yield possible negative biases in the effectiveness of transmitting information on job opportunities. For that, we assume that the probability of finding a job for a worker of type  $i$  is now given by:

$$\pi_i(x) \equiv \alpha \left[ n_{ii}(x) \frac{E_i}{P_i} + \gamma n_{ij}(x) \frac{E_j}{P_j} \right]$$

where  $\gamma \in (0, 1)$  is the negative bias in inter-group communication. This plays a role similar to the *preference bias* discussed in Currarini et al. (2009). This extended model obviously collapses to our benchmark model if  $\gamma \rightarrow 0$ . The worker's employment probability is still given by (2), i.e.  $e_i(x) = f[\pi_i(x)] \equiv \pi_i(x) / [\pi_i(x) + \beta]$ . The bid rent function is still given by the maximal land rent that the worker can afford and can now be written

as:

$$\Psi_i(x) = \max_{n_{ii}(x), n_{ij}(x)} [e_i(x)(w - t|x|) - n_{ii}(x)c_i(x) - n_{ij}(x)c_j(x)] - u_i$$

subject to  $n_{ii}(x) \geq 0$  and  $n_{ij}(x) \geq 0$ , where  $c_i(x)$  and  $c_j(x)$  are given by (25) and (26). The optimal number of social interactions is determined as follows. First, if  $c_i(x)/(E_i/P_i) < c_j(x)/(\gamma E_j/P_j)$ , the worker only chooses to interact with her own population so that

$$(E_i/P_i) f' [n_{ii}^*(x) E_i/P_i] = \frac{c_i(x)}{w - t|x|} \quad (37)$$

and  $n_{ij}^*(x) = 0$ . Obviously, this is equal to the optimal number of interactions  $n_i^*(x)$  that is chosen when there are no inter-group interactions and given by (28). Second, if  $c_i(x)/(E_i/P_i) > c_j(x)/(\gamma E_j/P_j)$ , the worker chooses to interact only with the other population  $j$  so that

$$(\gamma E_j/P_j) f' [n_{ij}^*(x) E_j/P_j] = \frac{c_j(x)}{w - t|x|} \quad (38)$$

and  $n_{ii}^*(x) = 0$ . Finally, if  $c_i(x)/(E_i/P_i) = c_j(x)/(\gamma E_j/P_j)$ , the worker chooses to interact with any mix of the two populations.

To solve this social-interaction choice, we can consider both the spatially-integrated and spatially-segregated city equilibria from Sections 5.1 and 5.2.

### Proposition 8

(i) Consider the **spatially-integrated city** described in Definition 2. In this city, workers have no incentives to interact with the other population.

(ii) Consider the **spatially-segregated city** described in Definition 5. If

$$\frac{2P_1 + P_2}{2\gamma P_1} > \frac{E_2/P_2}{E_1/P_1} > \frac{\gamma(2P_1 + P_2)}{2P_1} \quad (39)$$

hold, then no workers want to interact with other workers from the other population.

In the *spatially-integrated city*, the two populations are totally symmetric, in particular in terms of social-interaction costs and employment rates. In the presence of a bias  $\gamma$  in

the social interactions with the other population, it is clearly optimal not to interact with the other population.

In the *spatially-segregated city*, where population 1 lives close to jobs and population 2 further away, things are less straightforward. In that case, population 1 will not interact with population 2 if it has a strong employment advantage and/or if population 2 has a strong employment disadvantage, and/or if the inter-group communication is ineffective. The first inequality in (39) gives the condition for which this is true. The condition always holds because the LHS is larger than one while, by Proposition 7, the RHS is lower than one as the equilibrium aggregate employment rate of population 1 is always larger than the one of population 2 ( $E_1/P_1 > E_2/P_2$ ). This is not that surprising given that the benefit of reaching an individual of population 2 is less effective in terms of acquisition of job information and more costly in terms of travel cost because of its dispersion around the periphery of the city.

Similarly, population 2 will not interact with population 1 if the former has no strong employment disadvantage and/or if the latter has no strong employment advantage, and/or if the inter-group communication is ineffective. This is expressed by the second inequality in (39). Population 2 has no incentive to seek interactions with the other population if the effectiveness of inter-group communication is low enough. Population 2 has a clear benefit of “chasing” population 1 because the latter conveys more job information and spreads over a compact area. The negative bias in inter-group communication is therefore necessary to cut the incentives to interact with population 1. However, this bias needs not to be very strong. As an illustration, a 10%-minority population will not interact with a 90%-majority population for any bias  $\gamma$  lower than 0.93 when the aggregate employment rates are 94% and 92% for populations 1 and 2 (see Table 1 at the entry  $(P_1, P_2) = (1, 0.1)$ ). Finally, *ceteris paribus*, the absence of inter-group interactions holds provided that the population occupying the center is relatively large compared to the one at the periphery. This indeed keeps the RHS low enough in the second condition of (39). In this sense, the combination of spatial segregation and absence of inter-group

interactions - as we have studied above - is more likely to be consistent with the urban configuration where a minority group locates far away from the job center.<sup>19</sup>

## 6 Directed search interactions

Let us go back to the model where workers only interact with other workers from the same group. In the previous sections, workers chose the frequency of search interactions without knowing the location of the interaction partners (*random search*). We now consider that search interactions are *directed* in the sense that workers choose the frequency of interactions according to the location of their interaction partners. We first consider the homogenous-population case and then the heterogenous one. We show that the results under random and directed search are qualitatively similar.

### 6.1 Homogenous population

Suppose now that a worker located at  $x$  in the city support  $D = [-b, b]$  chooses the number of interactions  $n(x, y)$  with another individual located at  $y$ . Each interaction with a person located at  $y$  gives her a probability of finding a job equal to  $\pi(x, y)$ , which depends on the repetition of interaction,  $n(x, y)$ , and the employment likelihood of the person she meets,  $e(y)$ . That is, we now assume that the probability of finding a job for a worker located at  $x$  and meeting a worker located at  $y$  is given by:

$$\pi(x, y) = \alpha v [n(x, y)] e(y) \quad (40)$$

where  $\alpha > 0$  and where  $v [z] = 1 - \exp [-z]$ ,<sup>20</sup> which is increasing and concave, with  $v(0) = 0$ . Quite naturally, there are decreasing returns to the number of social interactions. Interestingly,  $n(x, y)$  now varies with  $y$  because of  $e(y)$ , which means that the individual

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<sup>19</sup>For instance, using the population entries of Table 1, we find that, for any  $\gamma \geq 0.3$ , population 2 has no incentives to interact with population 1 if it is a minority group ( $P_2 < P_1$ ), but do want to interact with population 1 if it is a majority group ( $P_2 > P_1$ ).

<sup>20</sup>It will be clear below why we choose an exponential function.

located in  $x$  may interact very often with a person located in  $y$  because her employment  $e(y)$  is high and less often with someone residing in  $x'$  because  $e(x')$  is low. This was not true in the previous section where  $n(x)$  was constant and independent of the location of the person visited because of random search. In that case, each location was visited as often as any other one. The probability of finding a job for a worker located at  $x$  now depends on the total set of interactions and is given by:

$$\pi(x) = \int_D \pi(x, y) dy = \int_D \alpha v [n(x, y)] e(y) dy = \int_D \alpha (1 - \exp[-n(x, y)]) e(y) dy \quad (41)$$

Indeed, instead of (3), we define  $\pi(x)$  as in (41) so that each contact with a person depends on her location (here  $y$ ) and her employment status ( $e(y)$ ). This is why we have an integral over locations  $y$  and why  $e(y)$  now replaces  $E/P$ , which did not vary with location. As before, the probability of being employed is equal to  $e(x) = \pi(x)/[\pi(x) + \beta]$ . For simplicity, we denote  $e(x) = g[\pi(x)]$  where  $g(z) \equiv z/(z + \beta)$ . It is easily verified that  $g(z)$  is an increasing and concave function of  $z$ .

The bid rent is given by the maximal land rent that the worker can afford given her chosen frequency of directed searches:

$$\Psi(x) = \max_{n(x, \cdot)} e(x) (w - t|x|) - \int_D \tau |x - y| n(x, y) dy - u \quad (42)$$

where  $\tau |x - y|$  is the travel cost for a single search interaction. By maximizing  $\Psi(x)$ , we obtain the following first order condition:

$$v' [n^*(x, y)] = \frac{1}{e(y)g'[\pi(x)]} \frac{\tau |x - y|}{w - t|x|} \quad (43)$$

which has a unique solution for  $n^*$  because  $v'(\cdot)$  is a decreasing function. The frequency of search interactions decreases with the distance to the visited individual  $|x - y|$  and with the distance  $|x|$  to the workplace while it increases with the employment likelihood  $e(y)$  of the visited agents. From a job search perspective, workers prefer to be closer to other employed workers.

Using the property of the exponential function,  $v[v'^{-1}(z)] = 1 - z$  and keeping the definition of average search cost, i.e.  $c(x) = \int_{-b}^b \tau |x - y| dy/P$ , the probability of finding

a job is then equal to:<sup>21</sup>

$$\pi(x) = \alpha E - \frac{P}{g'[\pi(x)]} \frac{c(x)}{w - t|x|} \quad (44)$$

where  $c(x)$  is given by (16). Observe that there exists very few  $v[\cdot]$  functions such that this integral has an explicit formulation because  $e(y)$  must aggregate adequately. This is why we chose an exponential function for  $v[\cdot]$ . Consider the equilibrium defined in Definition 2 but for directed search so that equation (18) is replaced by  $e(x) = \pi(x)/[\pi(x) + \beta]$ , where  $\pi(x)$  is given by (41), equation (20) is replaced by (43) and  $n^*(x)$  by  $n^*(x, y)$ . We have the following result.

**Proposition 9** *Consider a closed-city competitive spatial equilibrium with an homogeneous population and directed search. Assume that  $w$  is large enough. Then, if the population size  $P$  belongs to some interval  $[\underline{P}, \overline{P}]$ , there exists a unique high-employment level  $E^*$  such that  $e^*(x)$  is given by:*

$$e^*(x) = 1 - \frac{\beta + \sqrt{\beta^2 + 4\beta(\beta + \alpha E^*) \frac{\tau[(P/2)^2 + x^2]}{w + tx}}}{2(\beta + \alpha E^*)} \quad (45)$$

and  $E^*$  by

$$2(\beta + \alpha E^*) E^* = (\beta + 2\alpha E^*) P - F(P, E^*) \quad (46)$$

where

$$F(P, E^*) = 2 \int_0^{P/2} \sqrt{\beta^2 + 4\beta(\beta + \alpha E^*) \frac{\tau[(P/2)^2 + x^2]}{w + tx}} dx \quad (47)$$

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<sup>21</sup>Indeed,

$$\begin{aligned} \pi(x) &= \int_D \alpha v[n^*(x, y)] e(y) dy \\ &= \alpha \int_D v \left\{ v'^{-1} \left[ \frac{1}{\alpha e(y) g'[\pi(x)]} \frac{\tau|x-y|}{w-t|x|} \right] \right\} e(y) dy \\ &= \alpha \int_D \left\{ 1 - \frac{1}{\alpha e(y) g'[\pi(x)]} \frac{\tau|x-y|}{w-t|x|} \right\} e(y) dy \\ &= \alpha E - \frac{1}{g'[\pi(x)]} \frac{\tau \int_D |x-y| dy}{w-t|x|} \end{aligned}$$

which leads to (44).

*In this equilibrium, the employment probability  $\pi(x)$  and the frequency of search interactions  $n^*(x, y)$  decreases with the distance to the job center while the employment rate  $E^*$  decreases with larger commuting  $t$  and search costs  $\tau$  but increases with wages  $w$ .*

First, the employment rate  $e^*(x)$  decreases with higher distance  $x$  to the job center. Accordingly, workers residing away from the center and their own social network have less incentives to search a job and have therefore lower employment rates. Second, suppose that the travel cost parameter  $\tau$  is equal to zero. Then, we obtain the standard “frictional” employment and unemployment rates  $e^*(x) = \alpha E^* / (\beta + \alpha E^*)$  and  $1 - e^*(x) = \beta / (\beta + \alpha E^*)$ . Those values are constant across space because workers reach each other worker at no cost. They are also sensitive to the number of employed workers. Indeed, the probability  $\pi(x, y)$  that a worker located at  $x$  finds a job by interacting with someone at  $y$  is bounded given our assumption on  $v(z) < 1$ . As a result, the probability of finding a job - given all entertained interactions -  $\pi(x)$  increases with the number of employed workers that are visited. Intuitively, an increase in urban population improves the potential amount of job information and therefore raises more than proportionally the employment level. Therefore, search frictions have stronger effects in smaller cities where employment probabilities are lower. If the population is too small, there exists not enough job information to induce workers to search for a job and the equilibrium may therefore fail to exist. This is why Proposition 9 requires the population size to be higher than the threshold  $\underline{P}$ . Finally, the existence of travel cost exacerbates the effect of search frictions. It is represented in the second term of the square root of (45). Unsurprisingly the job search cost raises the frictional unemployment rate.

Even though we can understand the main properties of equation (46), it is difficult to solve it analytically. We thus run some numerical simulations for this equation. Figure 4 plots the locus of (46) in the space  $(E, P)$  (see solid curve). As stated in Proposition 9, this figure confirms the conclusions established in the case of random search. First, the city supports only small enough population (i.e.  $P < \bar{P}$ ). Second, there exist multiple equilibria as each population size  $P$  supports a high and low employment equilibrium. If

we focus on the high-employment equilibrium, then it can be seen that, as the population size rises, the employment level  $E^*$  first increases and then decreases. This is the result of two forces. On the one hand, when the city size is small, an increase in the population raises the employment rate more than proportionally because the frictional unemployment  $\beta/\alpha$  becomes a smaller portion of the workforce. On the other hand, when the city size becomes too large, commuting and search travel costs reduce the workers' net income (wages minus travel cost) and therefore their incentives to search for a job.

[Insert Figure 4 here]

Let us now investigate the case of two populations.

## 6.2 Heterogenous populations

As in Section 5, let us now consider two populations of sizes  $P_1$  and  $P_2$  that spread over the supports  $D_1 = [-b_1, b_1]$  and  $D_2 = [-b_2, -b_1) \cup (b_1, b_2]$ . Our analysis of Section 6.1 holds by substituting the parameters  $(P, D)$  and the functions  $(n, e, \pi, \gamma, c)$  respectively for  $(P_i, D_i)$  and  $(n_i, e_i, \pi_i, \gamma_i, c_i)$ ,  $i = 1, 2$ , where  $c_i(x)$  are defined by (25) and (26). The employment probability is given by

$$e_i^*(x) = 1 - \frac{\beta + \sqrt{\beta^2 + 4\beta(\beta + \alpha E_i^*) P_i \theta_i(x)}}{2(\beta + \alpha E_i^*)} \quad (48)$$

where

$$\theta_i(x) = \frac{c_i(x)}{w - t|x|}$$

It can be seen that  $e_i^*(x)$  increases when  $c_i(x)$  decreases with  $x$ . Therefore, within the same population, the employment rate rises when workers are located closer to the job center. The difference in a worker's employment probability between two populations not only depends on her location but also on the aggregate employment  $E_i$  and the size of her

population  $P_i$ . In equilibrium, as in (46), the labor market condition for each population  $i = 1, 2$  is determined by:

$$2(\beta + \alpha E_i^*) E_i^* = (\beta + 2\alpha E_i^*) P_i - F_i(E_i^*, P_1, P_2) \quad (49)$$

where

$$F_1(E_1^*, P_1, P_2) = 2 \int_0^{P_1/2} \sqrt{\beta^2 + 4\alpha(1 + \alpha E_1^*) P_1 \theta_1(x)} dx$$

$$F_2(E_2^*, P_1, P_2) = 2 \int_{P_1/2}^{(P_1+P_2)/2} \sqrt{\beta^2 + 4\alpha(1 + \alpha E_2^*) P_2 \theta_2(x)} dx$$

Since  $c_1(x)$  is equal to  $c(x)$ , it turns out that  $F_1(E_1^*, P_1, P_2)$  is equal to the function  $F(E_1^*, P_1)$  defined in (47) for a homogenous population. As a result, population 1 has an aggregate employment that only depends on its own size  $P_1$ . Figure 4 displays the locus of labor market equilibria for population 1 in terms of  $(P_1, E_1)$  using the same solid curve as for the homogenous population. The properties of population 1's labor market condition exactly replicates those of the homogenous population. In particular, the labor market condition is satisfied only for a population size of  $P_1$ , which is smaller than some upper bound  $\bar{P}_1$  and there exist two equilibria with high and low employment rates. We again focus on the high-level employment rates.

The equilibrium employment in the peripheral population 2 is determined by condition (49) for  $i = 2$  using the term  $F_2(E_2^*, P_1, P_2)$ . The dashed curves in Figure 4 represent the loci of those equilibria in  $(P_2, E_2)$  for several values of  $P_1$ . We obtain the same properties as in the case of random search. For a given  $P_1$ , there exists an upper envelop  $\bar{P}_2(P_1)$  such that the labor market condition has a solution. As in the case of random search, the labor market condition holds if  $P_1 < \bar{P}_1$  and  $P_2 < \bar{P}_2(P_1)$ . There also exist a lower and higher employment equilibrium, and we focus on the latter. As in the random-search case, the labor market can support equilibria with the larger or the smaller population residing close to the job center. Finally, Figure 4 shows that, if populations 1 and 2 have the same size, population 1 will have a higher aggregate employment rate  $E_i/P_i$  (that is, the ray from the origin  $(0, 0)$  to the equilibrium point has smaller angle for population 1).

This property remains true as long as population 1 is not too large and approaches the threshold level  $\bar{P}_1$ . Figure 4 also shows that there exists a minimum size for the peripheral population 2. This is because population 2 must benefit from sufficient social interactions to overcome its disadvantage in terms of job search and commuting to the employment center.

We can then close the model with the same land market conditions defined in (27) for the random-search case where  $\Psi(x, u_i)$  is still defined by (42) with the only difference that  $n_i(x)$  is now replaced by  $n_i(x, y)$ . It can easily be shown  $\Psi(x, u_i)$  is a decreasing function of  $x$  but it is difficult to show analytically that population 1 outbids population 2 in the interval  $[-b_1, b_1]$  and that the reverse is true in the intervals  $[-b_2, b_1]$  and  $[b_1, b_2]$ .<sup>22</sup> Therefore, the land market equilibrium must be numerically checked for each configuration of population. Figure 5 provides two examples where the land-market conditions do (left panel) and do not hold (right panel). In this figure, using the land market equilibrium conditions, we have plotted the bid-rent function for some population configurations. In the left panel, the bid rents cross only once so that both land and labor market conditions are simultaneously satisfied. In the right panel, the bid rents cross more than once so that the labor market conditions cannot support an urban equilibrium for which population 1 resides close to the job center and population 2 lives at the periphery of the city.

*[Insert Figure 5 here]*

Finally, Table 4 displays a set of population configurations for which both the labor and land market clear. It can be seen that equilibrium solutions exist only when the population sizes are neither too small nor too large. Also, the equilibrium employment rate for each population decreases with the size of each population or equivalently with the size of the city.

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<sup>22</sup>See the end of the Appendix where we partly show these results.

		$P_2$							
		6	8	12	16	20	24	28	32
$P_1$	6	(76, 58)	(76, 71)	--	--	--	--	--	--
	12	(80, 38)	(80, 63)	(80, 67)	(80, 69)	(80, 70)	--	--	--
	18	—	(80, 54)	(80, 61)	(80, 64)	(80, 65)	(80, 65)	(80, 64)	--
	24	—	—	(78, 53)	(78, 57)	(78, 59)	(78, 60)	(78, 59)	(78, 59)
	30	—	—	—	(76, 48)	(76, 51)	(76, 53)	(76, 53)	(76, 53)
	36	—	—	—	—	—	—	—	(74, 42)

Table 4: Aggregate employment rates (percent)  $E_1/P$  and  $E_2/P_2$

A “—” indicates that there is no solution for the labor market conditions.

A “--” indicates no land market equilibrium.

Parameters:  $(\alpha, \beta, t, \tau, w) = (0.1, 0.1, 0.1, 0.05, 20)$

## 7 Discussion and policy implications

In this paper, we develop a model where workers both choose their residential location (*geographical space*) and social interactions (*social space*). In equilibrium, we show under which condition the majority group resides close to the job center while the minority group lives far away from it. Even though the two populations are ex ante totally identical, we find that the majority group experiences a lower unemployment rate than the minority group and tends to socially interact more with other workers from her own group. Within each group, we demonstrate that workers residing farther away from the job center tend to search less for a job and are less likely to be employed. Indeed, workers from the majority group are less spread in the city than that of the minority group and thus have a better access to their social networks. This motivates them to search more actively for a job so that their aggregate employment rate is higher. This, in turn, makes social interactions more efficient since each visit to another worker from the majority group leads to a higher chance of obtaining a job. As a result, these workers will even interact more with other

workers from the majority group, which will increase the employment rate for this group, etc. This model is thus able to explain why ethnic minorities are segregated in the urban and social space and why this leads to adverse labor-market outcomes.

In first extension, we show that it can be optimal for the majority and the minority groups not to socially interact with each other. In a second extension, we analyze a model where workers can direct their search so that they interact more with workers who are more beneficial for them (in terms of employment) than others. In that case, the results are relatively close to the ones obtained with random search.

To wrap up, our main contribution is to develop a model where segregation in the urban and social space is endogenously determined and to explain why ex ante identical workers end up with very different labor-market outcomes because of this separation in the urban and social space. Our model puts forward the importance of the *direct interactions* between people in obtaining a job and why the majority-group social network is easier to access and of better quality than the one for the minority group.

Using the results of this paper, we can draw some policy implications that may improve the integration of minority workers in the city and help them find a job. We have shown that the neighborhood and distance to jobs are crucial in understanding labor-market outcomes of ethnic minorities. If residential segregation is the main culprit for the adverse labor-market outcomes of minority workers, then, following Boustant (2012), we can divide policy solutions to residential segregation into three categories: *place-based policies*, *people-based policies*, and *indirect approaches* to the problems of residential segregation.

*Place-based policies* either improve minority (poor) neighborhoods, rendering them more attractive to white and firm entrants, or require white (rich) suburbs to add housing options affordable to lower-income homeowners or renters.<sup>23</sup> Examples of such policies are the *neighborhood regeneration policies*. These policies have been implemented in the US and in Europe through the *enterprise zone programs* (Papke, 1994; Boarnet and Bogart,

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<sup>23</sup>For recent overviews on place-based policies, see Kline and Moretti (2014) and Neumark and Simpson (2015).

1996; Mauer and Ott, 1999; Bondonio and Engberg, 2000; Bondonio and Greenbaum, 2007; Givord et al., 2013; Briant et al., 2014) and the *empowerment zone programs* (Busso et al., 2013). For example, the enterprise zone policy consists in designating a specific urban (or rural) area, which is depressed, and targeting it for economic development through government-provided subsidies to labor and capital. The aim of the empowerment zone program is to revitalize distressed urban communities and it represents a nexus between social welfare policy and economic development efforts. By implementing these types of policies, one brings jobs to people and thus facilitates the flows of job information in depressed neighborhoods.

*People-based policies* assist homeowners or renters directly, through stronger enforcement of fair housing laws, offers of housing vouchers, or improved access to mortgage finance (such as the Community Reinvestment Act of 1977). Examples of such policies are the Moving to Opportunity (MTO) programs (Katz et al., 2001; Rosenbaum and Harris, 2001; Kling et al., 2005), which have only been implemented in the United States. By giving housing assistance (i.e. vouchers and certificates) to low-income families, the MTO programs help them to relocate to better and richer neighborhoods. The results of most MTO programs (in particular for Baltimore, Boston, Chicago, Los Angeles and New York) show a clear improvement of the well-being of participants and better labor market outcomes, especially in terms of labor-market participation (see, in particular, Katz et al., 2001, Kling et al., 2005, Rosenbaum and Harris, 2001).

Finally, *indirect approaches* target the symptoms of residential segregation, rather than the root causes—for example, by improving public transportation to reduce the isolation of black neighborhoods. Investments in public transport can have a substantial impact on the search activities of low-income minority workers and thus, on their unemployment rate. Indeed, if the labor participation for minority workers is affected by poor access to job locations and poor worker mobility, and if public transportation services are designed to effectively link workers with areas of concentrated employment, then increased access to public transit should yield higher levels of employment, in particular for African Americans

(Sanchez, 1998, 1999; Blumemberg and Manville, 2004).<sup>24</sup>

Which policy is the most effective clearly depends on the sense of causality between segregation and labor-market outcomes. If neighborhood segregation is the outcome – not the cause – of adverse labor-market outcomes of ethnic minorities, then people-based policies should be implemented. If segregation is the cause, then policies should focus on workers’ geographical location, as in the spatial mismatch literature, and place-based and transportation policies should be implemented. This is ultimately an empirical question of causality – whether people who experience high unemployment rate sort themselves to areas with bad access to jobs and poor social networks or people who are segregated spatially end up with high unemployment rates and a low access to social networks. In our model, labor-market outcomes, segregation and social interactions are determined simultaneously and we have highlighted the role of multiplier effects of both the social space and the geographical space on outcomes. In particular, we have seen that residence-based labor market networks can exacerbate the adverse effects of residential segregation on labor-market outcomes for ethnic minorities, especially when social networks are ethnically stratified. As a result, because of the multipliers that network effects create, the effects of the above-mentioned policies can be amplified, more so in areas with low employment.

## References

- [1] Akerlof, G. (1997), “Social distance and social decisions,” *Econometrica* 65, 1005-1027.
- [2] Ballester, C., Calvó-Armengol, A. and Y. Zenou (2006), “Who’s who in networks. Wanted: The key player,” *Econometrica* 74, 1403-1417.

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<sup>24</sup>Researchers studying the relationship between transportation and employment find that reliable transportation leads to increased access to job opportunity, higher earnings, and increased employment stability (Blumemberg, 2000; Cervero et al., 2002; Ong, 2002; Holzer and Ihlanfeldt, 1996).

- [3] Barber, A.E., Wesson, M.J., Roberson, Q.M. and S.M. Taylor (1999), “A tale of two job markets: Organizational size and its effects on hiring practices and job search behavior,” *Personnel Psychology* 52, 841-867.
- [4] Bartram, D., Lindley, P.A., Marshall, L. and J. Foster (1995), “The recruitment and selection of young-people by small businesses,” *Journal of Occupational Organizational Psychology* 68, 339-358.
- [5] Battu, H., Seaman, P. and Y. Zenou (2011), “Job contact networks and the ethnic minorities,” *Labour Economics* 18, 48-56.
- [6] Bayer, P., Ross, S.L. and G. Topa (2008), “Place of work and place of residence: Informal hiring networks and labor market outcomes,” *Journal of Political Economy* 116, 1150-1196.
- [7] Beckmann, M.J. (1976), “Spatial equilibrium and the dispersed city,” In: Y.Y. Papageorgiou (Ed.), *Mathematical Land Use Theory*, Lexington, MA: Lexington Books, pp. 117-125.
- [8] Behrens, K., Duranton, G. and F. Robert-Nicoud (2014), “Productive cities: Sorting, selection, and agglomeration,” *Journal of Political Economy*, forthcoming.
- [9] Berliant, M., Peng, S.-K. and P. Wang (2002), “Production externalities and urban configuration,” *Journal of Economic Theory* 104, 275–303.
- [10] Berliant, M. and P. Wang (2008), “Urban growth and subcenter formation: A trolley ride from the Staples Center to Disneyland and the Rose Bowl,” *Journal of Urban Economics* 63, 679–693.
- [11] Berthiaume, J. and W. Parsons (2006), “Referral bonuses: A popular way to bring in new talent,” Society for Human Resource Management, [http://moss07.shrm.org/hrdisciplines/compensation/Articles/Pages/CMS\\_016894.aspx](http://moss07.shrm.org/hrdisciplines/compensation/Articles/Pages/CMS_016894.aspx).

- [12] Blumemberg, E. (2000), "On the way to work: Welfare recipients and barriers to employment," *Economic Development Quarterly* 16, 314-325.
- [13] Blumemberg, E. and M. Manville (2004), "Beyond the spatial mismatch hypothesis: Welfare recipients and transportation policy," *Journal of Planning Literature* 19, 182-205.
- [14] Boarnet, M.G. and W.T. Bogart (1996), "Enterprise zones and employment: Evidence from New Jersey," *Journal of Urban Economics* 40, 198-215.
- [15] Bondonio, D. and J. Engberg (2000), "Enterprise zones and local employment: Evidence from the states' program," *Regional Science and Urban Economics* 30, 519-549.
- [16] Bondonio, D. and R.T. Greenbaum (2007), "Do local tax incentives affect economic growth? What mean impact miss in the analysis of enterprise zone policies," *Regional Science and Urban Economics* 37, 121-136.
- [17] Borjas, G.J. (1998), "To ghetto or not to ghetto: Ethnicity and residential segregation," *Journal of Urban Economics* 44, 228-253.
- [18] Bound, J. and Holzer, H.J. (1993), "Industrial shifts, skill levels, and the labor market for white and black males," *Review of Economics and Statistics* 75, 387-395.
- [19] Boustant, L.P. (2010), "Was postwar suburbanization white flight? Evidence from the black migration," *Quarterly Journal of Economics* 125, 417-443.
- [20] Boustant, L.P. (2012), "Racial residential segregation in American cities," In: N. Brooks, K. Donaghy and G.-J. Knaap (Eds.), *The Oxford Handbook of Urban Economics and Planning*, Oxford: Oxford University Press, pp. 318-339.
- [21] Briant, A., Lafourcade, M. and B. Schmutz (2014), "Can tax breaks beat geography? Lessons from the French EZ experience," *American Economic Journal: Economic Policy*, forthcoming.

- [22] Brueckner, J.K. and A.G. Largey (2008), “Social interaction and urban sprawl,” *Journal of Urban Economics* 64, 18-34.
- [23] Brueckner, J.K., Thisse, J-F. and Y. Zenou (2002), “Local labor markets, job matching and urban location,” *International Economic Review* 43, 155-171.
- [24] Brueckner, J.K. and Y. Zenou (2003), “Space and unemployment: The labor-market effects of spatial mismatch,” *Journal of Labor Economics* 21, 242-266.
- [25] Busso, M., Gregory, J. and P. Kline (2013), “Assessing the incidence and efficiency of a prominent place based policy,” *American Economic Review* 103, 897-947.
- [26] Calvó-Armengol, A. and M. Jackson (2004), “The effects of social networks on employment and inequality,” *American Economic Review* 94, 426-454.
- [27] Calvó-Armengol, A., Patacchini, E. and Y. Zenou (2009), “Peer effects and social networks in education,” *Review of Economic Studies* 76, 1239-1267.
- [28] Cervero, R., Sandoval, O. and J. Landis (2002), “Transportation as a stimulus of welfare-to-work. Private versus public mobility,” *Journal of Planning Education and Research* 22, 50-63.
- [29] Coulson, E., Laing, D. and P. Wang (2001), “Spatial mismatch in search equilibrium,” *Journal of Labor Economics* 19, 949-972.
- [30] Currarini, S., Jackson, M.O., and P. Pin (2009), “An economic model of friendship: Homophily, minorities, and segregation,” *Econometrica* 77, 1003-1045.
- [31] Cutler, D., Glaeser, E.L. and J.L. Vigdor (1999), “The rise and decline of the American ghetto,” *Journal of Political Economy* 107, 455-506.
- [32] Del Bello, C.L., Patacchini, E. and Y. Zenou (2014), “Social and geographical distance in the analysis of peer effects,” Unpublished manuscript, Stockholm University.

- [33] Fujita, M. and H. Ogawa (1980), “Equilibrium land use patterns in a nonmonocentric city,” *Journal of Regional Science* 20, 455-475.
- [34] Fujita, M. and J.-F. Thisse (2013), *Economics of Agglomeration. Cities, Industrial Location, and Regional Growth, 2nd Edition*, Cambridge: Cambridge University Press.
- [35] Galenianos, M. (2014), “Hiring through referrals,” *Journal of Economic Theory*, forthcoming.
- [36] Givord, P., Rathelot, R. and P. Sillard (2013), “Place-based tax exemptions and displacement effects : An evaluation of the Zones Franches Urbaines program,” *Regional Science and Urban Economics* 43, 151-163.
- [37] Glaeser, E.L. (1999), “Learning in cities,” *Journal of Urban Economics* 46, 254-277.
- [38] Glaeser, E.L., Sacerdote, B. and J.A. Scheinkman (1996), “Crime and social interactions,” *Quarterly Journal of Economics* 111, 508-548.
- [39] Gobillon, L., Selod, H. and Y. Zenou (2007), “The mechanisms of spatial mismatch,” *Urban Studies* 44, 2401-2427.
- [40] Granovetter, M.S. (1973), “The strength of weak ties,” *American Journal of Sociology* 78, 1360-1380.
- [41] Grauwin, S., Goffette-Nagot, F. and P. Jensen (2012), “Dynamic models of residential segregation: An analytical solution,” *Journal of Public Economics* 96, 124-141.
- [42] Hellerstein, J.K., McInerney, M. and D. Neumark (2011), “Neighbors and coworkers: The importance of residential labor market networks,” *Journal of Labor Economics* 29, 659-695.
- [43] Helsey, R.W. and W.C. Strange (2004), “Knowledge barter in cities,” *Journal of Urban Economics* 56, 327-345.

- [44] Helsey, R.W. and W.C. Strange (2007), “Urban interactions and spatial structure,” *Journal of Economic Geography* 7, 119-138.
- [45] Helsey, R.W. and W.C. Strange (2014), “Coagglomeration, clusters and the scale and composition of cities,” *Journal of Political Economy*, forthcoming.
- [46] Helsey, R.W. and Y. Zenou (2014), “Social networks and interactions in cities,” *Journal of Economic Theory* 150, 426-466.
- [47] Holzer, H.J. (1987), “Informal job search and black youth unemployment,” *American Economic Review* 77, 446-452.
- [48] Holzer, H.J. (1988), “Search method used by unemployed youth,” *Journal of Labor Economics* 6, 1-20.
- [49] Holzer H. J. (1998), “Why do small establishments hire fewer blacks than larger ones?” *Journal of Human Resources* 33, 896-814.
- [50] Holzer, H.J. and K.R. Ihlanfeldt (1996), “Spatial factors and the employment of blacks at the firm level,” *New England Economic Review* 65-86.
- [51] Holzer H.J. and Ihlanfeldt, K.R. (1998), “Customer discrimination and the employment outcomes of minority workers,” *Quarterly Journal of Economics* 113, 835-867.
- [52] Holzer, H. and Reaser, J. (2000), “Black applicants, black employees, and urban labor market policy,” *Journal of Urban Economics* 48, 365-387.
- [53] Ihlanfeldt, K.R. (2006), “A primer on spatial mismatch within urban labor markets,” In: R. Arnott and D. McMillen (Eds.), *A Companion to Urban Economics*, Boston: Blackwell Publishing, pp. 404-417.
- [54] Ihlanfeldt, K. R. and D. Sjoquist (1998), “The spatial mismatch hypothesis: A review of recent studies and their implications for welfare reform,” *Housing Policy Debate* 9, 849-892.

- [55] Ioannides, Y.M. (2012), *From Neighborhoods to Nations: The Economics of Social Interactions*, Princeton: Princeton University Press.
- [56] Ioannides, Y.M. and D.L. Loury (2004), “Job information networks, neighborhood effects, and inequality,” *Journal of Economic Literature* 42, 1056-1093.
- [57] Ioannides, Y.M. and G. Topa (2010), “Neighborhood effects: Accomplishments and looking beyond them,” *Journal of Regional Science* 50, 343-362.
- [58] Jackson, M.O. (2008), *Social and Economic Networks*, Princeton: Princeton University Press.
- [59] Jackson, M.O. and Y. Zenou (2014), “Games on networks,” In: P. Young and S. Zamir (Eds.), *Handbook of Game Theory, Vol. 4*, Amsterdam: Elsevier, forthcoming.
- [60] Kain, J. (1968), “Housing segregation, negro employment, and metropolitan decentralization,” *Quarterly Journal of Economics* 82, 175-197.
- [61] Katz, L.F., Kling, J.R. and J.B. Liebman (2001), “Moving to opportunity in Boston: Early results of a randomized mobility experiment,” *Quarterly Journal of Economics* 116, 607-654.
- [62] Kline, P. and E. Moretti (2014), “People, places, and public policy: Some simple welfare economics of local economic development programs,” *Annual Review of Economics*, forthcoming.
- [63] Kling, J.R., Ludwig, J. and L.F. Katz (2005), “Neighborhood effects on crime for female and male youth: Evidence from a randomized housing voucher experiment,” *Quarterly Journal of Economics* 120, 87-130.
- [64] Leonard, J. (1990), “The impact of Affirmative Action regulation and Equal Opportunity law on black employment,” *Journal of Economic Perspectives* 4, 47-54.

- [65] Lucas, R.E. and E. Rossi-Hansberg (2002), “On the internal structure of cities,” *Econometrica* 70, 1445-1476.
- [66] Mauer, D.C. and S.H. Ott (1999), “On the optimal structure of government subsidies for enterprise zones and other locational development programs,” *Journal of Urban Economics* 45, 421-450.
- [67] Mencken, F.C. and I. Winfield (1998), “In search of the “right stuff”: The advantages and disadvantages of informal and formal recruiting practices in external labor markets,” *American Journal of Economics and Sociology* 57, 135-153.
- [68] Mossay, P. and P.M. Picard (2011), “On spatial equilibria in a social interaction model,” *Journal of Economic Theory* 146, 2455–2477.
- [69] Mossay, P. and P.M. Picard (2013), “Spatial segregation and urban structure,” CREA Discussion Paper Series 13-03, Center for Research in Economic Analysis, University of Luxemburg.
- [70] Neumark, D. and H. Simpson (2015), “Place-based policies,” In: G. Duranton, V. Henderson and W. Strange (Eds.), *Handbook of Regional and Urban Economics, Vol. 5*, Amsterdam: Elsevier Publisher, forthcoming.
- [71] Ong, P. (2002), “Car ownership and welfare-to-work,” *Journal of Policy Analysis and Management* 21, 239-252.
- [72] Papke, L. (1994), “Tax policy and urban development: evidence from the Indiana enterprise zone program,” *Journal of Public Economics* 54, 37-49.
- [73] Patacchini, E. and Y. Zenou (2012), “Ethnic networks and employment outcomes,” *Regional Science and Urban Economics* 42, 938-949.
- [74] Pellizzari, M. (2010), “Do friends and relatives really help in getting a good job?” *Industrial and Labor Relations Review* 63, 494-510.

- [75] Rosenbaum, E. and L.E. Harris (2001), “Residential mobility and opportunities: Early impacts of the Moving to Opportunity demonstration program in Chicago,” 12, 321-346.
- [76] Ross, S.L. (2012), “Social interactions within cities: Neighborhood environments and peer relationships,” In: N. Brooks, K. Donaghy and G.-J. Knapp (Eds.), *The Oxford Handbook of Urban Economics and Planning*, Oxford: Oxford University Press, pp. 203-229.
- [77] Sanchez, T.W. (1998), “Equity analysis of personal transportation system benefits,” *Journal of Urban Affairs* 20, 69-86.
- [78] Sanchez, T.W. (1999), “The connection between public transit and employment. The cases of Portland and Atlanta,” *Journal of the American Planning Association* 65, 284-296.
- [79] Sato, Y. and Y. Zenou (2014), “How urbanization affect employment and social interactions,” CEPR Discussion Paper No. 9805.
- [80] Schelling, T.C. (1971), “Dynamic models of segregation,” *Journal of Mathematical Sociology* 1, 143-186.
- [81] Sigelman, L., Bledsoe, T., Welch, S., and M.W. Combs (1996), “Making contact? Black-white social interaction in an urban setting,” *American Journal of Sociology* 101, 1306-1332.
- [82] Topa, G. (2001), “Social interactions, local spillovers and unemployment,” *Review of Economic Studies* 68, 261-295.1.
- [83] Topa, G. and Y. Zenou (2015), “Neighborhood versus network effects”, In: G. Duranton, V. Henderson and W. Strange (Eds.), *Handbook of Regional and Urban Economics*, Vol. 5, Amsterdam: Elsevier Publisher, forthcoming.

- [84] Verdier, T. and Y. Zenou (2004), “Racial beliefs, location and the causes of crime,” *International Economic Review* 45, 731-760.
- [85] Wasmer, E. and Y. Zenou (2002), “Does city structure affect search and welfare?” *Journal of Urban Economics* 51, 515-541.
- [86] Wilson, W. (1996), *When Work Disappears: The World of the New Urban Poor*, New York: Alfred A. Knopf.
- [87] Zenou, Y. (2006), “Efficiency wages and unemployment in cities: The case of high relocation costs,” *Regional Science and Urban Economics* 36, 49-71.
- [88] Zenou, Y. (2009), *Urban Labor Economics*, Cambridge: Cambridge University Press.
- [89] Zenou, Y. (2013), “Spatial versus social mismatch,” *Journal of Urban Economics* 74, 113-132.
- [90] Zenou, Y. (2015), “A dynamic model of weak and strong ties in the labor market,” *Journal of Labor Economics*, forthcoming.
- [91] Zhang, J. (2004), “Residential segregation in an all-integrationist world,” *Journal of Economic Behavior and Organization* 54, 533-550.

## Appendix: Proofs

### Proof of Proposition 3

*Existence and uniqueness:* Denote

$$\Phi(E) \equiv \sqrt{\frac{\alpha}{\beta}} \left(1 - \frac{E}{P}\right) \sqrt{\frac{E}{P}} \quad (50)$$

which is the left-hand side of (22). It is easily checked that  $\Phi(0) = \Phi(P) = 0$  and that, by solving  $\Phi'(E) = 0$ , we obtain:  $E = P/3$  with

$$\Phi(P/3) = \sqrt{\frac{\alpha}{\beta}} \frac{2}{3} \sqrt{\frac{1}{3}} = 0.384 \sqrt{\frac{\alpha}{\beta}}$$

Since  $\Gamma(P)$  is constant and does not depend on  $E$ , then, as shown in Figure 1, there exists an equilibrium if only if  $\Gamma(P) < 0.384 \sqrt{\alpha/\beta}$ , which yields (24).

To ensure that all workers have positive employment probabilities, we must still check that the commuting and search costs of a worker at the city edge outweigh her probability of finding and taking a job. This is given by (13), which can now be written as:

$$\frac{c(b)}{w - tb} < \frac{\alpha E}{\beta P}$$

Observe that, using (16), we have:  $c(b) = c(P/2) = \tau P/2$ . Thus, this inequality is equivalent to:

$$\frac{\alpha E}{\beta P} (w - tb) > \frac{\tau P}{2}$$

Since  $E$  increases with  $w$ , this inequality is always true if  $w$  is large enough.

Finally, as shown in Figure 1, for a given  $P$ , equation (22) gives two solutions of  $E/P$  for which  $E/P > 0$ : one with a high employment rate,  $E^*/P > 1/3$ , and another with a low employment rate solution  $E^*/P < 1/3$ . Note that there is also a third equilibrium at  $E^*/P = 0$  where  $e_i^*(x) = n_i^*(x) = 0$ , which is ruled out by condition (13). The high employment equilibrium would be the one chosen by workers if they can coordinate on the equilibrium.<sup>25</sup> Because in most modern economies, the employment rate is above 33.33 percent, we focus on the equilibrium for which  $1/3 < E^*/P < 1$ .

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<sup>25</sup>Note that the low employment equilibrium can also be shown to be unstable in the context of migration (open city).

*Comparative statics:* Observe that the left-hand side of (22), i.e.  $\Phi(E)$ , is not affected by  $t$ ,  $\tau$ , and  $w$ . Using (16) in (23), one can write  $\Gamma(P)$  as:

$$\Gamma(P) = \frac{1}{P} \int_{-P/2}^{P/2} \sqrt{\frac{\frac{\tau}{P} \left(\frac{P^2}{4} + x^2\right)}{w - tx}} dx$$

It can be seen that  $\Gamma(P)$  increases with  $\tau$  and  $t$  but decreases with  $w$ . As a result, when  $t$ ,  $\tau$  increases,  $\Gamma(P)$  increases and the line of  $\Gamma(P)$  is shifted upward in Figure 1 and thus employment  $E^*$  decreases.

Observe also that the right-hand side of (22), i.e.  $\Gamma(P)$ , is not affected by  $\alpha/\beta$ . However,  $\Phi(E)$  increases with  $\alpha/\beta$ . Thus, when  $\alpha/\beta$  increases, the curve of  $\Phi(E)$  is shifted upward and thus  $E^*$  increases. ■

### Proof of Proposition 6

We can proceed as in the proof of Proposition 3. Since  $\Phi(E_i)$  is still defined by (50), with subscript  $i$  on the  $E$ s and the  $P$ s, then there is a unique equilibrium for which  $1/3 < E_1^*/P_1 < 1$  and  $1/3 < E_2^*/P_2 < 1$  if

$$\max\{\Gamma_1(P_1), \Gamma_2(P_1, P_2)\} < 0.384\sqrt{\frac{\alpha}{\beta}}$$

holds, which is (36). The first constraint,  $\Gamma_1(P_1) < 0.384\sqrt{\alpha/\beta}$ , puts an upper bound on population 1,  $\bar{P}_1 \equiv \Gamma_1^{-1}\left(0.384\sqrt{\alpha/\beta}\right)$  where  $\Gamma_1^{-1}$  is the inverse of the function  $\Gamma_1(P_1)$ . This bound is the same as for the homogenous-population case. The second constraint,  $\Gamma_2(P_1, P_2) < 0.384\sqrt{\alpha/\beta}$ , puts an upper bound for population 2,  $\bar{P}_2(P_1) \equiv \Gamma_2^{-1}\left(P_1, 0.384\sqrt{\alpha/\beta}\right)$  where  $\Gamma_2^{-1}$  is the inverse of the function  $\Gamma_2(P_1, P_2)$  w.r.t the second argument. The upper bound  $\bar{P}_2(P_1)$  falls with  $P_1$ , from  $\bar{P}_1$  at  $P_1 = 0$  to zero at some threshold population  $\underline{P}_1$ . One can check that  $\bar{P}_2(0) = \bar{P}_1$  where  $\underline{P}_1$  is such that  $\Gamma_2(\underline{P}_1, 0) = 0.384\sqrt{\alpha/\beta}$ . It can be checked that  $\underline{P}_1 < \bar{P}_1$ . Therefore, condition (36) holds if  $P_1 < \bar{P}_1$  and  $P_2 < \bar{P}_2(P_1)$ .

We also need to check that there are no corner solutions. The conditions are given by (32), which are:

$$\frac{c_1 (P_1/2)}{w - tP_1/2} < \frac{\alpha E_1}{\beta P_1}$$

and

$$\frac{c_2 ((P_1 + P_2) / 2)}{w - t(P_1 + P_2) / 2} < \frac{\alpha E_2}{\beta P_2}$$

Since the equilibrium employment level rises with higher wage  $w$ , the RHS of each condition rises with  $w$  while the LHS falls with it. The conditions are then satisfied for sufficiently high  $w$ .

We finally need to check when this urban structure is a spatial equilibrium in the city with populations  $P_1$  and  $P_2$ . As shown in Figure 2 and by condition (27), in equilibrium, the individuals from population 1 must bid for the highest land prices around the city center (for  $x$  such that  $-b_1 \leq x \leq b_1$ ) and population 2 must offer the highest land prices at the periphery (for  $x$  such that  $-b_2 \leq x \leq -b_1$  and  $b_1 \leq x \leq b_2$ ). Since the city is symmetric, we only need to check the following land market conditions:

$$\Psi_1(x, u_1) \geq \Psi_2(x, u_2) \quad \text{for } x \in [0, b_1]$$

and

$$\Psi_1(x, u_1) \leq \Psi_2(x, u_2) \quad \text{for } x \in [b_1, b_2]$$

The land market imposes the continuity of bid rents (see (27)) so that  $\Psi_1(b_1, u_1) = \Psi_2(b_1, u_2)$  and  $\Psi_2(b_2, u_2) = 0$ . These two conditions yield the equilibrium utility levels  $u_1^*$  and  $u_2^*$ . From (15) we know that the land gradient is given by

$$\Psi'_i(x, u_i) = -e_i^*(x)t - n_i^*(x) c'_i(x)$$

which is negative for any  $x \geq 0$ ,  $i = 1, 2$  because  $c'_i(x) > 0$ .

First, let us check that, on the interval  $[0, b_1]$ ,  $\Psi_1(x, u_1) \geq \Psi_2(x, u_2)$ . On the interval  $[0, b_1]$ , we can use the condition  $\Psi_1(b_1, u_1) = \Psi_2(b_1, u_2)$  to write the difference in bid rents as

$$\Psi_1(x, u_1) - \Psi_2(x, u_2) = - \int_x^{b_1} [\Psi'_1(x, u_1) - \Psi'_2(x, u_2)] dx$$

which is positive because the integrand

$$\Psi'_1(x) - \Psi'_2(x) = -[e_1^*(x) - e_2^*(x)]t - n_1^*(x)c'_1(x)$$

is negative since  $c'_2(x) = 0$  while  $c'_1(x) > 0$  and  $e_1^*(x) > e_2^*(x)$ . As a result,  $\Psi_1(x, u_1) \geq \Psi_2(x, u_2)$  on this interval. Population 2 is never able to bid away population 1 in the vicinity of the city center. This is because they lose access to their own members and therefore have higher interaction costs than population 1.

Second, let us check that, on the interval  $(b_1, b_2]$ ,  $\Psi_1(x, u_1) \leq \Psi_2(x, u_2)$ . This is equivalent to

$$\Psi_1(x, u_1) - \Psi_2(x, u_2) = \int_{b_1}^x [\Psi'_1(x, u_1) - \Psi'_2(x, u_2)] dx \leq 0$$

where

$$\Psi'_1(x, u_1) - \Psi'_2(x, u_2) = -[e_1^*(x) - e_2^*(x)]t - n_1^*(x)\tau + n_2^*(x)c'_2(x)$$

which is equivalent to

$$\Psi'_1(x, u_1) - \Psi'_2(x, u_2) = -[e_1^*(x) - e_2^*(x)]t - n_1^*(x)\tau + n_2^*(x)\tau \left( \frac{x - b_1}{b_2 - b_1} \right) \quad (51)$$

Since  $e_1^*(x) > e_2^*(x)$ , this expression is negative at  $x = b_1$  and, by continuity, it is also negative for slightly larger  $x$ . For this reason, the individuals from population 2 residing close to  $x = b_1$  will have no incentives to outbid population 1. This will occur if  $b_2$  is close to  $b_1$ , that is if the size of population 2,  $P_2$ , is small enough. Therefore, there exists a threshold  $\widehat{P}_2(P_1) > 0$  so that the land market conditions are satisfied if  $P_2 < \widehat{P}_2(P_1)$ . In general, we must write condition (51) for the closed city urban equilibrium under segregation to exist. ■

### Proof of Proposition 7

The equilibrium in  $E_i^*/P$  is defined by (33). If we look at the right-hand side of (33), note that, since  $c_1(x) \leq c_2(x)$  and since  $c_i(x)/(w-t|x|)$  increases with  $x$  for all  $x > 0$ , we automatically obtain that:

$$\Gamma_1(P_1, P_2) < \Gamma_2(P_1, P_2)$$

Commuting and search costs are smaller for population 1. Since the left-hand side of (33) is the same for the two populations, we can conclude that population 1's employment rate is higher than that of population 2, i.e.  $E_1^*/P_1 > E_2^*/P_2$ . From this result and expression (12), we deduce that the worker's employment probability  $e_i^*(x)$  decreases from the city center to the edge of the city, with a downward jump at the location  $b_1$  where the two populations swap.

From (11), we can also conclude that the number of social interactions within each population decreases with the distance to the center. However, because both sides of (11) fall with lower  $E_i/P_i$ , it is a priori unclear whether the members of population 2 chooses a lower number of interactions than those of population 1. One can show that, at location  $b_1$ , this number abruptly rises or falls according to whether the employment probability  $e_i^*(b_1)$  is larger or smaller than  $1/2$ . When workers have a high employability, they substitute their search effort for higher search effectiveness. At location  $b_1$ , population 2's workers exert stronger search effort than population 1's workers because it is population 2 that has a lower employment rate and is less effective in communicating job opportunities. These results hold true whatever the population sizes are.

Finally, we also observe the following properties of the labor market conditions. First, one can check that  $c_1(x) = c(x)$  on the interval  $[-b_1, b_1]$ . So, the search cost  $\Gamma_1(P_1, P_2)$  is equal to  $\Gamma(P_1)$  where  $\Gamma(\cdot)$  is the function obtained under homogenous population. As a result,  $\Gamma_1(P_1, P_2)$  increases with  $P_1$  and is independent of  $P_2$ . Therefore, properties of the central population 1 are similar as in the homogenous population model. In particular, a rise in population 1 should be accompanied by a fall in its aggregate employment rate. It can be shown that  $\Gamma_2(P_1, P_2)$  increases in both  $P_1$  and  $P_2$ . As a result, a rise in the size of

populations 1 and 2 should be accompanied by a fall in the aggregate employment rate of population 2. Intuitively, when  $P_1$  increases, population 2 must locate further away from the center and incur higher search and commuting costs. When  $P_2$  increases, population 2 is more dispersed and also incurs a higher search cost. ■

### Proof of Proposition 8

(i) The *integrated city* equilibrium exists only for population with symmetric sizes. In that case, the social-interaction costs are the same for the two populations so that  $c_i(x) = c_j(x) = c(x)$ . If the two populations would interact with each other, then, because of their symmetry, they would reach the same aggregate employment rate  $E_i/P_i$  and therefore would obtain the same value for  $c_i(x)/(E_i/P_i)$ . Since the latter value is lower than  $c_j(x)/(\gamma E_j/P_j)$ , i.e.  $c_i(x)/(E_i/P_i) < c_j(x)/(\gamma E_j/P_j)$ , the optimal choice is not to interact with each other so that  $n_{ij}^*(x) = 0$ .

(ii) Consider now the *segregated city* equilibrium where population 1 resides in the centered interval  $[-b_1, b_1]$  and population 2 in the periphery intervals  $[-b_2, -b_1] \cup (b_1, b_2]$  as defined in Definition 5. Let us show under which condition the two populations do not interact with each other.

Because of the symmetry, we can restrict our attention to  $x > 0$ . Population 1 does not interact with population 2 if

$$\frac{c_1(x)}{E_1/P_1} < \frac{c_2(x)}{\gamma E_2/P_2} \iff \frac{c_2(x)}{c_1(x)} > \frac{\gamma E_2/P_2}{E_1/P_1}, \forall x \in [0, b_1]$$

Population 2 does not interact with population 1 if

$$\frac{c_2(x)}{E_2/P_2} < \frac{c_1(x)}{\gamma E_1/P_1} \iff \frac{c_2(x)}{c_1(x)} < \frac{E_2/P_2}{\gamma E_1/P_1}, \forall x \in (b_1, b_2]$$

Those conditions imply

$$\min_{x \in [0, b_1]} \frac{c_2(x)}{c_1(x)} > \frac{\gamma E_2/P_2}{E_1/P_1} \quad \text{and} \quad \max_{x \in (b_1, b_2]} \frac{c_2(x)}{c_1(x)} < \frac{E_2/P_2}{\gamma E_1/P_1}$$

Given that  $c_2(x)/c_1(x)$  is monotonically decreasing in  $x$ , for all  $x > 0$ , we compute

$$\min_{x \in [0, b_1]} \frac{c_2(x)}{c_1(x)} = \max_{x \in (b_1, b_2]} \frac{c_2(x)}{c_1(x)} = \frac{c_2(b_1)}{c_1(b_1)} = \frac{2P_1 + P_2}{2P_1}$$

This yields the conditions

$$\frac{2P_1 + P_2}{2\gamma P_1} > \frac{E_2/P_2}{E_1/P_1} > \gamma \frac{2P_1 + P_2}{2P_1}$$

which are reported in (39).

### Proof of Proposition 9

As in (2) the employment probability  $e(x)$  is defined by:

$$e(x) = \frac{\pi(x)}{\pi(x) + \beta}$$

which can be inverted as

$$\pi(x) = \beta \frac{e(x)}{(1 - e(x))} \quad (52)$$

Using the definition  $\theta(x) \equiv c(x)/(w - t|x|)$  and the property  $g'(z) = \beta/(\beta + z)^2$ , expression (44) can be written as

$$\pi(x) + \frac{P}{\beta} (\pi(x) + \beta)^2 \theta(x) = \alpha E$$

Replacing  $\pi(x)$  by (52) we get

$$\alpha E (1 - e(x))^2 - \beta e(x) (1 - e(x)) - P\beta\theta(x) = 0$$

which finally can be written as

$$(\beta + \alpha E) [1 - e(x)]^2 - \beta [1 - e(x)] - P\beta\theta(x) = 0 \quad (53)$$

The first and last term of (53) are similar to the terms in expression (12), which is obtained in the case of random search. For  $c(x) > 0$  and  $\theta(x) > 0$ , the unique root such that  $e \in [0, 1]$  yields the following employment rate:

$$e^*(x) = 1 - \frac{\beta + \sqrt{\beta^2 + 4\beta(\beta + \alpha E) P\theta(x)}}{2(\beta + \alpha E)} \quad (54)$$

Replacing  $c(x)$  by (16) and  $\theta(x)$  by  $c(x)/(w - t|x|)$  in (54) gives (45). The employment rate  $e^*(x)$  decreases with larger  $\theta(x)$  and therefore with  $x$  since  $\theta(x)$  is an increasing

function of  $x$ . For the sake of analytical tractability, we assume  $\theta(x) \leq (\alpha/\beta)(E/P)$ , which is always true when  $w$  is large enough.

Using (54), the equilibrium aggregate employment  $E^* = \int_{-b}^b e^*(x)dx$  writes as

$$E^* = \frac{(\beta + 2\alpha E^*) P - \int_{-b}^b \sqrt{\beta^2 + 4\alpha(1 + \alpha E^*) P \theta(x)} dx}{2(\beta + \alpha E^*)} \quad (55)$$

From (55), we further obtain the implicit equation

$$2(\beta + \alpha E^*) E^* = (\beta + 2\alpha E^*) P - F(P, E^*) \quad (56)$$

where

$$F(P, E) = 2 \int_0^{P/2} \sqrt{\beta^2 + 4\beta(\beta + \alpha E) \frac{\tau [(P/2)^2 + x^2]}{w + tx}} dx \quad (57)$$

where the function  $F(P, E)$  is an increasing in both arguments. Those expressions yield (46) and (47).

The roots of equation (56) yields the equilibrium employment level  $E^*$  for a given population  $P$ . Note first that there exists no equilibrium when  $P$  is too large. Indeed, there exists a threshold  $\bar{P} > 0$  such that the equation accepts no positive root if  $P > \bar{P}$ . This is because the RHS of (46) falls to negative values whereas the LHS remains positive when  $P \rightarrow \infty$ . Indeed, for large enough  $P$ , the square root in (57) tends to a value larger than  $\sqrt{4\beta(\beta + \alpha E) \tau/w} \sqrt{(P/2)^2 + x^2}$  and the integral tends to a value larger than

$$\begin{aligned} & \sqrt{16\beta(\beta + \alpha E) \tau/w} (P/2)^2 \int_0^1 \sqrt{1 + z^2} dz \\ & = 1.1477 (P/2)^2 \sqrt{16\beta(\beta + \alpha E) \tau/w} \end{aligned}$$

which rises with larger  $P$ . Note secondly that there is no equilibrium for any too small  $P$ . To see this, note that  $F(P, E) > \beta P$ . So, by (56), we have

$$2(\beta + \alpha E^*) E^* > (\beta + 2\alpha E^*) P - \beta P$$

which implies

$$(\beta + \alpha(E^* - P)) E^* > 0 \iff E^* > P - \frac{\beta}{\alpha}$$

Therefore for any positive  $E^*$ , this imposes  $P > \beta/\alpha$ . Therefore, there exists a threshold  $\underline{P} \geq \beta/\alpha$  such that the equation accepts no positive root if  $P < \underline{P}$ .

Let us now show that the employment probability  $\pi(x)$  and the frequency of search interactions  $n^*(x, y)$  decreases with the distance to the job center. For that, consider (44). Because  $g(\cdot)$  is a concave function, the RHS of (44) is a decreasing function of  $\pi(x)$  while the LHS is an increasing function of  $\pi(x)$ . As a result, there is a unique solution for  $\pi(x)$ . Because the RHS decreases with larger ratio  $c(x)/(w - t|x|)$  and because this ratio increases with  $x$  and with  $\tau$  and  $t$ , the probability of finding a job  $\pi(x)$  decreases with  $x$  and with  $\tau$  and  $t$ . Since  $e(x)$  increases with  $\pi(x)$ , the same properties apply for the employment probability  $e(x)$ .

Finally, let us show that the employment rate  $E^*$  decreases with larger commuting  $t$  and search costs  $\tau$  but increases with wages  $w$ . Indeed, for  $c(x)$  and  $\theta(x)$  sufficiently close to zero, the employment probability tends to  $e^*(x) = \alpha E / (\beta + \alpha E)$  and the aggregate employment level is equal to  $E = P - \beta/\alpha$ . So, there exists a constant frictional unemployment of  $\beta/\alpha$  workers. Because  $\theta(x)$  increases with higher  $\tau$  and  $t$ , the aggregate employment  $E^*$  falls with higher travel cost  $\tau$  and commuting cost  $t$ . A similar argument also applies for  $w$ . ■

### Heterogenous population with directed search

The land conditions are similar to those obtained for random search. That is,  $\Psi_1(x) \geq \Psi_2(x)$  for  $x \in [0, b_1]$  and  $\Psi_1(x) \leq \Psi_2(x)$  for  $x \in [b_1, b_2]$ . The bid rents can be written as

$$\Psi_i(x, u_i) = e_i^*(x) (w - t|x|) - \int_{D_i} \tau |x - y| n_i^*(x, y) dy - u_i$$

The city border conditions  $\Psi_1(b_1, u_1) = \Psi_2(b_1, u_2)$  and  $\Psi_2(b_2, u_2) = 0$  yield the equilibrium utility levels  $u_1^*$  and  $u_2^*$ . Applying the envelop theorem, the land gradient is given by

$$\Psi_i'(x, u_i) = -e_i^*(x) t \text{sign}(x) - \int_{D_i} \tau n_i^*(x, y) \text{sign}(x - y) dy$$

Is the land rent of each population 1 and 2 is bell-shaped over the whole city support  $(-b_2, b_2)$ ? We here show that the land rent  $\Psi_1(x, u_1)$  of population 1 is bell-shaped over the

interval  $(-b_2, b_2)$ . We just need to show that the land gradient is negative for  $x \in (0, b_2)$ . By symmetry it is positive for  $x \in (-b_2, 0)$ . Note first that the first term of the last expression is negative for  $x \in (0, b_2)$ . Note second that, for  $x \in (b_1, b_2)$ , the integral in the second term is equal to  $\int_{-b_1}^{b_1} n_1^*(x, y) dy$ , which is positive. Therefore,  $\Psi_1'(x, u_1) < 0$  for any  $x \in (b_1, b_2)$ . Finally note that, for  $x \in (0, b_1)$ , the integral is proportional to  $\int_{-b_1}^x n_1^*(x, y) dy - \int_x^{b_1} n_1^*(x, y) dy$  and is also positive. Indeed, one can substitute the variable  $y$  by  $x - \varepsilon$  in the first integral, substitute the same variable  $y$  by  $x + \varepsilon$  in the second integral, inverse the boundaries of the first integral and change its sign to obtain

$$\int_0^{b_1+x} n_1^*(x, x - \varepsilon) d\varepsilon - \int_0^{b_1-x} n_1^*(x, x + \varepsilon) d\varepsilon$$

or equivalently

$$\int_{b_1-x}^{b_1+x} n_1^*(x, x - \varepsilon) d\varepsilon + \int_0^{b_1-x} [n_1^*(x, x - \varepsilon) - n_1^*(x, x + \varepsilon)] d\varepsilon$$

The first term is obviously positive while the second term is also positive because  $n_1^*(x, x - \varepsilon) > n_1^*(x, x + \varepsilon)$  holds for  $0 < \varepsilon < b_1 - x$ . The latter inequality indeed holds because as, by (48),  $e_1^*(y)$  falls with larger  $|y|$ , we have that  $e_1^*(x - \varepsilon) > e_1^*(x + \varepsilon)$ ,  $\varepsilon \in (0, b_1 - x)$ , and therefore  $n_1^*(x, x - \varepsilon) > n_1^*(x, x + \varepsilon)$  since, by (43),  $n_1^*(x, y)$  rises with larger  $e_1(y)$ . In other words, the land rent decreases with distance from the city center because workers lose access to those workers who simultaneously locate about the city center and who have higher employment propability and transmit more job opportunities.

For the population 2 located at the periphery, the land gradient may not be bell-shaped in  $x$  for  $x > 0$ . For instance, at  $x = b_1$ , it is equal to

$$\Psi_2'(b_1, u_2) = -e_2^*(b_1)t - \int_{-b_2}^{-b_1} \tau n_2^*(b_1, y) dy + \int_{b_1}^{b_2} \tau n_2^*(b_1, y) dy$$

which can be negative because the last term is larger than (the absolute value of) the second term. Hence, land rent may have a maximum on the district  $[b_1, b_2]$ . Indeed, workers have fewer incentives to interact with the half of their population located in the district  $[-b_2, -b_1]$ . In other words, when the peripheral districts are far away, a worker

located in  $[b_1, b_2]$  does not interact much with workers in the other district  $[-b_2, -b_1]$ . She rather wants to take advantage of a better access to the population in  $[b_1, b_2]$  by locating about at the centre of this interval. In this case, the land bid rent can have two modes over the city support  $[-b_2, b_2]$ .

Since  $\Psi_1(b_1) = \Psi_2(b_1)$ , we may write the difference in bid rents as

$$\Psi_1(x, u_1) - \Psi_2(x, u_2) = - \int_x^{b_1} [\Psi'_1(x, u_1) - \Psi'_2(x, u_2)] dx$$

where

$$\Psi'_1(x, u_1) - \Psi'_2(x, u_2) = - [e_1^*(x) - e_2^*(x)] t - \tau \int_{D_1} n_1^*(x, y) \text{sign}(x-y) dy + \tau \int_{D_2} n_2^*(x, y) \text{sign}(x-y) dy$$

However the latter expression is difficult to sign. ■

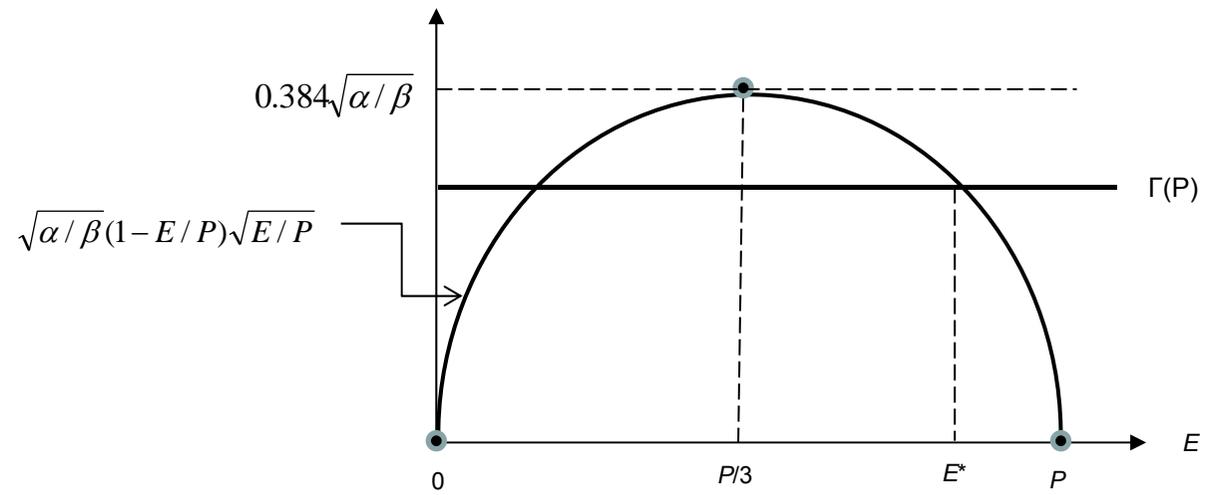


Figure 1: Urban equilibrium with homogeneous population

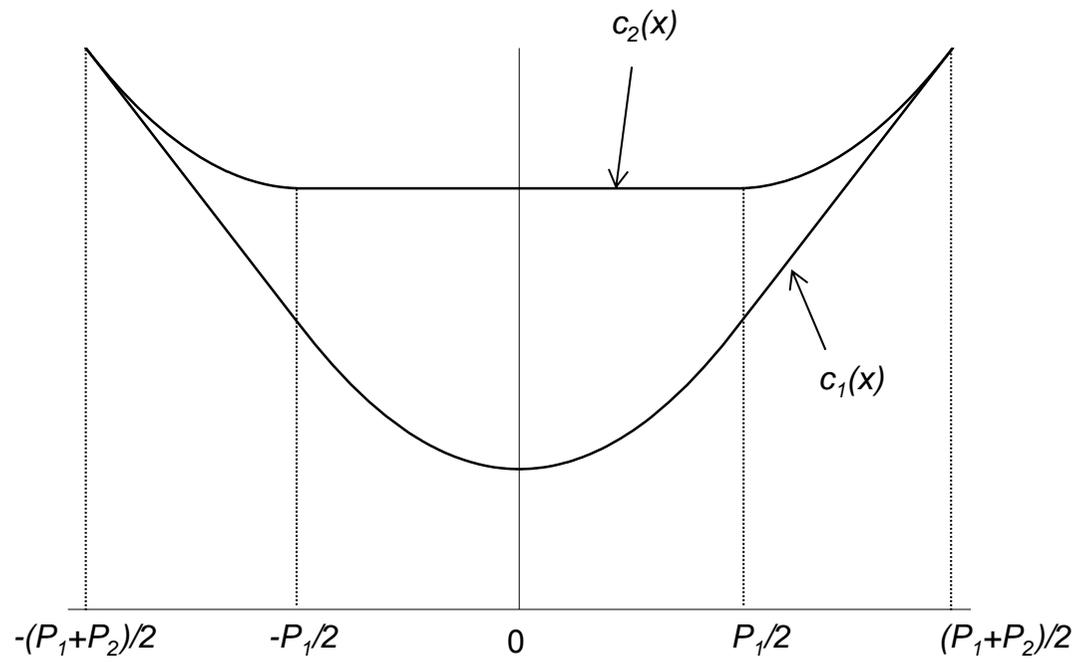
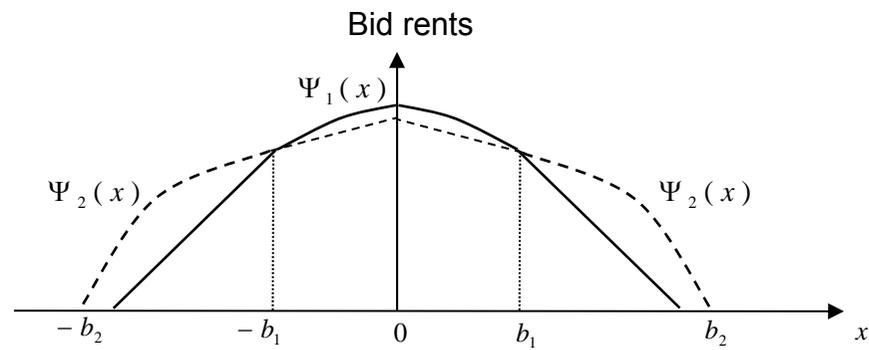
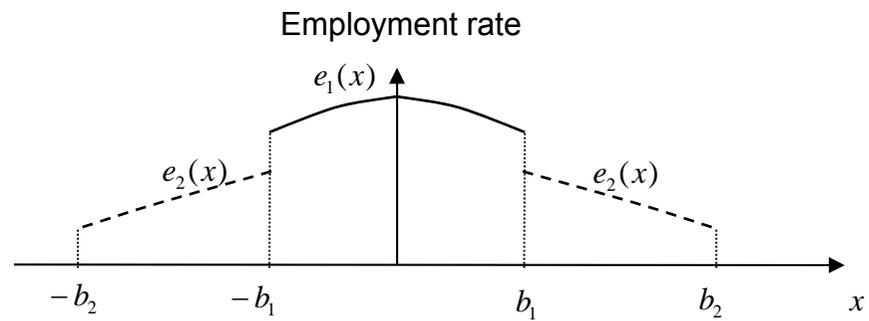
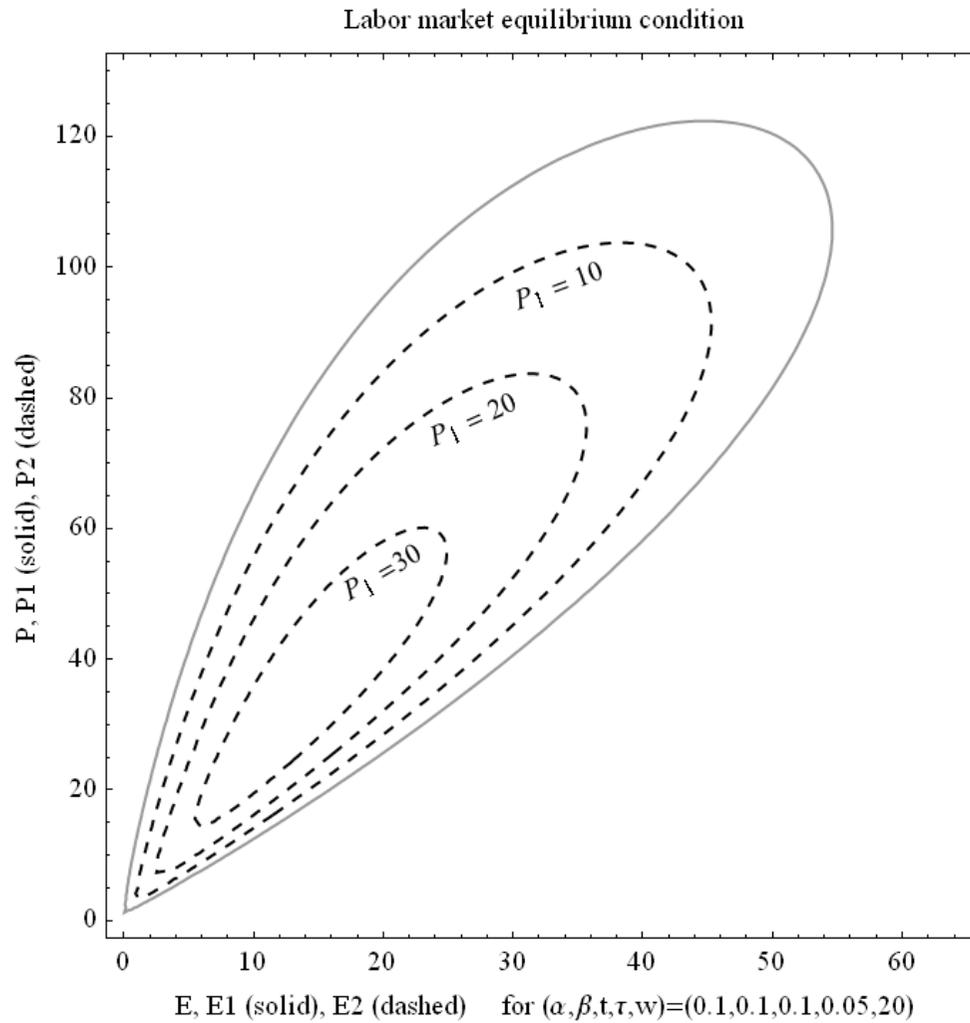


Figure 2: Travel cost functions in the segregated city



**Figure 3: Urban equilibrium with two populations**

Population 1 at the center (solid lines), population 2 at city edges (dashed lines)



**Figure 4: Labor market conditions with directed search**

Each curve displays the locus of aggregate employment and population size.

For one population the locus E-P is shown by the solid curve. For a population size  $P$ , the high employment equilibrium lies on the right hand branch of the curve. For two populations, the locus E1-P1 for the central population 1 is shown by the same solid curve while the locus E2-P2 for the peripheral population 2 is displayed by the dashed curve. Each dashed curve corresponds to a specific size of the population 1.

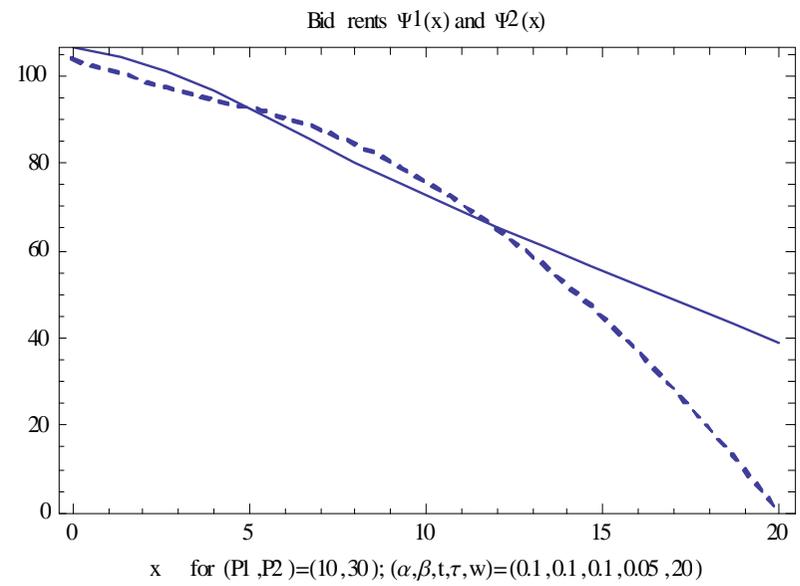
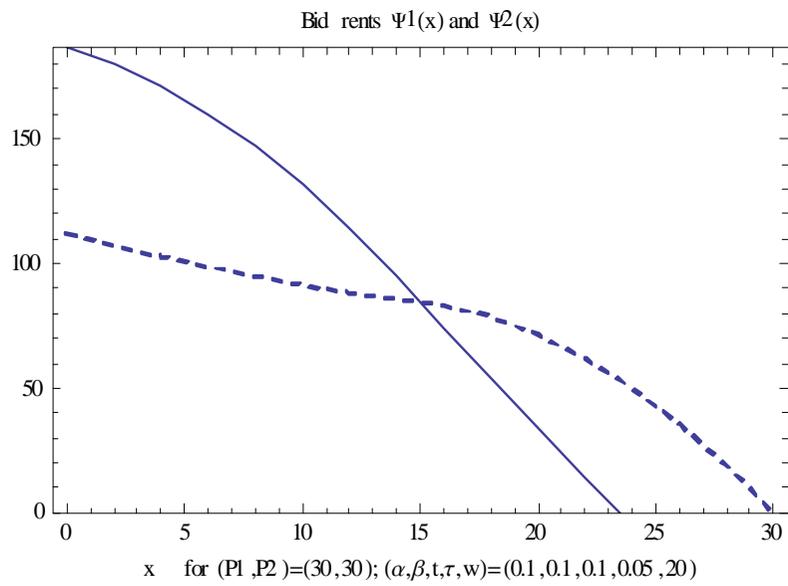


Figure 5: Bid rents may cross twice