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Regulation and Rational Banking Bubbles in Infinite Horizon

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October, 2016

Abstract

This paper develops a dynamic stochastic general equilibrium model in infinite horizon with a regulated banking sector where stochastic banking bubbles may arise endogenously. We analyze the conditions under which stochastic bubbles exist and their impact on macroeconomic key variables. We show that when banks face capital requirements based on Value-at-Risk, two different equilibria emerge and can coexist: the bubbleless and the bubbly equilibria. Alternatively, under a regulatory framework where capital requirements are based on credit risk only, as in Basel I, bubbles are explosive and, as a consequence, cannot exist. The stochastic bubbly equilibrium is characterized by positive or negative bubbles depending on the tightness of capital requirements based on Value-at-Risk. We find a maximum value of capital requirements under which bubbles are positive. Below this threshold, the stochastic bubbly equilibrium provides larger welfare than the bubbleless equilibrium. In particular, our results suggest that a change in banking policies might lead to a crisis without external shocks.

JEL classification: E2; E44; G01; G20

Key words: Banking bubbles; banking regulation; DSGE; infinitely lived agents; multiple equilibria; Value-at-Risk.

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1 Introduction

The Great Recession of 2007-2009 has highlighted the importance of the banking sector in the worldwide economy and its role in the propagation of the crisis. Valuation and liquidity problems in the U.S banking system are recognized to be a cause of the crisis (Miao and Wang, 2015). In particular, Miao and Wang (2015) argue that changes in agents' beliefs about stock market value of banks are suspected to explain sudden financial market crashes.

As a consequence, there has been a greater awareness among both academics and policy makers about the failure of banking regulation in preventing crises. The Basel committee on Banking Supervision was created in 1973 "to enhance understanding of key supervisory issues and improve the quality of banking supervision worldwide".[1] They released the first Basel Accord, called "Basel I" in 1988. The goal of Basel I was to create a framework for internationally active banks, in particular seeking, to prevent international banks from growing without adequate capital. Therefore, the committee imposed minimum capital requirements which were calculated based on credit risk weights of loans. Credit risk weights take into account possible losses on the asset side of a bank's balance sheet. The idea was that banks holding riskier assets had to hold more capital than other banks in order to ensure solvency. This approach has been criticized by researchers and regulatory agencies because it only considers credit risk and does not encompass market risk.[2] Market risk refers to the risk of losses from changes in market prices, which increases banks' default risk. The Basel committee has recognized this problem and released the Basel II Capital Accord.[3] This new accord also considers market values into the banking regulation framework in order to take into account market risk of the trading book. It allows banks to use an internal model based on *Value-at-Risk* to quantify their minimum capital requirements. The idea of capital requirements based on Value-at-Risk is to impose a solvency condition for banks which requires that the maximum amount of

[1] For more details, see The Basel Committee overview, https://www.bis.org/bcbs/.

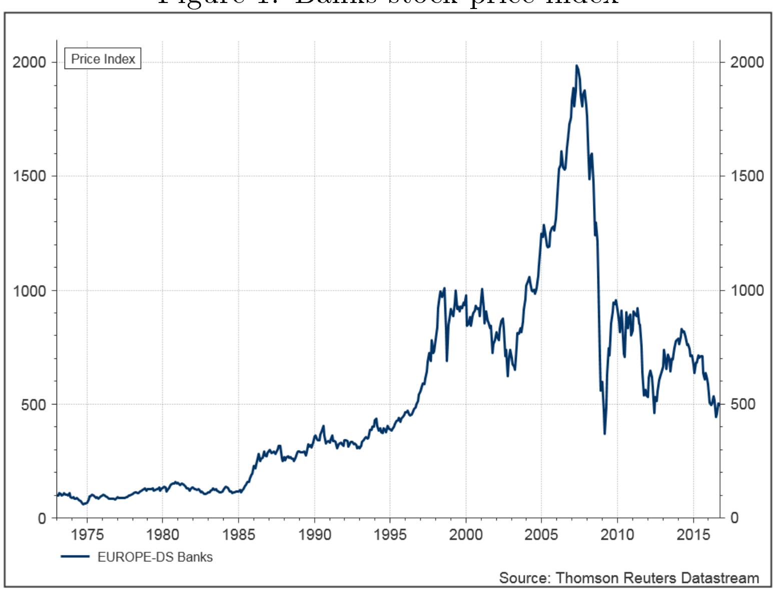
[2] For example, Dimson and Marsh (1995) analyze the relationship between economic risk and capital requirements using trading book positions of UK securities firms. They find that the Basel I approach leads only to modest correlation between capital requirements and total risk.

[3] See Basel Committee on Banking Supervision (2004).

debt that banks can hold, do not exceed the market value of banks assets in the worst case scenario.[4]

The aim of this study is to analyze the impact of banking regulation and in particular, Basel II, on the development of *stochastic bubbles* on banks' stock prices. A stochastic bubble on a bank's stock price is defined as a temporary deviation of the bank's stock price from the bank's fundamental value. Figure 1 plots the price index of 168 banks listed in Europe from 1973 to 2016. It shows that the price index has sharply increased from 2004, which coincides with the release of Basel II. Therefore, we suspect that the Basel II regulatory framework has allowed the existence of bubbles in the banking sector.

Figure 1: Banks stock price index



This paper also focuses on the effect of bubbles on macroeconomic key variables. Following Blanchard and Watson (1982) and Weil (1987), stochastic bubbles are bubbles that have an exogenous constant probability of bursting. Once they burst, they do not reemerge. We develop a dynamic stochastic general equilibrium model with three types of infinitely lived agents, banks, households, and

[4]Basel III, released in 2011, also proposes to use the Value-at-Risk to measure the minimum capital requirement. The difference with Basel II is that it is amended to include a Stressed-Value-at-Risk (SVaR). It aims at reducing pro-cyclicality of the market risk approach and insures that banks hold enough capital to survive long periods of stress.

firms, as well as a regulatory authority. Banks raise funds by accumulating net worth and demanding deposits (supplied by households) to provide loans to firms. Firms produce the consumption goods, invest and are subject to productivity shocks. The regulatory authority imposes two banking regulations. The first requires that banks keep a fraction of deposits as reserves. These reserves cannot be used to invest in loans (risky assets). The second measure requires banks to have an upper limit on the quantity of deposits based on Value-at-Risk capital requirements.

We show that bubbles emerge if agents believe that they exist. Thus, expectations of agents are self-fulfilling. Results suggest that when banks face capital requirements based on Value-at-Risk, two different equilibria emerge and can co-exist: the bubbleless and the stochastic bubbly equilibria. Capital requirements based on Value-at-Risk allow bubbles to exist. In contrast, under a regulatory framework where capital requirements are based on credit risk only, as in Basel I, banking bubbles are explosive and as a consequence cannot exist. The stochastic bubbly equilibrium before the bubble bursts is characterized by positive or negative bubbles depending on the tightness of capital requirements. A positive (resp. negative) bubble is a "persistent" overvaluation (resp. undervaluation) of the banking stock price. We find a maximum value of the capital requirement based on Value-at-Risk under which bubbles are positive. Below this value and until the bubble bursts, the stochastic bubbly equilibrium provides larger welfare than the bubbleless equilibrium. The intuition is that, when agents consider that a bubble exists, lower capital requirements lead to optimistic beliefs about bank valuation. Bubbles allow banks to relax the capital requirement constraint, and thus banks demand more deposits and make more loans. This effect reduces the lending rate and provides higher welfare. Profits of banks rise which increases the value of banks. As a consequence, initial beliefs about the value of banks are realized. In contrast, above this maximum capital requirement, bubbles are negative leading to a credit crunch and thus, reduce welfare. Therefore, our model shows that a change in regulation might lead to a crisis, by shifting the economy from higher to lower welfare. This can explain the existence of crises without external shocks. We also show that the equilibrium with positive stochastic bubbles exists if the probability that bubbles collapse is small. This is consistent with Weil

(1987) and Miao and Wang (2015). Moreover, as in Miao and Wang (2015), our results suggest that after the bubble bursts, consumption, welfare, and output fall. Consequently, a change in beliefs also modifies the equilibrium, from higher to lower welfare. Finally, we simulate impulse response functions to a negative productivity shock. The results show that bubbles do not amplify the effect of a negative productivity shock on the economy.

This paper is related to two strands of literature. First, it is related to the literature on banking regulation. Indeed, there is a very recent move towards macroeconomic models incorporating a banking sector (de Walque et al., 2010; Gertler and Kiyotaki, 2011; Gertler and Karadi, 2011; Gertler et al., 2012; He and Krishnamurthy, 2012; Brunnermeier and Sannikov, 2014). In particular, we focus on banking regulation and their impact on macroeconomic variables as in Dib (2010) and de Walque et al. (2010). As in Dangl and Lehar (2004) and Tomura et al. (2014), we study the impact of Value-at-Risk banking regulation on the economy. Dangl and Lehar (2004) compare the effect of capital regulation based on Basel I and Value-at-Risk internal model approach. They find that the latter regulation reduces risk in the economy. Tomura et al. (2014) introduce asset illiquidity in a dynamic stochastic general equilibrium model and show that capital requirements based on Value-at-Risk can lead banks to adopt macro-prudential behavior. We contribute to this literature by showing that capital requirements based on Value-at-Risk allow bubbles to exist. In contrast, under a regulatory framework where capital requirements are based on credit risk only, as in Basel I, bubbles are explosive and as a consequence cannot exist.

Second, this study is related to the literature on the existence and the effect of rational bubbles in infinite horizon and, in particular, on stochastic bubbles. The literature on the existence of bubbles in general equilibrium models with infinitely lived agents is scarce and marked with few important contributions (Miao, 2014). Therefore, the understanding of financial bubbles in infinite horizon models is still under explored. Tirole (1982) shows that bubbles under rational expectations with infinitely lived agents cannot exist. In addition, Blanchard and Watson (1982) argue that "the only reason to hold an asset whose price is above its fundamental value is to resell it at some time and to realize the expected capital gain. But if all agents intend to sell in finite time, nobody will be holding the asset thereafter, and this cannot be an equilibrium". Such behavior implies that agents over save so that they do not consume everything they could. This cannot be an equilibrium since agents would deviate to increase their consumption levels and, thus, the so called *transversality condition* (TVC) is not satisfied. In contrast, Kocherlakota (1992) demonstrates that bubbles may exist in an infinite horizon general equilibrium model with borrowing or wealth constraints. These constraints limit the agent arbitrage opportunities by introducing some portfolio constraints. Foremost, Kocherlakota (2008) shows that equilibrium in which the asset price contains a bubble can coexist with the bubbleless equilibrium in the presence of debt constraints. The only difference between the two states (bubbles and no bubbles) is that the bubbly one modifies the debt limit. The author calls this result the “bubble equivalence theorem”. We contribute to this literature by showing that banking bubbles may emerge with banking regulation based on Value-at-Risk in an infinite horizon general equilibrium framework.

Our study is mostly related to Miao and Wang (2015). They insert an endogenous borrowing constraint and show that bubbles can emerge in an infinitely lived general equilibrium framework without uncertainty. Bubbles are introduced through the bank problem. We borrow the same methodology to introduce bubbles. Nevertheless, our model contrasts with Miao and Wang (2015) regarding four major characteristics. First, our key idea is to introduce banking regulation in an infinitely lived agent model to analyze whether stochastic bubbles can arise. Second, our model is a stochastic general equilibrium. In contrast, Miao and Wang (2015) consider a deterministic model. Third, negative bubbles as well as positive bubbles can arise, while they only assume positive bubbles. Fourth, they consider an agency problem to justify a minimum dividend policy that links dividends to net worth. Our model does not impose a dividend policy.

The present paper is organized as follows. Section 2 presents the model. Section 3 and section 4 analyze, respectively, the bubbleless and the stochastic bubbly general equilibrium. Section 5 compares both equilibria. Section 6 presents the calibration, explores local dynamics and compares impulse response functions to a negative productivity shock for both equilibria. Finally, the last section concludes.

2 Model

We consider an economy with three types of infinitely lived agents, banks, households, and firms, as well as a regulatory authority. In this model, banking bubbles can arise. They emerge only if agents believe that banks' stock prices contain a bubble. The bubble is, thus, self-fulfilling. Banks, households, and firms are respectively represented by a continuum of identical agents of mass one. Households are shareholders of banks and owners of firms. It is assumed that banks have the necessary technology and knowledge to engage in lending activity while households do not. Thus, the latter do not lend directly to non-financial firms and have recourse to banks. At the end of each period, banks raise funds internally, using net worth, and externally, by taking deposits from households. Using raised funds, they lend to firms which produce consumption goods. In the model, a bubble is introduced through the bank problem, as in Miao and Wang (2015). We consider a bubble with an exogenous probability of burst, i.e., a *stochastic bubble* as in Blanchard and Watson (1982). Although a bubble can only arise if agents believe in its existence, it is not an agent choice. Agents are "bubble takers". The optimization problem of each agent is presented in this section.

2.1 Households

Households are represented by a continuum of identical agents of unit mass. Each household starts with an initial endowment of stocks s_0 and deposits D_0. At each period t, it receives net profits π_t generated by firms, it chooses its optimal consumption c_t, amount of stocks s_{t+1}, and deposits D_{t+1} for the next period. It also receives dividends d_t from the shares s_t it owns, sells its shares at price p_{t+1} and obtains an interest rate r_t on the amount deposited D_t in the previous period. There is no uncertainty on savings and thus r_t is the risk-free interest rate. We assume that preferences of households are represented by a linear utility function in consumption. Given their budget constraint (1), each household chooses the optimal amount of shares, deposits and consumption $\{s_{t+1}, D_{t+1}, c_t\}_{t=0}^{\infty}$ that maximizes its expected lifetime linear utility. Each household optimization problem is defined as follows:

$$Max_{\{s_{t+1},D_{t+1},c_t\}_{t=0}^{\infty}} E_t \sum_{t=0}^{\infty} \beta^t c_t,$$

subject to

$$D_t (1 + r_t) + s_t (p_{t+1} + d_t) + \pi_t = D_{t+1} + c_t + s_{t+1} p_{t+1}, \tag{1}$$

where $\beta \in]0,1[$ is the discount factor and E_t is the expectation operator.

The first order conditions with respect to D_{t+1} and s_{t+1}, are given by

$$\beta E_t (1 + r_{t+1}) = 1, \tag{2}$$

$$p_{t+1} = \beta E_t (d_{t+1} + p_{t+2}). \tag{3}$$

The combination of (2) and (3) gives the households no arbitrage condition, $E_t (d_{t+1} + p_{t+2}) / p_{t+1} = E_t (1 + r_{t+1})$. This last condition states that the return on stocks is equal to the return on deposits. If it is met, households are indifferent between both types of assets and both are held in the portfolio of agents. However, if this condition is not satisfied, the optimal solution of households yields to a corner solution, thus, only stocks or only deposits are held, depending on which has the highest return.

Since the optimization problem has an infinite horizon, consider also the transversality condition:

$$lim_{t \to \infty} \beta^t p_t s_t = 0. \tag{4}$$

Condition (4) ensures that the household spends all its budget and thus, does not hold positive wealth when $t \to \infty$. It is a necessary condition for an optimum choice of the household. Tirole (1982) shows that bubbles under rational expectations with infinitely lived agents cannot exist since the transversality cannot be satisfied. However, in our framework, banking bubbles satisfy this condition and therefore, may exist.

2.2 Firms

Firms are represented by a continuum of identical producers of unit mass. Each firm starts with an amount of loans L_0 to buy its initial capital K_0. Firms are subject to productivity shocks. The shock process is defined by an AR(1) process such that $A_t = A_{t-1}^{z_A} \exp(u_t)$, where z_A is a strictly positive persistence parameter and u_t is a normally distributed productivity shock with mean 0 and variance σ_z^2. After the shock, in each period t, firms produce y_t using capital K_t bought in the last period and reimburse their loans with interests r_t^l such that the total reimbursement is $L_t\left(1+r_t^l\right)$. Then, they distribute net profits to households and choose their optimal amount of total loans and capital for the next period $\{L_{t+1}, K_{t+1}\}_{t=0}^{\infty}$ to maximize their future expected discounted profits subject to their budget constraint (5) and the capital constraint (6). Note that we consider capital that fully depreciates. Each firm optimization problem is defined as follows:

$$Max_{\{L_{t+1},K_{t+1}\}_{t=0}^{\infty}} E_t \sum_{t=0}^{\infty} \beta^t \pi_t,$$

subject to

$$\pi_t = y_t - K_t\left(1+r_t^l\right), \tag{5}$$

$$y_t = A_t K_t^{\psi},$$

$$K_{t+1} = L_{t+1}, \tag{6}$$

$$\pi_t \geq 0 \text{ and } L_t, K_t > 0,$$

where $\psi \in]0,1[$ is the output elasticity of capital. Using the Lagrange method, the interior solution of the first order condition with respect to L_{t+1} is given by:

$$\psi E_t\left(A_{t+1} L_{t+1}^{\psi-1}\right) = E_t\left(1+r_{t+1}^l\right). \tag{7}$$

In the optimum, (7) shows that the marginal product of capital is equal to the marginal cost of loans.

2.3 Banks

The banking sector is represented by a continuum of identical banks of unit mass. To provide loans L_{t+1} to firms, banks raise funds by accumulating net worth N_{t+1} and demanding deposits D_{t+1}. The regulatory authority imposes that banks keep a fraction $\phi \in [0,1[$ of deposits as reserves[5]

$$R_t \equiv \phi D_t. \tag{8}$$

Each bank has a balance sheet composed of deposit D_t and net worth N_t on the liability side and of loans L_t and reserves R_t on the asset side such that

$$R_t + L_t = N_t + D_t. \tag{9}$$

Thus, at the end of each period t, each bank accumulates net worth using profits from assets earned in t net of deposit repayments and dividends. Let r_t^l be the lending rate earned in t and r_t the risk-free interest rate paid in t, so that

$$N_{t+1} = \left(1 + r_t^l\right) L_t + R_t - D_t \left(1 + r_t\right) - d_t - \mathbb{C}_t, \tag{10}$$

where $\mathbb{C}_t = \tau N_t$ represents operational costs paid by banks such as accounting and legal fees and management costs. The parameter $\tau \in]0,1]$ is the share of operational costs in net worth. One can think about initial public offering fees paid to a third party, for example to a business attorney or business service companies, to get listed on financial markets. Indeed, banks often use a third party such as large business service companies (KPMG, Deloitte) to prepare the legal and accounting side of public offerings. Specialized firms ensure that regulatory and legal compliance are met.

Banks are also subject to capital requirements based on Value-at-Risk as recommended by the Basel committee in Basel II.[6] This regulation imposes that banks hold a minimum level off capital which is calculated with the aim of avoiding banks becoming insolvent. The objective of the regulator is to preserve a

[5]Note that the reserve requirement ϕ is not crucial for the model nor for the bubble existence. However, it is of interest as it allows the derivation of additional policy implications.

[6]See the BIS publication, the First Pillar Minimum Capital Requirements, http://www.bis.org/publ/bcbs107.htm

safety buffer, such that the market value of banks' assets VA_t is sufficient to repay depositors. The market value of assets is given by

$$\mathrm{VA}_t = V_t(N_t) + D_t,$$

where $V_t(N_t)$ is banks' equity value. Therefore, the regulator imposes a solvency condition which requires that the maximum amount of deposits banks can hold does not exceed the market value of banks assets in the worst case scenario such that

$$D_t \leq (1-\mu)\,\mathrm{VA}_t,$$

where $\mu \in [0,1[$ is a regulatory parameter which captures the loss in market value of assets in the worst case scenario, as motivated by the Value-at-Risk regulation. This regulation, based on market values, is the same as in Dangl and Lehar (2004). The above equation is thus equivalent to

$$D_t \leq \eta V_t(N_t),$$

where $\eta = (1-\mu)/\mu > 0$ is the Value-at-Risk regulation parameter. It represents the maximum allowed leverage ratio in market value. We show in Appendix A that without capital requirements, if

$$\tau\beta(1-\phi) > \phi(1-\beta), \tag{11}$$

banks always hold the maximum amount of deposits. Indeed, when the marginal benefit from holding deposits exceeds its marginal cost, banks always want more deposits. From now on, we consider that (11) is always satisfied. Therefore, the above constraint always binds and becomes

$$D_t = \eta V_t(N_t). \tag{12}$$

For low values of η, the regulation is severe. Indeed, the amount of authorized deposits that banks can hold compared to banks' value is low. However, for high η, the regulation is considered as lenient.

The aim of our framework is to model the existence of stochastic banking bubbles as in Blanchard and Watson (1982), Weil (1987) and Miao and Wang (2015). In period t, agents may believe in a bubble or not. If agents do not believe a banking bubble exists in period t, a bubble can never emerge. In what follows, first, we present the problem of banks when agents do not believe a bubble exists. We then present the problem of banks when agents believe that it exists. In this latter case, following Blanchard and Watson (1982), we consider that the bubble may burst in the future with a probability $\xi \in]0,1[$. Note that once the bubble bursts, it never reappears.

Bubbleless path

At the end of period t, each bank chooses its optimal net worth to accumulate for next period $\{N_{t+1}\}$ in order to maximize its current dividends and expected present value of future dividends subject to the reserve requirement (8), the balance sheet (9), the budget constraint (10) and the capital requirement (12). If agents do not believe a bubble exists, the value of the bank in period t is denoted $V_t^*(N_t)$. The bank problem can be summarized by the following Bellman equation:

$$V_t^*(N_t) = Max_{\{N_{t+1}\}} \left\{ d_t + \beta E_t \left[V_{t+1}^*(N_{t+1}) \right] \right\},$$

subject to

$$d_t = \left(1 + r_t^l\right) N_t + D_t \left[r_t^l (1-\phi) - r_t \right] - \tau N_t - N_{t+1}, \quad (13)$$

$$D_t = \eta V_t^*(N_t), \quad (14)$$

$$N_t, D_t \geq 0 \text{ for all } t. \quad (15)$$

We show in Appendix B that the solution of the above maximization problem gives us the following form for the value function:

$$V_t^*(N_t) = q_t^* N_t, \quad (16)$$

where $q_t^* \geq 0$ is the marginal value of net worth. It can also be interpreted as the Tobin Q (Tobin, 1969). Define the bank's stock price in $t+1$ by

$$p_{t+1} = \beta E_t \left[V_{t+1}^* \left(N_{t+1} \right) \right].$$

Proposition 1. *When agents do not believe a bubble exists, the solution of each bank maximization problem is given by the following system of equations.*

$$E_t \left(q_{t+1}^* \right) = \frac{1}{\beta}, \tag{17}$$

$$q_t^* = \left(1 + r_t^l - \tau \right) + \eta q_t^* \left[r_t^l \left(1 - \phi \right) - r_t \right]. \tag{18}$$

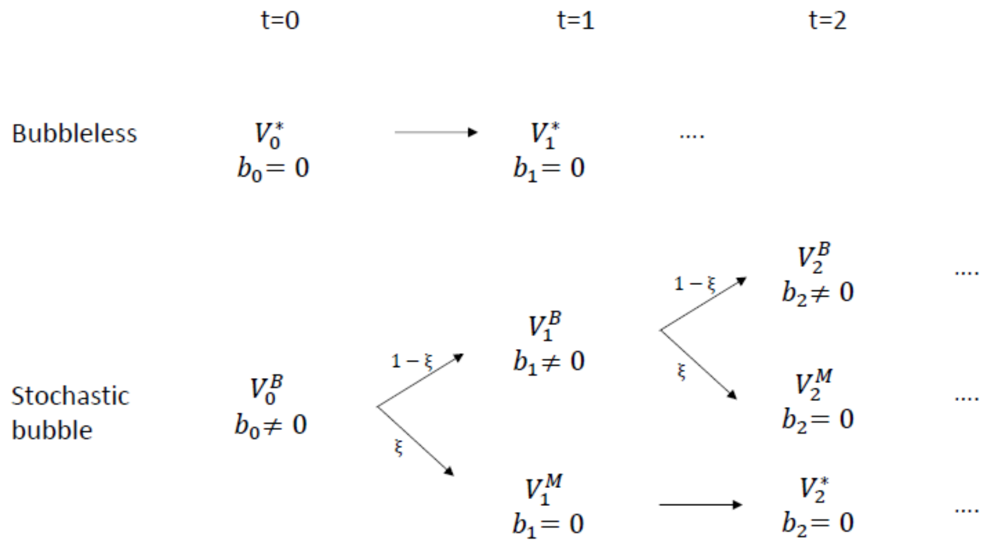
Proof of Proposition 1 is presented in Appendix B.

When agents do not believe a bubble exists, the expected marginal value of net worth given by (17) is constant. This comes from the fact the bank is risk-neutral. Thus, by increasing one unit of net worth today, the bank gets the expected discounted marginal value of net worth. Equation (18) shows that an additional unit of net worth today gives the discounted return due to the increase in loans minus operational costs. It also allows the bank to relax the constraint by taking η units of additional deposits (see equation (12)). Then, the bank earns an additional return of $\left[r_t^l \left(1 - \phi \right) - r_t \right]$. Using (17) and (18), results show that the lending rate is also constant, which is consistent with the risk neutrality assumption.

Bubbly path

When agents believe that a bubble exists in period t, the bank's value $V_t^B \left(N_t \right)$ contains a bubble $b_t \neq 0$. There exists a probability $\xi \in]0, 1[$ that the bubble bursts in $t+1$ such that $b_{t+1} = 0$ and thus, that the bank's value becomes $V_{t+1}^M (N_{t+1})$. Note that following Blanchard and Watson (1982), we assume that once the bubble bursts, it never reappears. Therefore, the bank's value can take two different possible values in $t+1$: $V_{t+1}^B \left(N_{t+1} \right)$ or $V_{t+1}^M \left(N_{t+1} \right)$, which occur, respectively, with a probability $(1 - \xi)$ and ξ. The timeline of events of the bubble and the value function are summarized in Figure 2.

Figure 2: Timeline of events



When a banking bubble exists in t, each bank chooses the optimal net worth $\{N_{t+1}\}$ in order to maximize its current dividends and expected present value of future dividends subject to the reserve requirement (8), the balance sheet (9), the budget constraint (10) and capital requirements (12).

$$V_t^B(N_t) = Max_{\{N_{t+1}\}}\left\{d_t + \beta E_t\left[V_{t+1}^B(N_{t+1})\right] + \xi\beta E_t\left[V_{t+1}^M(N_{t+1}) - V_{t+1}^B(N_{t+1})\right]\right\}, \tag{19}$$

subject to

$$d_t = \left(1 + r_t^l\right)N_t + D_t\left[r_t^l(1-\phi) - r_t\right] - \tau N_t - N_{t+1}, \tag{20}$$

$$D_t = \eta V_t^B(N_t), \tag{21}$$

$$N_t, D_t \geq 0 \text{ for all } t, \tag{22}$$

where $V_{t+1}^M(N_{t+1})$ is the value of the bank if the bubble bursts in $t+1$ and is defined by $V_{t+1}^*(N_{t+1})$ in the bubbleless equilibrium. Note that the difference between $V_{t+1}^M(N_{t+1})$ and $V_{t+1}^*(N_{t+1})$ lies in their initial values of net worth. The last term of (19) represents the change in values when the bubble bursts. Indeed, when the bubble bursts with a probability of ξ, the banks value shifts from $V_{t+1}^B(N_{t+1})$ to $V_{t+1}^M(N_{t+1})$.

We show in Appendix C that the solution of the bank maximization problem with a bubble gives the following value function, until the bubble bursts:

$$V_t^B(N_t) = q_t^B N_t + b_t, \tag{23}$$

where $q_t^B \geq 0$ is the marginal value of net worth and $b_t \neq 0$ is the bubble term on the bank's value. Variables q_t^B and b_t are to be endogenously determined. As it will become clear later, the bubble term is a self-fulfilling component that can be increasing, decreasing or explosive. Note that (23) is the same as in Miao et al. (2013). Define the stock price in $t+1$ when agents believe a bubble exists and before the bubble bursts by

$$p_{t+1} = \beta E_t \left[V_{t+1}^B(N_{t+1})\right] + \xi \beta E_t \left[V_{t+1}^M(N_{t+1}) - V_{t+1}^B(N_{t+1})\right].$$

Proposition 2. *When agents believe a bubble exists in t, until the bubble bursts, the solution of each bank maximization problem is given by the following system of equations.*

$$\begin{aligned} E_t\left(q_{t+1}^B\right) &= \frac{1 - \xi \beta E_t\left(q_{t+1}^M\right)}{\beta(1-\xi)}, & (24)\\ q_t^B &= \left(1 + r_t^l - \tau\right) + \eta q_t^B \left[r_t^l(1-\phi) - r_t\right], & (25)\\ (1-\xi)\beta E_t(b_{t+1}) &= b_t\left\{1 - \eta\left[r_t^l(1-\phi) - r_t\right]\right\}. & (26)\end{aligned}$$

From the regulation based on Value-at-Risk, the regulator forces the bank to satisfy (12) such that if $b_{t+1} = 0$, the value of q_{t+1}^M is given by

$$q_{t+1}^M = \frac{1}{\eta}\frac{D_{t+1}}{N_{t+1}}. \tag{27}$$

Proof of Proposition 2 is presented in Appendix C.

Equation (24) shows that, by increasing one unit of net worth today, the bank gets the expected discounted marginal value of net worth if the bubble lasts plus the expected discounted marginal value of net worth if the bubble bursts. The probability of a burst introduces a price distortion because it changes intertemporal arbitrage conditions. An increase in the marginal value of net worth if the bubble bursts, decreases the marginal value of net worth if the bubble stays.

Therefore, the bank's incentive to accumulate net worth if the bubble remains is reduced, and then, the bank distributes more dividends compared with when $b_t = 0$ for all t. Equation (25) has the same intuition than in the case where $b_t = 0$ for all t. However, here, the lending rate is not constant anymore and is positively correlated with the marginal value of net worth. The intuition is that the larger the lending rate is, the larger the incentive for banks to accumulate net worth is.

Equation (26) exists if and only if agents believe in the bubble such that $b_t \neq 0$. It represents the bubble growth rate. The idea is that the bubble allows the bank to relax the capital requirement constraint by raising the bank's value and thus increases deposits. In particular, the bubble allows to relax the capital requirement constraint while avoiding the operational costs. By increasing additional units of deposits, the growth of the economy becomes larger. Moreover, the larger the marginal gain from the bubble $\eta\left[r_t^l(1-\phi)-r_t^D\right]$ is, the smaller the growth rate of the bubble is. Finally, the bubble grows faster with ξ to compensate for the probability of bursting.

Proposition 3. *If*

$$\left\{1-\eta\left[r_t^l(1-\phi)-r_t\right]\right\}/\beta(1-\xi)<1/\beta, \tag{28}$$

the transversality condition of the household (4) is always satisfied.

Proof of Proposition 3 is presented in Appendix D.

Proposition 3 states that the transversality condition (TVC) is satisfied, i.e bubbles are not ruled out, if the growth rate of the bubble does not exceed the rate of time preference of households. The transversality condition insures that individuals do not hold positive wealth when $t \to \infty$. An important point to highlight here, is that without the capital requirement constraint the bubble growth is given by $E_t(b_{t+1})/b_t - 1 = 1/[\beta(1-\xi)] - 1$, which is ruled out by the TVC. Therefore, the bubble cannot exist. In addition, the combination of (24), (25) and (26) yield $E_t(b_{t+1})/b_t - 1 = \left(1+r_t^l-\tau\right)/\left(1-\beta\xi q_t^M\right)$. Thus, the growth rate is larger than $1/\beta$ when $\tau = 0$, which is ruled out by the TVC. The intuition is that operational costs ($\tau > 0$) reduce the growth rate of net worth and then, by no arbitrage, the growth rate of the bubble. Therefore the bubble is no longer explosive and is not ruled out. Analogously, Miao and Wang (2015) reduce the growth of net worth by assuming a minimum dividend policy as a function of net worth. It is also straightforward that under regulation based on book values as in Basel I, instead of on market values such that with the Value-at-Risk, bubbles cannot exist.[7]

The bubble return can be written as:

$$b_t \left(\frac{1}{\beta} - 1 \right) = \underbrace{\frac{1}{\beta (1-\xi)} \left\{ \eta \left[r_t^l (1-\phi) - r_t \right] - \xi \right\} b_t}_{\text{dividend yield}} + \underbrace{E_t (b_{t+1}) - b_t}_{\text{capital gain}}. \tag{29}$$

This equation shows that the return on the bubble is equal to a capital gain $E_t (b_{t+1}) - b_t$ plus a dividend yield. The dividend yield in the infinite horizon model guarantees that the transversality condition does not rule out the bubble. By relaxing the capital requirement, the bubble allows banks to raise η more units of deposits and earn a return $\left[r_t^l (1-\phi) - r_t \right]$ on it.

3 Bubbleless general equilibrium

This section defines and analyzes the bubbleless general equilibrium where variables are denoted x_t^*.

Definition 4. A competitive general bubbleless equilibrium with $b_t = 0$ for all t, is defined as sequences of allocations, prices and the shock process

$$\mathcal{E}_t^* = \left\{ d_t^*, N_{t+1}^*, K_{t+1}^*, L_{t+1}^*, D_{t+1}^*, \pi_t^*, y_t^*, c_t^*, s_{t+1,}^* q_t^*, r_t, r_t^{l*}, p_t^*, A_t \right\} \forall t,$$

such that taking prices as given, all agents maximize their future expected payoffs subject to their constraints and the transversality condition is satisfied. Finally, the market for loans, deposits, and stocks ($s_{t+1}^* = 1$) clear. The equilibrium consumption is given by the combination of the three budget constraints (1), (5) and (10), such that

$$c_t^* + \tau N_t^* = y_t^* - L_{t+1}^* - (R_{t+1}^* - R_t^*). \tag{30}$$

[7] The Basel ratio Tier 1 is based on book values and takes the following form: $N_t = \chi D_t$ where $\chi > 0$ is a regulation parameter.

Equation (30) is the condition on the goods market. The sum of households and banks consumption $c_t^* + \tau N_t^*$ is equal to output net of investment and variation in reserves. Households' consumption decreases with the investment which is represented by the amount of loans, the reserve variation and operational costs.

Bubbleless stationary equilibrium

Here, we analyze a stationary bubbleless equilibrium when variables are constant over time such that $\mathcal{E}_0^* = ... = \mathcal{E}_t^* = \mathcal{E}^*$ for all t. The equilibrium deposit rate is given by (2) such that $r = \frac{1}{\beta} - 1$. The marginal value of net worth in (17) is $q^* = \frac{1}{\beta}$. From the regulation based on Value-at-Risk in (12) and the value function (16),

$$\frac{D^*}{N^*} = \frac{\eta}{\beta}. \tag{31}$$

From (18), the lending rate is

$$r^{l*} = \frac{r(\eta + \beta) + \beta\tau}{\beta + \eta(1 - \phi)}. \tag{32}$$

Proposition 5. *The lending rate r^{l*} in a bubbleless stationary equilibrium increases with the reserves ϕ and operational costs τ. In contrast, it decreases with the Value-at-Risk regulation parameter η.*

Proof of Proposition 5 is presented in Appendix E. The intuition is that larger operational costs and reserves reduce the supply of loans, and as a consequence increase the lending rate. In contrast, a larger Value-at-Risk regulation parameter η allows banks to raise money using cheaply acquired funds, i.e deposits. This effect raises banks' size and reduces the lending rate.

For more insights, we also look at the interest rate spread, which is given by

$$\beta\left(r^{l*} - r\right) = \frac{(1-\beta)\eta}{\beta + \eta(1-\phi)}\phi + \frac{\beta^2}{\beta + \eta(1-\phi)}\tau.$$

The above equation shows that the discounted interest rate spread increases with operational costs τ and the fraction of reserves ϕ. For $\phi = 0$, the interest spread is only a function of operational costs. When there are no costs for the bank such that $\phi = \tau = 0$, the lending rate falls to the safe rate r.

The stationary level of loans is given by the first order condition (7) so that $L^* = \left[\left(1 + r^{l*}\right)/\psi\right]^{1/(\psi-1)}$. From the balance sheet constraint (9) and (31), $N^* = L^*/\left[1 + (1-\phi)(\eta/\beta)\right]$. Thus, the equilibrium consumption is given by $c^* = (L^*)^{\psi} - L^* - \tau L^*/\left[1 + (1-\phi)(\eta/\beta)\right]$. Denote W^* the welfare in a bubbleless stationary equilibrium. Therefore, $W^* = c^*$. The Appendix F shows that W^* and L^* are decreasing in the lending rate r^{l*}.

4 Stochastic bubbly general equilibrium

This section defines and analyzes the stochastic bubbly general equilibrium where variables before and after the bubble bursts at $t = T$ are, respectively, denoted x_t^B and x_t^M.

Definition 6. If a bubble exists in t such that $b_t \neq 0$, until the bubble bursts in T, a competitive stochastic bubbly general equilibrium is defined as

$$\mathcal{E}_t^B = \{d_t^B, N_{t+1}^B, K_{t+1}^B, L_{t+1}^B, D_{t+1}^B, \pi_t^B, y_t^B, c_t^B, b_t, s_{t+1}^B, q_t^B, q_t^{MB}, r_t, r_t^{lB}, p_t^B, A_t\}\ \forall t < T,$$

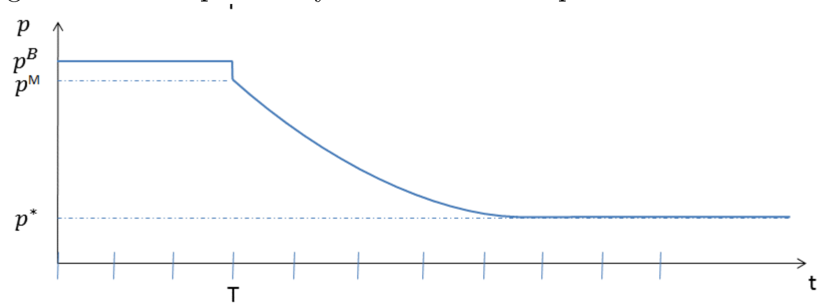
such that taking prices as given, all agents maximize their future expected payoffs subject to their constraints and the transversality condition is satisfied.[8] Finally, the market for loans, deposits, and stocks ($s_{t+1}^B = 1$) clear. At $t = T$, the bubble crashes such that $b_t = 0\ \forall t \geq T$, a competitive stochastic bubbly general equilibrium $\mathcal{E}_t^M$ is defined as $\mathcal{E}_t^*\ \forall t \geq T$ with $N_T^M = N_T^B$, such that taking prices as given, all agents maximize their future expected payoffs subject to their constraints and the transversality condition is satisfied. Finally, the market for loans, deposits, and stocks ($s_{t+1}^M = 1$) clear. As in the bubbleless equilibrium, the condition on the goods market is given by (30), where variables correspond to the ones from the stochastic bubbly general equilibrium.

For simplicity, as in Weil (1987) and Miao and Wang (2015), we study a stochastic bubbly equilibrium with the following properties. The equilibrium is

[8] Note that the bank marginal value of net worth q_t^B until the bubble bursts is a function of the marginal marginal value of net worth after the bubble collapses q_t^M. Therefore, this latter value is included in the equilibrium before the burst of the bubble and is called q_t^{MB}.

constant until the bubble collapses at $t = T$, such that $\mathcal{E}_0^B = ... = \mathcal{E}_{T-1}^B = \mathcal{E}^B$ with $b_0 = ... = b_{T-1} = b \neq 0$. We call it a semi-stationary equilibrium. At $t = T$, the banking bubble collapses such that $b_T = 0$ and the equilibrium is denoted by $\mathcal{E}_T^M$. Then, for all $t > T$, the equilibrium $\mathcal{E}_T^M$ converges to the bubbleless stationary equilibrium $\mathcal{E}^*$. Figure 3 shows the dynamic of the price when a positive banking bubble exists and then bursts.

Figure 3: Stock price's dynamic when the positive bubble bursts



At $t = T$, the bubble bursts such that $b_t = 0$ and stays at this value for all $t \geq T$. The price p_t^B falls to p_T^M. Then, the bank maximizes dividends and expected discounted future dividends such that the bubble is over and will never reappear. Therefore, the price converges to p^* for all $t > T$.

The semi-stationary equilibrium, i.e until the bubble bursts, is characterized by the following values. As in the bubbleless stationary equilibrium, the deposit rate is given by (2) $r = \frac{1}{\beta} - 1$. The lending rate before the bubble collapses is defined by (26) such that

$$r^{lB} = \frac{r(\beta + \eta) + \beta\xi}{(1 - \phi)\,\eta}. \tag{33}$$

Proposition 7. *In a semi-stationary bubbly equilibrium, the lending rate increases with the reserves ϕ and the probability of burst ξ. In contrast, it decreases with the Value-at-Risk regulation parameter η.*

Compared to the bubbleless lending rate given by (32), the lending rate is independent of operational costs τ. This characteristic will be explained later.

The interest rate spread between the lending rate and the risk-free deposit rate, until the bubble collapses, is

$$\beta\left(r^{lB}-r\right)=\frac{1-\beta}{(1-\phi)}\phi+\frac{\beta\left(1-\beta\right)}{(1-\phi)}\frac{1}{\eta}+\frac{\beta^2}{(1-\phi)}\frac{\xi}{\eta}.$$

Hence, the spread is a function of the bank's costs. It is increasing with a large probability of burst to compensate for the risk and with high fraction of reserves ϕ. In contrast, it decreases with less stringent capital requirement, which is represented by a high η. If $\xi=\phi=0$, then the interest rate spread is is equal to $\beta\left(1-\beta\right)/\eta$, which is proportional to the tightness of the regulatory constraint.

The marginal value of net worth while the bubble lasts and when the bubble collapses are, respectively, given by (24)

$$q^B=\frac{1-\beta\xi q^M}{\beta(1-\xi)}=\frac{1-\tau+r^{lB}}{\beta(1-\xi)}, \tag{34}$$

and

$$q^{MB}=\frac{\tau-r^{lB}}{\beta\xi}. \tag{35}$$

From (27), the leverage ratio is

$$\frac{D^B}{N^B}=\eta q^{MB}. \tag{36}$$

From the first order condition of firms (7), we obtain the equilibrium quantity of loans

$$L^B=\left[\frac{1}{\psi}\left(1+r^{lB}\right)\right]^{\frac{1}{\psi-1}}. \tag{37}$$

From (8), (9), (36) and (37),

$$N^B=\frac{L^B}{1+\left(1-\phi\right)\eta q^{MB}}.$$

It can be shown that N^B is strictly positive if and only if $q^{MB}>0$ which is equivalent to

$$\tau>\left[r(\beta+\eta)+\beta\xi\right]/\left(1-\phi\right)\eta. \tag{38}$$

Equation (38) is called the "non negative net worth condition". In what follows, we consider that this condition always holds. From the regulation (12) and the value function when the bubble exists (23),

$$b = \frac{D^B}{\eta} - q^B N^B. \tag{39}$$

Using (34), (36) and (39), the bubble term can be re-written as

$$\begin{aligned} b &= \left(q^{MB} - q^B\right) N^B \\ &= \left[\frac{\eta\left(\tau - \xi\right)\left(1 - \phi\right) - r\left(\eta + \beta\right) + \beta\xi}{\beta\xi(1-\xi)\left(1-\phi\right)\eta}\right] N^B. \end{aligned} \tag{40}$$

The equation above shows that the bubble increases with large operational costs. An increase in operational costs τ should, without bubble, raise the lending rate. However, in the presence of a bubble, the increase in τ enlarges the bubble, which relaxes the capital requirement constraint. Thus, loan supply increases, canceling out the effect of τ on the lending rate. From (12) and (1), the equilibrium consumption is $c^B = \left(L^B\right)^{\psi} - L^B - \tau L^B / \left[1 + (1-\phi) D^B/N^B\right]$. Finally, we define the bubbly semi-stationary welfare as $W^B = c^B$. Compared to the bubbleless stationary equilibria, the welfare has the same form. However, it now depends on the bubble. Indeed, the bubble modifies the value of lending rate by affecting the capital requirement constraint and thus, the equilibrium quantity of loans. The stationary bubbleless and the semi-stationary stochastic bubbly equilibrium are compared in the next section.

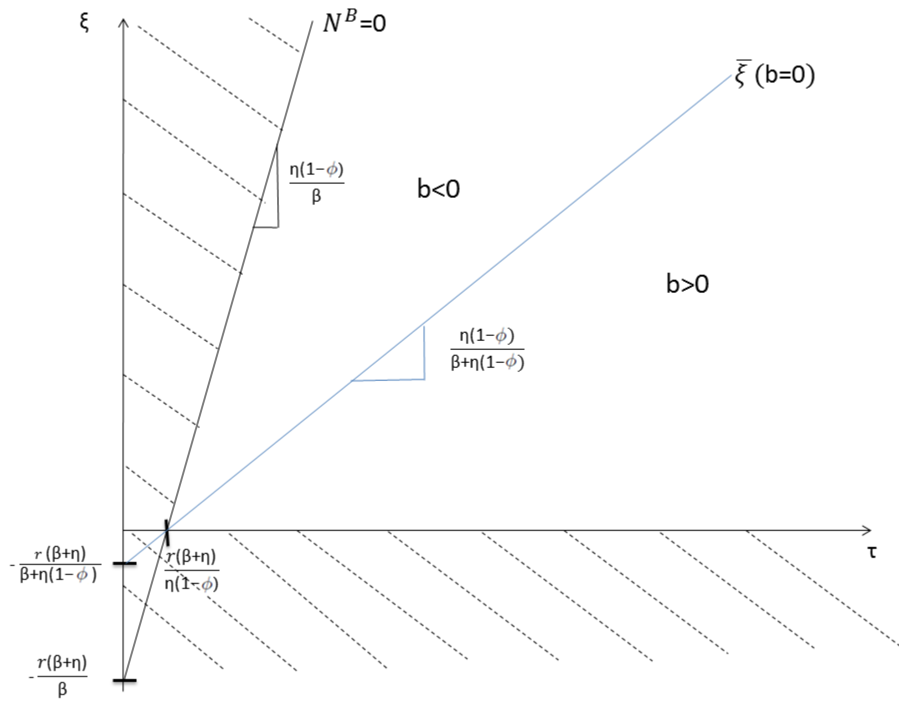
Using (40), the condition under which a semi-stationary stochastic bubbly equilibrium exists can be written as $\xi \neq \bar{\xi}$, where

$$\bar{\xi} = \frac{\eta\left[(1-\phi)\tau - r\right] - (1-\beta)}{\beta + \eta(1-\phi)}. \tag{41}$$

Therefore, the semi-stationary stochastic bubbly equilibrium exists if the probability of burst is $\xi \neq \bar{\xi}$. It can be shown that a positive bubble exists for small values of the probability of burst, $\xi < \bar{\xi}$. This is consistent with Weil (1987) and Miao and Wang (2015) who also find that positive bubbles exist only for small values of the bursting probability. Suppose the bubble is positive. Hence, a change in beliefs concerning the probability of burst might modify the equilibrium, from a positive semi-stationary bubbly equilibrium to a bubbleless stationary equilibrium.

Figure 4 displays the bubble's value in the parameter space (ξ,τ), for a given η and ϕ. At $\xi = \bar{\xi}$, the bubble term is zero. For $\xi < \bar{\xi}$ (resp. $\xi > \bar{\xi}$), the bubble is positive (resp.negative). The slope of the line $\bar{\xi}$ increases with large values of the Value-at-Risk regulation parameter η. Thus, the parameter space for the positive bubble widens. As the regulator becomes more lenient such that η is high, the economy can enter a state in which bubbles are positive, increasing welfare in the economy. As explained above, the space where $\xi > [\tau\,(1-\phi)\,\eta - r(\beta+\eta)]\,/\beta$ does not exist as $N^B > 0$.

Figure 4: Bubble's value in the parameter space



Alternatively, we can also write the existence condition of a stochastic semi-stationary bubbly equilibrium in terms of the regulation parameter based on Value-at-Risk η such that $\eta \neq \bar{\eta}$, where

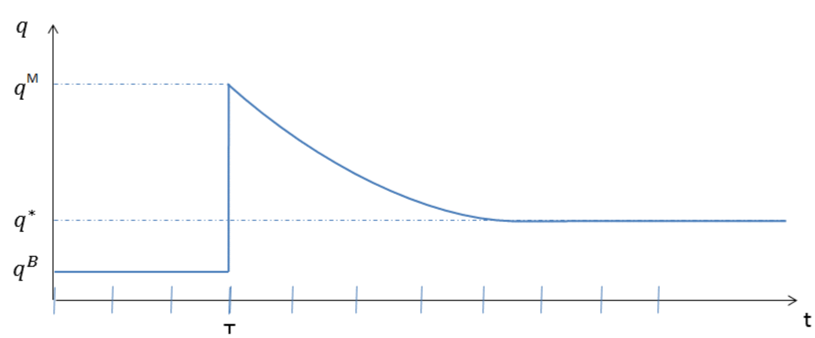
$$\bar{\eta} = \frac{1-\beta(1-\xi)}{(\tau-\xi)(1-\phi)-r}.$$

Proposition 8. *Under (11), (28), (38) and $\eta \neq \bar{\eta}$, a stochastic semi-stationary bubbly equilibrium exists ($b \neq 0$). For $\eta > \bar{\eta}$, the bubble is positive. In contrast, for $\eta < \bar{\eta}$, it is negative.*

Proposition 8 suggests that the semi-stationary equilibrium with a stochastic bubble exists if the regulation parameter based on Value-at-Risk is $\eta \neq \bar{\eta}$. Indeed, under the conditions described in Proposition 8, the transversality condition is satisfied. As a result, a positive bubble exists only for large values of the regulation parameter η. Thus, a reduction of the regulation parameter η might modify the equilibrium, from the positive bubbly equilibrium to the bubbleless equilibrium. Another important policy implication, here, is that the reserve requirement parameter ϕ affects negatively the threshold $\bar{\eta}$. As a consequence, when ϕ is large, the regulation parameter η should be even greater for the economy to be in the positive bubbly semi-stationary equilibrium.

Figure 5 shows the dynamics of the positive stochastic bubbly equilibrium for the marginal value of net worth q_t, before and after the bubble bursts at $t = T$. Suppose $b_t > 0$ for all $t < T$.

Figure 5: Transition path when the positive bubble bursts



At $t = T$, the bubble bursts such that $b_t = 0$ and stays at this value for all $t \geq T$. Since deposits and net worth are pre-determined variables, from (27), the marginal value of net worth q^B goes straight to q_T^M. Thus, the value of the bank and the price become, respectively, $V_T^M(N_T^B)$ and p_T^M. Then, the bank maximizes dividends and expected discounted future dividends such that the bubble is over and will never reappear. Therefore, the bank net worth converges from N_T^B to the net worth value in the stationary bubbleless equilibrium N^* on the path N_t^M and the marginal value from q_t^M to the bubbleless stationary equilibrium marginal value of net worth q^*. Thus, the price p_t^M converges to p^* for all $t > T$.

5 Comparison of both equilibria

This section compares the stationary bubbleless and the stochastic semi-stationary bubbly equilibria.

Proposition 9. *If $\eta \neq \bar{\eta}$ both equilibria with and without a bubble on stock prices coexist.*

Proposition 10. *If $\eta > \bar{\eta}$, the bubbly equilibrium lending rate before that the bubble collapses is lower than the bubbleless lending rate. Thus, welfare is larger with a positive bubble. In contrast, a negative bubble ($\eta < \bar{\eta}$) reduces welfare.*

Proof of Proposition 10 is in Appendix G. Both stochastic bubbly and bubbleless equilibria co-exist for all values of the Value-at-Risk regulation parameter η except at the point $\bar{\eta}$. This point can be viewed as a point of reversal at which you may move from a positive bubbly equilibrium to a negative bubbly stochastic semi-stationary equilibrium. At this reversal point, the equilibrium can move from from higher to lower welfare. For $\eta > \bar{\eta}$, the capital requirement based on Value-at-Risk is less stringent. In that case, the stochastic semi-stationary bubbly equilibrium provides larger welfare than the bubbleless equilibrium. The intuition is that, when agents consider that the bubble exists, a lower capital requirement leads to optimistic beliefs on banks value. The bubble allows banks to relax the capital requirement constraint, and thus banks demand more deposits, which raises their leverage, and make more loans. This effect reduces the lending rate and provides better welfare. In contrast, for more stringent capital requirement $\eta < \bar{\eta}$, the bubble is negative leading to a credit crunch and thus, reducing the welfare compared to the bubbleless equilibrium. An important point to highlight here is that a change in banking regulation may modify the equilibrium and leads to crises, by reducing welfare levels. This effect can explain the occurrence of crises without any external shocks. In addition, using (41), results also show that a change in beliefs about the probability of burst may also lead to a crisis, as in Miao and Wang (2015).

The following table summarizes and compares the main results discussed in this section.

Table 1: Policy implication

	$\eta > \bar{\eta}$	$\eta<\bar{\eta}$
variables		
b	$b>0$	$b<0$
r^l	$r^{lB} < r^{l*}$	$r^{lB} > r^{l*}$
L	$L^B>L^*$	$L^B<L^*$
$\frac{D}{N}$	$\frac{D^B}{N^B} > \frac{D^*}{N^*}$	$\frac{D^B}{N^B}<\frac{D^*}{N^*}$
W	$W^B>W^*$	$W^B<W^*$

Table 1 shows that, when agents believe a bubble exists, a positive bubble arises for lenient regulatory Value-at-Risk constraints, $\eta > \bar{\eta}$. It leads to the highest equilibrium welfare level, highest equilibrium quantity of loans and leverage levels. On the opposite, a negative bubble arises when capital requirement based on Value-at-Risk are more stringent. The negative bubbly semi-stationary equilibrium is characterized by the lowest equilibrium level of welfare, credit and leverage.

6 Local dynamics and simulations

The present section, first, presents the calibration. Second, it analyzes local dynamics around the bubbleless stationary equilibrium and the semi-stationary stochastic bubbly equilibrium. Finally, we simulate and compare a negative productivity shock from both equilibria.

6.1 Calibration

Here, we calibrate the parameters and we report the implied values for variables in the bubbleless stationary and bubbly semi-stationary equilibria. We present a numerical example. We calibrate the discount factor $\beta = 0.99$, the capital share $\psi = 0.33$, the probability of burst $\xi = 0.1$. The regulatory parameter is $\mu = 0.09$, which implies that $\eta = 10.11$. This calibration for μ allows us to have a tier 1 ratio around 8% as recommended by the Basel committee.[9] This ratio is 8.99% for the bubbleless stationary equilibrium and 7.12% for the semi-stationary stochastic bubbly equilibrium. The reserve parameter $\phi = 0.01$ is set as required by the European Central Bank.[10] Finally, we set operational costs to a proportion $\tau = 0.15$ of net worth. Under these values of parameters, Propositions 9 and 10 show that the bubbly and the bubbleless stationary equilibria, until the bubble bursts coexist and that the stochastic bubbly semi-stationary equilibria has a positive bubble ($\eta > \bar{\eta}$). Moreover, under this calibration, the marginal value of net worth in T, once the bubble has burst is $q_T^M = 1.3021$.

Table 2: Bubbleless and bubbly equilibria

	Bubbly > 0	Bubbleless
Variables		
N	0.0132024	0.0166121
D	0.173818	0.169664
d	0.000171925	0.000167799
L	0.185282	0.184579
p	0.0170206	0.0166121
q	0.977657	1.0101
r^l	0.0210922	0.0236939
b	0.00428355	0
W	0.386042	0.385514

Table 2 confirms results summarized in Table 1. Compared to the bubbleless steady state, the quantity of loans supplied by banks is larger in the stochastic bubbly semi-stationary equilibrium. This gives a relatively lower lending rate r^l, leading to a higher welfare W.

6.2 Local dynamics

To analyze the stability and uniqueness properties of the system, we log-linearize the system around the stationary and the semi-stationary equilibria. This results in a system of stochastic linear difference equations under rational expectations.

[9] This ratio is defined as total net worth over risky assets.

[10] See https://www.ecb.europa.eu/mopo/implement/mr/html/calc.en.html.

When agents do not believe a bubble exists, $b_t = 0$ for all t, as well as when agents believe a bubble exists, $b_t > 0$ for $t = 0, ...T$, until the bubble bursts, the eigenvalues associated with the linearized system around, respectively, the stationary bubbleless and the stochastic semi-stationary bubbly equilibria, show that the number of unstable eigenvalues (eigenvalues that lie outside the unit circle) is equal to the number of forward looking variables.[11] Thus, under this calibration, the system of equations when $b_t = 0$ for all t and when $b_t > 0$ for all $t < T$, is determined and both the bubbleless and the bubbly equilibria are stable and unique. This implies that given an initial value of N_t^* in the neighborhood of the stationary bubbleless equilibrium, there exists a unique value of q_t^* such that the system of linear difference equations converges to the unique stationary bubbleless equilibrium along a unique saddle path (see Blanchard and Kahn, 1980). Similarly, given an initial value of N_t^B in the neighborhood of the stochastic semi-stationary bubbly equilibrium, there exists a unique value of q_t^B such that the system of linear difference equations converges to the unique stochastic semi-stationary bubbly equilibrium along a unique saddle path, for all $t < T$.

6.3 Simulations

As an illustration, Figure 6 displays the impulse response functions of a 1% negative productivity shock from the stationary bubbleless and the semi-stationary positive stochastic bubbly equilibria until the bubble bursts (for all $t < T$). To that end, we calibrate the persistence of the productivity shock z_A to 0.95. This is standard in the real business cycle literature.

From the bubbleless stationary equilibrium, a negative productivity shock decreases firms profits and thus also the demand for loans. By the balance sheet, the reduction in assets of banks leads to a fall in net worth accumulation, which increases dividends (see equation (10)). Moreover, the fall in net worth reduces the ability of banks to raise deposits. The reduction in loans leads to a decrease in production and welfare. Since there is no uncertainty about the bank's value, the marginal value of net worth and the lending rate are constant. Finally, the

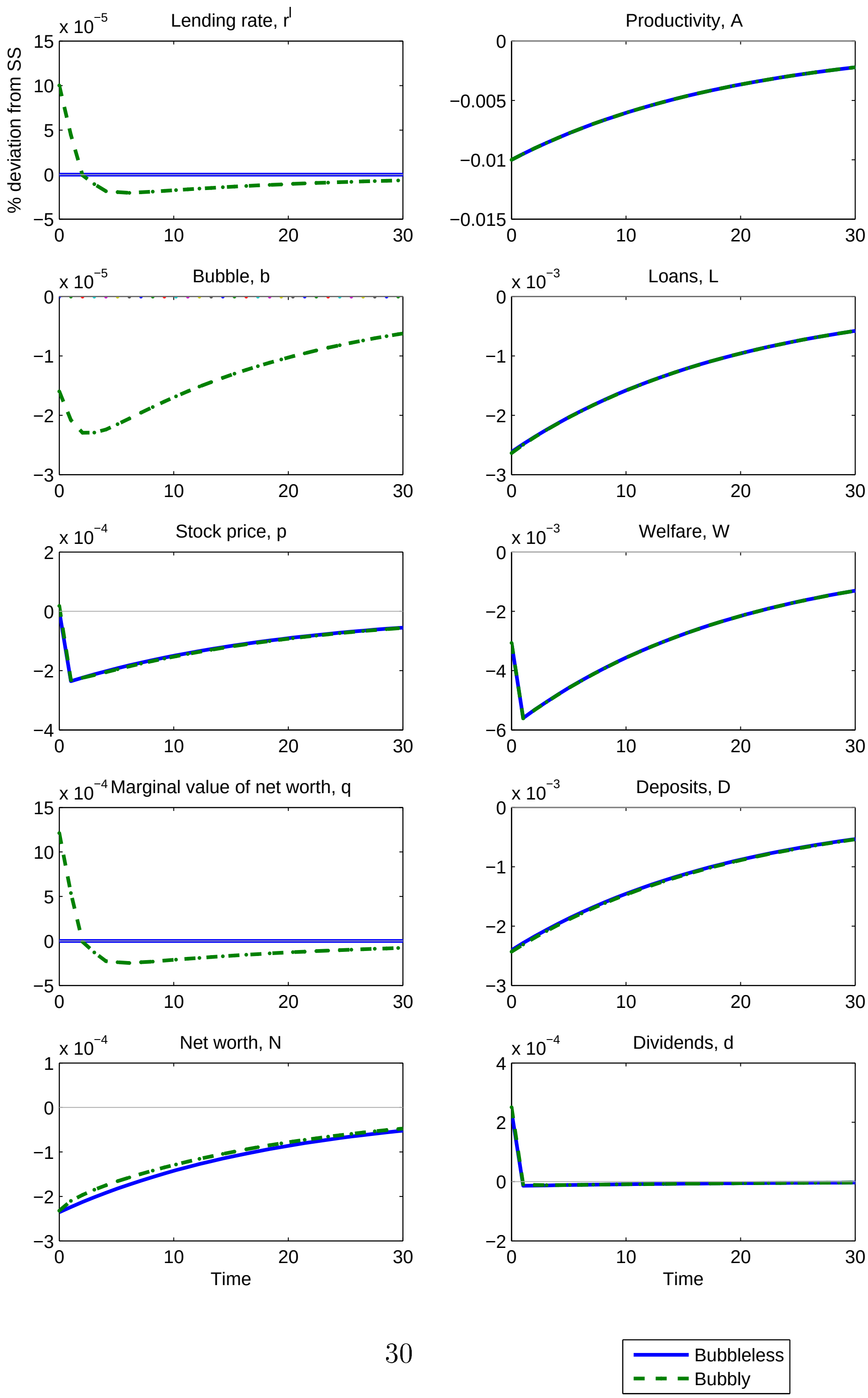
[11] Eigenvalues are reported in Appendix H.

stock price falls following the decrease in net worth.

The impulse response functions from the semi-stationary stochastic bubbly equilibrium are similar to the ones from the bubbleless equilibrium. The main difference lies in the fact that the uncertainty on the burst of the bubble changes the inter-temporal substitution between net worth and dividends. A negative productivity shock that decreases loans demand and decreases net worth raises the marginal value of net worth. Indeed, a fall of net worth below its steady state value raises the incentive to increase net worth, reducing the value of holding investment in the bubble, and thus the bubble growth diminishes. Therefore, net worth from the bubbly equilibrium falls by less than from the bubbleless equilibrium.

In conclusion, impulse response functions from both equilibria show that the effect of a productivity shock are similar. This suggests that the bubble does not amplify the effect of shocks on real economic variables.

Figure 6: Negative productivity shock



7 Conclusion

In this paper, we develop a stochastic general equilibrium model in infinite horizon with a regulated banking sector where a stochastic banking bubble may arise endogenously. We show that a bubble emerges if agents believe that it exists. Thus, expectations of agents are self-fulfilling. Results suggest that when banks face capital requirements based on Value-at-Risk, two different equilibria emerge and can coexist: the bubbleless and the bubbly equilibria. Capital requirements based on Value-at-Risk allow the bubble to exist. Alternatively, under a regulatory framework where capital requirements are based on credit risk only as specified in Basel I, a bubble is explosive and as a consequence cannot exist. The stochastic bubbly equilibrium is characterized by a positive or a negative bubble depending on capital requirements based on Value-at-Risk. We find a maximum capital requirement under which the bubble is positive. Below this threshold, the stochastic bubbly equilibrium provides larger welfare than the bubbleless equilibrium. Therefore, this result suggests that a change in banking policies might lead to a crisis. This can explain the existence of crises without any external shocks. We also show that a semi-stationary equilibrium with a positive (resp. negative) stochastic bubble exists if the probability that the bubble collapses is small (resp. high). Consequently, a change in beliefs about the bubble's probability of burst also modifies the equilibrium, from a higher to a lower welfare.

Our model can be extended by the addition of different elements. Risk aversion of households and endogenous labor choice can be considered. However, endogenous labor choice will complicate the model without changing our main results. Risk aversion can be introduced by a quadratic utility function for households and thus, the emergence of bubbles can be studied in this context. One can also add a probability of default on loans repayments in order to model credit risk in the economy and analyze its impact on key macroeconomic variables.

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Appendices

Appendix A

Here, we show that without capital requirement, each bank chooses to hold the maximum amount of deposits.
Each bank maximization problem without capital requirement is given by

$$V_t\left(N_t, D_t\right) = Max_{\{N_{t+1}, D_{t+1}\}}\left[d_t + \beta E_t V_{t+1}(N_{t+1}, D_{t+1})\right],$$

subject to

$$d_t = \left(1 + r_t^l\right) N_t + D_t\left[r_t^l(1-\phi) - r_t\right] - \tau N_t - N_{t+1},$$

$$N_t, D_t \geq 0 \ \text{ for all } t.$$

From the problem described above,

$$\begin{aligned} V_t(N_t, D_t) = & \\ & Max_{\{N_{t+1}, D_{t+1}\}}\left\{\left(1 + r_t^l\right) N_t + D_t\left[r_t^l\left(1-\phi\right) - r_t\right] - \tau N_t \right. . \\ & \left. - N_{t+1} + \beta E_t V_{t+1}(N_{t+1}, D_{t+1})\right. \end{aligned} \tag{42}$$

equation The marginal value from an increase in net worth and deposits are given by

$$\frac{\partial V_t\left(N_t, D_t\right)}{\partial N_{t+1}} = -1 + \beta E_t \frac{\partial V_{t+1}(N_{t+1}, D_{t+1})}{\partial N_{t+1}}, \tag{43}$$

and

$$\frac{\partial V_t\left(N_t, D_t\right)}{\partial D_{t+1}} = \beta E_t \frac{\partial V_{t+1}(N_{t+1}, D_{t+1})}{\partial D_{t+1}}.$$

Using the envelop theorem,

$$\frac{\partial V_t\left(N_t, D_t\right)}{\partial N_t} = 1 + r_t^l - \tau,$$

and

$$\frac{\partial V_t\left(N_t, D_t\right)}{\partial D_t} = r_t^l\left(1-\phi\right) - r_t.$$

Banks decide to hold an infinite amount of deposits if $\partial V_t(N_t, D_t)/\partial D_{t+1} > 0$, which is equivalent to

$$r_t^l(1-\phi) - r_t > 0. \tag{44}$$

The interior solution for the net worth is given by $\partial V_t(N_t, D_t)/\partial N_{t+1} = 0$. Equation (43) becomes

$$1 + r_t^l - \tau = \frac{1}{\beta}. \tag{45}$$

From equation (45), we get the following lending rate

$$r_t^l = \frac{1}{\beta} - 1 + \tau.$$

Putting (43) in (44), we get the following condition

$$\tau\beta(1-\phi) > \phi(1-\beta).$$

If the above condition holds, banks always choose the maximum amount of deposits, and consequently the capital requirement regulation always binds.

Appendix B

This appendix presents the proof of Proposition 1. From the bank bubbleless maximization problem,

$$V_t^*(N_t) = Max_{\{N_{t+1}\}}\left\{d_t + \beta E_t\left[V_{t+1}^*(N_{t+1})\right]\right\},$$

subject to

$$d_t = \left(1 + r_t^l\right)N_t + D_t\left[r_t^l(1-\phi) - r_t\right] - \tau N_t - N_{t+1},$$

$$D_t = \eta V_t^*(N_t),$$

$$N_t, D_t \geq 0 \text{ for all } t.$$

The Bellman equation becomes

$$V_t^*(N_t) = Max_{\{N_{t+1}\}}\left(1 + r_t^l - \tau\right)N_t + \eta V_t(N_t)\left[r_t^l(1-\phi) - r_t\right] - N_{t+1} + \beta E_t\left[V_{t+1}^*(N_{t+1})\right] \quad .$$

The marginal value from a net worth increase is given by

$$E_t\left[\frac{\partial V_t^*\left(N_t\right)}{\partial N_{t+1}}\right]=-1+\beta E_t\left[\frac{\partial V_{t+1}^*\left(N_{t+1}\right)}{\partial N_{t+1}}\right].$$

By the envelop theorem,

$$\frac{\partial V_t^*\left(N_t\right)}{\partial N_t}=\left(1+r_t^l-\tau\right)+\eta\frac{\partial V_t^*\left(N_t\right)}{\partial N_t}\left[r_t^l\left(1-\phi\right)-r_t\right].$$

The interior solution for the net worth is given by $\partial V_t\left(N_t\right)/\partial N_{t+1}=0$. Therefore,

$$E_t\left[\frac{\partial V_{t+1}^*\left(N_{t+1}\right)}{\partial N_{t+1}}\right]=\frac{1}{\beta}.$$

Since the problem is linear in N, we get

$$V_t^*\left(N_t\right)=q_t^*N_t. \qquad (46)$$

Replacing (46) in the maximization problem, the solution is given by the following system:

$$\begin{aligned}E_t\left(q_{t+1}^*\right)&=\frac{1}{\beta},\\ q_t&=\left(1+r_t^l-\tau\right)+\eta q_t\left[r_t^l\left(1-\phi\right)-r_t\right].\end{aligned}$$

Appendix C

This appendix proves Proposition 2. From the bank maximization problem when agents believe in a bubble such that $b_t\neq 0$, we have

$$V_t^B\left(N_t\right)=Max_{\{N_{t+1}\}}\left\{d_t+\beta E_t\left[V_{t+1}^B\left(N_{t+1}\right)\right]+\xi\beta\left\{E_t\left[V_{t+1}^M\left(N_{t+1}\right)\right]-E_t\left[V_{t+1}^B\left(N_{t+1}\right)\right]\right\}\right\},$$

subject to

$$d_t=\left(1+r_t^l\right)N_t+D_t\left[r_t^l\left(1-\phi\right)-r_t\right]-\tau N_t-N_{t+1},$$

$$D_t=\eta V_t^B\left(N_t\right),$$

$$N_t, D_t \geq 0 \text{ for all } t,$$

where $V^M_{t+1}(N_{t+1})$ is the value of the bank if the bubble bursts in $t+1$ and is defined as $V^*_{t+1}(N_{t+1})$ for the bubbleless maximization problem.

The Bellman equation becomes

$$V^B_t(N_t) = Max_{\{N_{t+1}\}} \left\{ \left(1 + r^l_t - \tau\right) N_t + \eta V_t(N_t) \left[r^l_t (1-\phi) - r_t \right] - N_{t+1} \right\}.$$

The marginal value from a net worth increase is given by

$$E_t \left[\frac{\partial V^B_t(N_t)}{\partial N_{t+1}} \right] = -1 + \beta E_t \left[\frac{\partial V^B_{t+1}(N_{t+1})}{\partial N_{t+1}} \right] + \xi \beta E_t \left[\frac{\partial V^M_{t+1}(N_{t+1})}{\partial N_{t+1}} - \frac{\partial V^B_{t+1}(N_{t+1})}{\partial N_{t+1}} \right].$$

By the envelop theorem,

$$\frac{\partial V^B_t(N_t)}{\partial N_t} = \left(1 + r^l_t - \tau\right) + \eta \frac{\partial V^B_t(N_t)}{\partial N_t} \left[r^l_t (1-\phi) - r_t \right].$$

The interior solution for the net worth is given by $\partial V^B_t(N_t) / \partial N_{t+1} = 0$. Therefore,

$$E_t \left[\frac{\partial V^B_{t+1}(N_{t+1})}{\partial N_{t+1}} \right] = \frac{1 - \xi \beta E_t \left[\frac{\partial V^M_{t+1}(N_{t+1})}{\partial N_{t+1}} \right]}{(1-\xi)\beta}.$$

Since the problem is linear in N, we get

$$V^B_t(N_t) = q^B_t N_t + b_t. \tag{47}$$

Replacing (47) in the maximization problem, the solution is given by the following system:

$$\begin{aligned} E_t\left(q^B_{t+1}\right) &= \frac{1 - \xi \beta E_t\left(q^M_{t+1}\right)}{\beta(1-\xi)}, \\ q^B_t &= \left(1 + r^l_t - \tau\right) + \eta q^B_t \left[r^l_t (1-\phi) - r_t \right], \\ (1-\xi)\beta E_t(b_{t+1}) &= b_t \left\{ 1 - \eta \left[r^l_t (1-\phi) - r_t \right] \right\}. \end{aligned}$$

Appendix D

This appendix presents the proof of Proposition 3. We show the condition to ensure that the stochastic bubbly equilibrium until the bubble bursts satisfies the transversality condition. The following transversality condition is required:

$$lim_{t\rightarrow\infty} p_t \beta^t = lim_{t\rightarrow\infty} E_{t-1}\left[\xi\left(q_t^M N_t\right) + (1-\xi)\left(q_t^B N_t + b_t\right)\right]\beta^t = 0.$$

It is satisfied if

$$lim_{t\rightarrow\infty} E_{t-1}\left[\xi\left(q_t^M N_t\right) + (1-\xi) N_t q_t^B\right]\beta^t = lim_{t\rightarrow\infty} E_{t-1}(1-\xi) b_t \beta^t = 0.$$

Since the bubble growth rate is

$$\frac{E_t(b_{t+1})}{b_t} = \frac{1}{\beta(1-\xi)}\left\{1-\eta\left[r_t^l(1-\phi)-r_t\right]\right\},$$

the TVC requires that

$$\frac{1}{\beta(1-\xi)}\left\{1-\eta\left[r_t^l(1-\phi)-r_t\right]\right\} < \frac{1}{\beta}.$$

Thus, the condition to allow a bubble to exist is

$$\eta\left[r_t^l(1-\phi)-r_t\right] > \xi.$$

Appendix E

This appendix proves Proposition 5. Here, we prove that the interest rate of loans in the bubbleless stationary equilibrium is negatively correlated with the Value-at-Risk regulation parameter η. Using (32), we have that

$$r^{l*} = \frac{r(\eta+\beta)+\beta\tau}{\beta+\eta(1-\phi)}.$$

Therefore,

$$\frac{\partial r^{l*}}{\partial \eta} = \frac{(1-\beta)-[1-\beta(1-\tau)](1-\phi)}{[\beta+\eta(1-\phi)]^2} < 0.$$

The numerator is negative if and only if $\tau\beta(1-\phi) > \phi\,(1-\beta)$, which is always satisfied (see Appendix A).

Appendix F

The stationary bubbleless steady state welfare is given by the consumption such that

$$W = L^{\psi} - \left(1 + \frac{\tau}{1+(1-\phi)\frac{D}{N}}\right) L.$$

Therefore, the marginal impact of the lending rate on welfare is

$$\frac{dW}{dr^l} = \psi\frac{dL}{dr^l}L^{\psi-1} - \frac{dL}{dr^l}\left(1 + \frac{\tau}{1+(1-\phi)\frac{D}{N}}\right).$$

Thus, $\frac{dW}{dr^l} < 0$ if and only

$$\psi L^{\psi-1} < \left(1 + \frac{\tau}{1+(1-\phi)\frac{D}{N}}\right). \tag{48}$$

Since $L = \left[(1+r^l)/\psi\right]^{\frac{1}{\psi-1}}$,

$$r^l > \frac{\tau}{1+(1-\phi)\,\frac{D}{N}}. \tag{49}$$

In the stationary bubbleless equilibrium, the lending rate is $r^{l*} = \frac{r(\eta+\beta)+\beta\tau}{\beta+\eta(1-\phi)}$. Therefore the condition (49) becomes

$$r^{l*} = \frac{r\,(\eta+\beta)+\beta\tau}{\beta+\eta\,(1-\phi)} > \frac{\tau}{\beta+(1-\phi)\,\eta}.$$

It is equivalent to

$$r\,(\eta+\beta) > 0.$$

which is always verified.

Appendix G

Here, we display the proof of Proposition 10. The spread between the bubbly and the bubbleless lending rate is

$$r^{lB} - r^{l*} = \frac{r\eta + 1 - \beta(1-\xi)}{\eta(1-\phi)} - \frac{1 - \beta\,(1-\tau) + \eta r}{\beta + \eta\,(1-\phi)}.$$

Therefore, $r^{lB} - r^{l*} > 0$ if

$$\eta < \frac{1 - \beta(1-\xi)}{(\tau - \xi)\,(1-\phi) - r} = \bar{\eta}.$$

Hence, the bubbly lending rate is higher than then bubbleless lending rate if and only if a negative bubble exists. For a positive bubble, we have $r^{lB} - r^{l*} < 0$.

As a consequence, it can be shown that the welfare is higher in the presence of a positive bubble. In contrast, it is lower with a negative bubble.

Appendix H

Table 3 displays eigenvalues associated with the linearized system around the stationary bubbleless and the semi-stationary bubbly equilibrium.

Table 3: Eigenvalue of the bubbly and bubbleless equilibria

bubbly ($b_t > 0$)	bubbleless ($b_t = 0$)
values	values
0	0
0	2.236e-55
0	3.012e-36
0	3.452e-36
1.456e-19	4.408e-19
9.661e-18	1.321e-17
9.161e-17	1.472e-17
0.95	0.95
1.01	1.01
1.038	1.915e+39
Inf	Inf
Inf	Inf
Inf	