Mathematics Seminar
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2 October 2007, at 5 pm **Room 3.04 bs**

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Deformation quantization for actions of non-abelian Lie groups and hyperbolic geometry

Abstract

We define a universal deformation formula (UDF) for the actions of the affine group on Frechet algebras. More precisely, starting with any associative Frechet algebra which the affine group acts on in a strongly continuous and isometrical manner, the UDF produces a family of topological associative algebra structures on the space of smooth vectors of the action deforming the initial product. The deformation field obtained is based over an infinite dimensional parameter space naturally associated with the space of pseudo-differential operators on the real line. We will also present some geometrical aspects of the UDF and in particular its relation with hyperbolic geometry.

9 October 2007, at 5 pm **Room 3.04 bs**

Pierre Mathonet
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A review of equivariant and invariant quantizations

Abstract

In the framework of geometric quantization, one can think of a quantization procedure as a linear bijection from the space of classical observables (also called symbols) to a space of differential operators acting on wave functions.

It is known that there is no natural quantization procedure. In other words, the spaces of symbols and of differential operators are not isomorphic as representations of $Diff(M)$.

However, when there is a Lie group G acting on M by local diffeomorphisms, P. Lecomte and V. Ovsienko defined a G-equivariant quantization as a linear bijection from the space of symbols to the space of differential operators that exchanges the actions of G on these spaces.

The first example of such equivariant quantization was given by Lecomte and Ovsienko, who considered the case of the projective group $PGL(m+1,\mathbb{R})$ acting on the manifold $M=\mathbb{R}^m$ by linear fractional transformations.

Together with C. Duval, they then considered the conformal group $SO(p+1,q+1)$ acting on the space $\mathbb{R}^{p+q}$ or on a manifold endowed with a flat conformal structure.

In both cases, they obtained results of existence and uniqueness of equivariant quantizations, up to normalization.

These results were followed by a series of papers dealing with other types of differential operators or other groups of equivariance.

The first generalization of these works was formulated by P. Lecomte. He conjectured the existence of a quantization procedure depending on a torsion-free connection, that would be natural (in all arguments) and that would be left invariant by a projective change of connection : a projectively invariant quantization.

The existence of such a quantization procedure was proved by M. Bordemann for differential operators acting on densities, using the notion of Thomas-Whitehead connections.

These results were generalized by S. Hansoul using Thomas whitehead connections. At the same time, together with F. Radoux, we provided new and easier proofs of the existence results using Cartan connections.

In this talk, I will give a review of equivariant and invariant quantizations starting from the very beginning. Then I will focus on recent developments that use the theory of Cartan connections in order to settle the existence problem of projectively invariant quantizations.