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Where and When to Invest in Infrastructure

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Abstract

This paper analyzes an irreversible "where-and-when" investment decision, in which a government must decide not only when to invest in income-increasing infrastructure but also where to make the investment, doing so under imperfect observability of the investment gains. The two models considered in the paper differ in the source of the imperfection. In the signal model, the imperfection comes from imperfect observability of initial income gains from the investment, while in the option model, it comes from the stochastic nature of the income gains in the second period. In addition to providing the first treatment of this type of problem, the analysis shows that the influences of underlying parameters on whether or not the government waits to invest are similar in the two models.

Where and When to Invest in Infrastructure

by

Jan K. Brueckner and Pierre M. Picard*

1. Introduction

Starting with Aschauer (1989), a large literature has developed studying the productivity effects of public infrastructure investment. Most recently, Duranton and Turner (2012) focus on the effect of transportation infrastructure, exploring the impact of highway investment on regional growth in a study that builds on earlier work.[1] The related connectivity benefits provided by airports can also stimulate local economies, and papers measuring this effect include Brueckner (2003), Sheard (2013) and others.[2] For earlier contributions to the infrastructure literature whose focus is broader than simply transportation investment, see the survey paper by Munnell (1992).[3]

All of this prior work has generated a broad consensus that public investment stimulates regional economies, and this view provides the starting point for the present paper. The paper, however, considers a question that has received no attention (to our knowledge) in the infrastructure literature. Suppose that a government, facing a constraint on funds, can make only a single infrastructure investment and seeks to maximize the gain from investment. The question is: when faced with two location choices with different investment gains, as well as a timing choice (invest in period 1 or period 2), *where* and *when* should a government make its infrastructure investment? In other words, if the government can make one irreversible investment, which of the regions it serves should get the investment? Moreover, should the investment be made now, or should it be deferred until a later period?

These where-and-when questions are potentially intertwined because the regional impacts of the investment may be only partly unobservable, raising the possibility that the wrong location (with inferior gains) is chosen. Waiting to invest, however, may fully reveal the different regional gains from the investment, which allows the best location to be selected.

The downside from waiting, however, is the foregone (but perhaps suboptimal) benefit from investing immediately.

There are two natural ways of portraying this lack of observability, which in turn lead to two different models of the government's decision problem. In the first model, the initial gains from the investments in the two regions are observable. But region-specific random shocks shift the subsequent gains in an unpredictable fashion, possibly reversing the initial ranking. The realizations of these random shocks are observable, however, if the government waits to invest, allowing a better location (from the perspective of subsequent gains) to be chosen. This version of the decision problem is called the "option model" since it bears some connection to a standard investment problem under uncertainty, where waiting helps to resolve future risks.

The presence of two investment choices, however, creates some notable differences between the present option model and the standard one. Although greater uncertainty delays the investment date in the standard option model with a single potential investment, a higher return variance in the current option model need not making waiting (analogous to a delay in continuous time) more desirable. However, the benefit from waiting does depend on the covariance between the two random influences that help determine the second period's investment gains in the regions. If the covariance is high, the future is still uncertain but the gains from waiting are low because the random effects are unlikely to reverse the advantage of the region with the higher initial investment gain. This type of outcome, where waiting may not be optimal despite high future uncertainty, is not present in models with only a single investment opportunity.

Under the second model, the regional gains from the investment are initially unobservable, although they become observable if the government waits to invest. Despite their first-period unobservability, the gains are partly revealed by random signals received by the government in that period, which provide partial information about the business climates in the two regions. The government must decide whether to invest based on this (possibly misleading) signal information or to wait and act using full information. This version of the decision problem is called the "signal model." The option and signal models can be derived as special cases of a more-general model. Note that, while the randomness in the option model lies in the investment returns, randomness in the signal model is due to noise in the signal.

Like in the signal model, the role of information acquisition in determining the timing of investment has been studied by Cukierman (1980), Demers (1991), and Thijssen, van Damme, Huisman and Kort (2001), although in contexts very different from the current one. Similarly, the option model is connected to previous work on investment decisions because both analyze the question of "when" to invest (see Dixit and Pindyck (1994) and the references therein).[4] However, the existence of two different investment locations introduces a departure from the standard option model, making the question not only when but also where to invest. This departure is like the one studied by Dixit (1993) and Décamps, Mariotti and Villeneuve (2006), where the investor decides when to invest and which among a menu of production technologies to use, faced with stochastic evolution of the output price.

Several transportation-investment examples serve to illustrate the option and signal models. The first example concerns the Green Line, a portion of the Los Angeles light rail system whose routing was chosen based on job location patterns that had changed dramatically by the time the system was complete, impairing ridership and making a different routing look better with hindsight. This outcome illustrates the option model, with the job-pattern change corresponding to an unfavorable realization of future uncertainty for one investment location. The relevant details are presented in the following excerpt from Wikipedia:

> Construction on the Green Line began in 1987. One of the reasons for construction was that the Green Line would serve the aerospace and defense industries in the El Segundo area. Construction of the line cost $718 million. By the time the Green Line opened in 1995, the Cold War was over, and the aerospace sector was hemorrhaging jobs. ... As a result, ridership has been below projected estimates, averaging approximately 44,000 daily weekday boardings in June 2008.
>
> The Green Line's western alignment was originally planned and partially constructed to connect with LAX [Los Angeles International Airport], but the airport was planning a major remodeling during the line's construction. Los Angeles World Airports wanted the connection to LAX to be integrated with this construction, but there were concerns that the overhead lines of the rail would interfere with the landing paths of airplanes. In addition, citizens of neighboring communities to LAX opposed the expansion of the airport. ...
>
> The Green Line's eastern terminus also suffers from the fact that it stops two miles (3 km) short of the heavily used Norwalk/Santa Fe Springs Metrolink station, where several Metrolink lines operate. Because of this, and the Green Line's re-routed western alignment away from LAX, critics have labeled the Green Line as a train that goes "from nowhere to nowhere."

This discussion shows that, while the initial employment pattern made a Green Line routing to El Segundo look attractive relative to a routing to LAX, shocks to the economy (analogous to the random future influences in the option model) reduced aerospace employment and made the routing inferior ex post. If the future had been predictable, the LAX routing would presumably have been chosen despite the hurdles it faced, which appear relatively minor in retrospect. In the absence of such foresight, the poor Green Line routing decision could have been avoided by waiting to make the choice.[5]

Several other transportation examples illustrate the signal model. Both involve privately financed tollways designed to extend existing highway networks. The Dulles Greenway was completed in 1995 as an extension of the Dulles Toll Road, which connects Dulles International Airport to central Washington, D.C. The Greenway extended 12 miles beyond the airport, serving Virginia's Loudoun county, and initial traffic was projected at 20,000 vehicles per day (Jain (2010)). As explained by Jain, the outcome was different:

> Within six months of opening in late 1995, the project was in financial distress. Average daily traffic demand was an abysmally low 10,500. Toll rates were reduced from an initial $1.75 to $1.00 by March 1996, and future toll hikes were deferred in an attempt to increase ridership.... By July 1996, road usage increased to 21,000 daily travelers, averaging 1% to 2% monthly growth. However, the net effect on projected revenues was marginal, as decreased toll rates offset the increase in ridership.

The result was default on the project's debt, with the owners beginning "discussions with the ... creditors in the summer of 1996 to work out a plan for deferring debt payments and restructuring loan contracts..." (Jain (2010)).

This outcome can be viewed in the context of the signal model, with the project planners relying on signals that proved to be faulty predictors of latent transportation demand in the area, either because of low quality or randomly favorable realizations. Waiting for more demand information could have led to a different decision, with the developers choosing a project designed to increase freeway capacity elsewhere in the highly congested Washington region.

A similar example involves the State Route 125 tollway in San Diego, built to extend an existing highway network in the inland part of the region closer to the Mexican border. Like the Dulles Greenway, traffic on the SR 125 fell seriously short of projections, leading to bankruptcy of its developer in 2010 (see Wikipedia). Moreover, misleading signals appeared to have played a role, with the toll road built partly in anticipation of relocation of the San Diego International Airport to an inland location near its route, an event that never took place.[6] Again, waiting to invest (allowing resolution of the airport issue) might led have the developers to a different decision, building elsewhere in a region that, like the Washington area, is highly congested and in need of extra freeway capacity.

Another more-dramatic example of infrastructure located in wrong place is the Ciudad Real Central Airport in Spain. Built at a cost of 1.1 billion euros and located on the high speed Madrid-Seville rail line, the airport was intended to serve as an overflow facility for Madrid's Barajas airport, 150 miles distant. Completed in 2009, the airport closed in 2012 after the last of a handful of carriers offering service terminated their operations due to low passenger volumes. The construction of the airport, which is now in receivership, reflected "poor planning and overoptimism on the part of large financial investors" (Wikipedia). From the perspective of the model, the investors evidently relied on faulty signals or misinterpreted more-reliable ones, although the failure of the airport also coincided with Great Recession, complicating the picture. Waiting to invest could have led to a better decision, perhaps involving a different location for the airport.

Building on these examples, the paper analyzes a government's investment problem using the option and signal frameworks. The main contribution of the paper is to draw attention to the existence of "where-and-when" investment decisions and to show how they might be analyzed. Since such decisions have received almost no treatment in the literature, this contribution is significant, especially given the real world importance of these where-and-when decisions.

A more specific objective of the paper to understand the effect of the various parameters of the model on the waiting decision. These parameters include the cost of the investment, the income gains from investing in the two regions, the discount factor, the length of the period-2 income stream, and the parameters governing the stochastic elements of the models (the signal variance, and the variances and covariance of period-2 incomes in the option model). Although the variance and covariance effects in the option model (mentioned above) are noteworthy, the rest of the paper's comparative-static results are mostly natural and unsurprising. Thus, the paper's real contribution lies not in these particular results but rather in exposing the issues involved in where-and-when investment decisions. In doing so, the analysis offers a general lesson for policy makers by showing that precipitous infrastructure-investment decisions may have a downside, with waiting being potentially beneficial.

Section 2 of the paper presents the general framework, which yields the two models as special cases. Section 3 analyzes the option model, while section 4 analyzes the signal model. Section 5 presents conclusions.

2. The General Framework

Consider an economy with two regions, a and b and two time periods, 1 and 2. Let I_a and I_b represent the investment decisions in regions a and b, which are mutually exclusive and irreversible. These variables satisfy

$$\begin{aligned} I_a &= 0 \text{ or } 1 \\ I_b &= 1 - I_a. \end{aligned} \tag{1}$$

The investment can be made in either period 1 or period 2, and the variable W indicates whether the decision maker waits until period 2 to make the investment. $W = 1$ holds if the investment occurs in period 2 (if the decision maker waits), and $W = 0$ holds if it occurs in period 1.

The investment entails a one-time cost of c, and it raises a region's income. The investment is productive in one region and less productive in the other, but the identity of the productive region may not be initially observable. The income gain equals $\theta + \delta$ in the productive region and θ in the unproductive region, where $\theta, \delta > 0$ (θ is referred to below as the "base" income gain). The identity of the productive region is indicated by the variable D, with $D = 1$ holding when a is the productive region and $D = 0$ holding when the productive region is b.

While the income gains in period 1 contain only these nonstochastic elements, the gains in period 2 include stochastic components, which consist of random variables that multiply the nonstochastic expressions. If region a is the productive region, its income gain in period 2 is $(\theta+\delta)\epsilon_a$, where ϵ_a is a positive random variable. If region a is instead the unproductive region, its period-2 income gain is $\theta\epsilon_a$. Corresponding income gains for region b in these two cases are $\theta\epsilon_b$ and $(\theta+\delta)\epsilon_b$.

Note that, even if region a is productive, region b could have the higher period-2 income due to stochastic influences, an outcome that arises if $(\theta+\delta)\epsilon_a < \theta\epsilon_b$ holds. However, since the realizations of ϵ_a and ϵ_b are observed once period 2 is reached, it follows that if government waits to invest, the investment can be made in the region where it yields the highest gain.

Let y denote regional income in the absence of an investment, which is supplemented by the above income gains. Then, incomes in the two regions in the periods 1 and 2 depend on where the investment is made (I_a), when it occurs (W) and which region is productive (D). Using the above information, these incomes (denoted by y_{a1}, y_{a2}, y_{b1}, and y_{b2}) are given by

$$y_{a1} = y + (1-W)I_a(\theta+D\delta) \tag{2}$$

$$y_{a2} = y + I_a(\theta+D\delta)\epsilon_a \tag{3}$$

$$y_{b1} = y + (1-W)(1-I_a)(\theta+(1-D)\delta) \tag{4}$$

$$y_{b2} = y + (1-I_a)(\theta+(1-D)\delta)\epsilon_b. \tag{5}$$

From (2) and (3), if the investment occurs immediately ($W=0$) and if region a is chosen ($I_a=1$), then $y_{a1}=y+\theta+\delta$ and $y_{a2}=y+(\theta+\delta)\epsilon_a$ hold if a is the productive region ($D=1$). If instead region b is productive ($D=0$), then $y_{a1}=y+\theta$ and $y_{a2}=y+\theta\epsilon_a$ hold. In either case, $y_{b1}=y_{b2}=y$ holds from (4) and (5) since $1-I_a=0$. If, on the other hand, region b is chosen, then the regional incomes are gotten by swapping the a and b subscripts in the previous expressions. If waiting occurs ($W=1$), then period-1 incomes equal y in both regions, while the previous expressions continue to apply for period-2 incomes.

The identity of the productive region is indicated by signals, which may reveal this identify imperfectly. Concretely, the signals are pieces of information about the business climate that contain evidence about the likely income gains from the investment in the two regions. Note that these signals are not produced by some other optimizing economic agent whose goal is to influence the government's decision; they are generated instead by "nature." Let s_a and s_b denote these random signals, which are received in period 1 and are indicators of the nonstochastic portion of the income from investment in the two regions. The signals take the form

$$s_a = \theta + D\delta + \beta v_a \tag{7}$$

$$s_b = \theta + (1-D)\delta + \beta v_b, \tag{8}$$

where v_a and v_b are random variables and where β equals 0 or 1. Note that the difference between the nonstochastic parts of the signals (equal to $(2D-1)\delta$) is larger in absolute value the greater is the income gain δ from making the investment in the productive region. Thus, a large productivity difference is more readily revealed by the signals than a smaller one. When $\beta = 0$, this revelation is perfect, with the signals fully revealing the identity of the productive region, but it is imperfect when $\beta = 1$, with the signals not fully informative.

The ensuing analysis considers two cases. The "option" case is characterized by the restriction $\beta = 0$, so that the signal perfectly reveals the productive region but income uncertainty is present in period 2. In this case, waiting to invest means sacrificing guaranteed income in period 1 to ensure that the highest possible income is earned in period 2.

In the "signal" case, $\beta = 1$ holds, so that the signal contains noise. But ϵ_a and ϵ_b are constant and equal to 1, so that the income gains have no random element in period 2. Since the identity of the productive region is not fully revealed until period 2, waiting is necessary to ensure that the investment occurs there, but at a cost of lost income in period 1. Figures 1 and 2 show the time lines for the option and signal models.

The next section analyzes the option model, pointing out differences relative to the standard option framework, while section 4 analyzes the less-familiar and more-complex signal model.

3. The Option Model

3.1. The setup

Consider first the discounting of future income. Let $\rho < 1$ be the discount factor used to value period 2 income in period 1. Next, let period 2 be viewed as a sequence of possibly multiple (and stationary) future periods over which an income flow given by (3) or (5) is earned, and let λ be the factor used to discount this income stream back to the beginning of period 2. With multiple future periods $\lambda > 1$ will hold, while λ would equal 1 if period 2 encompasses just a single period. Thus, λ is an indicator of the length of the period-2 income stream. Note that $\rho\lambda$ is the factor that discounts the period-2 income stream income back to period 1.

Recall that in the option model, the signal is perfectly informative rather than noisy, with $\beta = 0$ in (7) and (8). But income earned in period 2 is stochastic, governed by the random variables ϵ_a and ϵ_b in (3) and (5). To simplify the analysis, these variables are assumed to have the same mean, denoted $\mu > 0$. Using (2) with $W = 0$ as well as (3), the discounted expected income from investing in region a in period 1 is then

$$E(y_{a1} + \rho\lambda y_{a2}) \;=\; y \,+\, \theta \,+ D\delta \,-\, c \,+\, \rho\lambda[y \,+\, (\theta \,+\, D\delta)\mu], \tag{9}$$

where c again gives the cost of the investment. Since $E(\epsilon_b)$ also equals μ, the only change required to generate the analogous expression for investing in region b is to replace D in (9) with $1 - D$. Since the investment then yields higher expected income in the region with $D = 1$ (whose identity is observable), if the government makes the investment in period 1, it will invest in that region, which is assumed without loss of generality to be region a. From (9), the resulting expected income equals

$$(1 + \rho\lambda)y \,-\, c \,+\, (\theta + \delta)(1 + \rho\lambda\mu). \tag{10}$$

By waiting to invest and thus observing the realizations of ϵ_a and ϵ_b, the government can secure the higher of the two future income streams, which may be offered by region b, not region a, if ϵ_a is small relative to ϵ_b. Since a is the (ex-ante) productive region, the discounted expected income from waiting is given by

$$y \;+\; \rho E[max\{\lambda(y+(\theta+\delta)\epsilon_a)-c, \;\; \lambda(y+\theta\epsilon_b)-c\}], \tag{11}$$

The choice of making no investment in period 2 is assumed to be unattractive, which requires a sufficiently small c (footnote 7 below gives the relevant condition). This assumption marks a key difference relative to the standard option model, where a single investment is available and where that investment may turn out to be undesirable once future uncertainty is resolved. Therefore, in the standard model, waiting may result in no investment being undertaken, in contrast to the present model, where some investment always occurs, either in period 1 or 2.

Waiting to invest is optimal when (10) is less than (11). Cancelling the common $(1+\rho\lambda)y$ terms, waiting is then optimal when

$$\theta \;+\; \delta \;-\; c \;+\rho\lambda(\theta+\delta)\mu \;\;<\;\; \rho E[max\{\lambda(\theta+\delta)\epsilon_a - c, \;\; \lambda\theta\epsilon_b - c\}]. \tag{12}$$

To generate the formula for the expected value in (12), note that the first term is maximal when $\epsilon_a > g\epsilon_b$, where

$$g \equiv \frac{\theta}{\theta+\delta} \;\;<\;\; 1 \tag{13}$$

is the relative gain from investing in the unproductive region, and that the second term is maximal when $\epsilon_a < g\epsilon_b$. Letting $t(\epsilon_a, \epsilon_b)$ denote the joint density of ϵ_a and ϵ_b, and assuming these random variables both have support $[\underline{\epsilon}, \overline{\epsilon}]$, with $\overline{\epsilon} > \underline{\epsilon} > 0$,[7] the RHS of (12) can be written

$$\rho \int_{\epsilon_b=\underline{\epsilon}}^{\overline{\epsilon}} \left[\int_{\epsilon_a=g\epsilon_b}^{\overline{\epsilon}} \lambda(\theta+\delta)\epsilon_a t(\epsilon_a,\epsilon_b)d\epsilon_a \;\;+\;\; \int_{\epsilon_a=\underline{\epsilon}}^{g\epsilon_b} \lambda\theta\epsilon_b t(\epsilon_a,\epsilon_b)d\epsilon_a \right] d\epsilon_b \;-\; \rho c. \tag{14}$$

Noting that the $\rho\lambda(\theta+\delta)\mu$ term in (12) equals

$$\rho\lambda(\theta+\delta) \int_{\epsilon_b=\underline{\epsilon}}^{\overline{\epsilon}} \int_{\epsilon_a=\underline{\epsilon}}^{\overline{\epsilon}} \epsilon_a t(\epsilon_a,\epsilon_b) d\epsilon_a d\epsilon_b, \tag{15}$$

and subtracting (15) from the integral expression in (14), that expression reduces to

$$\rho \int_{\epsilon_b=\underline{\epsilon}}^{\overline{\epsilon}} \left[\int_{\epsilon_a=\underline{\epsilon}}^{g\epsilon_b} -\lambda(\theta+\delta)\epsilon_a t(\epsilon_a,\epsilon_b) d\epsilon_a \quad + \quad \int_{\epsilon_a=\underline{\epsilon}}^{g\epsilon_b} \lambda\theta\epsilon_b t(\epsilon_a,\epsilon_b) d\epsilon_a \right] d\epsilon_b \quad =$$

$$\rho\lambda(\theta+\delta) \int_{\epsilon_b=\underline{\epsilon}}^{\overline{\epsilon}} \int_{\epsilon_a=\underline{\epsilon}}^{g\epsilon_b} (g\epsilon_b - \epsilon_a) t(\epsilon_a,\epsilon_b) d\epsilon_a d\epsilon_b. \tag{16}$$

This expression gives the option value of waiting to invest, which equals the gain from putting the investment in the region where it earns the highest period-2 income, with the gain measured relative to the expected value in period 1 of the period-2 income from investing in region a (given by $\rho\lambda(\theta+\delta)\mu$ in (12)). It is important to note that this option value differs from that in a standard option framework because it reflects the ability to choose between *two* investment locations once future conditions becomes clear. While waiting in the usual model gives the investor a choice between investing or not investing once the future is revealed, the choice here is between two alternate investment locations.

After moving ρc in (14) to the LHS of (12), that expression reduces to $\theta+\delta-(1-\rho)c$. Since ρ is the factor for discounting period 2 income back to period 1, it embodies a discount rate r satisfying $\rho = 1/(1+r)$. Substituting, the previous expression then reduces to $\theta+\delta-rc/(1+r)$. The last term equals the period-1 present value of the interest earned in period 2 on a bank deposit of c made in period 1 as an alternative to the infrastructure investment. Since the first two terms capture forgone period-1 income gain from not investing in period 1, the new LHS expression in (12) equals the net period-1 income loss from waiting.

Therefore, waiting to invest is desirable when the option value of waiting from (16) exceeds the net period-1 income loss due to waiting. Dividing both sides of the resulting inequality by $\rho\lambda(\theta+\delta)$, the waiting condition reduces to

$$\frac{1}{\rho\lambda}\left[1 \ - \ \frac{(1-\rho)c}{\theta+\delta}\right] \quad < \quad \int_{\epsilon_b=\underline{\epsilon}}^{\overline{\epsilon}} \int_{\epsilon_a=\underline{\epsilon}}^{g\epsilon_b} (g\epsilon_b - \epsilon_a) t(\epsilon_a,\epsilon_b) d\epsilon_a d\epsilon_b. \tag{17}$$

Note that if the largest possible value of $g\epsilon_b$, which equals $g\overline{\epsilon} < \overline{\epsilon}$, is less than $\underline{\epsilon}$, then no ϵ_a values lie in the range of the inner integral in (17), making the RHS equal to zero. To rule out this case, so that waiting has a chance to be optimal, $g\overline{\epsilon} > \underline{\epsilon}$ is assumed to hold.

3.2. The effects of parameter changes on the waiting decision

Any parameter change that makes the LHS of (17) smaller without affecting the RHS favors waiting. Such changes include an increase in the investment cost c or the length of the period-2 income stream, as captured by λ, either of which increases the incentive to make the right investment decision. However, an increase in the discount factor ρ has an ambiguous effect on the LHS of (17) and thus on the desirability of waiting. This is a sensible conclusion given that a higher valuation of the future applies to both income gains and investment costs. An increase in δ, indicating a larger gain from investing in the productive region, raises the LHS of (17), but it also reduces g, decreasing both the upper limit of integration and the integrand and thus lowering the magnitude of integral. As a result, the desirability of waiting decreases when δ rises. However, the same conclusion does not apply to an increase in the base income gain θ, which (by increasing g) raises both sides of (17), making the waiting effect ambiguous. While increases in δ and θ have identical effects on the lost income from waiting, they have opposite effects on the relative gain from investing in the unproductive region (as measured by g), on which the option value depends. This difference accounts for the contrast between the parameters' effects. Summarizing yields

Proposition 1. *In the option model, an increase in c or λ or a decrease in δ raises the desirability of waiting.*

Intuition suggests that a higher variance in the present option model may have no clearcut effect on the value of waiting, a conclusion that contrasts with the outcome in the usual option framework. The reason is that greater variability in both ϵ's need not raise the likelihood that ϵ_b is large enough relative to ϵ_a to reverse region a's initial productivity advantage. As a result, the gain from waiting to observe the actual outcome may be no higher with a larger variance.

Although the normal distribution is a natural choice in analyzing the effect of variance changes, both here and in the signal model below, using it to address the effect of a higher variance for the ϵ's proves to be intractable. As a result, the uniform distribution is employed instead to evaluate the above intuition, with $t(\epsilon_a, \epsilon_b) \equiv 1/\tau^2$, $\overline{\epsilon} = k + \tau/2$, and $\underline{\epsilon} = k - \tau/2 > 0$. Then, ϵ_a and ϵ_b are independent with variances of $\tau^2/12$. The intuition is confirmed by computing the value of the integral in (17) for the uniform case and evaluating the derivative with respect to τ, which is ambiguous in sign, as predicted.[8]

In contrast to this ambiguity regarding the variance, intuition suggests that a greater covariance between ϵ_a and ϵ_b should reduce the desirability of waiting. When the covariance between the ϵ's is higher, the random influences move more nearly in step with one another, so that the period-1 income gain from investing in the productive region is less likely to be reversed in period 2. The higher covariance thus lowers the benefit from waiting. To investigate this question, the following analysis compares cases where the correlations between ϵ_a and ϵ_b are $+1$ and -1, showing that waiting is not desirable in the first case but may be desirable in the second, as intuition would predict. The demonstration applies generally, not relying on the uniform distribution.

The case where ϵ_a and ϵ_b have a correlation of $+1$ can be generated by starting with an arbitrary marginal distribution for ϵ_b, denoted $t_b(\epsilon_b)$, and then assuming that $\epsilon_a = \epsilon_b$. The joint distribution of ϵ_a and ϵ_b then satisfies $t(\epsilon_a, \epsilon_b) = t_b(\epsilon_b)$ for $\epsilon_a = \epsilon_b$ and $t(\epsilon_a, \epsilon_b) = 0$ otherwise. Alternatively, a correlation of -1 is generated by assuming $\epsilon_a = \overline{\epsilon} + \underline{\epsilon} - \epsilon_b$, which correspondingly alters the definition of the joint distribution. Note that these two alternate cases preserve the maintained assumptions that ϵ_a and ϵ_b have the same support and the same means (an outcome that would not obtain if the coefficients relating ϵ_a to ϵ_b were not $+1$ and -1).

In the $+1$ case, all realizations of ϵ_a and ϵ_b lie outside the range of integration for the integral in (17). The reason is that $\epsilon_a < g\epsilon_b$ holds over this range, implying $\epsilon_a < \epsilon_b$ given $g < 1$, while a $+1$ correlation requires $\epsilon_a = \epsilon_b$. As a result, the integral equals zero, indicating that there is no benefit from waiting. This conclusion highlights the difference between the current setup and the standard option framework. Even though future uncertainty is still present, the option to wait is worthless because the returns from the two location choices are perfectly correlated. Because of this correlation, no additional information about the best investment location is gained by waiting.

Note that with perfect correlation, the model effectively contains just a single investment opportunity, with region a dominating region b since it is initially more productive and cannot become less productive in period 2. As a result, the model reduces to the standard option framework augmented by the auxiliary assumption that investment in period 2 is always worthwhile once that period is reached. While waiting is suboptimal in this setting, it may become desirable when the auxiliary assumption is dropped (allowing truly bad investments), with waiting then providing the opportunity to entirely avoid such an investment once resolution of uncertainty reveals its quality.

By contrast, if the correlation equals -1, so that $\epsilon_a = \overline{\epsilon} + \underline{\epsilon} - \epsilon_b$, then as ϵ_b ranges from $\underline{\epsilon}$ to $\overline{\epsilon}$, ϵ_a ranges from $\overline{\epsilon}$ to $\underline{\epsilon}$. Since some of the resulting ϵ_a values satisfy $\epsilon_a < g\epsilon_b$ under the maintained assumptions (recall the discussion following (17)), the integral in (17) is positive rather than zero, indicating a benefit to waiting.

This argument is illustrated in Figure 3, where the shaded area shows the range of integration for the integral in (17). With a correlation of $+1$, the possible combinations of ϵ_a and ϵ_b lie along the dotted portion of the 45 degree line. Because this segment has no overlap with the shaded area, the value of the integral in (17) equals zero. With a correlation of -1, the possible ϵ combinations lie along the dotted line with slope -1 in Figure 3. Since this segment passes through the shaded area, it gives a positive value for the integral in (17). Summarizing yields

Proposition 2. *If the correlation between ϵ_a and ϵ_b in the option model equals $+1$, then waiting to invest is undesirable. But waiting may be desirable when the correlation equals -1.*

Numerical examples suggest that the message of Proposition 2 applies more generally. Although analytical results are not available, calculations show that if $t(\epsilon_a, \epsilon_b)$ is bivariate normal, then the integral in (17) monotonically decreases as the distribution's correlation coefficient increases over the $(-1, 1)$ range, regardless of the values of g and the other distribution parameters. Thus, the benefit from waiting falls as ϵ_a and ϵ_b become more closely associated.

4. The Signal Model

4.1. Form of the investment decision rule

In the signal model, the period-2 investment returns are nonstochastic rather than random, with $\epsilon_a \equiv \epsilon_b \equiv 1$, but the identity of the productive region is unobservable in period one though partly revealed by signals. The government makes decisions based on the difference between the signals from the two regions, $s_a - s_b$. Using (7) and (8) and setting $\beta = 1$, the signal difference is given by

$$z \equiv s_a - s_b = v_a - v_b + (2D-1)\delta. \tag{18}$$

If z takes a large value, pointing toward higher productivity in region a, the government invests in region a in period 1. If z takes a small value, pointing toward higher productivity in region b, the government again invests in period 1 but chooses region b. However, if z takes an intermediate value, the signals are less clear about the identity of the higher productivity region. In this case, the government waits, deferring the investment until period 2. This assumed behavior on the part of the government, under which z is compared to critical values to reach an investment decision, is a convenient heuristic representation of optimizing behavior that can be properly derived from first principles, as explained in the appendix. That behavior leads to the same final decision rule as the one implied by the heuristic approach, as the appendix demonstrates.

Letting $\overline{z}$ and $\overline{z} \geq \underline{z}$ denote the upper and lower critical values for z, the decision rule is to invest in region a if $z > \overline{z}$, invest in b if $z < \underline{z}$, and wait if $\underline{z} \leq z \leq \overline{z}$. The government's goal is to choose $\overline{z}$ and $\underline{z}$ in optimal fashion, so as to maximize the expected income gain from the investment. Note that if the constraint $\overline{z} \geq \underline{z}$ were to bind at the solution, no values of the signals would lead the government to wait before investing.

The previous inequalities imply particular ranges for the value of $v_a - v_b$, the difference between the signals' random elements, in the three cases. Letting $x \equiv v_a - v_b$ denote this difference, the decision rule implies that the government will

$$\begin{aligned} &\text{Invest in } a \text{ in period 1 } (W=0,\ I_a=1) \quad \textit{when} \quad x > (1-2D)\delta + \overline{z} \\ &\text{Wait } (W=1) \quad \textit{when} \quad (1-2D)\delta + \underline{z} \leq x \leq (1-2D)\delta + \overline{z} \\ &\text{Invest in } b \text{ in period 1 } (W=0,\ I_a=0) \quad \textit{when} \quad x < (1-2D)\delta + \underline{z} \end{aligned} \tag{19}$$

4.2. Objective function

Using the decision rule in (19), the expected income gain from the investment can be computed conditional on $\overline{z}$ and $\underline{z}$, and it serves as the government's objective function. This computation involves a number of different steps. To begin, let G_a^1 denote the discounted value of the income gain from investing in region a in period 1 when a is the productive region ($D = 1$). Similarly, let G_a^0 denote the discounted income gain from investing in region a when b is the productive region ($D = 0$), and let G_b^1 and G_b^0 denote the analogous discounted income gains from investing in region b when it is, respectively, unproductive and productive. Finally, let G_w denote the discounted income gain from waiting, which is not region specific. Since these expressions give income gains from the investment and thus exclude the y terms in (2)–(5), they are given by

$$\begin{aligned} G_a^1 &= G_b^0 = \theta + \delta - c + \rho\lambda(\theta+\delta) \\ G_a^0 &= G_b^1 = \theta - c + \rho\lambda\theta \\ G_w &= \rho[\lambda(\theta+\delta) - c] \end{aligned} \tag{20}$$

To interpret (20), note first that, when the government does not wait to invest (lines 1 and 2), the present value of the income gain equals $(1+\rho\lambda)(\theta+\delta)$ if the investment is made in the productive region but equals $(1+\rho\lambda)\theta$ otherwise. Second, since productivity can be observed when the government waits, the investment is always then made in the productive region, generating a discounted income gain of $\rho\lambda(\theta+\delta)$ (no period-1 gain occurs). Third, note that the cost c is incurred in period 1 in the first two cases and is thus not discounted, while with waiting, the cost appears in period 2 and is thus discounted by ρ. Finally, in order for the investment to be worth undertaking after waiting until period 2, the inequality

$$\lambda(\theta+\delta) > c \tag{21}$$

must hold, indicating that the present value of the stream of subsequent income gains must be larger than the cost of the investment. This inequality is assumed to be satisfied.

With this background, the overall discounted expected income gain from the investment can be computed. Letting $P(E)$ denote the probability of the event E, this expression is given by

$$
\begin{aligned}
& P(D=1)\cdot P(x > (1-2D)\delta + \overline{z}|D=1)\cdot G_a^1 \;\; + \\
& P(D=0)\cdot P(x > (1-2D)\delta + \overline{z}|D=0)\cdot G_a^0 \;\; + \\
& P(D=1)\cdot P(x < (1-2D)\delta + \underline{z}|D=1)\cdot G_b^1 \;\; + \\
& P(D=0)\cdot P(x < (1-2D)\delta + \underline{z}|D=0)\cdot G_b^0 \;\; + \\
& P(D=1)\cdot P((1-2D)\delta + \underline{z} \leq x \leq (1-2D)\delta + \overline{z}|D=1)\cdot G_w \\
& P(D=0)\cdot P((1-2D)\delta + \underline{z} \leq x \leq (1-2D)\delta + \overline{z}|D=0)\cdot G_w
\end{aligned} \tag{22}
$$

Note that $P(D=1)$ and $P(D=0)$ give the government's prior probabilities that the productive region is a (b). Since the government has no knowledge prior to receipt of the signal, these probabilities equal 1/2. To interpret (22), observe that the first line equals the probability that region a is productive times the probability that region a is chosen under the decision rule, given that it is productive, times the income gain from this choice. Region a could be chosen, however, when it is unproductive, and the second line of (22) gives the probability of this occurrence times the associated income gain. The remaining lines of (22) are interpreted in an analogous fashion.

To rewrite (22) in a usable form, the prior probabilities can be suppressed since they are all 1/2, and the second probability expressions can be rewritten using the cumulative distribution function of the signals' noise difference x, denoted $F(\cdot)$. Then, after inserting the G expressions from (20) into (22) and ignoring the 1/2 factor, the government's objective function can be rewritten as

$$
\begin{aligned}
\Phi(\overline{z}, \underline{z}) \;\; = \;\; & [1 - F(-\delta + \overline{z})]\cdot[\theta + \delta - c + \rho\lambda(\theta+\delta)] \;\; + \\
& [1 - F(\delta + \overline{z})]\cdot[\theta - c + \rho\lambda\theta] \;\; +
\end{aligned}
$$

$$
\begin{aligned}
&F(-\delta+\underline{z})\cdot[\theta-c+\rho\lambda\theta] \;\; + \\
&\qquad F(\delta+\underline{z})\cdot[\theta+\delta-c+\rho\lambda(\theta+\delta)] \;\; + \\
&[F(-\delta+\overline{z})-F(-\delta+\underline{z})]\cdot\rho[\lambda(\theta+\delta)-c] \;\; + \\
&\quad F[(\delta+\overline{z})-F(\delta+\underline{z})]\cdot\rho[\lambda(\theta+\delta)-c]. \qquad (23)
\end{aligned}
$$

The lines of (23) correspond to the lines of (22). Note that, in writing the second probabilities in (22) in terms of F, the conditioning factors $D=0,1$ are used in evaluating the $(1-2D)\delta$ terms, which then equal either δ or $-\delta$.

4.3. Optimization problem

The government's goal is to maximize Φ in (23) by choice of $\overline{z}$ and $\underline{z}$ subject to the constraint $\overline{z}\geq\underline{z}$. Letting f denote the density corresponding to F, and derivative of (23) with respect to $\overline{z}$ is

$$\frac{\partial\Omega}{\partial\overline{z}} \;=\; -f(-\delta+\overline{z})(\theta+\delta-(1-\rho)c) \;+\; f(\delta+\overline{z})(\rho\lambda\delta+(1-\rho)c-\theta). \qquad (24)$$

The derivative $\partial\Omega/\partial\underline{z}$ is given by a similar expression. The expression in (24) equals zero at an interior solution for $\overline{z}$, but the noninterior solutions may also exist.

To make the analysis manageable, the distribution of $x=v_a-v_b$ is assumed to be symmetric and unimodal with mean zero. This outcome emerges if the distributions of v_a and v_b are themselves identical, symmetric and unimodal with zero means. In this case, the critical $\overline{z}$ and $\underline{z}$ values will be symmetric around zero, as can be seen by comparing (24) and the analogous derivative for $\underline{z}$. Symmetry allows the $\overline{z}$ and $\underline{z}$ to be replaced by u and $-u$, with the constraint $\overline{z}\geq\underline{z}$ reducing to $2u\geq 0$ or $u\geq 0$. Thus, $[-u,u]$ is the signal range over which waiting is optimal.

With this substitution, only one first-order condition is needed, and (24) can be used with $\overline{z}$ replaced by u. Setting (24) equal to zero, the first-order condition for an interior solution can then be written as

$$\Big[(\rho\lambda\delta+(1-\rho)c-\theta)f(-\delta+u)\Big]\left[-R \;+\; \frac{f(\delta+u)}{f(-\delta+u)}\right] \;=\; 0, \qquad (25)$$

where

$$R \quad = \quad \frac{\theta + \delta - (1-\rho)c}{\rho\lambda\delta + (1-\rho)c - \theta}. \tag{26}$$

A fully general analysis of (25) and (26) is complex because the numerator and denominator of R can take either sign, making additional assumptions necessary. Specifically, the base income gain θ is assumed to be small enough relative to δ and c that the denominator of (26) is positive:[9]

$$\rho\lambda\delta \;+\; (1-\rho)c \;-\; \theta \quad > \quad 0. \tag{27}$$

Using (24), an interior solution for u satisfies

$$-R \;+\; \frac{f(\delta+u)}{f(-\delta+u)} \quad \equiv \quad -R \;+\; H(u,\delta) \quad = \quad 0, \tag{28}$$

where $H(u,\delta)$ denotes the density ratio in (28). To insure that the second-order condition holds at an interior solution, (27) must be decreasing in u at the solution, with the expression changing sign from positive to negative. For an interior solution to be unique, this sign change must only occur once. This requirement in turn implies that $H(u,\delta)$ changes sign just once, which means that H is (weakly) monotonically decreasing in u, with $\partial H/\partial u \leq 0$. This condition constitutes a third maintained assumption, along with (22) and (27).

To understand the behavior of the $H(u,\delta)$ function, consider Figure 4. Recalling that $H(u,\delta)$ is the density ratio from above, note that H equals 1 when $u=0$, and that as u rises above zero, H decreases until u equals δ, at which point $-\delta+u=0$ holds and density's mode is reached. But further increases in u, which put both $-\delta+u$ and $\delta+u$ on the downward-sloping part of the density, have an ambiguous effect on H, although it remains below 1. However, for several familiar densities, including the normal and triangular cases, H continues to decrease as u increases beyond δ.[10] In the normal case with variance σ^2, which is considered further below,

$$H(u,\delta) \quad = \quad \frac{exp[-(\delta+u)^2/2\sigma^2]}{exp[-(-\delta+u)^2/2\sigma^2]} \quad = \quad exp[-2\delta u/\sigma^2], \tag{29}$$

a decreasing function of u, and a calculation for the triangular density yields the same conclusion. These examples lend plausibility to the assumption $\partial H/\partial u \leq 0$. Note that the weak inequality covers the case of a uniform distribution, where H is constant at 1 over the density's support (see below).

4.4. Non-interior solutions

Eventually, a comparative-static analysis is carried out to show how parameter changes affect the value of u at an interior solution. But non-interior solutions are of considerable interest, and they are considered first. To begin, observe that if

$$\theta \ + \ \delta \quad \leq \quad (1-\rho)c, \tag{30}$$

then $R \leq 0$ holds given (27), and (25) is then positive for all u. In this case, an infinite u is desirable. Thus, waiting is always optimal, with no signal values inducing the investor to invest in period 1.[11]

Substituting $rc/(1+r)$ for $1-\rho$ as before, (30) reduces to $\theta + \delta < rc/(1+r)$. Recall that the RHS of this inequality equals the period-1 present value of the interest earned in period 2 on a bank deposit of c made in period 1 as an alternative to the infrastructure investment. If this present value is greater than the forgone period-1 income gain from the investment, equal to $\theta + \delta$, then waiting is preferable regardless of the signal values.

By contrast, suppose that $R \geq 1$. Then, since $H \leq 1$, (25) is negative or zero for $u \geq 0$, implying that $u = 0$ holds at the optimum. In this case, no values of the two signals lead the government to wait: it always invests in period 1, choosing region a if $u > 0$ and region b if $u < 0$. Rearrangement of (17) shows that $R \geq 1$ holds when

$$(\theta+\delta)(1-\rho\lambda) \quad \geq \quad 2(1-\rho)c. \tag{31}$$

Note that this condition cannot be satisfied if $\rho\lambda \geq 1$, which rules out no waiting as an optimal choice.[12] This inequality states that the future income stream is sufficiently long (λ is sufficient large) that an extra dollar of stream income has a present value that equals or exceeds 1. This high present value amplifies the loss from investing in the unproductive region, which makes waiting to invest optimal for at least some range of signal values, ruling out $u = 0$.

Using (30) and (31), a simple statement about the conditions leading to noninterior solutions can be made, as follows:

> **Proposition 3.** *In addition to the maintained assumptions in the signal model, suppose that $\rho\lambda < 1$ holds, so that no waiting may be an optimal choice. Then, unconditional waiting (an infinite u) is optimal if and only if $\theta + \delta \leq (1-\rho)c$, while no waiting (a zero u) is optimal if and only if $\theta + \delta \geq [2(1-\rho)/(1-\rho\lambda)]c$.*

Intuitively, the proposition says that unconditional (no) waiting is optimal when the forgone period-1 income from investing in the productive region $(\theta + \delta)$ is sufficiently small (large). Note that the critical $\theta + \delta$ above which no waiting is optimal is more than twice as large as the critical value below which unconditional waiting is optimal.

4.5. Interior solutions and comparative statics

In contrast to these non-interior solutions, an interior solution to (25) may exist when $0 < R < 1$. To rule out inessential complications, a fourth assumption (which is satisfied in the normal and triangular cases) is imposed. This assumption is $H(\infty, 0) = 0$, which states that the limit of the monotonically decreasing H function as u increases without bound is zero. Under this assumption, an interior solution for u is guaranteed to exist since $H(u, \delta)$ starts at 1 when $u = 0$ and decreases to zero as u increases without bound. Therefore, H must equal R at some interior u when $0 < R < 1$. This conclusion, along with the previous results for noninterior solutions, is illustrated in Figure 5. The figure graphs the H function, showing how it lies between zero and 1 for $u \geq 0$ and thus cannot yield an interior solution when R is outside this interval.

Comparative-static analysis showing the effect of individual parameters on u can be carried out. The analysis relies on the assumption that $H(u, \delta)$ is decreasing in u, which implies that parameter changes that raise R (but do not directly affect H) serve to reduce u.

Inspection of (26) shows that $\partial R/\partial c,\ \partial R/\partial\lambda < 0$ and $\partial R/\partial\theta > 0$ hold, implying

$$\frac{\partial u}{\partial c}, \ \frac{\partial u}{\partial \lambda} \ > \ 0, \quad \frac{\partial u}{\partial \theta} < 0. \tag{32}$$

Therefore, the signal range over which waiting is optimal widens when the investment cost or the length of the future income stream rises or when the base income gain θ from investing in either region falls. Note that the first two effects parallel those in the option model, but that the effect of θ, which was previously ambiguous, is now determinate. The effect on R of a higher ρ (and thus the effect on u) is ambiguous, as in the option model.

A higher δ affects both R and H, and differentiation of (26) shows that

$$\frac{\partial u}{\partial \delta} = -\frac{\partial H/\partial \delta - \partial R/\partial \delta}{\partial H/\partial u}. \tag{33}$$

The sign of $\partial H/\partial \delta$ is ambiguous, and

$$sign\frac{\partial R}{\partial \delta} = sign[(1-\rho)c - \theta], \tag{34}$$

which is also ambiguous. Although the sign of $\partial u/\partial \delta$ is thus ambiguous, consideration of the normal case provides an answer. Setting H for the case of a normal probability distribution, given by (29), equal to R and solving for u yields

$$u = -\frac{\sigma^2}{2\delta}\, logR. \tag{35}$$

Note that $logR$ is negative given $R < 1$, making (35) positive. Differentiation of (35) shows that $\partial u/\partial \delta$ has the sign of $logR - (\delta/R)(\partial R/\partial \delta)$, which is negative provided that θ is small (making $\partial R/\partial \delta$ in (34) positive). Under these assumptions, the effect of a higher δ matches that of a higher θ, narrowing the range of signals over which waiting is optimal. Note that, while this impact matches that in the option model, it cannot be derived without auxiliary assumptions, unlike in that model.

Intuition would suggest that a greater signal variance should have the opposite effect, widening the signal range over which waiting is optimal. Using (35), this intuition in confirmed, with differentation yielding

$$\frac{\partial u}{\partial \sigma^2} > 0. \tag{36}$$

Recall that, while the variance effect was ambiguous in the option model (matching intuition), the variance effect in the signal model captures a different type of impact, which makes its sign determinate.[13] Summarizing yields

Proposition 4. *Using the maintained assumptions and focusing on the range of interior solutions in the signal model, an increase in c or λ or a decrease in θ widens the range of signal values over which waiting to invest is optimal. When u is normally distributed, the same effect occurs when σ^2 increases or (assuming θ is small) when δ decreases.*

It is also interesting to ask whether, starting at an interior solution, divergence in the values of the parameters c, λ, θ, and δ is capable of pushing the solution to one of the non-interior cases (where $u = 0$ or infinity). Assuming $\rho\lambda < 1$, a zero solution for u ($R \geq 1$) is ensured when c is sufficiently small, given (31), and an infinite u becomes optimal ($R \leq 0$) when c is sufficiently large, given (30). But a zero u need not become optimal when λ approaches its lower bound of 1 (see (31)), and an infinite u need not become optimal as λ increases since λ does not appear in (30). A zero δ need not make an infinite u optimal (see (30)), but (provided $\rho\lambda < 1$) increasing δ eventually makes $u = 0$ optimal, given (31) (the same conclusions apply to θ). Since σ^2 plays no role in (30) or (31), changes in its value cannot produce satisfaction of one of these inequalities. The upshot of this discussion is that, in only a few cases are extreme values of the parameters capable of pushing an interior u solution to either zero or infinity.

4.6. Signal enhancement by local governments

A natural question is how the model would change if the local government in a region could send its own productivity signal (possibly based on superior information) that could influence the investment choice of the super-regional government. Suppose that the local governments send signals of q_a and q_b that augment nature's signal in an additive fashion, and that the costs of sending the signals are kq_a and kq_b, where $k > 0$. Focusing on region a, z in (18) is then replaced by $v_a - v_b + q_a - q_b$, and modifying (19), region a is chosen when $x > (1 - 2D)\delta + q_b - q_a + \overline{z}$.

Let α_a denote the prior probability of region a's government that its region is productive. Then, with the income gain being zero if the investment is not carried out in region a, the expected gain from the local government's viewpoint is

$$\Gamma_a \equiv \alpha_a[1 - F(-\delta + q_b - q_a + \overline{z})](\theta + \delta) + (1 - \alpha_a)[1 - F(\delta + q_b - q_a + \overline{z})]\theta - kq_a, \quad (37)$$

using the original variable $\overline{z}$ instead of u. A analogous expression applies to region b.

It is not clear how the standard signaling analysis could be applied to this model, given the difference between current structure and the usual signaling context. However, one limited conclusion can be reached under atypical assumptions. Suppose that the local governments are leaders with respect to the super-regional government but Cournot competitors with one another. In other words, each local government anticipates the response of the super-regional government's u choice to a change in its own q, while treating the other local government's q as parametric. Under these assumptions, it is easy to see that $\partial\overline{z}/\partial q_a = 1$,[14] which means from (37) that $\partial\Gamma_a/\partial q_a = -k < 0$ holds and that $q_a = 0$ is optimal. With the same conclusion holding for region b, neither local government finds it optimal to send a signal. Because government signals are fully offset in the choice of u, it is not worthwhile to incur the cost of sending them.

Thus, adding local government signaling to the "nature's signal" model has no effect given that signals are not sent. Since this conclusion rests on atypical assumptions, however, further work that attempts to wed the current model to a standard signaling framework would be useful, possibly being a subject for future research.

5. Conclusion

This paper has analyzed an irreversible "where-and-when" investment decision, in which a government must decide not only when to invest in income-increasing infrastructure but also where to make the investment, doing so under imperfect observability of the investment gains. The two models considered in the paper differ in the source of the imperfection. In the signal model, the imperfection comes from initially imperfect observability of the income gains from the investment, while in the option model, it comes from the stochastic nature of the income gains in the second period. In addition to providing the first treatment of this type of problem, the analysis shows that the influences of underlying parameters on the waiting decision are similar in the two models. Waiting to invest is more likely when the investment cost is high or the income stream beyond the initial period is long, and it is less likely when the income-gain differential between the productive and unproductive regions is large. While greater uncertainty makes waiting more likely in the signal model, the variance effect in the option model is ambiguous, reflecting the availability of two investment choices in period 2 rather than a single choice. However, a greater covariance of returns between the two choices makes waiting less likely.

Although the paper attempts to incorporate local-government signaling in a highly restricted fashion, recasting the signal model in the tradition of the standard signaling framework remains a (possibly challenging) task for future research. Given the importance of the issues raised by the paper, this and other extensions would appear to be fruitful paths for additional work.

The paper's main contribution is to draw attention to "where-and-when" investment decisions, which have received virtually no treatment in the literature. In doing so, the paper carries a lesson for policymakers. By demonstrating the potential benefit of waiting, the analysis highlights the possible downside from precipitous infrastructure-investment decisions. Before deciding where to invest in infrastructure, policymakers should seriously consider whether adequate information has been accumulated about the available options.

Appendix

This appendix shows that the heuristic approach to analysis of the signal model, which relies on critical values for z, is equivalent to an approach that proceeds from first principles. To carry out this approach, imagine that rather than being discrete in nature, the investment can be "divided" between three choices: investing in region a or b in period 1 or waiting to invest. Let $A(z)$ and $B(z)$ denote the "fraction" of the investment made in periods a and b in period 1, respectively, and let $C(z)$ denote the fraction of the investment that involves waiting, made in period 2. Each of these fractions is conditional on the signal difference z, and they must satisfy $A(z)+B(z)+C(z)=1$ as well as $0 \leq A(z) \leq 1$, $0 \leq B(z) \leq 1$, and $0 \leq C(z) \leq 1$. The fractions will be chosen optimally, eventually taking values of either 0 or 1. Recalling that $f(z \pm \delta)$ gives the density of z when $D = 0, 1$, the objective function, equal to the expected income gain from the investment, can be written as

$$\Phi = \frac{1}{2} \int \begin{bmatrix} G_a^1 \cdot A(z) \cdot f(z-\delta) \\ + G_a^0 \cdot A(z) \cdot f(z+\delta) \\ + G_b^1 \cdot B(z) \cdot f(z-\delta) \\ + G_b^0 \cdot B(z) \cdot f(z+\delta) \\ + G_w \cdot C(z) \cdot f(z-\delta) \\ + G_w \cdot C(z) \cdot f(z+\delta) \end{bmatrix} dz \equiv \frac{1}{2} \int \phi(z) dz, \qquad (a1)$$

where $\phi(z)$ denotes the integrand in $(a1)$.

The government's problem is to maximize Φ subject to the previous constraints, and point-wise optimization with respect to $A(z)$, $B(z)$ and $C(z)$ can be use for each z. Note that since the objective function and constraints are linear in $A(z)$, $B(z)$ and $C(z)$, the solutions must lie at the borders of the set defined by the constraints, yielding solutions of 0 or 1, as noted above.

To simplify the analysis, the first constraint is used to replace $C(z)$ by $1 - A(z) - B(z)$. So, for each z, the point-wise problem is to maximize the following expression subject to the constraints $A(z) + B(z) \leq 1$, $0 \leq A(z) \leq 1$, and $0 \leq B(z) \leq 1$:

$$\phi(z) \equiv \phi_0(z) + \phi_A(z)A(z) + \phi_B(z)B(z), \qquad (a2)$$

where $\phi_0(z)$ is independent of $A(z)$ and $B(z)$ while

$$\phi_A(z) \equiv \frac{d\phi(z)}{dA(z)} = \frac{1}{2}[(G_a^1 - G_w)f(z-\delta) + (G_a^0 - G_w)f(z+\delta)] \qquad (a3)$$

$$\phi_B(z) \equiv \frac{d\phi(z)}{dB(z)} = \frac{1}{2}[(G_b^1 - G_w)f(z-\delta) + (G_b^0 - G_w)f(z+\delta)]. \qquad (a4)$$

The solution of this constrained linear optimization problem is

$$(A(z), B(z)) = \begin{cases} (0,0) & \text{if } \phi_A(z) < 0 \text{ and } \phi_B(z) < 0 \\ (1,0) & \text{if } \phi_A(z) \geq 0 \text{ and } \phi_A(z) \geq \phi_B(z) \\ (0,1) & \text{if } \phi_B(z) \geq 0 \text{ and } \phi_B(z) > \phi_A(z) \end{cases}. \qquad (a5)$$

Using (12),

$$\phi_A(z) = \frac{1}{2}f(z-\delta)\{[\theta + \delta - c(1-\rho)] + [\theta - \lambda\delta\rho - c(1-\rho)]H(z)\} \qquad (a6)$$

$$\phi_B(z) = \frac{1}{2}f(z-\delta)\{[\theta - \lambda\delta\rho - c(1-\rho)] + [\theta + \delta - c(1-\rho)]H(z)\}, \qquad (a7)$$

where

$$H(z) = \frac{f(z+\delta)}{f(z-\delta)} \qquad (a8)$$

is a decreasing function on the interval $[-\delta, \delta]$ as before and where $H(0) = 1$. As before, the assumption in (27) is imposed and R is defined by (26). Then, using $(a6)$ and $(a7)$,

$$\begin{aligned} \phi_A(z) &\geq (<) \; 0 \text{ as } R \geq (<) \; H(z) \\ \phi_B(z) &\geq (<) \; 0 \text{ as } RH(z) \geq (<) \; 1 \\ \phi_A(z) &\geq (<) \; \phi_B(z) \text{ as } 1 \geq (<) \; H(z). \end{aligned} \qquad (a9)$$

Combining $(a9)$ and $(a5)$ then yields

$$(A(z), B(z)) \;=\; \begin{cases} (0,0) & \text{if } \; R < H(z) \text{ and } RH(z) < 1 \\ (1,0) & \text{if } \; R \geq H(z) \;\text{ and }\; 1 \geq H(z) \\ (0,1) & \text{if } \; RH(z) \geq 1 \;\text{ and }\; 1 < H(z). \end{cases} \qquad (a10)$$

Defining $\underline{z}$ and $\overline{z}$ such that

$$H(\underline{z}) \;=\; R \;\text{ and }\; H(\underline{z}) \;=\; 1/R \qquad (a11)$$

and using (a9), the following decision rule emerges:

$$(A(z), B(z)) \;=\; \begin{cases} (0,0) & \text{if } (R < 0) \text{ or } (0 < R < 1 \text{ and } \underline{z} < z < \overline{z}) \\ (1,0) & \text{if } \; (0 < R < 1 \text{ and } z > \overline{z}) \text{ or } (R \geq 1 \;\text{ and } z > 0) \\ (0,1) & \text{if } \; (0 < R < 1 \text{ and } z < \underline{z}) \text{ or } (R \geq 1 \text{ and } z < 0)). \end{cases} \qquad (a12)$$

After imposing symmetry, so that $\overline{z} = u$ and $\underline{z} = -u$, it can be seen that $(a12)$ is the same as the decision rule developed in section 4.

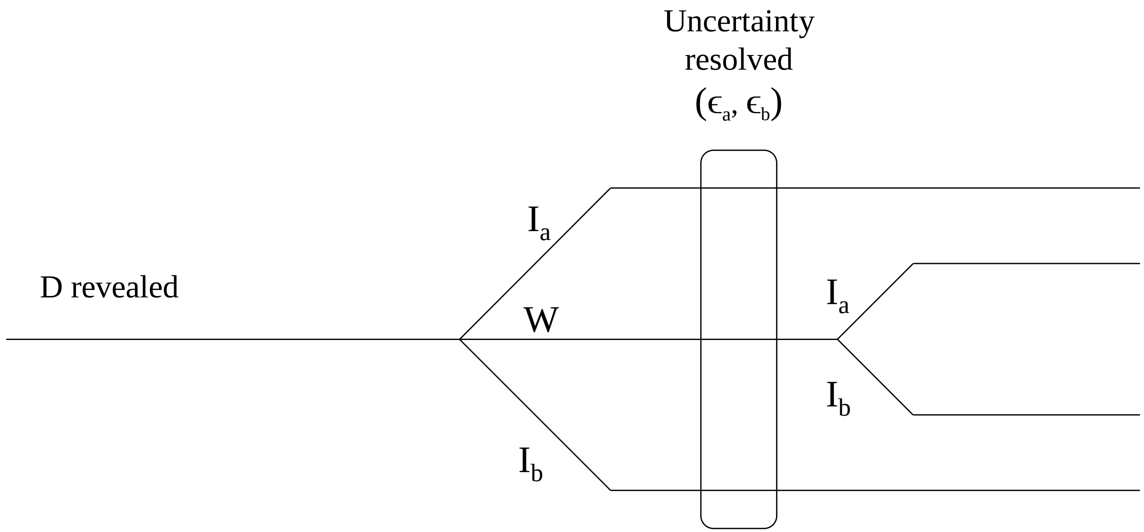

Figure 1: Option framework

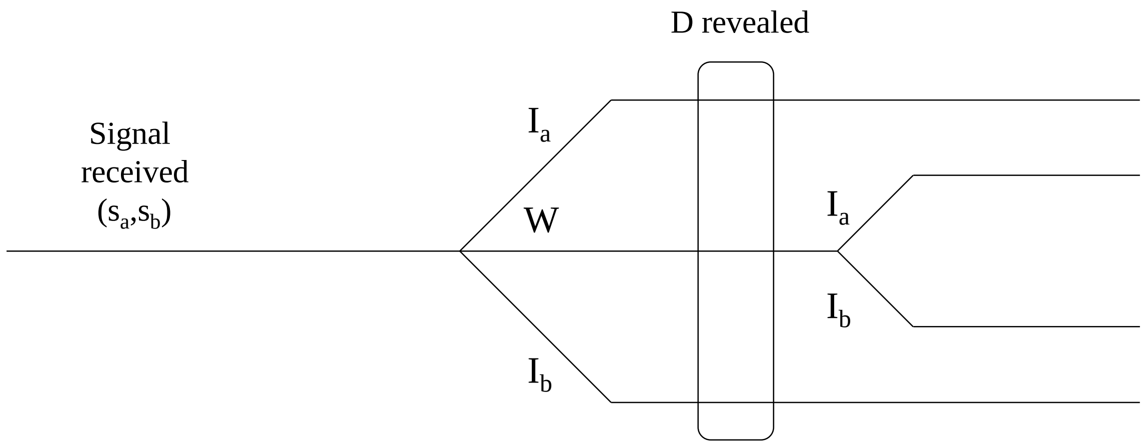

Figure 2: Signaling framework

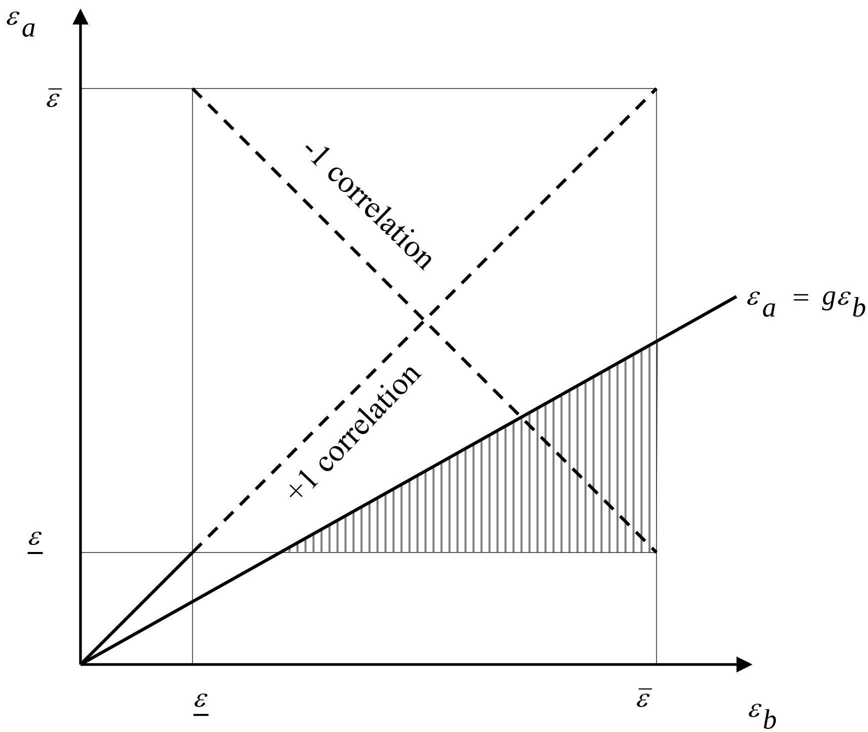

Figure 3: Option model

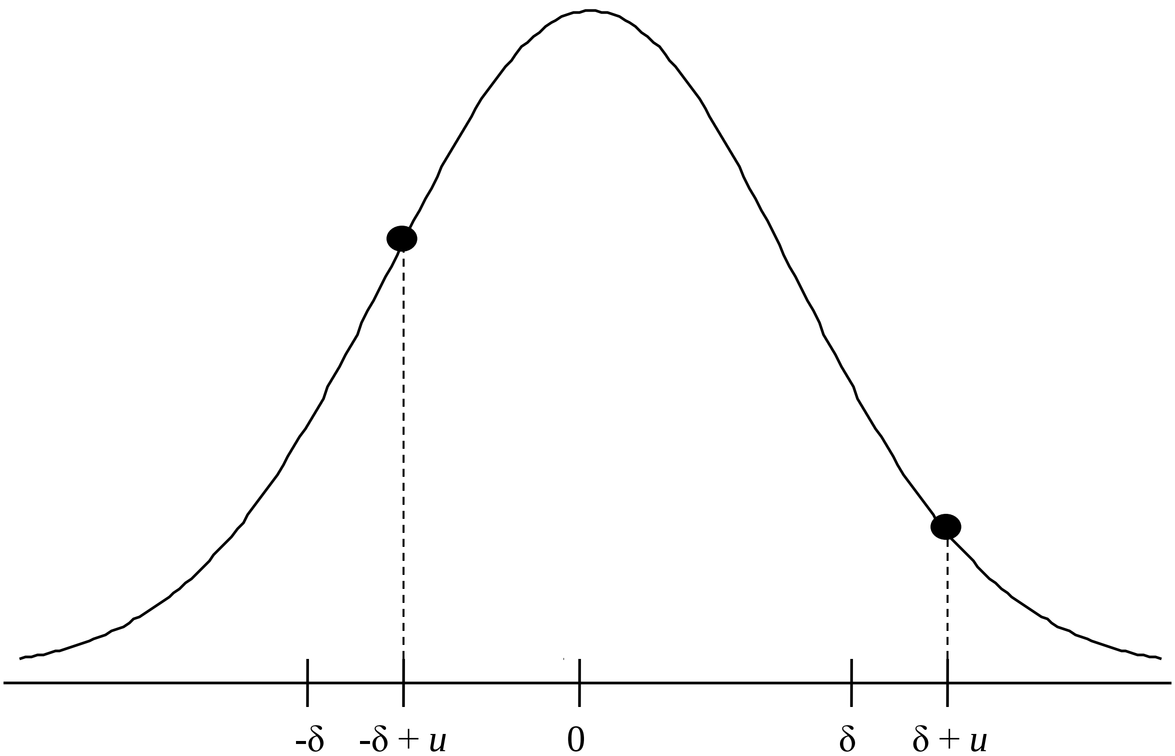

Figure 4: Elements of Density Ratio

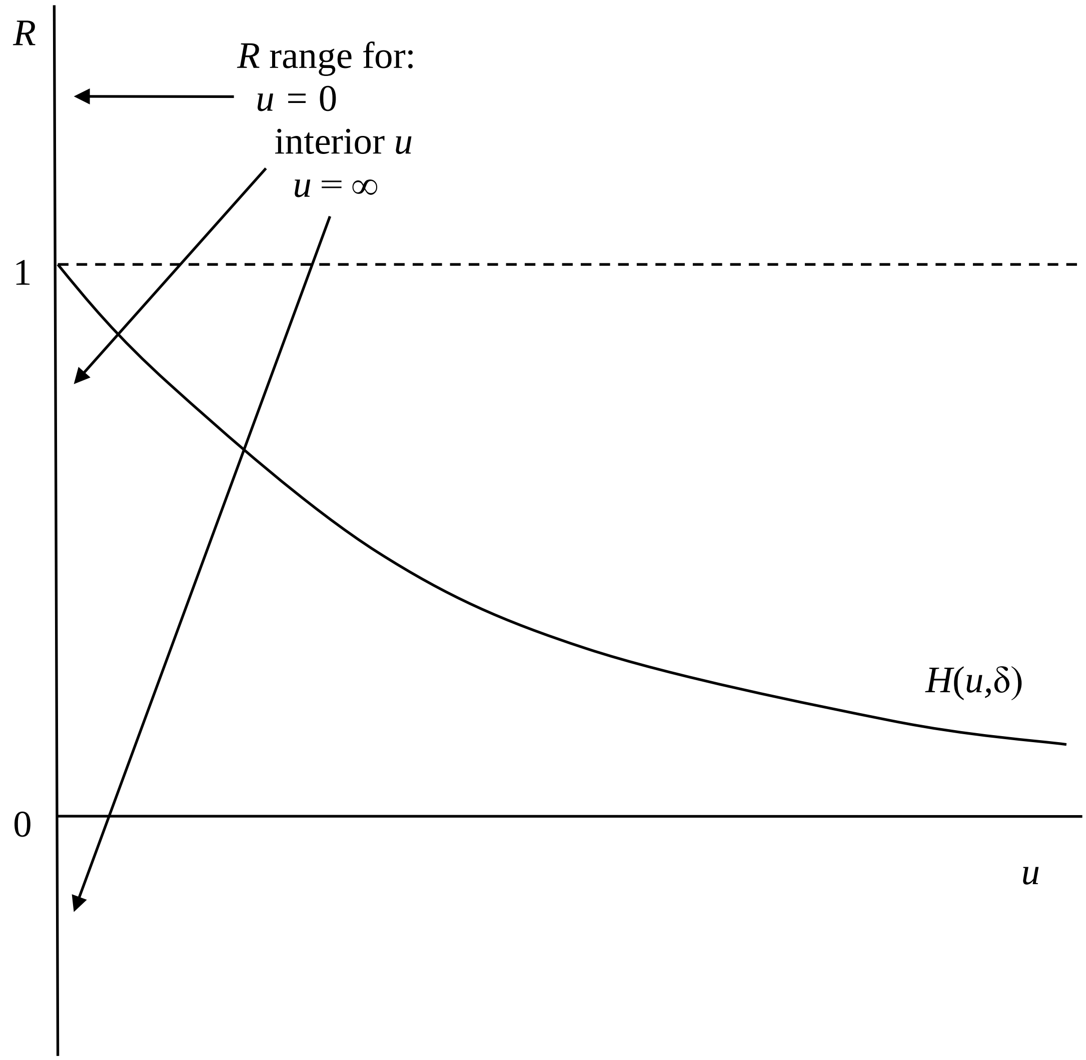

Figure 5: First-order condition

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Footnotes

*We thank David Brownstone, Kangoh Lee, and Ken Small for comments and suggestions. Any errors, however, are ours.

[1]See Fernald (1999) and Chandra and Thompson (2000). In addition, Baum-Snow (2007) studies the effects of highway investment on suburbanization within metro areas.

[2]See Green (2007) and Tittle, McCarthy and Xiao (2012).

[3]See also Morrison and Schwartz (1996) and Haughwout (2002).

[4]For studies that use the option approach to land development, see Capozza and Helsley (1990) and Li and Capozza (1994).

[5]Redding, Sturm and Wolf (2011) show that the location of misplaced infrastructure may be hard to alter, focusing on the location of the major German hub airport in Frankfurt. The hub would have been located in Berlin had the country not been divided prior to the 1990s, but irreversibility of the investment means that relocation of the airport to Berlin in the current unified country is impractical.

[6]This view is due to Professor Gordon J. Fielding, a noted expert on transportation policy in the Southern California region (expressed in private conversation).

[7]Given this support, the condition required for suppression of λy in (11) is $\underline{\epsilon} > c/(\lambda\delta)$.

[8]Extensive manipulation shows that the integral equals

$$\frac{1}{2\tau}[g^2(3k^2 + \tau^2/4) \; + \; g(2\tau k - 4k^2) \; + \; (k - \tau/2)^2],$$

an expression whose τ derivative is ambiguous in sign.

[9]It is easily seen that satisfaction of (22) carries no implication regarding satisfaction of (17), which thus constitutes an independent condition.

[10]When $\delta + u$ is outside the support of the density but $-\delta + u$ is inside it, H equals zero. When both points are outside the support, H is undefined but is set at zero for consistency.

[11]It can be shown that satisfaction of (22) carries no implication regarding satisfaction of (30). In addition, while satisfaction of (30) implies satisfaction of (27), the reverse is not true. Thus, (30) imposes a further condition beyond the maintained assumptions in (22) and (27).

[12]It can be shown that satisfaction of (27) carries no implication regarding satisfaction of (31). In addition, although satisfaction of (31) implies satisfaction of (22), the reverse is not true. Thus, like (30), (31) imposes a further condition beyond the maintained assumptions in (22) and (27).

[13]In the uniform case, $f(x) = 1/\tau$ for $x \in [-\tau/2, \tau/2]$ and zero elsewhere (recall from (10) that x is the signal difference). Then, assuming $\delta < \tau/2$, $H(u, \delta) = 1$ for $u \in [\delta - \tau/2, -\delta + \tau/2]$, $H(u, \delta) = 0$ for $u \in (\tau/2 - \delta, \tau/2 + \delta]$, $H(u, \delta) = \infty$ for $u \in (-\tau/2 - \delta, -\tau/2 + \delta]$ and is undefined elsewhere. Therefore, the optimal u is zero for $R > 1$, infinite for $R \leq 0$, lies anywhere in the interval $[-\tau/2 + \delta, \tau/2 - \delta]$ for $R = 1$, and equals $\tau/2 - \delta$ for $0 < R < 1$. In the latter case, note that u is increasing the variance of the uniform distribution, which rises with τ.

[14]This conclusion can be seen from (28), which would be written as $-R + H(\overline{z} + q_b - q_a, \delta)$ in the presence of government signals (with $\overline{z}$ replacing u). Since the first H argument must be constant, it follows that $\partial\overline{z}/\partial q_a = 1$ and $\partial\overline{z}/\partial q_b = -1$.