General Mathematics Seminar

Of the

University of Luxembourg

In cooperation with the

Luxembourg Mathematical Society

January 2011

Tuesday, January 4, 2011, at 17:00 **Campus Kirchberg, room A02**

Walter van Suijlekom

(Radboud University, Nijmegen)

Gauge theories and noncommutative manifolds

Abstract:
In this talk, we will discuss some aspects of the intrinsic gauge theoretical nature of noncommutative manifolds. Following Connes, we describe a noncommutative (Riemannian, spin) manifold by its fundamental class in K-homology. Among other functional analytical data, such a K-cycle consists of a (noncommutative) C*-algebra. As a consequence of noncommutativity, there might exist non-trivial inner automorphisms; these will be referred to as gauge transformations.

The key example that motivates this terminology from physics is when one replaces the algebra of functions on a manifold by matrix-valued functions. The resulting Morita equivalence describes ordinary Yang-Mills theory as formulated in terms of vector bundles and connections thereon. If time permits, we will consider a second class of examples, so-called toric noncommutative manifolds.

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Tuesday, January 25, 2011, at 17:00 **Campus Kirchberg, room A02**

Arkady Onishchik
(Yaroslavl University, Russia)

Homogeneous complex supermanifolds associated to compact Hermitian symmetric spaces

Abstract:

I would like to discuss the following classification problem: given a complex flag homogeneous space $M = G/P$ (G is a semi-simple complex Lie group, P its parabolic subgroup), to describe all homogeneous complex supermanifolds $(M;\mathscr{O})$ with reduction M. In the case when $M = Gr_{4;2}$ is the Grassmann manifold of 2-planes in $\mathbb{C}^4$ this problem was formulated by Yu.I. Manin; it was motivated by certain physical models. He also gave an example of a non-split homogeneous complex supermanifold with this reduction; this is the so-called Π- symmetric supergrassmannian $\Pi Gr_{4|4;2|2}$. My goal is to discuss certain general approaches to this classification problem and to formulate certain results in the case when M is an irreducible compact Hermitian symmetric space.

First, I consider the case when the supermanifold $(M;\mathscr{O})$ is split and $\mathscr{O}$ is determined by the homogeneous vector bundle over M induced by a representation ϕ of P. If ϕ is completely reducible, then homogeneous supermanifolds of this sort may be described in terms of the highest weights of ϕ.

The most difficult is the case of a non-split supermanifold $(M;\mathscr{O})$. There is the following conjecture: if, in our situation, the retract of this supermanifold is determined by an irreducible representation of P, then this retract is isomorphic to $(M;\Omega)$, where Ω is the sheaf of holomorphic differential forms on M. It is also possible to describe non- split homogeneous supermanifolds with this retract. The conjecture is proved for the following symmetric spaces:$M = Gr_{n;k}, 3 \leq k \leq n-k; Gr_{2n+1;2}, n \geq 2;$ $Gr_{4;2}, Gr_{6;2}; Sp_{2n}/U_n, n \geq 2; E_6/D_5 \times T; E_7/E_6 \times T$.