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Nikolai Neumaier

1 Introduction

It is a sad task to remind of the scientific work of one of our most valuable colleague, coworker and truly outstanding friend: Nikolai Neumaier. His life and his scientific career ended after a long but in the end hopeless fight in 2010: though struggling with cancer for almost two years, Nikolai kept working till the very last days of his only too short life.

In this note we try to collect the main results and achievements of Nikolai's scientific oeuvre, knowing well that a personality like his is much too rich to be recalled or described in such an insufficient way. All his research centered around the mathematical description of quantization using the notions of star products and deformation theory. Instead of commenting on his papers separately, we try to group them under certain topics: quantization of cotangent bundles, classification results, quantization and symmetries, as well as applications of deformation quantization to various topics in mathematical physics.

When Nikolai joined the little deformation quantization group around Martin Bordemann and Stefan Waldmann within the research division chaired by Hartmann Römer at the Physics department in Freiburg University in 1997 to work on his Master thesis, he already had a reputation as an excellent physics student who also was mathematically well-trained, in particular in differential geometry. He also seemed to have liked the unique atmosphere and 'style' in that group: in those days those unfortunate mathematical physicists having stranded in a physics department constantly had to defend their 'funny' approaches as still being 'physical' while on the same time feeling of course superior to their true physics colleagues: for the former 'understanding' meant to establish deep truths about physics in utmost mathematical precision, whereas the latter were despised since they were apparently content with parroting their teachers' half-wisdoms. Of course this was a desperate battle doomed to be lost –there is presently no more mathematical physics in Freiburg–, but on the other hand highly entertaining and enjoyable, in particular for people having a refined sense of humour like Nikolai. Very quickly Nikolai had developed his own characteristic style of research: in his problem solving mode he would silently listen with great attention, and in case the problem seemed interesting, murmuring "Ich glaube, das muss man einfach mal tun" he would dive away and resurface, say two weeks later with the problem completely solved. Nikolai always needed a silent retreat where he could concentrate on his work: for a long time he did everything to keep his office on the top of the ten storey physics building within a rather draughty, in the wintertime almost unheatable penthouse. Nikolai's theory making mode functioned similarly: carefully studying interesting literature on which he –rather occasionally– had very precise and never trivial questions, he would finally emerge from his retreat with an interesting problem to look at. He found both subjects of his PhD-thesis in his own very autonomous way.

Beside his research Nikolai will always be remembered as a talented university teacher: already during is PhD thesis he supervised exercise groups at the physics department of the University of Freiburg. His group was known to be challenging and sometimes certainly more difficult than others, Nevertheless, it was always overcrowded since the students liked his clear way of explaining, his patience with everybody as long as he was willing to learn, and his supportive help whenever needed. Later on in his career, his own lectures were well accepted by students and not few profited essentially from his clever choice of topics. It was certainly one of the very characteristics of Nikolai that he intended to make his point clear without compromises: in his lectures, he had the very ambition to present every proof in such a detail that students really were able to understand. A cheap excuse like "this is easy to see" was never heard from him. His teaching had an enormous importance for him, in particular during his illness: Only a few weeks before his death, when he was not able to give his lecture personally, he still prepared notes for the course such that Svea Beiser could substitute him and give the lecture at his place.

Above all, and everybody knowing Nikolai will agree, he was a very honourable person; modest and sometimes almost shy, but generous and helpful in any possible way.

2 Quantization of Cotangent Bundles

After starting as a diploma student in the group of Hartmann Römer in Freiburg, Nikolai was given the task to investigate the relation of a standard-ordered quantization based on a chosen connection on a configuration space manifold Q with the still to be found Weyl-ordered analog. This was part of a project about the deformation quantization of cotangent bundles using a connection and a smooth volume density as geometric ingredient. Very typical for him, he disappeared for a couple of days and came back with the complete solution. He computed the adjoint of the standard-ordered quantized symbol with respect to a not necessarily covariantly constant integration density explicitly in terms of his operator N, being the exponential of a d'Alembert-like operator on the cotangent bundle T^*Q associated to the canonical, maximally indefinite metric originating from the choice of the connection on Q, see eqn (20) in [3] and eqn (1.8) of [2]. From there on it was clear that Nikolai was not just another student to be supervised, but that we have found a collaborator on equal footing. In the course of his diploma thesis [13] this resulted in two papers on star products on cotangent bundles [3, 4].

Throughout his career, Nikolai came back to cotangent bundles once in a while and transferred many ideas and techniques to other more general situations. Started as a follow-up project we joined forces with Markus Pflaum in [2], where various generalizations including magnetic terms on cotangent bundles (in order to incorporate the Aharonov-Bohm effect and magnetic monopoles) as well as analytic aspects were investigated. One main theme of this paper was to view a (global) symbol calculus for differential operators as a representation of a star product algebra and thus the study of the representation theory of the star product algebras appeared to be of crucial importance. Nikolai came back to this point of view at several other occasions.

In the work [10], the interplay of quantization on cotangent bundles on the one hand and phase space reduction of cotangent bundles with respect to symmetries of the configuration space on the other hand were investigated. The result was a very explicit construction of a star product on the cotangent bundle of the quotient (with respect to a symplectic form involving magnetic terms in the case of a non-vanishing value of the momentum map) supporting the meta-theorem of "quantization commutes with reduction".

Also the paper [19] on the deformation quantization of Lie algebroids can be seen as originating from Nikolai's early investigations on cotangent bundles: here the essence of the Fedosov construction used in[3, 4] is distilled in such a way that it becomes applicable to the dual of a Lie algebroid as a generalization of a cotangent bundle. Even though the Poisson structure is far from being symplectic, the homogeneity arguments used in the case of cotangent bundles can be transferred to this situation.

3 Classification Results

Being part of his PhD thesis [14], Nikolai used the Fedosov construction of star products on symplectic manifolds and adapted it to various more specific situations. In particular, the question was raised how the different classification schemes of symplectic star products by formal series in the second de Rham cohomology fit together. Transferring ideas of Deligne (1995) and in particular the ν-Euler derivations from Gutt and Rawnsley (1999) to the Fedosov framework, Nikolai provided an elegant proof for a fact stated by Deligne, namely that the Deligne class (now called the characteristic class $c(\star)$) of a symplectic star-product coincides with the Fedosov class appearing in the Fedosov construction, up to a trivial and convention-depending rescaling [15]. Moreover, he showed how certain additional requirements important in physics, like the existence of a *-involution, can be understood in terms of corresponding properties of the characteristic class $c(\star)$.

On Kähler manifolds there were at that time two essentially different constructions: the local & glueing construction by Karabegov and the Fedosov-inspired construction using the Kähler connection. In both constructions a characteristic class was implementable, and Nikolai showed how the two resulting parametrizations by formal series of closed two-forms of type $(1, 1)$ match together [16].

In a very remarkable paper [17] deformation quantizations of the convolution algebra associated to a proper étale Lie groupoid were studied. This is one of the main tools to understand the deformation quantization of symplectic spaces with not too bad singularities like symplectic orbifolds. Among many other things, a major result is the classification of trace functionals for this quantization in terms of cyclic (co)homology computations.

Still within the main theme of classification results one can also locate the work [18]. Here the setting was again the realm of Kähler manifolds on which particular Morita equivalence bimodules were constructed. The bimodule structure has the property of "separation of variables". Moreover, the relation between the canonical Wick, Weyl, and anti-Wick star product on a Kähler manifold are revealed.

4 Symmetries and Quantization

Symmetries in form of Lie group actions or Lie algebra actions attracted Nikolai's imagination from the very first steps, still in his diploma thesis. There he related the general quantization of a cotangent bundle with connection for the case of a Lie group $Q = G$ with Gutt's construction of a star product on T^*G, showing that the two star products coincide once the $\frac{1}{2}$-commutator connection is chosen. Apart from his thesis these results have appeared in [3].

Later on, together with Michael Müller-Bahns he investigated invariant star products on symplectic manifolds with invariant connection by means of the Fedosov construction. In [12, 11] they found several existence and classification results on quantum momentum maps. The ideas also influenced the work on reduction [10].

Being already seriously ill, Nikolai kept working on his last project, the classification of invariant star products on symplectic manifolds up to equivariant Morita equivalence. The resulting paper will appear now posthumous [8].

Encoding the gauge symmetries in gauge field theory, principal bundles are the core ingredient. For many reasons one is interested in finding analogs of principal bundles in a more noncommutative setting. In [5] the deformation quantization of principal bundles over a Poisson manifold with star product was defined to be a (right) module deformation of the functions on the total space, invariant under the principal action of the structure group. More generally, a deformation quantization of a surjective submersion was defined as a (right) module deformation. Then the main result is that such a deformation always exists and is unique up to equivalence. The main step in the proof is an explicit computation of the relevant Hochschild cohomologies: they simply vanish, making the deformation problem trivially solvable.

5 Applications to Mathematical Physics

Even though this section title does not seem to be very specific, it should emphasize that it always was Nikolai's main concern not just to produce fine and interesting mathematical results, but also mathematics relevant for physics. In this sense he was a true mathematical physicist. His main motivation for his work came from the quest of a good mathematical description of the quantization problem. In this spirit several papers can be seen as applications of the techniques from deformation quantization to more physical questions.

The papers [6, 7] focus on the idea that the spacetime we are living in is not a smooth manifold at all length scales. Instead, moving to smaller scales it becomes "noncommutative" and thus a much more complicated object. Inspired by noncommutative geometry, many people have tried to formulate a noncommutative spacetime using, among other techniques, star products. In [6, 7] a C^*-algebraic approach to locally noncommutative spacetimes is proposed and investigated. As a follow-up project in [9] the deformation of states was studied: for a C^*-algebraic deformation quantization a la Rieffel it is shown that a state, i.e. a positive linear functional, of the undeformed algebra can be deformed into a state for all the deformed algebras, depending continuously on the deformation parameter $\hbar$. From a physical point of view this is a crucial observation as only this way a classical limit is consistent: every classical state is the classical limit of (non-unique) quantum states.

Finally, in [1] open systems are studied by techniques of deformation quantization. Here an open system is understood as arising from a Cartesian product of the system and the "bath" with a coupled dynamics. The important question was then how the positivity of states and the complete positivity of the time evolution can be restored by deformation. It turns out that there is an affirmative answer.

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