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Income Effects and Vertical Differentiation in International Trade

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Income Effects and Vertical Differentiation in International Trade

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Abstract

We analyse a trade model with non-homothetic preferences and different quality versions of each product. Income effects drive the quality composition of consumption, production and trade flows. We show that a rise in local population fosters local asymmetric specialization in high-quality production and exports while it harms low income groups. By contrast, an increase in local productivity may generate specialization in high quality production, which in turn may trigger an immiserizing growth process. Weaker comparative advantages induce firm to move and make a local productivity improvement more likely to increase production of higher quality goods everywhere.

Keywords: Heterogeneous firms, vertical differentiation, horizontal differentiation, trade, income heterogeneity.

JEL codes: F12, F16, L11, L15.

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1 Introduction

There is substantial evidence that product quality matters in the trade patterns within and between sectors.¹ In the traditional view of product quality under vertical differentiation, goods are versioned under different quality levels, consumers agree on the quality ranking of the versions of goods and purchase one unit of each good.² A small set of trade applications investigate the case of a single good that is versioned in numerous qualities and produced in different countries (Flam and Helpman, 1987; Stokey, 1991; Matsuyama, 2000). The non-homothetic preferences naturally allow incomes to play a role in the consumption and production location of higher quality goods. Because they consider a single good, those models explain the intra-product trade but are not well adapted to highlight intra-industry trade. To do this, one must consider a set of differentiated products with different quality versions of each product.

To discuss product quality in intra-industry trade, the recent trade literature has developed models of horizontally differentiated goods augmented to quality features. In most approaches, product quality is modelled as an idiosyncratic parameter (shifter) on the demand of each specific good.³ In this view, however, consumers value different products in a different way and do not agree on which product has a higher value for them. Also, with the exception of few contributions with non-homothetic preferences, this literature is unable to explain the role of income in consumers' switch to higher quality products.

In this paper, we present an international trade model where each variety is vertically differentiated. Firms produce a fixed set of differentiated varieties that can be produced each at a high and low quality and sold under perfect competition. Varieties are heterogenous in their production costs and quality levels. As usual in the vertical differentiation literature, consumers need to purchase a single unit of each variety in one version only. Preferences are non-homothetic and permit the discussion of income effect on the consumption baskets of high and low quality goods. We sequentially discuss the patterns of international trade between two countries hosting populations with homogeneous and heterogeneous income and firms that are firstly immobile and then mobile. We first examine the effects of small changes in term of population and productivity when countries are symmetric. We then propose and discuss a class of vertical differentiation models with countries that are asymmetric in size and productivity.

¹See e.g Schott (2004), Hummels and Klenow (2005), Verhoogen (2008), Khandelwald (2010), Hallack and Schott (2011), Crozet *et al.* (2012), etc.

²See Mussa and Rosen 1978; Gabszewicz and Thisse 1979; Shaked and Sutton 1982.

³See e.g. Fajgelbaum *et al.*, (2011); Verhoogen (2008); Baldwin and Harrigan (2012); Jaimovic and Merella (2012, 2015), Picard (2014), Comite *et al.* (2014), etc.

We show that a larger domestic population fosters the home specialization in high-quality exports and the foreign specialization in low-quality exports. Also, a positive growth in domestic productivity fosters home specialization in high-quality export. It entices the foreign country to specialize in high-quality exports if and only if relative prices are not very sensitive to productivity changes. Much of this process goes through the change in the domestic relative wages. If they are not much reactive, a higher productivity makes domestic high-quality goods cheaper and hence more attractive abroad while at the same time local population becomes rich enough to import more foreign high-quality goods. If they are reactive, a higher productivity makes domestic high-quality goods more attractive abroad and foreign high-quality goods less attractive at home. At some point, the rise in domestic productivity may generate an immiserizing growth process in which the welfare of domestic workers falls (Bhagwati, 1958). This process is similar to the “dutch disease” in which an improvement of domestic resource may harm the country (Jones, Neary and Ruane, 1987). The main mechanism here lies in the reaction of relative wages and the consumers’ choice to shift from higher to lower quality goods.

When the incomes of local populations are heterogenous, we show that a rise in a population harms its lowest income group, that ends up consuming lower quality goods. A productivity improvement of the highest skilled groups also harms this lowest skill group but benefits more to the high skilled group when the latter is smaller in size and has larger productivity advantage. Finally, when firms are mobile, they move according to the comparative advantages obtained in each location. When comparative advantages are correlated with the additional costs for quality upgrades, the equilibrium reaches a similar structure and leads to the same properties. However, productivity shocks trigger more relocation of production under weaker comparative advantages so that local consumers are less likely to be unable to benefit from their productivity gains.

The paper relates to several strands of the economic literature. As explained above it closely relates to the studies of vertical differentiation in general equilibrium with trade (Flam and Helpman, 1987; Stokey, 1991; Matsuyama, 2000). Although its research agenda is of high relevance, this strand of literature is constrained by the difficulty to model vertical differentiation in a general equilibrium because the former relies on non-homothetic preferences. The present contribution differs from this literature as follows. First, while Flam and Helpman (1987) and followers focus on an endogenous and continuous quality spectrum of the same good, this paper considers an exogenous spectrum of horizontally differentiated goods with only two quality levels (which limits the dimensionality of the problem). This paper therefore adds some more

realism about the set of goods under study. Second, this paper considers trade partners with comparable conditions rather than the typical North-South situation where South suffers from an exogenous disadvantage. Production conditions and product characteristics are the same in both countries and, even when we add comparative advantages, the latter are independent of the preferences for goods produced in a specific country (which departs from Matsuyama, 2000). Third, while the above authors assume that qualities can be produced anywhere, (the first two sections of) the present paper focuses on a specific factor model where all versions of a same variety are produced in the same country (e.g. high and low quality grape in California, high and low quality watches in Switzerland, etc.). This setting allows us to discuss a less extreme quality composition of trade flows where exports include mixes of high and low quality goods in all trade directions. The last section of this paper considers the possibility of moving the production of the differentiated goods according the comparative advantage à la Dornbush *et al.* (1977).

Because of the recent interest in explaining product quality in micro-trade data, Fieler (2012) and Jaimovich and Merella (2012, 2015) have revived the study of vertical differentiation in trade. In contrast to our approach, Fieler (2012) and Jaimovich and Merella (2015) study the properties of relative price of imported and exported varieties under various quality versions as well as the import penetration and export compositions. Their comparative statics exercises mostly imply three countries and cannot be discussed in our two-country framework. In addition, because they consider infinitely small countries, a change in national economic conditions does not impact global demand and supply. Their models thus feature the properties of small open economies. Jaimovich and Merella (2012, 2015) also contrast by considering the dividible goods rather than indivisible goods as in the usual traditional differentiation literature with unit purchases. As in this paper, Jaimovich and Merella (2012) highlight that a higher global productivity can increase the production and consumption of high quality goods across borders (Linder hypothesis). Given their convenient result, we do not emphasize this point in this paper.

Fieler's (2012) and our paper complement the discussion of product quality in horizontal product differentiation frameworks. As shown in Fajgelbaum *et al.* (2011), the relevance of income levels and distributions can be captured in models with non-homothetic preferences. Frameworks with homothetic preferences are indeed invariant to income distribution (Verhoogen, 2008; Baldwin and Harrigan, 2011, etc.) while models with non-homothetic preference and perfectly tradeable homogenous good may yield no interesting effects of relative prices (e.g. Picard 2014). Comite *et al.* (2014) propose to reconcile vertical and horizontal differentiation by

discussing a spatial metaphor involving continuums of heterogeneous consumers. In any case, all those models consider that each variety is associated with one quality, which makes them stand away from the typical literature on quality versioning and vertical differentiation. The distinguishing point is whether two products are sold at different prices because one embeds a characteristic that is valuable to some consumers and not others (e.g. white versus red light bulb) or because one embeds additional characteristics that are valuable to all consumers (e.g. long-life versus short life-light bulb). The issue is to avoid a possible misinterpretation in the sale-price relationship and the confusion of quality with consumer taste. Comite et al (2014) show that prices strongly correlate across markets because they may reflect cost and vertical differentiation characteristics, which are specific to each variety, whereas sales do not because they depend on consumer taste, which varies across markets. Trade flows include both types of goods and effects so that both views deserve investigation. Finally, while horizontal differentiation models augmented to quality permit the discussion of monopolistic competition settings, the current knowledge of vertical product differentiation and general equilibrium unfortunately constrains us to limit our analysis to perfect competition. This contribution may be considered a ground base for further research with monopolistic competition.

The remainder of the paper is organized as follows. Section 2 describes the model. Section 3 develops the case with international trade when population are homogeneous, while Section 4 extends the analysis to populations being heterogeneous in income. In Section 5 we allow for firms mobility across countries. Section 6 concludes.

2 Model

We consider two trading countries (domestic and foreign) respectively populated by a mass M and M^* of individuals endowed with s and s^* labor units (skill), where the asterisk denotes foreign variables. The skills are distributed according to the cumulative distribution functions G and $G^* : R^+ \rightarrow [0, 1]$. Countries are endowed with two different sets of perfectly differentiated varieties $i \in \Omega = [0, N]$ and $j \in \Omega^* = [0, N^*]$, that can each be versioned with high or low quality, denoted by H and L .

Individuals consume a single unit of all domestic and foreign varieties $i \in \Omega$ and $j \in \Omega^*$. A domestic individual maximizes her utility

$$U = \sum_{k=H,L} \left(\int_{\Omega} q_k(i) x_k(i) di + \int_{\Omega^*} q_k^*(j) x_k(j) dj \right),$$

satisfying her budget constraint

$$\sum_{k=H,L} \left(\int_{\Omega} p_k(i) x_k(i) di + \int_{\Omega^*} p_k^*(j) x_k(j) dj \right) = ws,$$

where $q_k(i) > 0$ and $q_k^*(j) > 0$ are the product qualities and $x_k(i)$ and $x_k(j) \in \{0, 1\}$ are the individual's unitary consumption decisions ($x_H(i) + x_L(i) = x_H(j) + x_L(j) = 1$). For every individual, there exists a positive scalar μ such that she buys the high quality H of a home variety i if

$$q_H(i) - \frac{1}{\mu} p_H(i) \geq q_L(i) - \frac{1}{\mu} p_L(i), \quad (1)$$

and the low quality L otherwise, the same applying for the purchase of foreign varieties. This scalar μ measures the inverse of the marginal utility of income and is equal to the inverse of the Lagrange multiplier of the budget constraint. It is a function of the individual's skill s (see below). Foreign consumers have the same utility and expenditure, but are endowed with an income w^*s^* , which yields an inverse marginal utility of income μ^* .

At home, the production of each variety unit requires $\theta_k(i)$ labor units with $\theta_H(i) > \theta_L(i)$. In the foreign country, it requires $\theta_k^*(j)$ labor units, $\theta_H^*(j) > \theta_L^*(j)$. Domestic varieties cannot be produced abroad and *vice versa* (until Section 5). Under perfect competition, the price of variety is equal to its unit cost:

$$p_k(i) = \theta_k w \quad \text{and} \quad p_k^*(j) = \theta_k^* w^*, \quad k \in \{H, L\} \quad (2)$$

where w and w^* are the domestic and foreign wage per labor unit. By (1), the set of high-quality domestic varieties consumed by the domestic individuals with μ is given by

$$\mathcal{H}\left(\frac{\mu}{w}\right) = \left\{ i : \frac{\mu}{w} \geq \Delta(i) \right\}, \quad \text{where} \quad \Delta(i) = \frac{\theta_H(i) - \theta_L(i)}{q_H(i) - q_L(i)}, \quad (3)$$

denotes the home additional cost per quality unit of upgrading variety i . For brevity we call this here the home ‘‘cost per quality’’. Likewise, the sets of high-quality foreign varieties consumed by this domestic individual are equal to

$$\mathcal{H}^*\left(\frac{\mu}{w^*}\right) = \left\{ j : \frac{\mu}{w^*} \geq \Delta^*(j) \right\}, \quad \text{where} \quad \Delta^*(j) = \frac{\theta_H^*(j) - \theta_L^*(j)}{q_H^*(j) - q_L^*(j)}. \quad (4)$$

The sets of low-quality purchases are the complements of those: $\mathcal{L}(\mu/w) = \Omega \setminus \mathcal{H}(\mu/w)$ and $\mathcal{L}^*(\mu/w^*) = \Omega^* \setminus \mathcal{H}^*(\mu/w^*)$. Similarly, a foreign individual with marginal utility of income μ^* purchases the set $\mathcal{H}(\mu^*/w)$ and $\mathcal{H}^*(\mu^*/w^*)$ of home and foreign high-quality varieties and the complement sets of low-quality goods $\mathcal{L}(\mu^*/w) = \Omega \setminus \mathcal{H}(\mu^*/w)$ and $\mathcal{L}^*(\mu^*/w^*) = \Omega^* \setminus \mathcal{H}^*(\mu^*/w^*)$.

In this paper we make three reasonable assumptions. We first assume that there exists no measurable mass of varieties with the same cost per quality $\Delta(i)$ and $\Delta^*(j)$. In this case,

we can rank the varieties so that the costs per quality $\Delta(i)$ and $\Delta^*(j)$ are strictly increasing functions. Then, the identities $\mu = \Delta(i)$ and $\mu = \Delta^*(j)$ accept unique solutions in i and j and their inverse functions Δ^{-1} and Δ^{*-1} are well defined, increasing functions. Second, we assume that every individual buys a mix of high and low qualities. For this, it should be that μ/w and μ^*/w lie in the interval $[\Delta(0), \Delta(N)]$ while μ/w^* and μ^*/w^* lie in $[\Delta^*(0), \Delta^*(N)]$, $\forall s, s^*$. In other words, the upper bound eliminates satiation issues. The two assumptions guarantee that the identities $\mu = \Delta(i)$ and $\mu = \Delta^*(j)$ have unique interior solutions on the sets Ω and Ω^* . Finally, to guarantee purchases of every varieties, we assume that the consumers are productive (rich) enough. This implies that μ/w and μ^*/w is larger than $\theta_k(i)/q_k(i)$, while μ/w^* and μ^*/w^* are larger than $\theta_k^*(j)/q_k^*(j)$, $\forall s, s^*, i, j, k$. This assumption is generally labelled as “full market coverage” and is standard in vertical differentiation literature. Also, it is in line with most quality-augmented horizontal differentiation trade models where consumers purchase all varieties.⁴

The left and right panels of Figure 1 depict the cost-per-quality of foreign and domestic varieties. Consumptions of high and low quality varieties can be inferred from this figure. A domestic individual with inverse marginal utility μ consumes the sets of high and low-quality domestic varieties, $\mathcal{H}(\mu/w)$ and $\mathcal{L}(\mu/w)$ (intersection of black curves in right panel) and the sets of high and low-quality imports, $\mathcal{H}(\mu/w^*)$ and $\mathcal{L}(\mu/w^*)$ (intersection of black curves in left panel). Similarly, a foreign consumer with inverse marginal utility μ^* consumes the sets of high- and low-quality foreign varieties, $\mathcal{H}^*(\mu^*/w^*)$ and $\mathcal{L}^*(\mu^*/w^*)$ (intersection with gray line, left panel), and imports the sets of high- and low-quality varieties, $\mathcal{H}(\mu^*/w)$ and $\mathcal{L}(\mu^*/w)$ (intersection with gray line, right panel). The second assumption imposes the equilibrium to lie below the highest value of $\Delta(N)$ in the right panel while the third one constrains the equilibrium to lie above the highest curve $\theta_H(i)/q_H(i)$ and $\theta_L(i)/q_L(i)$. Symmetric conditions apply in the left panel.

⁴See Crampes and, Hollander, 1995, Boom, 1995, Ecchia and Lambertini 1997, Maxwell, 1998, Wang and Yang, 2001, and Wang, 2003, *inter alia*.

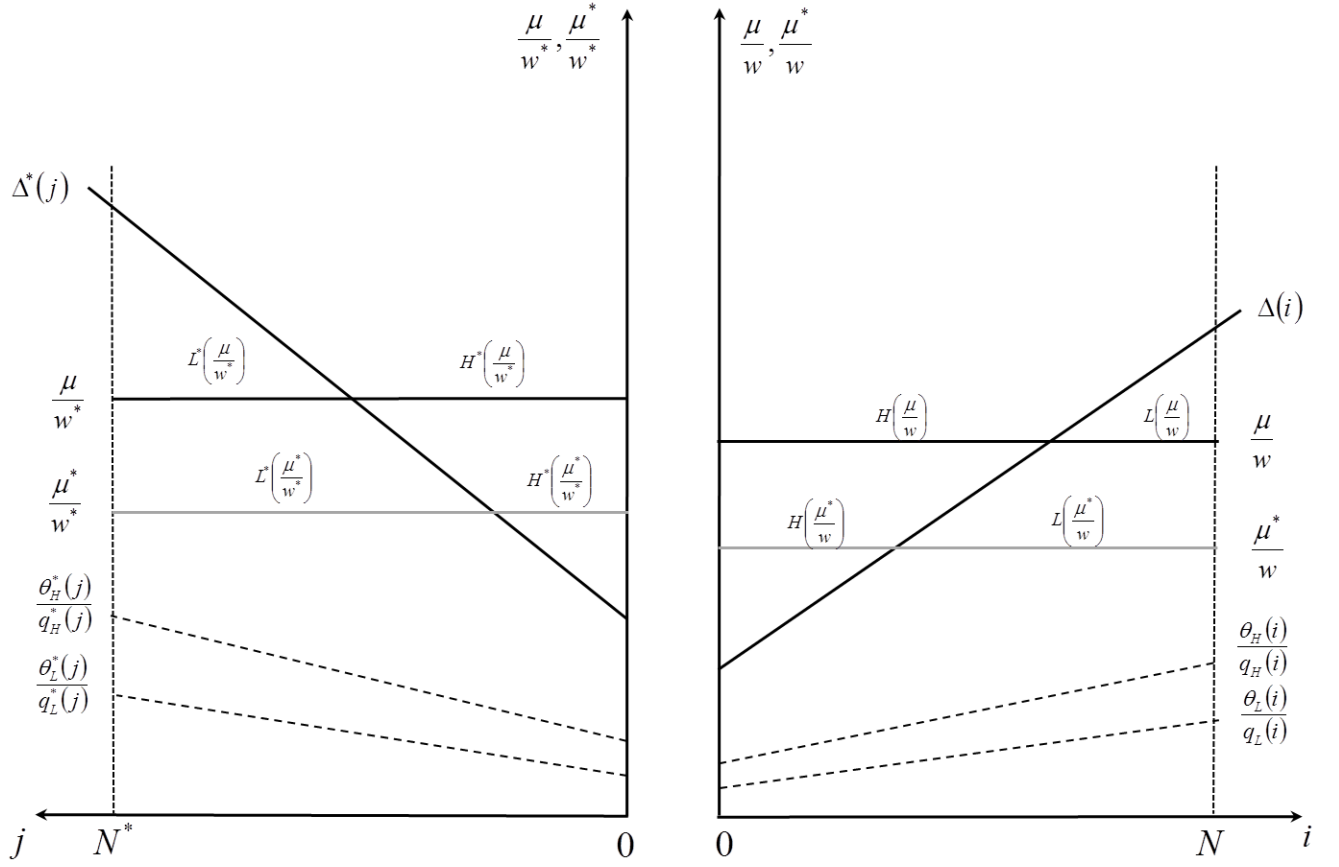


Figure 1: Two country equilibrium

Note: The right hand panel compares the “cost-per-quality” $\Delta(i)$ of each domestic variety $i \in \Omega$ to the inverse marginal utilities μ/w and μ^*/w of home and foreign consumers. The sets of high-quality varieties consumed by those consumers are respectively denoted by $H(\mu/w)$ and $H(\mu^*/w)$. The sets of consumed low-quality varieties are denoted by $L(\mu/w)$ and $L(\mu^*/w)$. The left hand panel gives a symmetric representation for the foreign varieties $j \in \Omega^*$.

The expenditure of a domestic and foreign individual can be written in terms of local wages as

$$e(\mu; w, w^*) \equiv F\left(\frac{\mu}{w}\right) + \frac{w^*}{w} F^*\left(\frac{\mu}{w^*}\right) \quad \text{and} \quad e^*(\mu^*; w, w^*) \equiv F^*\left(\frac{\mu^*}{w^*}\right) + \frac{w}{w^*} F\left(\frac{\mu^*}{w}\right), \quad (5)$$

where

$$F(z) \equiv \int_{\mathcal{H}(z)} \theta_H(i) di + \int_{\mathcal{L}(z)} \theta_L(i) di \quad \text{and} \quad F^*(z) \equiv \int_{\mathcal{H}^*(z)} \theta_H^*(j) dj + \int_{\mathcal{L}^*(z)} \theta_L^*(j) dj. \quad (6)$$

The terms $F(\mu/w)$ and $(w^*/w) F^*(\mu/w^*)$ express the domestic individual’s expenditure on local goods and imports using the local labor units as numéraire. Also, the term $F(\mu/w)$

indicates the labor content of her domestic local consumption while $F^*(\mu/w^*)$ expresses that on her imports.

At given wages w and w^* , a higher inverse marginal utility μ implies that the consumer chooses wider sets of high quality goods, $\mathcal{H}(\mu/w)$ and $\mathcal{H}^*(\mu/w^*)$, and narrower sets of low quality, $\mathcal{L}(\mu/w)$ and $\mathcal{L}^*(\mu/w^*)$. Because $\theta_H(i) > \theta_L(i)$ and $\theta_H^*(j) > \theta_L^*(j)$, this raises $F(\mu/w)$ and $F^*(\mu/w^*)$. So, e is a monotonically increasing function of μ . The inverse marginal utility of income can then be defined as $\hat{\mu}(s; w, w^*)$, which solves

$$e(\mu; w, w^*) = s.$$

We therefore have $\partial\hat{\mu}/\partial s > 0$. The inverse marginal utility of income rises with higher skills. For the sake of conciseness we dispense the functions $\hat{\mu}$ and e with the references to the arguments (w, w^*) whenever this brings no confusion.

The set of domestic consumers purchasing the high-quality domestic variety i is given by $\mathcal{X}_H(i) = \{s : \hat{\mu}(s) \geq \Delta(i)\} = \{s : s \geq e(\Delta(i))\}$. In similar fashion, the set of foreign consumers purchasing the high-quality domestic variety i is given by $\mathcal{X}_H^*(i) = \{s : \hat{\mu}^*(s) \geq \Delta(i)\} = \{s : s \geq e^*(\Delta(i))\}$. The product demand for the domestic high-quality variety i writes as

$$X_H(i) = M \{1 - G[e(\Delta(i))]\} + M^* \{1 - G^*[e^*(\Delta(i))]\}.$$

In a similar way the demand for domestic low-quality variety i is

$$X_L(i) = MG[e(\Delta(i))] + M^*G^*[e^*(\Delta(i))].$$

The demands for foreign varieties can be written in a similar way.

To close the model, we impose the labor market clearing in one country (the one in the second country being satisfied by Walras law). This is equivalent to impose the following trade balance condition, which equates the value of domestic export to import. Using the consumers' expenditures, this can be written as

$$\int w^* F^* \left(\frac{\hat{\mu}(s)}{w^*} \right) dG(s) = \int w F \left(\frac{\hat{\mu}^*(s^*)}{w} \right) dG^*(s^*) \quad (7)$$

This identity determines the domestic relative wage w/w^* . Normalizing the foreign wage to unity, the term of trade is given by w , the domestic price of labor.

To sum up, a trade equilibrium is defined by the profiles of prices $(p_k(\cdot), p_k^*(\cdot))$ that make firms break even (condition (2)), inverse marginal utility of income $(\mu(\cdot), \mu^*(\cdot))$ that match individuals' optimal consumption choices at given prices (condition (5)) and domestic wage w that balances trade (condition (7)). Since prices are directly derived from wages, it is sufficient to check conditions (5) and (7). We first study the case where each country hosts a homogenous population.

3 Homogenous populations

In this section we assume that each home and foreign individual is endowed respectively with s and s^* labor units, where s and s^* are two scalars. That is, the domestic skill distributions reduces to $G(s') = 0$ if $s' < s$ and $G(s') = 1$ otherwise, while the foreign distribution to $G^*(s') = 0$ if $s' < s^*$ and $G^*(s') = 1$ otherwise. Letting $m = M/M^*$ and using the above wage normalization, the equilibrium is given by the inverse marginal inverse utilities (μ, μ^*) and the relative wage w that satisfy the expenditure and balance trade identities (5) and (7). That is,

$$s = F\left(\frac{\mu}{w}\right) + \frac{1}{w}F^*(\mu), \quad (8)$$

$$s^* = F^*(\mu^*) + wF\left(\frac{\mu^*}{w}\right), \quad (9)$$

$$mF^*(\mu) = wF\left(\frac{\mu^*}{w}\right). \quad (10)$$

In the absence of effects on relative prices, consumers in the richer country buy higher-quality products. Indeed, for a given w , μ increases as domestic productivity s rises so that domestic consumers purchase more of all high-quality goods. In this text, the issue is about how relative prices impact this process.

This non-linear system of equations can be studied only in particular settings. As in Matsuyama (2000), we first study the properties of the symmetric equilibrium. We then analyze the class of economic environments where the functions F and F^* are linear.

3.1 About the symmetric equilibrium

When countries are symmetric with respect to skill, varieties and population, we have $s = s^*$, $F = F^*$ and $m = 1$. Then, the symmetric configuration with $w = 1$, $\mu = \mu^* \equiv \mu^0$ and $s = 2F(\mu^0)$ is an equilibrium if our assumptions hold: $\Delta(N) \geq \mu^0 \geq \max_{i,k} \{\Delta(0), \theta_k(i)/q_k(i)\}$. This occurs if $\int_0^N \theta_L(i)di \leq s/2 \leq \int_0^N \theta_H(i)di$ and $\theta_k(i)/q_k(i) < [\theta_H(0) - \theta_L(0)] / [q_H(0) - q_L(0)]$, $k = H, L$, which is satisfied for many classes of primitives $\theta_k(\cdot)$ and $q_k(\cdot)$.

We study the impact of infinitely small exogenous changes in domestic population $\tilde{m} = dm/m > 0$ and productivity $\tilde{s} = ds/s > 0$ on the consumption of high and low-quality goods (for details, see Appendix A, B). Totally differentiating and simplifying the above system of equations around the symmetric equilibrium yields the following endogenous changes in the

wage and inverse marginal utility of domestic and foreign individuals:

$$\begin{aligned}\tilde{w} &= -\frac{1}{\eta}\tilde{m} - \frac{1}{\eta}\tilde{s}, \\ \tilde{\mu} &= -\frac{\eta+1}{2\eta^2}\tilde{m} - \frac{1-\eta}{2\eta^2}\tilde{s}, \\ \tilde{\mu}^* &= -\frac{\eta-1}{2\eta^2}\tilde{m} - \frac{\eta-1}{2\eta^2}\tilde{s}\end{aligned}\tag{11}$$

where $\tilde{x} = dx/x$ denotes the change in the variable x and where

$$\eta \equiv \left[\frac{d \ln F(z)}{d \ln z} \right]_{z=\mu^0} = \left[\frac{\theta_H(i) - \theta_L(i)}{\Delta'(i)} \right]_{i=\Delta^{-1}(\mu^0)} > 0,$$

measures the elasticity of expenditure on local goods with respect to marginal utility of income at the symmetric equilibrium. Since $\mu^0 = F^{-1}(s/2)$, this measure is exogenously given by the economic parameters. A larger η entails that the consumer's expenditure (measured in labor units) rises more rapidly when she purchases more high-quality varieties. This can be due to price or quantity effects. The expenditure may rise faster because of higher cost differences between high and low-quality varieties (larger $\theta_H(i) - \theta_L(i)$) or because of stronger incentives to substitute towards high quality (lower $\Delta'(i)$). The latter reason occurs when the additional quality rises faster than its additional cost ($q'_H(i) - q'_L(i) > \theta'_H(i) - \theta'_L(i)$). All this depends on the product characteristics, production technology and on the skill level of the population because the latter affects the cutoff variety $i = \Delta^{-1}(\mu^0)$. Finally, the elasticity of expenditure closely relates to the income elasticity in demand. More formally, we have

$$\eta = \frac{\left[\frac{d \ln F(z)}{d \ln s} \right]_{z=\mu^0}}{\left[\frac{d \ln \mu}{d \ln s} \right]_{z=\mu^0}}\tag{12}$$

where the numerator is the income elasticity of demand for local goods and the denominator the income elasticity of inverse marginal utility of income. As Matsuyama (2000) emphasizes, the point of non-homothetic models is to study the cases where this elasticity varies with incomes s . However, the above formula also points out the importance of the income effect on marginal utility, which is hard to measure.

From (11), we compute

$$\begin{aligned}\tilde{\mu} - \tilde{w} &= -\frac{1-\eta}{2\eta^2}\tilde{m} - \frac{1-3\eta}{2\eta^2}\tilde{s}, \\ \tilde{\mu}^* - \tilde{w} &= \frac{1+\eta}{2\eta^2}\tilde{m} + \frac{1+\eta}{2\eta^2}\tilde{s}.\end{aligned}$$

The impact on high- and low-quality consumption sets is reported in Table 1. Consumption patterns differ according to whether the elasticity η belongs to the interval $(0, 1/3)$, $(1/3, 1)$ or $(1, \infty)$. When they do, we report the condition on η .

| | Relative wage | Home local | Home imports | Foreign local | Foreign imports |
|--------------|---------------|---------------------|---------------------|-----------------------|-----------------------|
| | $w \searrow$ | $H(\mu/w) \searrow$ | $H^*(\mu) \searrow$ | $H^*(\mu^*) \nearrow$ | $H(\mu^*/w) \nearrow$ |
| $m \nearrow$ | True | $\eta < 1$ | True | $\eta < 1$ | True |
| $s \nearrow$ | True | $\eta < 1/3$ | $\eta < 1$ | $\eta > 1$ | True |

Table 1: Comparative statics around the symmetric equilibrium: homogenous populations.

Notes: w is domestic wage; m is the size of home to foreign country; s is the home productivity; H is the set of high quality varieties. $H(\mu/w) \searrow$ reads as the set of home local high-quality varieties shrinks.

3.1.1 Population growth

Consider a rise in domestic population (first row). Since a larger domestic population increases domestic labor supply, it depresses domestic wages (lower w , first column). Domestic consumers purchase less high-quality imports ($\mathcal{H}^*(\mu)$ shrinks, third column) because their relative wage and prices fall while their productivity and income do not change. By the same token, it raises the relative wage of foreign consumers who import more high-quality varieties ($\mathcal{H}(\mu^*/w)$ expands, fifth column). The effect on the trade patterns is unambiguous: as the domestic population increases, the home country exports a larger number of high-quality varieties and imports a smaller number of high-quality goods. To sum up, *a growth in domestic population size fosters specialization in domestic high-quality production and export.*

Population growth affects local consumptions to the extent of relative price movements and elasticity of expenditure. From (11), the smaller the elasticity of expenditure η , the stronger the relative price movements. If $\eta < 1$, the rise in home population size strongly deteriorates domestic relative wages so that consumers shift their local expenditure toward local low-quality goods ($\mathcal{H}(\mu/w)$ shrinks, first column). That is, they substitute high-quality imports for low-quality imports and local goods. In sum, they consume more low-quality products and are worse off with certainty. If $\eta > 1$, the domestic relative income is not much affected so that domestic consumers substitute high-quality imports for low-quality imports and high-quality local products ($\mathcal{H}(\mu/w)$ expands, first column). In this case, the home production of high quality goods increases. Foreigners do exactly the opposite ($\mathcal{H}^*(\mu^*)$ expands if and only if $\eta < 1$, fourth column).

We summarize those results in the following proposition:

Proposition 1 *Consider initially symmetric countries with homogenous populations. A rise in a country population fosters its specialization in high-quality exports. If the expenditure elasticity is low enough, domestic consumers are worse off.*

Several remarks can be made. First, this result differs from the effect of a population growth in a closed economy. Indeed, it is easy to show that a larger population size has no effect on consumption in the closed economy version of this model. Because of the constant returns to production scale, additional individuals proportionally contribute to their own consumption of each type of varieties. In the trade version of the model, a rise in domestic population decreases the domestic costs of high- and low- quality goods in the same proportion so that the local production of those goods is given no relative advantages. However, it decreases the domestic “cost-per-quality” relative to the other country, which entices specialization in higher quality goods. Second, the result about specialization parallels Fajgelbaum *et al.*'s (2011, Proposition 2(i)) finding that bigger countries become net exporters of high-quality goods. Yet, whereas they explain this property through the existence of a ‘home market effect’, the current result is based on the relative price movements.⁵

3.1.2 Productivity growth

Consider a rise in domestic productivity (second row). This inflates the domestic labor supply (in terms of labor units) and depresses the domestic wage (lower w , first column). However, although foreign goods become less attractive at home, the larger home productivity raises domestic workers’ income. Their consumption choices then relate again to the movements in relative prices and elasticity of expenditure. If $\eta > 1$, the domestic relative prices do not fall so as to strongly affect the workers’ purchasing power stemming from their productivity gain. Domestic consumers then purchase more high-quality goods from both local and foreign production ($\mathcal{H}(\mu/w)$ and $\mathcal{H}^*(\mu)$ expand, second and third columns). If $1/3 < \eta < 1$, the domestic relative wage falls so much as to decrease the attractiveness of high-quality foreign goods. Domestic consumers substitute high-quality imports for low-quality imports and high-quality local production. Finally, if $\eta < 1/3$, the domestic relative prices diminish more than domestic productivity gains. Thus consumers are enticed to consume less high-quality goods from both local and foreign production. Since the set of varieties is given, those consumers clearly have a lower welfare. It occurs in circumstances of low income elasticity of demand (low

⁵In Fajgelbaum *et al.* (2013), there exists no relative wage effect stemming from the trade of differentiated goods because of the assumption of a perfectly tradeable homogenous good.

η). This is an instance of “immiserizing growth” whereby economic expansion harms a country due to adverse changes in relative wage (Bhagwati 1958).⁶ In contrast to the literature, the effect occurs with initially symmetric countries and no exogenous export bias in the production feasibility set. The originality here lies in the mechanism where the welfare loss results from the domestic individuals’ choices to shift to their consumptions for lower quality goods. As $\mathcal{H}(\mu^*/w)$ expands and $\mathcal{H}^*(\mu)$ shrinks, an export bias endogenously emerges not in terms of larger export quantities but in terms of higher export quality. This stems from the absence of individuals’ responses in terms of quantity of each variety. Relative prices alter the intensive margins of trade at a product classification level that includes both quality and variety (e.g. Jerseys, pullovers,... of Kashmir goats, nomenclature code HS611012) but does not affect the intensive margin of trade at the product classification level of the variety (e.g., Jerseys, pullovers,..., code HS6110). Compared to other models where consumers purchase many units of the same variety, here, the consumption rigidity at the variety level generates a stronger impact of relative prices. However, economic observers come with few examples of immiserizing trade processes so that the elasticity η might be assessed to lie above 1/3 for most relevant trade economies. The point of this discussion is to show that adverse welfare effects may also occur when countries trade vertically differentiated goods.

The rise in domestic productivity benefits foreign consumers because their relative wages and prices become higher. Their imports become less expensive so that they purchase more numerous high-quality goods ($\mathcal{H}(\mu^*/w)$ expands, last column). If $\eta < 1$, their relative wages improve so much that they substitute their local high-quality goods for high-quality imports ($\mathcal{H}^*(\mu^*)$ shrinks while $\mathcal{H}(\mu^*/w)$ expands, fourth and fifth columns). Otherwise, they increase their expenditures on both local and imported high-quality goods at the expense of low-quality varieties ($\mathcal{H}^*(\mu^*)$ and $\mathcal{H}(\mu^*/w)$ expand, fourth and fifth columns).

The effect of the productivity increase on trade patterns depends on consumers’ reaction to movements in relative wages. If $\eta < 1$, a rise in domestic productivity leads to a higher number of high-quality exports and a lower number of high-quality imports. In other words, *when relative prices are responsive to the shocks, a better domestic productivity fosters home specialization in high-quality export and entices the foreign country to specialize in low-quality*

⁶In contrast to this paper, Bhagwati (1958) and followers discuss on two-goods two-country model with representative consumers and divisible goods. The result also shares similarities with the “dutch disease” in which an improvement of domestic resource unfavorably affects terms of trade and harms the country (Jones, Neary and Ruane 1987). Note that, in this literature as in this paper, the ‘growth’ refers to an exogenous change in the productivity or factor endowments.

export. By contrast, if $\eta > 1$, a rise in domestic productivity yields a higher number of both high-quality exports and imports. That is, *when relative prices play a less important role, a better domestic productivity fosters the specialization of all countries in high-quality exports.*

Proposition 2 *Consider initially symmetric countries with homogenous populations. Then, a positive growth in domestic productivity fosters home specialization in high-quality export. It entices the foreign country to specialize in high-quality exports if and only if relative prices do not respond much ($\eta > 1$). If they are strongly responsive ($\eta < 1/3$), domestic consumers end up consuming lower-quality goods and are exposed to an immiserizing growth effect.*

A rise in domestic productivity proportionally decreases costs of all domestic goods and gives a domestic advantage in terms of cost-per-quality relative the other country, which entices the home specialization in high quality goods. The point here is that the productivity growth also raises the purchasing power of domestic consumers who demand higher quality goods from abroad if and only their relative wages do not significantly deteriorate ($\eta > 1$). Such a conclusion may explain the two views on the relationship between country per-capita income and the quality of export contents. On the one hand, if $\eta > 1$, the above result is congruent to the empirical fact that the quality of exports correlates with the incomes of the destination countries. Fieler (2012) shows that quality levels (measured by unit prices) rise with importers' per-capita incomes. Verhoogen's (2008) and Iacovone and Javorcik's (2009) give empirical evidence about the export of higher-quality versions from Mexico to richer countries (like the US). Hallak and Schott's (2011) provide empirical evidence that poorer economies export higher quality goods to richer markets. On the other hand, if $\eta < 1$, it is congruent with the view that developed countries specialize in higher quality goods and less developed in lower quality goods (Helpman and Flam, 1987). Yet, our result is not based on the assumption of any exogenous absolute or comparative advantage of the richer country in the production of product quality. It is also consistent with the idea that richer countries become net exporters of high-quality goods and poorer countries net importers of those (Fajgelbaum *et al.*, 2011, Proposition 2(ii)). However, as said before, in this paper the economic force lies on the response of relative prices rather than in the presence of a home market effect.

3.1.3 Linder's hypothesis

This model matches the empirical fact that two richer countries trade more high quality products. Indeed, extending the above analysis to equal increases in *each* country's productivity $\tilde{s} = \tilde{s}^*$, one gets $\tilde{w} = 0$ and $\tilde{\mu} = \tilde{\mu}^* = \tilde{\mu} - \tilde{w} = \tilde{\mu}^* - \tilde{w} = \tilde{s}/\eta$. Therefore, when both countries

become equally more productive, relative wages do not change but consumers purchase more numerous high-quality imports and local varieties. Therefore, *the two high income countries specialize in the production of higher quality goods and trade more high-quality goods*. Conversely, two low income countries will specialize in low-quality varieties and trade more of those with each other. This confirms Linder's (1961) hypothesis about the nature of trade and the similarities of countries' demands.

Proposition 3 *With homogenous and symmetric populations, two high income countries specialize in the production of higher quality goods and trade more of those.*

As said above, when global productivity rises, consumers purchase more numerous high-quality goods. As a result, the average quality of their consumption baskets rise and the expenditures shift towards goods with higher quality.

The study of variations around the symmetric equilibrium may constitute too a limited framework of analysis. We extend the above results for equilibria with asymmetric countries and productivity levels.

3.2 Linear expenditures

To give analytical tractability for the system of equations (8), (9) and (10), we focus on the class of economic environments that yield linear expenditure functions, $F(z) = F^*(z) = a + bz$, where a and b are two positive scalars. While those functions F and F^* are assumed to be symmetric across countries for the sake of clarity, the population ratio m and the productivity levels s and s^* are asymmetric. The main advantage of this approach is to present and discuss analytical solutions of asymmetric situations.

To construct linear expenditure functions, we equate the above definition to the previous definition $F(z) = \int_0^{\Delta^{-1}(z)} (\theta_H(i) - \theta_L(i)) di + \int_0^N \theta_L(i) di$. As a result, the constant coefficients a and b must be satisfy

$$a = \int_0^N \theta_L(i) di \quad \text{and} \quad b = \frac{1}{z} \int_0^{\Delta^{-1}(z)} (\theta_H(i) - \theta_L(i)) di,$$

which respectively measure the labor input for *all* low-quality varieties and the average labor input needed for the upgrade of the goods that are produced at an higher quality. The latter identity puts a constraint on the cost and quality profiles, $\theta_k(\cdot)$ and $q_k(\cdot)$, $k = L, H$. Several classes of profiles satisfy this constraint. One of them is given by the following:⁷

⁷Other primitives satisfying linear expenditure can include cost and quality functions ($\theta_k(i)$, $q_k(i)$) that embed exponential, linear, quadratic, hyperbolic functions of i .

Cost and quality profiles for linear expenditures: $\theta_k(i) = \theta_k^*(i) \equiv \bar{\theta}_k i^{\alpha-1}$ and $q_L(i) = q_L^*(i) \equiv q_0/i$ with $k = L, H$, $i \in [0, N]$, $\alpha > 1$, $q_0 > 0$ and $\bar{\theta}_H > \bar{\theta}_L \geq 0$.

In this class of primitives, the cost-per-quality functions is equal to $\Delta(i) = i^\alpha$ and $\Delta^*(j) = j^\alpha$ while the quality upgrades are given by $q_H(i) - q_L(i) = (\bar{\theta}_H - \bar{\theta}_L)/i$ and $q_H^*(j) - q_L^*(j) = (\bar{\theta}_H - \bar{\theta}_L)/j$. The varieties with longer quality ladders, $q_H(i) - q_L(i)$, have a more than proportionate spread in cost, $\theta_H(i) - \theta_L(i)$. The coefficients of the expenditure function are then equal to $a = \bar{\theta}_L N^\alpha / \alpha$ and $b = (\bar{\theta}_H - \bar{\theta}_L) / \alpha$. Such a class of primitives includes asymmetric configurations that support an equilibrium under the non satiation and full market coverage assumptions. To make this clear, Figure 2 displays an example where an equilibrium is supported for substantial differences in population sizes and productivities.

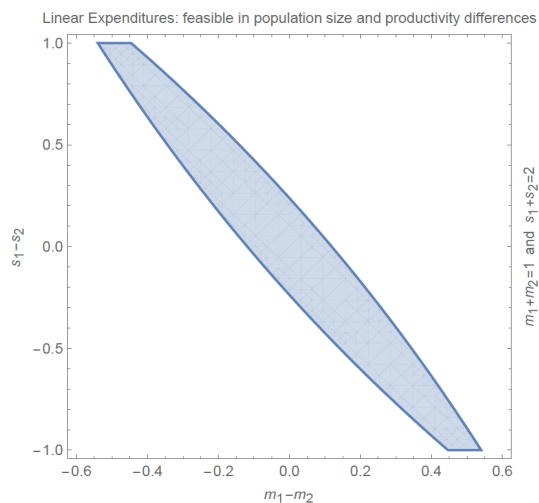


Figure 2: Linear expenditures: population size and productivities supporting an equilibrium.

Note: The darker region depicts the set of differences in population sizes $m_1 - m_2$ and productivity $s_1 - s_2$ such that $m_1 + m_2 = 1$ and $s_1 + s_2 = 2$. The point at the center represents the populations $m_1 = m_2 = 1/2$ and $s_1 = s_2 = 1$. Other parameters are equal to $\{\alpha, n, \bar{\theta}_L, \bar{\theta}_H, q_0\} = \{2, 1.3, 0.3, 0.8, 10\}$.

Equilibrium equations (8), (9) and (10) accept the following unique solution (see Appendix C):

$$w = \frac{s^* - a(1+m)}{ms - a(1+m)}, \quad (13)$$

$$\mu = \frac{ss^* - a[s(1+2m) + s^*] + 2a^2(1+m)}{2b[ms - a(1+m)]}, \quad (14)$$

$$\mu^* = \frac{mss^* - a[s^*(2+m) + ms] + 2a^2(1+m)}{2b[ms - a(1+m)]}, \quad (15)$$

These values are positive for sufficiently small a : $a < a^{\max}$ with $a^{\max} > 0$, which we assume. One may finally compute μ/w and μ^*/w from those expressions.⁸

The impact on high- and low-quality consumption sets is presented in the Table 2. Consumption patterns match those obtained in the previous sub-section in the case where the elasticity of expenditure η lies below one. This is not surprising as the expenditure elasticity can here be computed as

$$\frac{d \ln F(z)}{d \ln z} = \frac{bz}{a + bz} < 1. \quad (16)$$

More interesting is that those results apply for asymmetric population sizes and skill endowments.

| | Relative wage | Home local | Home imports | Foreign local | Foreign imports |
|--------------------------------|---------------|-----------------------|-----------------------|-----------------------|-------------------------|
| | $w \searrow$ | $H(\mu/w) \searrow$ | $H^*(\mu) \searrow$ | $H^*(\mu^*) \nearrow$ | $H(\mu^*/w) \nearrow$ |
| $m \nearrow$ | True | True | True | True | True |
| $s \nearrow$ | True | $s^*/(1 + 2m) \leq a$ | True | True | True |
| $a \searrow (a \rightarrow 0)$ | $s^* \geq ms$ | False | $s^* \geq ms(1 + 2m)$ | True | $s^* \geq m^2s/(2 + m)$ |

Table 2: Comparative statics under linear expenditure functions ($a > 0$).

3.2.1 Population, productivity growth and Linder hypothesis

Because of the adverse effect of relative prices, a larger domestic population, be it initially large or small, decreases its domestic consumption of both high-quality local goods and imports. The rise in domestic productivity raises the foreign consumption of all high-quality goods and reduces the imports of high-quality goods at home. As a result, *more productive - or richer - countries become a net exporters of high-quality goods and a net importer of low-quality commodities*. However, the rise in domestic productivity may increase or decrease the domestic consumers' consumption of local high-quality goods $\mathcal{H}(\mu/w)$. In particular, it increases it if and only if $a < s^*/(1 + 2m)$. Under this condition, the domestic relative wages and prices fall and diminish the attractiveness of foreign goods so that domestic consumers substitute high-quality imports for low-quality imports and high-quality local goods. Otherwise, they substitute for

⁸Appendix C shows the restrictions on the cost and quality profiles that guarantee the existence of such an equilibrium. Depending on the parameters the set of the size and productivity differences between countries can be large.

low-quality imports and local goods. This parallels the discussion of the symmetric equilibria when $\eta < 1$. The present class of linear expenditure functions therefore also supports the case for “immiserizing growth”. The domestic workers see their relative income fall and consume less high-quality goods from both local and foreign production.

In this model, richer countries trade more high quality products. For countries of same productivity ($s = s^*$) but different country sizes, an infinitesimal increase in productivity $ds = ds^* > 0$ leaves the relative prices unaffected while it raises μ , μ^* , $\mu - w$ and $\mu^* - w$ by the amount $ds/(2b) > 0$ for a close to zero. By a continuity argument, it raises μ , μ^* , $\mu - w$ and $\mu^* - w$ for sufficiently small a . Therefore, high income countries also specialize in the production of higher quality goods and trade more of those goods.⁹ Hence, *most of the conclusions for symmetric countries apply with asymmetric countries.*

3.2.2 Impact of technology

The assumption of linear expenditure allows us to highlight the effect of the product quality technology embedded in the parameters a and b .

A fall in parameter a corresponds to a downward shift of the cost of all low quality goods $\theta_L(\cdot)$. To keep b constant, we temporarily fix the cost and utility gains of quality upgrades, $\theta_H(i) - \theta_L(i)$ and $q_H(i) - q_L(i)$ (which keeps $\Delta^{-1}(\cdot)$ invariant). Such a change can be interpreted as the technological progress or diffusion on the basic attributes embedded in high- and low-quality goods. For example, the production of hard and soft cover books may both benefit from faster printing machines. The comparative statics on a is displayed in the last line of Table 2 for small enough a . In the case of symmetric countries ($m = 1, s = s^*$), a fall in a decreases costs and prices proportionally everywhere so that the relative prices are unaffected (first column) and consumers purchase more high-quality products (second and third columns are false while fourth and last columns are true). So, technological progress on basic attributes boosts the sales of high-quality goods.

In the case of asymmetric countries (say $s^* \neq ms$), relative wages increase in the country with the lowest labor supply (first column). Given the general fall in labor demand, this country faces a lower pressure on wages and incomes. In any case, the price of goods decreases in both countries and consumers substitute for more high-quality (second and fourth columns). They also substitute for more high-quality imports if the terms of trade effects do not play a too important role, which is likely to occur if countries are not too asymmetric (third column is

⁹Things may differ when a is large enough, but this is left for future research.

false and fifth one is true if $ms(1+2m) \geq s^* \geq m^2s/(2+m)$). This extends the previous conclusion to the case of not too asymmetric countries. Otherwise, changes in relative prices may entice the consumers to purchase more low-quality imports.

The parameter a also reflects the mass of varieties in the economy. Since N raises a , the introduction of new varieties has the opposite effect as the one described above. Therefore, for not too asymmetric countries, consumers purchase more low-quality products. Because they have preference for product diversity, they cut on the quality of their purchases to consume more diverse goods.

A rise in the parameter b corresponds to the combination of a proportional increase in the quality upgrading costs, $\theta_H(i) - \theta_L(i)$, and the utility gains, $q_H(i) - q_L(i)$. Under the above cost and quality profiles for linear expenditures, this is realized by an increase in the parameter difference $\bar{\theta}_H - \bar{\theta}_L$ because $\theta_H(i) - \theta_L(i) = (\bar{\theta}_H - \bar{\theta}_L) i^{\alpha-1}$ and $q_H(i) - q_L(i) = (\bar{\theta}_H - \bar{\theta}_L) / i$. To keep a constant, we maintain the low quality cost profile $\theta_L(\cdot)$. It can be seen from (13) (14) and (15) that a higher b has no effect on relative prices and no effect on the factors $b\mu$ and $b\mu^*$. Hence, at the equilibrium, it does not affect the expenditure function $F = a + b\mu$ and $F^* = a + b\mu^*$ and has thus no impact on consumption baskets.

4 Within-country income inequalities

In this section, we consider heterogenous workers. Suppose that the masses M and M^* of individuals are respectively divided in K domestic skill groups, $k \in \{1, \dots, K\}$ and foreign skill groups, $l \in \{1, \dots, K^*\}$. Home individuals in group k are endowed with s_k labor units and distributed according to the probability g_k ($\sum_k^K g_k = 1$), while foreign individuals in groups l are endowed with s_l^* labor units and distributed according to g_l^* ($\sum_l^{K^*} g_l^* = 1$). Again we set M/M^* to m and normalize w^* to unity. At the equilibrium, the wage and the inverse marginal utility in group k and in group l are given by the scalars and vectors $(w, \{\mu_k\}_{k \in K}, \{\mu_l^*\}_{l \in K^*})$ that satisfy the expenditure identities and the balanced trade condition:

$$\begin{aligned} s_k &= F\left(\frac{\mu_k}{w}\right) + \frac{F^*(\mu_k)}{w}, \\ s_l^* &= F^*(\mu_l^*) + wF\left(\frac{\mu_l^*}{w}\right), \\ m \sum_{k=1}^K F^*(\mu_k) g_k &= w \sum_{l=1}^{K^*} F\left(\frac{\mu_l^*}{w}\right) g_l^*. \end{aligned}$$

In each country richer consumers buy higher-quality products. Since F is an increasing function, the inverse marginal utility rises with the skill level: $s_k \geq s_{k'} \iff \mu_k \geq \mu_{k'}$. As a

result, more skillful individuals purchase large sets of high-quality varieties.

We study the variations around the symmetric equilibrium and for the class of linear expenditure functions.

4.1 About the symmetric equilibrium

In this case, we consider symmetric countries such that $F = F^*$, $m = 1$, $K = K^*$, and $g_k = g_l^*$ for $k = l$. At the equilibrium, we obtain the symmetric configuration where $w = 1$ and $\mu_k = \mu_l^* \equiv \mu_k^0$ for $k = l$ with $s_k = 2F(\mu_k^0)$.

The consumption of high- and low-quality goods around this symmetric equilibrium varies according to the exogenous changes of various factors. We consider the infinitesimal changes in domestic population $\tilde{m} = dm/m > 0$, keeping the group distribution equal, and the changes in group k 's domestic productivity $\tilde{s}_k = ds_k/s_k > 0$ keeping population sizes fixed (for details, see Appendix A, B). We also consider how the change in domestic productivity of group k affects other domestic groups $k' \neq k$. Finally, we consider the changes in the domestic skill distribution $\tilde{g}_k = dg_k/g_k$, $k \in \{1, \dots, K\}$, keeping this population constant.

Totally differentiating and simplifying the above system of equations around the symmetric equilibrium yields the following endogenous changes in the wage and inverse marginal utility of domestic and foreign individuals (see Appendix A and B):

$$\begin{aligned}\tilde{w} &= -\frac{\tilde{m} + \varphi_k \tilde{s}_k + \sum_k \varphi_k \tilde{g}_k}{\sum_k \varphi_k \eta_k}, \\ \tilde{\mu}_k &= -\frac{(\eta_k + 1)}{2\eta_k \sum_k \varphi_k \eta_k} \left[\tilde{m} - \frac{2 \sum_k \varphi_k \eta_k}{(\eta_k + 1)} \tilde{s}_k + \sum_k \varphi_k \tilde{s}_k + \sum_k \varphi_k \tilde{g}_k \right], \\ \tilde{\mu}_{k'} &= -\frac{(\eta_{k'} + 1)}{2\eta_{k'} \sum_k \varphi_k \eta_k} [\tilde{m} + \varphi_k \tilde{s}_k + \sum_k \varphi_k \tilde{g}_k], \text{ for all } k' \neq k, \\ \tilde{\mu}_k^* &= \frac{(1 - \eta_k)}{2\eta_k \sum_k \varphi_k \eta_k} [\tilde{m} + \sum_k \varphi_k (\tilde{s}_k + \tilde{g}_k)].\end{aligned}\tag{17}$$

where all sums are over the set $\{1, \dots, K\}$. In those expressions, $\eta_k \equiv \eta(\mu_k^0) = \mu_k^0 F'(\mu_k^0) / F(\mu_k^0)$ is the elasticity of expenditure in group k while

$$\varphi_k \equiv \frac{F(\mu_k^0) g_k}{\sum_l F(\mu_l^0) g_l} = \frac{s_k g_k}{\sum_l s_l g_l}\tag{18}$$

is the share of group k in the total domestic consumption. For $K = 1$, this equilibrium naturally collapses to the one obtained under homogenous populations. One easily computes μ_k/w and μ_k^*/w from those expressions.

The consumption patterns may differ according to the equilibrium value of the expenditure elasticity of the consuming groups, η_k and $\eta_{k'}$ at home and η_l^* abroad. To shorten the discussion

suppose that less skillful individuals have lower expenditure elasticities. For instance, this is the case under a linear expenditure function $F(z) = a + bz$ where $a > 0$ and $b > 0$ (see (16)).

Monotone expenditure elasticity: $s_k \leq s_{k'} \iff \eta_k \leq \eta_{k'}$.

By (12), this assumption requires that the ratio of income elasticity of demand and inverse marginal utility of income falls with income. While it is standard to associate poorer individuals with lower income elasticities of demand, no such statement exists for the inverse marginal utility of income. The above assumption then implicitly requires to assume that the inverse marginal utility of income does become too much inelastic when income falls.

4.1.1 Population growth

The impact on high- and low-quality consumption sets is reported in Table 3. Consider first a rise in the home population keeping the same group distribution (first row). As this raises domestic labor supply, the domestic relative wage falls (first column). If all elasticities are simultaneously larger or lower than one, the interpretation of this table is the same as for Table 1. The main difference lies in the distributional effects caused by the discrepancies of expenditure elasticities across skill groups. In particular, if $\eta_k < 1$, domestic consumers with skill s_k substitute high-quality imports for both low-quality imports and local production (second and fourth columns). They substitute for low-quality imports but high-quality local production otherwise. If $\eta_k < 1$, the population growth has an immiserizing effect whereby this group k ends up consuming lower quality goods and therefore enjoying lower utility. Obviously, if all groups have $\eta_k < 1$, they all get harmed as in the case of homogenous population (Section 3). The discussion then sums to pinpoint the ranking of elasticity of expenditure across skill groups. Under the above assumption of monotone expenditure elasticity, *domestic lower-skill individuals are likely to end up with a consumption basket with more low-quality goods and to suffer more from a rise in home population. By contrast, lower-skill foreigners are more likely to purchase more high-quality goods and benefit more for this population change.*

To fix ideas, consider a “balanced set of monotone expenditure elasticity” that we define as $s_K < \dots < s_1$ and $\eta_K < \dots < 1 < \dots < \eta_1$. Then, a rise in home population forces the lowest skill domestic group K to decrease its consumption of all high-quality goods whereas the highest skill group 1 still raises its consumption of local high-quality goods. The lowest skill then surely suffer from its country population growth. Conversely, the lowest skill foreigners increase their consumption of all high-quality goods and certainly benefit from the population change.

| | Terms of trade $w \searrow$ | Home group k | | Home group $k' \neq k$ | | Foreign group l | |
|----------------|-----------------------------------|-------------------------------|-----------------------------|--------------------------|--------------------------|-------------------------|-------------------------|
| | | Local cons. | Imports | Local cons. | Imports | Local cons. | Imports |
| | | $H(\mu_k/w) \searrow$ | $H^*(\mu_k) \searrow$ | $H(\mu_{k'}/w) \searrow$ | $H^*(\mu_{k'}) \searrow$ | $H^*(\mu_l^*) \nearrow$ | $H(\mu_l^*/w) \nearrow$ |
| $m \nearrow$ | True | $\eta_k < 1$ | True | $\eta_{k'} < 1$ | True | $\eta_l < 1$ | True |
| $s_k \nearrow$ | True | $\eta_k < \widehat{\eta}_k/3$ | $\eta_k < \widehat{\eta}_k$ | $\eta_{k'} < 1$ | True | $\eta_l < 1$ | True |
| $ds > 0$ | True | $\eta_k < 1$ | True | $\eta_{k'} < 1$ | True | $\eta_l < 1$ | True |

Table 3: Comparative statics around the symmetric equilibrium: heterogenous populations.

Notes: w is domestic wage; m is the size of home to foreign country; s is the home productivity; H is the set of high quality varieties. $H(\mu/w) \searrow$ reads as the set of local high-quality varieties consumed at home shrinks. The home group k is the group with an increase in productivity s_k while the home group k' is the group with no productivity change. dS is the change in domestic aggregate labor supply.

Proposition 4 *Consider a symmetric equilibrium with multiple skill groups and balanced set of monotone expenditure elasticity. Then, a rise in home population harms the lowest skill domestic group while it benefits to the lowest skill foreigners.*

4.1.2 Productivity growth

Consider again a rise in productivity s_k of group k . As under homogenous population, it improves the foreign relative prices and incomes. The latter thus unambiguously import more high-quality goods from home ($\mathcal{H}(\mu_l^*/w)$ expands, sixth column). Domestic consumers purchase more numerous high-quality local goods and imports if $\eta_l < 1$ and more numerous low-quality local products and high-quality imports otherwise ($\mathcal{H}^*(\mu_l^*)$ expands iff $\eta_l < 1$, fifth column). However, the rise in productivity deteriorates domestic terms of trade and reduces the home relative income. On the one hand, a worker in a group $k' \neq k$ keeps the same productivity and is negatively affected only by the less favorable terms of trade. She always reduces her imports of high-quality goods while she purchases fewer local high-quality goods if and only if her relative income falls ($\mathcal{H}^*(\mu_{k'})$ shrinks while $\mathcal{H}(\mu_{k'}/w)$ shrinks or expands if $\eta_{k'} < 1$, fourth and fifth columns). Under monotone expenditure elasticity, *lower domestic skill groups suffer more from a rise in the productivity productivity of other domestic groups.*

On the other hand, a worker in group k faces a rise in income as the same time as less favorable terms of trade. If her expenditure elasticity η_k is lower than the threshold

$$\widehat{\eta}_k = 1 - \frac{2 \sum_{l \neq k} \varphi_l \eta_l}{\varphi_k} < 1,$$

she imports fewer high-quality goods (third column). Furthermore, if this elasticity is lower than $\widehat{\eta}_k/3$, she also purchases fewer local high-quality goods (fourth column). The difference with the homogenous population case lies in the fact that the thresholds $\widehat{\eta}_k$ is group dependent. Hence, according to the equilibrium values of η_k and $\widehat{\eta}_k$, it can be that some skill groups expand their high-quality purchases after an increase in own productivity while others reduce them. Using $\varphi_k = s_k g_k / \sum_l s_l g_l$ one can check that

$$\eta_k < \widehat{\eta}_k \iff (1 - \eta_k) s_k g_k > 2 \sum_{l \neq k} \eta_l s_l g_l$$

and

$$\eta_k < \widehat{\eta}_k/3 \iff (1 - 3\eta_k) s_k g_k > 2 \sum_{l \neq k} \eta_l s_l g_l$$

Those conditions are never satisfied for a group with $\eta_k > 1$. This group therefore purchases more numerous high-quality imports and local goods and benefits from its own productivity increase. Similarly, take the highest skilled group k with $s_k > s_{k'}$, $k' \neq k$. Those conditions are less likely to be satisfied if this group has a higher elasticity of expenditure η_k and smaller (effective) labor supply $s_k g_k$. Conversely, take the lowest skilled group k with $s_k < s_{k'}$, $k' \neq k$. Then, those conditions are more likely to be satisfied for low elasticity η_k and high share $s_k g_k$. To sum up, *a productivity increase in a high skill group is more likely to benefit to this group if it has a higher skill and smaller size. The same productivity increase in a low skill group is more likely to harm to this group the lower its skill level and the larger its size.*

To clarify this point we can restrict the analysis to two skilled groups with $s_1 > s_2$ and $1 > \eta_1 > \eta_2$ and to the effect of a skill-biased technological change (like in the OECD since the 1980ies). We thus consider an increase of the domestic high skill $ds_1 > 0 = ds_2$. Figure 2 displays three cases A , B and C in the plane (η_1, η_2) . In any case the low skilled ends up purchasing fewer high-quality varieties and is worse off. By contrast, the high skilled purchases more numerous high-quality of all goods in region A and fully benefits from her productivity improvement. This occurs for high enough elasticities (η_1, η_2) . She purchases more local high-quality goods and more low-quality imports in region B . Finally, she ends up with more numerous low-quality of all goods in region C for low enough elasticities (η_1, η_2) . The immiserizing growth applies to both groups. The new element is that the region A expands as $s_1 g_1 / (2s_2 g_2)$ falls. That is, the high skilled group is more likely to improve her consumption basket if it has a small size (small g_1/g_2) and if the skill and income difference is not too high (low s_1/s_2). Indeed, in such a case, it has a small impact on total labor supply and relative prices. Whereas the effect of skill bias technological change has received much attention in the

literature (e.g. Acemoglu and Autor (2011) for a recent review), our analysis is the first to show how it can be related to the production and consumption of vertically differentiated goods.

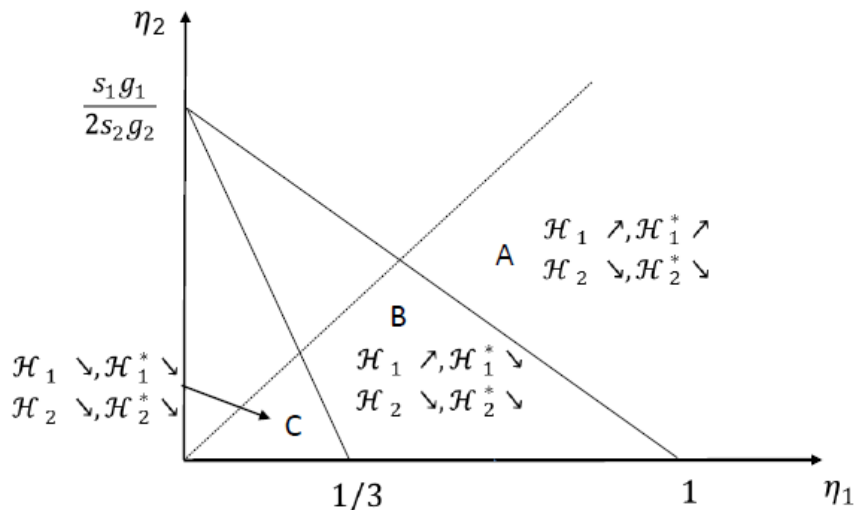


Figure 3: Changes in consumption sets of two domestic groups.

Note: The high skill group increases its productivity s_1 . The low skill group has productivity $s_2 < s_1 < 1$.

The sets of high-quality local and imported purchases of group k are H_k and H_k^* .

We summarize this discussion in the following proposition:

Proposition 5 *Suppose multiple skill groups in two countries. Then, a productivity increase of the high skill group benefits to this group for a larger set of parameters if it has higher skills and smaller size; it harms to the low skill group more if the latter has lower skills and larger size.*

4.1.3 Income redistribution

Our last discussion is about income redistribution across individuals. Since the individual's income is proportional to her skill level or labor supply s , redistributing income is equivalent to shifting individuals across skill groups. Let thus us consider the infinitesimal change of the skill group shares in the domestic population: $\tilde{g}_k = dg_k/g_k$, $k \in \{1, \dots, K\}$. Since $\sum_k g_k = 1$, we need to impose $\sum_k g_k \tilde{g}_k = 0$. However, from (17), the effect of skill redistribution depends on the sum $\sum_k \varphi_k \tilde{g}_k$, which is equal to $ds / (\sum_l s_l g_l)$, where $ds \equiv \sum_k s_k dg_k$ measures the marginal increase in average domestic skill or, equivalently, the marginal increase in the labor supply (in terms of labor units).¹⁰ If the latter is constant, there are no effects on relative prices.

¹⁰Indeed, $\sum_k \varphi_k \tilde{g}_k = (\sum_k s_k g_k \tilde{g}_k) / (\sum_l s_l g_l) = (\sum_k s_k dg_k) / (\sum_l s_l g_l)$.

Skill redistribution has no effect on purchases of high-quality goods within each group in any country. A mean-preserving redistribution of income has then no impact on the purchases of the individuals who keep the same skills. Skill redistribution, of course, benefits those individuals who improve their skills.

Things change when redistribution increases the average skill and labor supply. This occurs, for instance, if countries achieve redistribution using better education programs. In this case, $dS > 0$ and the impact of redistribution is displayed in the last row of Table 3. Foreign relative wages rise and benefit foreign consumers who import more high-quality varieties and may consume more local high-quality varieties (if $\eta_l < 1$). Lower domestic relative wages harm domestic consumers who import less high-quality goods and may even reduce their purchases of local high-quality goods, possibly immiserizing the low skill workers.

Income redistribution encompasses a simpler effect than in the literature. Flam and Helpman (1987) and Matsuyama (2000) assume that high- and low-quality varieties are produced in different countries. As a result, a local redistribution of income changes the international demands for low- and high-quality goods and thus the trade patterns and terms of trade. This cannot be the case here because countries' production sets are alike and induce symmetric trade patterns and relative price effects.¹¹¹²

We now show that the above results extend to the case of asymmetric countries using the framework of linear expenditure functions.

4.2 Linear expenditures

Again, we focus on the class of cost and quality profiles that support a linear expenditure function $F(z) = a + bz$ where $a, b > 0$. Countries differ in population masses, productivity and skill groups. The equilibrium conditions (17) can be written as a system of linear equations. Defining here the average skills as $s \equiv \sum_k s_k g_k$ and $s^* \equiv \sum_l s_l^* g_l^*$, we determine the unique

¹¹Note also that Flam and Helpman (1987) do not consider mean preserving changes in income distribution, which prohibits a neat comparison with our result.

¹²Observe that, because our analysis does not encompass trade costs, it cannot replicate Fajgelbaum *et al.*'s (2011, Proposition 2(iii)) according to whom countries with larger income inequality specialize in high quality exports. If one introduced trade costs in present model, the home-produced varieties would be more valuable to domestic consumers and entice them to purchase less foreign high-quality varieties, which may switch the foreign country's specialization to low quality goods.

solution

$$\begin{aligned}
w &= \frac{s^* - a(m+1)}{ms - a(m+1)}, \\
\mu_k &= \frac{s_k s^* - a[s_k(1+m) + ms + s^*] + 2a^2(m+1)}{2b(ms - a(m+1))}, \\
\mu_l^* &= \frac{ss_l^* m - a[s_l^*(1+m) + ms + s^*] + 2a^2(m+1)}{2b(ms - a(m+1))}.
\end{aligned} \tag{19}$$

if $ms > a(m+1)$. It can be shown that those expressions are positive for low enough a so that they determine a unique equilibrium. The impact on salary and high- and low-quality consumption sets is reported in Table 4.

| | Terms of trade | Home group k | | Home group k' | | Foreign group l | |
|----------------|----------------|-------------------------------------|----------------------------|--------------------------|--------------------------|-------------------------|-------------------------|
| | | Local cons. | Imports | Local cons. | Imports | Local cons. | Imports |
| | $w \searrow$ | $H(\mu_k/w) \searrow$ | $H^*(\mu_k) \searrow$ | $H(\mu_{k'}/w) \searrow$ | $H^*(\mu_{k'}) \searrow$ | $H^*(\mu_l^*) \nearrow$ | $H(\mu_l^*/w) \nearrow$ |
| $m \nearrow$ | True | True | True | True | True | True | True |
| $s_k \nearrow$ | True | $a < \frac{m(s-g_k s_k)}{1+m-mg_k}$ | $a < \frac{s^*}{1+m+mg_k}$ | True | True | True | True |
| $ds > 0$ | True | True | True | True | True | True | True |

Table 4: Comparative statics under linear expenditure functions: heterogenous populations

These results are consistent with Table 3 whenever the elasticity of expenditure is lower than one. As expressed above, this is because $\eta < 1$ under linear expenditure functions. They are also consistent with Figure 2.

5 Firm mobility and comparative advantages

In the previous sections, we have shown that countries can partially specialize in the production of different quality of goods. The location of the production of varieties being fixed, differences in labor supply had a strong impact on relative prices and may harm a more productive population. In reality the production of varieties may change location according to local factor advantages. We here discuss the endogenous choice of production location in a context of comparative advantages *à la* Dornbush et al. (1977). We therefore propose a Ricardian framework that encompasses comparative advantages in production, perfect competition and non-homothetic preferences as is the literature quoted in the introduction. However, we emphasize two distinctive features. First, we forbid any *a priori* country advantage in producing the goods with either the higher quality levels or the lower cost per quality. This contrasts with

Matsuyama (2000) and Jaimovich and Merella (2012) who focus on North-South patterns. Second, we model multiple quality versions for each variety in order to unveil the role of vertical differentiation within product lines.

To be consistent with the previous analysis, we assume the same preferences over the same sets of domestic varieties $i \in \Omega = [0, N]$ and foreign varieties $j \in \Omega^* = [0, N^*]$ but permit that those varieties can be produced at home and abroad with labor requirements $\theta_k(i)$ and $\theta_k^*(i)$ for $i \in \Omega$ and with $\theta_k(j)$ and $\theta_k^*(j)$ for $j \in \Omega^*$. Hence, varieties keep their national label whereas they can be produced in anywhere (say, Greek Feta cheese produced in Germany). In addition, we add the following assumption about comparative advantages:

Comparative advantages: $\gamma(i) \equiv \frac{\theta_H^*(i)}{\theta_H(i)} = \frac{\theta_L^*(i)}{\theta_L(i)}$ and $\gamma^*(j) \equiv \frac{\theta_H^*(j)}{\theta_H(j)} = \frac{\theta_L^*(j)}{\theta_L(j)}$ with $\gamma'(i) > 0$ and $\gamma^{*'}(j) < 0$.

This assumption implies two things. First, as in Fieler (2012), specialization occurs in terms of variety but not quality. In other words, the production location of a same variety with high and low quality occurs within the same country. This is because firms produce a domestic variety i with quality $k = L, H$ at home if it costs less there: $w\theta_k(i) \leq \theta_k^*(i)$ (where w^* is again normalized to 1). Under the above assumption, we have $\theta_H^*(i)/\theta_H(i) = \theta_L^*(i)/\theta_L(i)$ so that the production decision is independent of the quality. In other words, if there exists a cost advantage to produce Feta cheese in Germany compared to Greece, this advantage equally applies for the high and low quality cheeses. This can be because Germany has the knowledge, skills and equipments to produce cheese more efficiently. Domestic and foreign varieties i and j are then produced at home if and only if $\gamma(i) \geq w$ and $\gamma^*(j) \geq w$, respectively. If we denote the cut-off varieties \hat{i} and \hat{j} such that $\gamma(\hat{i}) = \gamma^*(\hat{j}) = w$. Then, home and foreign varieties i and j are respectively produced at home if $i \leq \hat{i}$ and $j \geq \hat{j}$. Flatter functions γ and γ^* indicate less pronounced comparative advantages and stronger incentives to change production location as relative prices move. Finally, the combination of the assumptions $\gamma'(i) < 0$, $\gamma^{*'}(j) > 0$, $\Delta'(i) > 0$ and $\Delta^{*'}(j) > 0$ implies varieties with low cost per quality (low $\Delta(i)$ and $\Delta(j)$) have weaker comparative advantages.¹³¹⁴

Second, a domestic consumer chooses to purchase a high-quality domestic good if her utility

¹³The same results would apply if low "cost per quality" (low $\Delta(i)$ and $\Delta(j)$) has stronger comparative advantages. Indeed, if $\gamma'(i) < 0$ and $\gamma^{*'}(j) > 0$, higher domestic wage increases the cutoff variety \hat{i} and decreases \hat{j} . One can then replace η_2 and ψ below by $-\eta_2$ and $-\psi$, which will not alter the final result.

¹⁴The equilibrium properties depend on how the profiles of comparative advantages θ_k^*/θ_k , $k = H, L$, map to the profile of "cost per quality" Δ . The standard North-South framework can be modeled by setting $\gamma'(i) > 0$ and $\gamma^{*'}(j) > 0$ (see Matsuyama 2002 and Jaimovich and Merella 2012). In this case, the domestic country has a comparative advantage in all goods with low "cost per quality" and will attract the production of those

gain $\mu [q_H(i) - q_L(i)]$ is higher than the cost of the quality upgrade, either $w\theta_H(i) - w\theta_L(i)$ or $\theta_H^*(i) - \theta_L^*(i)$, depending on where the good is produced. That is, if $\mu/w \geq \Delta(i)$ or $\mu \geq \Delta(i)$ where $\Delta(i)$ is again defined in (3). The domestic consumer purchases the set of home-produced high-quality varieties

$$\mathcal{H}\left(\frac{\mu}{w}, \hat{i}, \hat{j}\right) = \left\{i : i \geq \hat{i}, \frac{\mu}{w} \geq \Delta(i)\right\} \cup \left\{j : j \leq \hat{j}, \frac{\mu}{w} \geq \Delta^*(j)\right\},$$

and imports the sets foreign-produced high-quality varieties

$$\mathcal{H}^*\left(\mu, \hat{i}, \hat{j}\right) = \left\{i : i < \hat{i}, \mu \geq \Delta(i)\right\} \cup \left\{j : j > \hat{j}, \mu \geq \Delta^*(j)\right\}.$$

When the cutoffs \hat{i} and \hat{j} rises, domestic consumers purchase more numerous home-made high-quality goods but less high-quality imports. The sets of low-quality goods follow the same definitions with reverse inequalities for $\Delta(i)$ and $\Delta^*(j)$ while the consumption sets of foreign consumers are defined in a symmetric way with μ^* instead of μ . The expenditure functions become

$$F\left(z, \hat{i}, \hat{j}\right) \equiv \int_{\mathcal{H}(z, \hat{i}, \hat{j})} \theta_H(i) di + \int_{\mathcal{L}(z, \hat{i}, \hat{j})} \theta_L(i) di,$$

$$F^*\left(z^*, \hat{i}, \hat{j}\right) \equiv \int_{\mathcal{H}^*(z^*, \hat{i}, \hat{j})} \theta_H^*(i) di + \int_{\mathcal{L}^*(z^*, \hat{i}, \hat{j})} \theta_L^*(i) di,$$

which both increase with z . The expenditure function F rises while F^* falls with smaller \hat{i} and larger \hat{j} . The equilibrium is then given by the individuals' expenditures, balance trade and firm location conditions:

$$s = F\left(\frac{\mu}{w}, \hat{i}, \hat{j}\right) + \frac{1}{w} F^*\left(\mu, \hat{i}, \hat{j}\right), \quad (20)$$

$$s^* = F^*\left(\mu^*, \hat{i}, \hat{j}\right) + w F\left(\frac{\mu^*}{w}, \hat{i}, \hat{j}\right), \quad (21)$$

$$mF^*\left(\mu, \hat{i}, \hat{j}\right) = w F\left(\frac{\mu^*}{w}, \hat{i}, \hat{j}\right), \quad (22)$$

$$w = \gamma\left(\hat{i}\right) = \gamma^*\left(\hat{j}\right) \quad (23)$$

Since γ and γ^* are respectively ncreasing and decreasing functions, it directly follows that the numbers of home-produced varieties ($i \leq \hat{i}$ and $j \geq \hat{j}$) fall with higher domestic wages. A higher w entices firms with weak comparative advantage to relocate abroad. As a result, local product diversity falls with higher relative wages.

goods with low Δ . This paper however explores the case where countries are symmetric with respect to their comparative advantages.

Quite intuitively, when comparative advantages are strong (steep γ), the relocation of production is negligible and has no important impact on the terms of trade. The results of Section 3 then apply. By contrast, when comparative advantages are weak (flat γ), firms' relocation reduces movements of relative prices so that w remains almost constant. A larger domestic population or productivity shifts the labor supply upwards and entices production to relocate at home. As a result, *the domestic product diversity rises when the home country becomes larger and more productive*. The impact on consumption baskets however depends on how the expenditures on local goods and imports vary with the set of home- and foreign-made varieties, which in turn depends on the functional forms of F and F^* . It is a priori unclear whether the effect of firm relocation is stronger or weaker than the effect of relative price movements in the absence of firm mobility.

For the sake of conciseness, we concentrate on the variations about the equilibrium with symmetric countries such that $m = 1$, $s = s^*$, $F = F^*$ while $\gamma = 1/\gamma^*$ and $\gamma(0) = \gamma^*(0) = 1$. The first assumption on γ and γ^* implies that the comparative advantages of home varieties exactly are symmetric to those of foreign varieties. The second assumption calibrates the comparative advantages such that the equilibrium maps to the one discussed in Section 3. In particular, the symmetric equilibrium here implies that factor price equalization and home production of home varieties: $w = 1$ and $\hat{i} = \hat{j} = \gamma^{-1}(1) = 0$. Consumption patterns are again symmetric and inverse marginal utility given by $s = 2F(\mu^0, 0, 0)$.

Let $\eta_1 = \partial \ln F / \partial \ln z > 0$, $\eta_2 = -\partial \ln F / \partial \ln \hat{i} > 0$ and $\psi = d \ln \gamma / dw > 0$, evaluated at the symmetric equilibrium. While η_1 measures the elasticity of expenditure w.r.t. to individual income, η_2 measures the elasticity of expenditure w.r.t. the production location of the varieties. As production relocates at home, the expenditure on the local varieties rises. Those effects depends on the cost per quality. On the other hand, the inverse of the elasticity ψ measures the effect of comparative advantages and therefore the incentives of marginal firms to shift production to home as the domestic wage falls. When comparative advantages are strong (steep γ and thus low ψ), domestic wages have a negligible impact on the relocation of domestic varieties. When they are weak (flat γ and high ψ), a small decrease in domestic wages leads to large relocation of foreign production at home. The combination of those effects is measured by $\psi\eta_2$.

Comparative statics are displayed in Table 5. They are identical to those obtained without firms' relocation (Table 1) when $\psi\eta_2 = 0$. The equilibrium properties are qualitatively maintained: the impact of a domestic population increase is similar as without mobility of firms and comparative advantages (first line). A rise in home productivity has favorable effects on

the consumption of high-quality goods by the foreign population as before but the domestic population can be induced to increase or decrease their consumption of high and low quality goods (second line). In addition, the larger population triggers relocation of some foreign firms at home ($\hat{j} \nearrow$) while domestic firms stay put ($\hat{i} = 0$).

| | Terms of trade | Home local | Home imports | Foreign local | Foreign imports | Relocation |
|--------------|----------------|----------------------------------|----------------------------|-----------------------|-----------------------|--------------------|
| | $w \searrow$ | $H(\mu/w) \searrow$ | $H^*(\mu) \searrow$ | $H^*(\mu^*) \nearrow$ | $H(\mu^*/w) \nearrow$ | $\hat{j} \nearrow$ |
| $m \nearrow$ | True | $\eta_1 < 1$ | True | $\eta_1 < 1$ | True | True |
| $s \nearrow$ | True | $\eta_1 < (1 - 4\psi\eta_2) / 3$ | $\eta_1 < 1 - 4\psi\eta_2$ | $\eta_1 < 1$ | True | True |

Table 5: Comparative statics around the equilibrium with symmetric countries.

When comparative advantages are very strong ($\psi \rightarrow 0$), firms have very weak incentives to relocate according to changes in domestic wages. As in Section 3, an increase in domestic productivity induces domestic consumers to reduce their purchases of both high-quality home production and imports if they have low expenditure elasticities, respectively, if $\eta_1 < 1/3$ and $\eta_1 < 1$, which harms them with certainty. However, when, *ceteris paribus*, comparative advantages weaken (higher ψ), the conditions in second and third columns of Table 5 become more stringent. Domestic consumers increase their purchases of high-quality local production and imports for a larger set of economic parameters. Immiserating growth is then less likely to occur. To the contrary, they access a wider set of local products and are able to purchase more high-quality local goods and imports. Firm mobility mitigates the negative impact of relative prices on domestic consumption and makes it more likely the gain from a productivity increase.

Proposition 6 *Suppose two symmetric countries with mobile firms and comparative advantages. Then, domestic productivity growth is less likely to harm the home country under weaker comparative advantages. Domestic individuals access to a wider product diversity and are more likely to buy more high-quality local goods and imports.*

6 Concluding remarks

This paper discusses a trade model with vertical differentiation embodied in horizontally differentiated varieties. Individuals have non-homothetic preferences and purchase a unit of each

differentiated good versioned in a few qualities. In contrast the previous literature studying asymmetric production capabilities of product quality (e.g. North-South trade), this paper focuses on countries with similar production capabilities. In this case, larger population and productivity may lead to the specialization in high-quality production and exports and to the specialization of trading partners in low-quality exports. Much of the process results from changes in relative prices. If the latter are reactive, the rise in domestic productivity may generate an immiserizing growth process. If they are not, a better productivity in one country leads to the specialization of all countries in high-quality exports. As predicted by Linder (1961), two richer countries then consume, produce and trade more high quality goods. When the incomes of local populations are heterogenous, improvements in local population and productivity may harm this population and, if so it does harm more lowest income group. Finally, firm mobility mitigates the negative effects of terms of trade and makes local population more like to benefit full from their productivity increases.

References

- [1] Acemoglu, D., & Autor, D. 2011. Skills, tasks and technologies: Implications for employment and earnings. *Handbook of labor economics*, **4**: 1043-1171.
- [2] Baldwin, R. and Harrigan, J. 2011. Zeros, quality and space: Trade theory and trade evidence. *American Economic Journal: Microeconomics* **3**: 60-88.
- [3] Boom, A. 1995. Asymmetric international minimum quality standards and vertical differentiation. *Journal of Industrial Economics* **43**:101-19.
- [4] Choi, Y. C., Hummels, D., and Xiang, C. 2009. Explaining import quality: the role of the income distribution, *Journal of International Economics*, **78**: 293–303.
- [5] Crampes, C. and Hollander, A. 1995. Duopoly and quality standards. *European Economic Review* **39**: 71-82.
- [6] Crozet, M., Head K. and Mayer, Th. 2012. Quality sorting and trade: firm-level evidence for French wine, *Review of Economic Studies*, **79**(2): 609-644.
- [7] Di Comite, F., Thisse, J-F. and Vandenbussche, H. 2014. Verti-zontal differentiation in export markets, *Journal of International Economics*: **93**(1), 50-66.

- [8] Dornbusch, R., Fischer, S., & Samuelson, P. A. 1977. Comparative advantage, trade, and payments in a Ricardian model with a continuum of goods. *American Economic Review* **67(5)**: 823-839.
- [9] Ecchia, G. and Lambertini, L. 1997. Minimum quality standards and collusion. *Journal of Industrial Economics* **45**: 101-113.
- [10] Fajgelbaum, P., Grossman, G. M., & Helpman, E. 2011. Income distribution, product quality, and international trade. *Journal of Political Economy*, **119(4)**, 721-65.
- [11] Flam, H. and Helpman, E. 1987. Vertical product differentiation and north-south trade. *American Economic Review* **77**: 810-822.
- [12] Fieler, A.C. 2012, Quality differentiation in international trade: theory and evidence. Working paper, University of Pennsylvania.
- [13] Gabszewicz, J. and Thisse, J.-F. 1979. Price competition, quality and income distribution. *Journal of Economic Theory* **20**: 340-359.
- [14] Hallak, J.C., and Schott, P.K. 2011. Estimating cross-country differences in product quality. *The Quarterly Journal of Economics* **126(1)**: 417-474.
- [15] Hummels, D. and Klenow, P. J. 2005. The variety and quality of a nation's exports", *American Economic Review*, **95**: 704-723.
- [16] Jaimovich E. and Merella V. 2012. Quality ladders in a Ricardian model of trade with nonhomothetic preferences, *Journal of the European Economic Association*, **10(4)**: 908-937, 08.
- [17] Jaimovich, E. and Merella V. 2015. Love for quality, comparative advantage, and trade, *Journal of International Economics*, **97(2)**: 376-391.
- [18] Jones R., Neary JP. and Ruane F. 1987. International capital mobility and the Dutch disease. In Henryk Kierzkowski ed. Protection and competition in international trade: Essays in honor of W.M.Corden. Oxford: Blackwell.
- [19] Khandelwal A. 2010. The long and short (of) quality ladders, *Review of Economic Studies*, **77(4)**: 1450-1476.
- [20] Linder, S., 1961. An Essay on Trade and Transformation. Stockholm: Almqvist & Wiksell.

- [21] Manova K. and Zhang Z. 2012. Export Prices across Firms and Destinations, *Quarterly Journal of Economics*, **127**: 379-436.
- [22] Maxwell, J. W. 1998. Minimum quality standards as a barrier to innovation. *Economics Letters* **58**: 355-60.
- [23] Matsuyama, K. 2000. A Ricardian model with a continuum of goods under nonhomothetic preferences: demand complementarities, income distribution, and north-South trade. *Journal of Political Economy* **108**: 1093-1120.
- [24] Neven, D. and Thisse, J-F. 1989. On quality and variety competition. Discussion paper at Université catholique de Louvain, Center for Operations Research and Econometrics (CORE).
- [25] Schott, P. K. 2004. Across-product versus within-product specialization in international trade", *Quarterly Journal of Economics*, **119**: 647-678.
- [26] Shaked, A., Sutton, J., 1982. Relaxing price competition through product differentiation. *Review of Economic Studies* **49**: 3-13.
- [27] Stokey, N. 1991. The volume and composition of trade between rich and poor countries. *Review of Economic Studies* **58**: 63-80.
- [28] Verhoogen, E. 2008. Trade, quality upgrading and wage inequality in the Mexican manufacturing sector, *Quarterly Journal of Economics*, **123**: 489-530.
- [29] Wang, X. H. 2003. A note on the high-quality advantage in vertical differentiation models. *Bulletin of Economic Research* **55**: 91-9.

Appendices

Appendix A

In this Appendix we examine infinitesimal changes around the symmetric equilibrium in a general setting where populations are heterogenous. The results are then be applied to the case with homogenous and heterogeneous population, either around the symmetric equilibrium (Appendix B) or assuming linear expenditure functions (Appendix C).

Let the home country host the groups $k \in \{1, \dots, K\}$ and the foreign one host the groups $l \in \{1, \dots, K^*\}$. So, the inverse marginal utility of income are given by μ_k and μ_l^* . The equilibrium conditions are:

$$s_k = F\left(\frac{\mu_k}{w}\right) + \frac{1}{w}F^*(\mu_k), \quad (24)$$

$$s_l^* = F^*(\mu_l^*) + wF\left(\frac{\mu_l^*}{w}\right), \quad (25)$$

$$m \sum_k F^*(\mu_k) g_k = w \sum_l F\left(\frac{\mu_l^*}{w}\right) g_l^*. \quad (26)$$

where the sum of k is over $\{1, \dots, K\}$ and the sum of l over $\{1, \dots, K^*\}$.

First, total differentiating the identity (24) yields

$$ds_k = F'_k \frac{d\mu_k}{w} - F'_k \frac{\mu_k}{w^2} dw + \frac{F_k^{*'} d\mu_k}{w} - \frac{F_k^*}{w^2} dw, \quad (27)$$

where

$$\begin{aligned} F_k &= F\left(\frac{\mu_k}{w}\right) & \text{and} & & F_k^* &= F^*(\mu_k), \\ F'_k &= F'\left(\frac{\mu_k}{w}\right) & \text{and} & & F_k^{*'} &= F^{*'}(\mu_k). \end{aligned}$$

Using a tilda to denote the change in the log of the variables, $\tilde{x} = d \ln x = dx/x$, we can write

$$ds_k = (F'_k + F_k^{*'}) \frac{\mu_k}{w} \tilde{\mu}_k - (F'_k \mu_k + F_k^*) \frac{1}{w} \tilde{w}.$$

Define

$$\eta_k \equiv \frac{\mu_k}{w} \frac{F'_k}{F_k} \quad \text{and} \quad \eta_k^* \equiv \mu_k \frac{F_k^{*'}}{F_k^*},$$

so that we can shortly write $F'_k = \eta_k \frac{w}{\mu_k} F_k$ and $F_k^{*'} = \eta_k^* \frac{1}{\mu_k} F_k^*$. Define also

$$\phi_k = \frac{F_k}{F_k + F_k^*/w} \quad \text{and} \quad \phi_k^* = \frac{F_k^*/w}{F_k + F_k^*/w},$$

with $\phi_k + \phi_k^* = 1$. Dividing the LHS of (27) by s_k and its RHS by $F_k + F_k^*/w$ (equation (24)) and plugging those definitions yields

$$\tilde{s}_k = (\eta_k \phi_k + \eta_k^* \phi_k^*) \tilde{\mu}_k - (\eta_k \phi_k + \phi_k^*) \tilde{w}. \quad (28)$$

For the domestic individuals belonging to a group $k' \neq k$ that keeps the same productivity, $\tilde{s}_{k'} = 0$, this identity becomes:

$$0 = (\eta_{k'} \phi_{k'} + \eta_{k'}^* \phi_{k'}^*) \tilde{\mu}_{k'} - (\eta_{k'} \phi_{k'} + \phi_{k'}^*) \tilde{w}.$$

Second, total differentiating (25) yields

$$ds_l^* = \mu_l^* F_l'^* \frac{d\mu_l^*}{\mu_l^*} + w F_l \frac{dw}{w} + \mu_l^* F_l' \frac{d\mu_l^*}{\mu_l^*} - \mu_l^* F_l' \frac{dw}{w}.$$

where

$$\begin{aligned} F_l &= F\left(\frac{\mu_l^*}{w}\right) & \text{and} & & F_l^* &= F^*(\mu_l^*) \\ F_l' &= F'\left(\frac{\mu_l^*}{w}\right) & \text{and} & & F_l'^* &= F'^*(\mu_l^*) \end{aligned}$$

Using the same procedure as before, we get

$$\tilde{s}_l^* = \tilde{\mu}_l^* (\eta_l^* \phi_l^* + \eta_l \phi_l) + \phi_l \tilde{w} (1 - \eta_l)$$

where

$$\begin{aligned} \eta_l &\equiv \frac{\mu_l^* F_l'}{w F_l} & \text{and} & & \eta_l^* &\equiv \mu_l^* \frac{F_l'^*}{F_l^*} \\ \phi_l &= \frac{w F_l}{F_l^* + w F_l} & \text{and} & & \phi_l^* &= \frac{F_l^*}{F_l^* + w F_l}, \end{aligned}$$

with $\phi_l + \phi_l^* = 1$.

Third, totally differentiating (26) we get

$$\begin{aligned} &\frac{dm}{m} m \sum_k F_k^* g_k + m \sum_k F_k'^* \frac{d\mu_k}{\mu_k} \mu_k g_k + m \sum_{k=1}^K F_k^* dg_k \\ &= \\ &\frac{dw}{w} w \sum_l F_l g_l^* + w \sum_l F_l' \left(\frac{\mu_l^*}{w} \frac{d\mu_l^*}{\mu_l^*} - \frac{\mu_l^*}{w^2} dw \right) g_l^* + w \sum_{l=1}^{K^*} F_l dg_l^*, \end{aligned}$$

where we denote F_k^* , $F_k'^*$, F_l and F_l' are defined above. Using $\tilde{g}_k = dg_k/g_k$ and $\tilde{g}_l^* = dg_l^*/g_l^*$ and substituting for $F_k'^* = F_k^* \frac{\eta_k^*}{\mu_k}$ and $F_l' = F_l \eta_l \frac{w}{\mu_l^*}$, this can be re-written as

$$\tilde{m} + \sum_k \varphi_k^* \eta_k^* \tilde{\mu}_k + \sum_k \varphi_k^* \tilde{g}_k = \tilde{w} + \sum_l \varphi_l \eta_l (\tilde{\mu}_l^* - \tilde{w}) + \sum_l \varphi_l \tilde{g}_l^*,$$

where

$$\varphi_k^* = \frac{F_k^* g_k}{\sum_{k'} F_{k'}^* g_{k'}} \quad \text{and} \quad \varphi_l = \frac{F_l g_l^*}{\sum_{l'} F_{l'} g_{l'}^*},$$

where $\sum_k \varphi_k^* = 1$ and $\sum_l \varphi_l = 1$.

Finally, the equilibrium changes are given by

$$\begin{aligned} \tilde{s}_k &= (\eta_k \phi_k + \eta_k^* \phi_k^*) \tilde{\mu}_k - (\eta_k \phi_k + \phi_k^*) \tilde{w}, \\ \tilde{s}_l^* &= \tilde{\mu}_l^* (\eta_l^* \phi_l^* + \eta_l \phi_l) + \phi_l \tilde{w} (1 - \eta_l), \\ \tilde{m} + \sum_k \varphi_k^* (\eta_k^* \tilde{\mu}_k + \tilde{g}_k) &= \tilde{w} + \sum_l \varphi_l [\eta_l (\tilde{\mu}_l^* - \tilde{w}) + \tilde{g}_l^*]. \end{aligned} \tag{29}$$

Appendix B

In this Appendix, we apply results of Appendix A to the case of homogenous and heterogenous populations around the symmetric equilibrium. We assume symmetric countries so that $F(z) = F^*(z)$, $M = M^*$, $m = 1$. By symmetry, we have $w = 1$ and $\mu_k = \mu_k^*$.

Homogeneous populations

For homogeneous populations, we consider that each population belongs to a single group: $K = K^* = 1$. Also there is no change in the skill distribution so that $\tilde{g}_k = 0$. We consider changes in population and domestic productivity, \tilde{m} , and \tilde{s} , so that $\tilde{s}^* = 0$. Finally, symmetry implies $\eta \equiv \eta_k = \eta_k^* = \eta_l^* = \eta_l$ while $\phi_k = \phi_k^* = \frac{1}{2}$ and $\varphi_k^* = \varphi_k^* = 1$, $\forall k \in \{1, \dots, K\}$ and $l \in \{1, \dots, K^*\}$. As a consequence we can omit the subscripts k and l . Equations (29) become

$$\begin{aligned} \tilde{s} &= \eta \tilde{\mu} - \frac{1}{2} (1 + \eta) \tilde{w}, \\ 0 &= \eta \tilde{\mu}^* + \frac{1}{2} (1 - \eta) \tilde{w}, \\ \tilde{m} &= (1 - \eta) \tilde{w} + \eta (\tilde{\mu}^* - \tilde{\mu}). \end{aligned}$$

Solving for $\tilde{\mu}$, $\tilde{\mu}^*$ and \tilde{w} yields

$$\begin{aligned} \tilde{w} &= -\frac{1}{\eta} \tilde{m} - \frac{1}{\eta} \tilde{s}, \\ \tilde{\mu} &= -\frac{\eta + 1}{2\eta^2} \tilde{m} - \frac{1 - \eta}{2\eta^2} \tilde{s}, \\ \tilde{\mu}^* &= -\frac{\eta - 1}{2\eta^2} \tilde{m} - \frac{\eta - 1}{2\eta^2} \tilde{s}. \end{aligned}$$

Furthermore,

$$\begin{aligned} \tilde{\mu} - \tilde{w} &= -\frac{1 - \eta}{2\eta^2} \tilde{m} - \frac{1 - 3\eta}{2\eta^2} \tilde{s}, \\ \tilde{\mu}^* - \tilde{w} &= \frac{1 + \eta}{2\eta^2} \tilde{m} + \frac{1 + \eta}{2\eta^2} \tilde{s}. \end{aligned}$$

The study of those expressions yields Table 1.

Heterogenous populations

We here discuss the case of heterogeneous populations and symmetric groups across country: that is, $K = K^*$ while $s_k = s_l^*$ and $g_k = g_l^*$ if $k = l$. Given this symmetry, the equilibrium values are given by $\eta_k = \eta_k^* = \eta = \eta_l^*$, $\phi_k = \phi_k^* = \phi_l = \phi_l^* = 1/2$ and $\varphi_k^* = \varphi_l$ for any $k = l$. We consider changes in population and domestic productivity, \tilde{m} , and \tilde{s}_k , so that $\tilde{s}_l^* = 0$. We also consider changes in domestic distribution $\tilde{g}_k \neq 0$ while $\tilde{g}_l^* = 0$. Hence conditions (29) become

$$\begin{aligned}\tilde{s}_k &= \eta_k \tilde{\mu}_k - \frac{1}{2}(\eta_k + 1) \tilde{w}, \\ 0 &= \eta_l \tilde{\mu}_l^* + \frac{1}{2} \tilde{w} (1 - \eta_l), \\ \tilde{m} + \sum_k \varphi_k^* (\eta_k \tilde{\mu}_k + \tilde{g}_k) &= \tilde{w} + \sum_l \varphi_l \eta_l (\tilde{\mu}_l^* - \tilde{w}).\end{aligned}$$

Solving this with respect to \tilde{w} , $\tilde{\mu}_k$ and $\tilde{\mu}_l^*$ yields

$$\begin{aligned}\tilde{w} &= -\frac{\tilde{m} + \sum_n \varphi_n (\tilde{g}_n + \tilde{s}_n)}{\sum_n \varphi_n \eta_n} \\ \tilde{\mu}_k &= -\frac{(\eta_k + 1)}{2\eta_k \sum_n \varphi_n \eta_n} \tilde{m} + \frac{\tilde{s}_k}{\eta_k} - \frac{(\eta_k + 1)}{2\eta_k \sum_n \varphi_n \eta_n} \sum_n \varphi_n \tilde{s}_n - \frac{(\eta_k + 1)}{2\eta_k \sum_n \varphi_n \eta_n} \sum_n \varphi_n \tilde{g}_n \\ \tilde{\mu}_l^* &= \frac{(1 - \eta_l)}{2\eta_l \sum_n \varphi_n \eta_n} \tilde{m} + \frac{(1 - \eta_l)}{2\eta_l \sum_n \varphi_n \eta_n} \sum_n \varphi_n \tilde{s}_n + \frac{(1 - \eta_l)}{2\eta_l \sum_n \varphi_n \eta_n} \sum_n \varphi_n \tilde{g}_n\end{aligned}$$

where n is element of the set $\{1, \dots, K\}$. From this, we can derive

$$\begin{aligned}\tilde{\mu}_k - \tilde{w} &= \frac{(\eta_k - 1)}{2\eta_k \sum_n \varphi_n \eta_n} \left[\tilde{m} + \left(\varphi_k - \frac{2 \sum_n \varphi_n \eta_n}{(\eta_k - 1)} \right) \tilde{s}_k + \sum_n \varphi_n \tilde{g}_n \right] \\ \tilde{\mu}_l^* - \tilde{w} &= \frac{(\eta_l + 1)}{2\eta_l \sum_n \varphi_n \eta_n} \left(\tilde{m} + \varphi_k \tilde{s}_k + \sum_n \varphi_n \tilde{g}_n \right).\end{aligned}$$

The study of those expressions yields Table 3.

Appendix C

In this section we examine the equilibria with the class of preferences and costs supporting linear expenditure functions, $F(z) = a + bz$, $a, b > 0$.

We first determine a class of primitives satisfying the constraints imposed by this linearity. We need to spot the primitive such that $F(z) = \int_0^{\Delta^{-1}(z)} (\theta_H(i) - \theta_L(i)) di + \int_0^N \theta_L(i) di = a + bz$ where a and b is independent of z . For this, we impose $F'(z) = b$. This amounts to define the cost and quality profiles $(\theta_H, \theta_L, q_H, q_L)$ so that

$$\frac{d}{dz} \int_0^{\Delta^{-1}(z)} (\theta_H(i) - \theta_L(i)) di = \left[\frac{(\theta_H(i) - \theta_L(i))}{\Delta'(i)} \right]_{i=\Delta^{-1}(z)} = b.$$

As we have four profiles for one equality constraint, many classes of profiles can be constructed. The class of linear expenditure satisfies this requirement. Indeed, since this yields $\Delta(i) = i^\alpha$, $\Delta^{-1}(z) = z^{1/\alpha}$ and $\theta_H(i) - \theta_L(i) = (\bar{\theta}_H - \bar{\theta}_L) i^{\alpha-1}$, we get $b = (\bar{\theta}_H - \bar{\theta}_L) / \alpha$.

Homogeneous populations

In what follows we show the equilibrium conditions and comparative statics for the case with homogenous populations. Using $F(z) = F^*(z) = a + bz$, the equilibrium conditions become

$$\begin{aligned} s &= a + \frac{b\mu}{w} + \frac{a + b\mu}{w}, \\ s^* &= a + b\mu^* + w \left(a + \frac{b\mu^*}{w} \right), \\ m(a + b\mu) &= w \left(a + \frac{b\mu^*}{w} \right). \end{aligned}$$

Notice that $s, s^* > a$. For non-trivial solution ($w \neq 0$), this can be written as a system of linear equations, which accepts the following unique solution

$$w = \frac{s^* - a(1 + m)}{ms - a(1 + m)} \quad (30)$$

$$\mu = \frac{s^*s - a[s^* + s(1 + 2m)] + 2a^2(m + 1)}{2b[ms - a(1 + m)]} \quad (31)$$

$$\mu^* = \frac{mss^* - a[s^*(2 + m) + ms] + 2a^2(m + 1)}{2b[ms - a(1 + m)]} \quad (32)$$

if $ms \neq a(1 + m)$. Thus, we obtain the solution (13-14). The sufficient conditions of equilibrium existence are provided in the supplementary appendix.

Comparative statics Using the equilibrium sufficient condition because $a < \frac{sm}{m+1}$ and $a < \frac{s^*}{1+m}$, we check the variations of w , μ , μ^* , μ/w and μ^*/w with respect to m and s . We compute

$$\begin{aligned} \frac{\partial w}{\partial m} &= -\frac{s^*s - a(s^* + s)}{[ms - a(m + 1)]^2} < 0, & \frac{\partial \mu}{\partial m} &= \frac{s - a}{2b} \frac{\partial w}{\partial m} < 0 & \text{and} & \frac{\partial \mu/w}{\partial m} = \frac{a}{2b} \frac{\partial w}{\partial m} < 0, \\ \frac{\partial \mu^*}{\partial m} &= -\frac{a}{2b} \frac{\partial w}{\partial m} > 0 & \text{and} & \frac{\partial \mu^*/w}{\partial m} = -\frac{s^* - a}{2b} \frac{\partial w}{\partial m} > 0 \end{aligned}$$

while

$$\frac{\partial w}{\partial s} = -m \frac{s^* - a(m + 1)}{[ms - a(m + 1)]^2} < 0, \quad \frac{\partial \mu}{\partial s} = \frac{a}{2bm} \frac{\partial w}{\partial s} < 0 \quad \text{and} \quad \frac{\partial \mu^*}{\partial s} = -\frac{a}{2b} \frac{\partial w}{\partial s} > 0$$

and

$$\begin{aligned} \frac{\partial \mu^*/w}{\partial s} &= \frac{m(s^* - a)}{2b[s^* - a(m + 1)]} > 0 \\ \frac{\partial \mu/w}{\partial s} &= \frac{s^* - a(2m + 1)}{2b[s^* - a(m + 1)]} < 0 \quad \iff \quad a < \frac{s^*}{1 + 2m}. \end{aligned}$$

Finally, at $a \rightarrow 0$, we get

$$\begin{aligned} \frac{\partial w}{\partial a} &= \frac{(s^* - ms)(m+1)}{m^2 s^2}, & \frac{\partial \mu}{\partial a} &= \frac{s^* - 2sm^2 - sm}{2bm^2 s}, & \frac{\partial \mu^*}{\partial a} &= -\frac{s^* + ms}{2bms} \\ \frac{\partial \mu/w}{\partial a} &= -\frac{s^* + ms}{2bs^*} & \text{and} & & \frac{\partial \mu^*/w}{\partial a} &= \frac{sm^2 - s^*m - 2s^*}{2bs^*} \end{aligned}$$

Those expressions yield Table 2.

Heterogeneous populations

Using $F(z) = a + bz$, the equilibrium conditions (24), (25) and (26) are now:

$$\begin{aligned} s_k &= a + 2b \frac{\mu_k}{w} + \frac{a}{w}, \\ s_l^* &= a + aw + 2b\mu_l^*, \\ am + mb \sum_k \mu_k g_k &= aw + b \sum_l \mu_l^* g_l^*. \end{aligned}$$

For $w \neq 0$, this is a system of linear equations with the unique solution given in (19) if $ms \neq a(m+1)$. This solution is an equilibrium if w , μ and μ^* are all positive. Note that from the two first equations it must be that $s_k > a$ and $s_l^* > a$. Using the same argument as for homogenous populations, the equilibrium exists if $a < a^{\max} \equiv \min_{k,l} \{a_k, a_l^*, \frac{sm}{m+1}, \frac{s^*}{1+m}\}$ where a_k and a_l^* are the lowest (but strictly positive) roots of the numerators of μ_k and μ_l^* in (19).

We now check the changes in the variables under the equilibrium condition $a < a^{\max}$. This implies $a < \frac{sm}{m+1}$ and $a < \frac{s^*}{1+m}$. Also we have $s_k > a$ and $s_l^* > a$. We first compute

$$\begin{aligned} \frac{\partial w}{\partial m} &= -\frac{ss^* - a(s+s^*)}{[a(1+m) - ms]^2} < 0, & \frac{\partial \mu_k}{\partial m} &= (s_k - a) \frac{1}{2b} \frac{\partial w}{\partial m} < 0 & \text{and} & \frac{\partial \mu_l^*}{\partial m} = -\frac{1}{2} \frac{a}{b} \frac{\partial w}{\partial m} > 0 \\ \frac{\partial \mu_k/w}{\partial m} &= -\frac{1}{2} \frac{a}{b} \frac{ss^* - a(s+s^*)}{(a+am - s^*)^2} < 0 & \text{and} & \frac{\partial \mu_l^*/w}{\partial m} = -\frac{s_l^* - a}{a} \frac{\partial \mu/w}{\partial m} > 0 \end{aligned}$$

Second,

$$\begin{aligned} \frac{\partial w}{\partial s} &= -m \frac{s^* - a(m+1)}{(ms - a - am)^2} < 0, & \frac{\partial \mu_k}{\partial s} &= \frac{s_k - a}{2b} \frac{\partial w}{\partial s} < 0 & \text{and} & \frac{\partial \mu_l^*}{\partial s} = -\frac{a}{2b} \frac{\partial w}{\partial s} > 0 \\ \frac{\partial \mu_k/w}{\partial s} &= -\frac{1}{2} \frac{a}{b} \frac{m}{s^* - a - am} < 0 & \text{and} & \frac{\partial \mu_l^*/w}{\partial s} = -\frac{s_l^* - a}{a} \frac{\partial \mu_k/w}{\partial s} > 0 \end{aligned}$$

Also, since k and k' ($k' \neq k$) are affected the same way, $\frac{\partial \mu_k}{\partial m} = \frac{\partial \mu_{k'}}{\partial m}$ and $\frac{\partial \mu_k}{\partial s} = \frac{\partial \mu_{k'}}{\partial s}$. Finally,

$$\begin{aligned}\frac{\partial w}{\partial s_k} &= -mg_k \frac{s^* - a(1+m)}{(ms - a - am)^2} < 0, & \frac{\partial \mu_i^*}{\partial s_k} &= -\frac{a}{2b} \frac{\partial w}{\partial s_k} > 0 & \text{and} & \frac{\partial \mu_i^*/w}{\partial s_k} = g_k m \frac{s_i^* - a}{2b(s^* - a - am)} > 0, \\ \frac{\partial \mu_k}{\partial s_k} &= -\frac{m(s - g_k s_k) - a(1+m - mg_k)}{2bmg_k} \frac{\partial w}{\partial s_k} < 0 & \text{iff} & m(s - g_k s_k) < a(1+m - mg_k), \\ \frac{\partial \mu_k/w}{\partial s_k} &= \frac{s^* - a(1+m + mg_k)}{2b(s^* - a - am)} < 0 & \text{iff} & s^* < a(1+m + mg_k), \\ \frac{\partial \mu_{k'}}{\partial s_k} &= \frac{\partial \mu_{k'}}{\partial s} \frac{ds}{ds_k} = \frac{\partial \mu_{k'}}{\partial s} g_k = \frac{\partial \mu_k}{\partial s} g_k < 0, \\ \frac{\partial \mu_{k'}/w}{\partial s_k} &= \frac{\partial \mu_{k'}/w}{\partial s} \frac{ds}{ds_k} = \frac{\partial \mu_{k'}/w}{\partial s} g_k = \frac{\partial \mu_k/w}{\partial s} g_k < 0.\end{aligned}$$

Appendix D: Firm mobility and comparative advantages

In this section we examine the symmetric equilibrium when firms decide whether to locate production. We have $s = s^*$, $m = 1$ and $N = N^*$, $F = F^*$ while $\gamma = 1/\gamma^*$ and $\gamma(0) = \gamma^*(0) = 1$.

We first characterize the symmetric equilibrium. One readily gets that $\hat{i} = \hat{j} = \hat{i}^0 \equiv \gamma^{-1}(w)$. Then, the system of equilibrium conditions is equivalent to the system (8)-(10) in Section 3 where $F(z)$ is replaced by $F(z, \hat{i}^0, \hat{i}^0)$. The solution is thus the same $w = 1$ and $\mu = \mu^* \equiv \mu^0$ with $s = 2F(\mu^0, \hat{i}^0, \hat{i}^0)$. Therefore $\hat{i} = \hat{j} = \hat{i}^0 = \gamma^{-1}(1) = 0$ and $\mu = \mu^* = \mu^0$ with $s = 2F(\mu^0, 0, 0)$. Varieties i and j respectively produced at home and abroad.

We now examine the impact of infinitely small exogenous changes in population sizes $\tilde{m} = dm/m$, domestic productivity $\tilde{s} = ds/s$ and cut-off variety $\tilde{\hat{i}} = d\hat{i}/\hat{i}$ on the consumption of high- and low-quality goods. Note that, about this symmetric equilibrium, $d\hat{i} = dw/\gamma'(0)$ and $d\hat{j} = 0$ if $dw \geq 0$ while $d\hat{i} = 0$ and $d\hat{j} = -dw/\gamma'(0)$ if $dw < 0$. In the sequel analyze the changes such that $dw > 0$, $d\hat{i} = dw/\gamma'(0)$ and $d\hat{j} = 0$. The symmetric analysis holds for $dw < 0$.

Total differentiating the identity

$$s = F\left(\frac{\mu}{w}, \hat{i}, \hat{j}\right) + \frac{F^*\left(\mu, \hat{i}, \hat{j}\right)}{w}$$

yields (omitting the arguments for brevity):

$$ds = F'_1 \frac{d\mu}{w} - F'_1 \frac{\mu}{w^2} dw + F'_2 d\hat{i} + \frac{F_1^{*'} d\mu}{w} + \frac{F_2^{*'} d\hat{i}}{w} - \frac{F^*}{w^2} dw$$

where we define

$$F'_1 \equiv \frac{\partial F}{\partial z} > 0, \quad F'_2 \equiv \frac{\partial F}{\partial \hat{i}} < 0 \quad \text{and} \quad F_2^{*'} \equiv \frac{\partial F^*}{\partial \hat{i}}$$

So,

$$ds = (F'_1 + F'_1) \frac{\mu}{w} \tilde{\mu} - (F'_1 \mu + F^*) \frac{1}{w} \tilde{w} + \left(F'_2 + \frac{F_2^{*'}}{w}\right) \hat{i} \tilde{\hat{i}}.$$

where tilda denotes the change in the log of the variables. Define

$$\eta_1 \equiv \frac{zF_1'}{F} > 0, \quad \eta_1^* \equiv \frac{zF_1^{*'}}{F} > 0, \quad \eta_2 \equiv -\frac{\widehat{i}F_2'}{F} > 0 \quad \text{and} \quad \eta_2^* \equiv -\frac{\widehat{i}F_2^{*'}}{F} < 0$$

evaluated at $z = \mu^0$, $w = 1$ and $\widehat{i} = \widehat{j} = \widehat{i}^0$. We can then shortly write $F_1' = \eta_1 F w / \mu$, $F_1^{*'} = \eta_1^* F^* / \mu$, $F_2' = -\eta_2 F / \widehat{i}$ and $F_2^{*'} = -\eta_2^* F^* / \widehat{i}$. Substituting we get

$$\widetilde{s} = (\eta_1 \phi_1 + \eta_1^* \phi_1^*) \widetilde{\mu} - (\eta_1 \phi_1 + \phi_1^*) \widetilde{w} - (\eta_2 \phi_2 + \eta_2^* \phi_2^*) \widetilde{\widehat{i}}$$

where

$$\phi_1 = \frac{F}{F + F^*/w}, \quad \phi_1^* = \frac{F^*/w}{F + F^*/w}, \quad \phi_2 = \frac{F}{F + F^*/w} \quad \text{and} \quad \phi_2^* = \frac{F^*}{F + F^*/w}$$

with $\phi_1 + \phi_1^* = \phi_2 + \phi_2^* = 1$.

Total differentiating

$$s^* = F^* \left(\mu^*, \widehat{i}, \widehat{j} \right) + wF \left(\frac{\mu^*}{w}, \widehat{i}, \widehat{j} \right)$$

yields

$$ds^* = 0 = \mu^* F_1^{*'} \frac{d\mu}{\mu^*} + wF \frac{dw}{w} + \widehat{i} F_2^{*'} \frac{d\widehat{i}}{\widehat{i}} + \mu^* F_1' \frac{d\mu^*}{\mu^*} - \mu^* F_1' \frac{dw}{w} + w \widehat{i} F_2' \frac{d\widehat{i}}{\widehat{i}}.$$

Using the same procedure as above we find

$$0 = \widetilde{\mu}^* (\eta_1^* \phi_1^* + \eta_1 \phi_1) + \phi_1 \widetilde{w} (1 - \eta_1) - (\eta_2^* \phi_2^* + w \eta_2 \phi_2) \widetilde{\widehat{i}}.$$

Total differentiation of

$$mF^* \left(\mu, \widehat{i}, \widehat{j} \right) = wF \left(\frac{\mu^*}{w}, \widehat{i}, \widehat{j} \right).$$

yields

$$\widetilde{m} + \widetilde{\mu} \eta_1^* - \widetilde{\widehat{i}} \eta_2^* = \widetilde{w} + \widetilde{\mu}^* \eta_1 - \widetilde{w} \eta_1 - \widetilde{w} \widetilde{\widehat{i}} \eta_2.$$

At the symmetric equilibrium, we have $w = 1$ and $\mu = \mu^*$. Then, we infer that $\eta_1 = \eta_1^*$, $\eta_2 = -\eta_2^*$, and $\phi_1 = \phi_1^* = \phi_2 = \phi_2^* = 1/2$. Also, we have

$$\widetilde{\widehat{i}} = \widetilde{w} \psi \quad \text{where} \quad \psi \equiv \frac{1}{\gamma'(0)} > 0.$$

Then,

$$\begin{aligned} \widetilde{s} &= \eta_1 \widetilde{\mu} - \frac{1}{2} (\eta_1 + 1) \widetilde{w}, \\ 0 &= \eta_1 \widetilde{\mu}^* + \frac{1}{2} \widetilde{w} (1 - \eta_1), \\ \widetilde{m} &= \widetilde{w} (1 - \eta_1) + \eta_1 (\widetilde{\mu}^* - \widetilde{\mu}) - 2 \widetilde{\widehat{i}} \eta_2. \end{aligned}$$

Solving for $\tilde{\mu}$, $\tilde{\mu}^*$ and \tilde{w} yields

$$\begin{aligned}
\tilde{w} &= \frac{-\tilde{m} - \tilde{s}}{\eta_1 + 2\psi\eta_2}, \\
\tilde{\mu} &= -\frac{(\eta_1 + 1)}{2\eta_1(\eta_1 + 2\psi\eta_2)}\tilde{m} + \frac{\eta_1 - 1 + 4\eta_2\psi}{2\eta_1(\eta_1 + 2\psi\eta_2)}\tilde{s}, \\
\tilde{\mu}^* &= \frac{(1 - \eta_1)}{2\eta_1(\eta_1 + 2\psi\eta_2)}\tilde{m} + \frac{(1 - \eta_1)}{2\eta_1(\eta_1 + 2\psi\eta_2)}\tilde{s}, \\
\tilde{\mu} - \tilde{w} &= -\frac{(1 - \eta_1)}{2\eta_1(\eta_1 + 2\psi\eta_2)}\tilde{m} + \frac{3\eta_1 - 1 + 4\eta_2\psi}{2\eta_1(\eta_1 + 2\psi\eta_2)}\tilde{s} \\
\tilde{\mu}^* - \tilde{w} &= \frac{(1 + \eta_1)}{2\eta_1(\eta_1 + 2\psi\eta_2)}\tilde{m} + \frac{(1 + \eta_1)}{2\eta_1(\eta_1 + 2\psi\eta_2)}\tilde{s}, \\
\tilde{\hat{i}} &= -\frac{\tilde{m} + \tilde{s}}{\eta_1 + 2\psi\eta_2}\psi \quad \text{and} \quad \tilde{\hat{j}} = 0,
\end{aligned}$$

where

$$\psi \equiv \frac{1}{\gamma'(0)} > 0, \quad \eta_1 = \frac{\mu^0 \partial F / \partial z}{F} > 0 \quad \text{and} \quad \eta_2 = -\frac{\hat{i}^0 \partial F / \partial \hat{i}}{F} > 0,$$

evaluated at $(\mu, \hat{i}) = (\mu^0, \hat{i}^0)$. Those solutions naturally match the case without firm mobility when one sets $\eta_1 = \eta$ and $\eta_2 = 0$. Those changes such that $dw > 0$, $d\hat{i} = dw/\gamma'(0)$ and $d\hat{j} = 0$ if and only if $\tilde{m} + \tilde{s} < 0$.

For the case where $dw \leq 0$, $d\hat{i} = 0$ and $d\hat{j} = dw/\gamma'(0)$, we need to swap \hat{i} , η_2 and ψ by \hat{j} , η_3 and ψ^* where $\eta_3 = -(\hat{j}^0/F) \partial F / \partial \hat{j}$ and $\psi^* = 1/\gamma^{*'}(0)$. By symmetry, $\eta_3 = -\eta_2$ and $\psi^* = -1/\gamma'(0) = -\psi$. Therefore, the above expressions of the changes of $(\tilde{w}, \tilde{\mu}, \tilde{\mu}/\tilde{w}, \tilde{\mu}^*, \tilde{\mu}^*/\tilde{w})$ are unchanged because $\psi\eta_2$ is equal to $\psi^*\eta_3$. Only the change becomes

$$\tilde{\hat{j}} = \frac{\tilde{m} + \tilde{s}}{\eta_1 + 2\psi\eta_2}\psi \quad \text{and} \quad \tilde{\hat{i}} = 0$$

This condition applies if $\tilde{m} + \tilde{s} \geq 0$. Table 5 displays the comparative statics for this case where $\tilde{m} > 0$ or $\tilde{s} > 0$.