FACULTY OF LAW, ECONOMICS AND FINANCE

CREA
Discussion
Paper
2016-19

Economics

Centre for Research in Economics and Management
University of Luxembourg

Variety and Quality in Trade Dynamics

available online : http://wwwfr.uni.lu/recherche/fdef/crea/publications2/discussion_papers

Hamano Masashige, Sophia University, Tokyo
Research Fellow CREA, Université du Luxembourg

August, 2016

For editorial correspondence, please contact: crea@uni.lu
University of Luxembourg
Faculty of Law, Economics and Finance
162A, avenue de la Faïencerie
L-1511 Luxembourg

UNIVERSITÉ DU LUXEMBOURG

Variety and Quality in Trade Dynamics*

Hamano Masashige†

First Draft: August 2016

Abstract

This paper provides a simple two-country DSGE model that includes endogenous determination of both the number of traded varieties and the traded products' quality with heterogeneous firms. By introducing product quality explicitly, the theoretical model sheds light on the cyclical properties of quality in international trade. We find that aggregate product quality of exports and imports are both negatively correlated with real exports and imports in the US data. Our model can replicate a wedge-shaped pattern of cross-correlations, together with a number of statistics on US trade dynamics. We also perform several impulse response analyses and find that trade liberalization induces quality downgrading of exports and imports, which negatively impacts consumer welfare.

Keywords: *product quality, product variety, firm entry, firm heterogeneity*

JEL classification: *F12, F41, F43*

*I would like to thank Philippe Martin for providing helpful comments. This project is supported by JSPS KAKENHI Grant Number 25885071 and the National Research Fund, Luxembourg. Of course, all remaining errors are my own.

†Sophia University and CREA research fellow, University of Luxembourg. Contact address: Sophia University, Yotsuya Campus, 2 go-kan, 1123, 7-1 Kioi-cho, Chiyoda-ku Tokyo 102-8554 Japan. E-mail: masahige.hamano@gmail.com

1 Introduction

The last decades have witnessed a significant increase in trade all over the world. For both exports and imports, this rise has been driven by the so-called "extensive margins" of trade, i.e., the number of exported and imported varieties (Hummels and Klenow (2005), Broda and Weinstein (2004) and Broda and Weinstein (2006)). At the same time, the heterogenous quality of exported and imported products has been featured as an important stylized fact in international trade (Schott (2004), Khandelwal (2010) and Hallak (2006)). This latter strand of the literature focuses on either cross country differences in trade or its growth over time (Hallak and Schott (2011) and Feenstra and Romalis (2014)), but little is known about business cycles of quality in trade dynamics. To what extent are trade dynamics followed by those in quality of exports and imports? Does the international real business cycle model that embeds both endogenous quality and the number of varieties account for the observed trade dynamics of quality and the extensive margins of trade? This paper addresses these issues.

To grasp the significance of product quality in the business cycle context, we present a brief sketch based on the data in Feenstra and Romalis (2014).[1] The first column of Figure 1 presents the relationship between the variety of exports and the value of exports, as well as between the variety of imports and the value of imports. On the horizontal axis, the logged number of product varieties exported (imported) by a country in a specific year is presented, while the logged aggregate value of exports (imports) of that country in that year is reported on the vertical axis. We see clear positive relationships for both imports and exports. Trade expansion is followed by a significant increase in the extensive margins

[1] Feenstra and Romalis (2014) estimate the quality of exports and imports at the four-digit SITC level from 1984 to 2011 for countries all over the world. They use both demand- and supply-side information to estimate product quality. Their estimates of quality are highly correlated with those of other studies, such as Khandelwal (2010) and Hallak and Schott (2011). In their data set, the quality of a product is defined with respect to the world average, which is normalized to unity. In Figure 1, the annual export (import) quality of a country is defined as the weighted average. Specifically, we weight the quality of each product by its share of total exports (imports). The data set is available on Feenstra's website: http://www.robertfeenstra.info/data/.

of trade, both across countries and within countries over time. On the other hand, the second column of Figure 1 shows the relationship between the quality of exports and the value of exports and that between the quality of imports and the value of imports for the same sample of countries and the same study period. A simple pooled OLS estimation shows a weak but highly statistically significant *negative* relationship between the quality and value of aggregate trade. In our estimation, an increase in exported quality of one percent is associated with a decrease in aggregate exports of 0.9 percent. Additionally, an increase in imported quality of one percent is associated with a decrease in aggregate imports of 4.9 percent. When a country expands its exports, it typically follows by downgrading the average quality of the product categories, both across countries and within countries over time. The same is true for imports.[2]

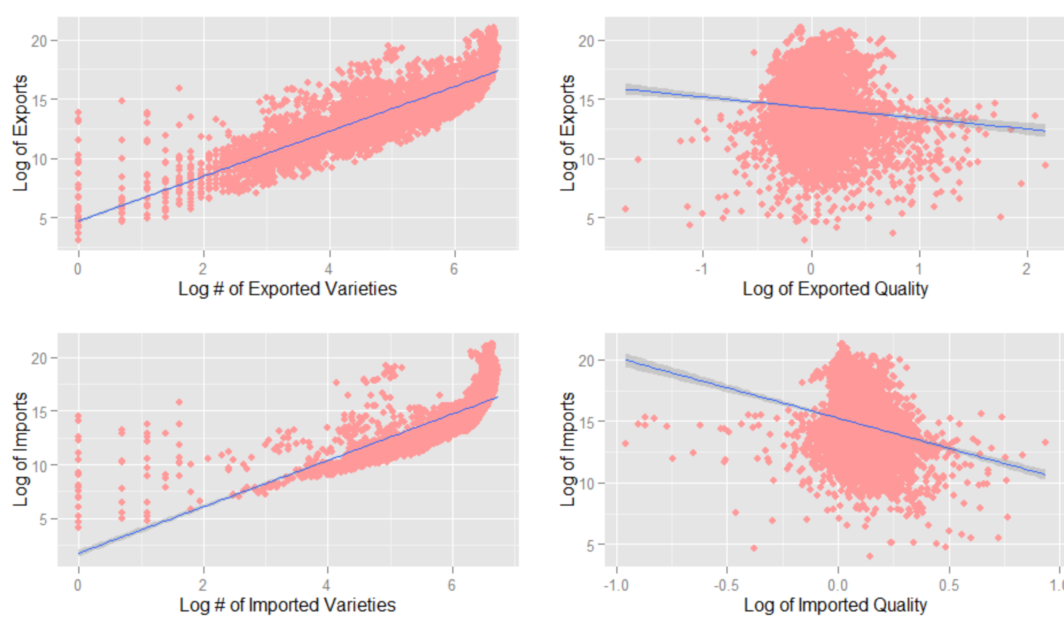

Figure 1: Variety and Quality of Exports and Imports

[2]For the extensive margins of trade, when the number of exported varieties increases by one percent, the export value increases by 1.9 percent. When the number of imported varieties increases by one percent, the import value increases by 4.2 percent. Both figures are statistically significant at more than the one percent level.

Note: Variety and quality of exports and imports are computed at the four-digit SITC level from 1984 to 2011 for countries all over the world based on Feenstra and Romalis (2014). The blue lines in each panel show regression lines, and the shaded parts are 95% confidence regions.

Starting from these observations, we build a stylized two-country DSGE model that features heterogenous firms, allowing them to upgrade or downgrade their product quality. Assuming that goods of higher quality require higher costs to produce, firms choose their optimal quality level based on their specific technology.[3] Such a parsimonious extension of Ghironi and Melitz (2005) successfully generates the observed wedge-shaped pattern of cross-correlations between trade flows and quality of exports and imports. As the impulse response analysis following a positive, transitory labor productivity shock reveals, a key to generating these results is quality upgrading among a smaller number of more productive firms. Our model can also account for a number of other US trade statistics, such as the counter-cyclicality of the trade balance, the pro-cyclicality of both exports and imports and the positive correlation between exports and imports.

We also perform an impulse response analysis with respect to worldwide permanent reductions in iceberg trade costs and fixed export costs. In particular, our results underestimate the gains from trade liberalization, since a permanent reduction in either trade-related cost induces a simultaneous degradation in quality. Without taking into account changes in the quality level, we may overstate the gains from trade liberalization.

Product quality is a key element in explaining a number of puzzling features of the data, such as the positive correlation between the export unit price and distance (see, for instance, Baldwin and Harrigan (2011) and Manova and Zhang (2012)). Several papers built theoretical models that ground endogenous product quality on such observations (Kugler and Verhoogen (2012), Johnson (2012), Antoniades (2012), Feenstra and Romalis (2014), Picard (2015) and Rodrigue and Tan (2015)). These papers develop a more complex mechanism than ours, typically determining product quality with sunk costs and often with endogenous price markups. However, the functional form that positively relates a firm-specific productivity level and a firm-specific quality level eventually comes close

[3] The positive correlation between quality and marginal cost is supported by empirical studies. See, for instance, Verhoogen (2008) and Kugler and Verhoogen (2012).

to ours. Importantly, these papers often lack dynamics and do not unravel the cyclical properties of quality in trade dynamics.

Our paper is also related to the international real business cycle (IRBC) literature that started with Backus et al. (1992), sharing a number of its characteristics. Allowing current accounts to fluctuate with internationally held state non-contingent bonds, our theoretical model can account for several empirical features of trade as does the standard IRBC model.[4] Ghironi and Melitz (2007) discuss the business cycle properties of exported and imported varieties for the US and successfully generate the observed cross-correlations in the number of exported varieties, whereas they do not address product quality explicitly. Arespa and Gruber (2016) embed endogenous product quality in an otherwise standard IRBC model and explore its implications for relative price dynamics. Beyond the difference in focal point, they lack endogenous determination of the number of product variates, whereas we consider endogenous determination of both variety and quality based on heterogenous firms.[5]

This paper is organized as follows. In the next section, we present the model with endogenous quality choice based on heterogenous firms. In Section 3, we calibrate the theoretical model and document the impulse response functions with respect to a productivity shock and to two types of trade liberalization shocks. In the following subsection, second moments and cross-correlations of trade statistics are shown and compared with the actual US data. In the last section, we conclude.

[4] As documented in Heathcote and Perri (2002) and Engel and Wang (2011), the standard IRBC model with state non-contingent bonds can account for counter-cyclical trade balances and pro-cyclical exports and imports as in the data, but it fails to replicate the high volatility of exports and imports, together with that of real exchange rates.

[5] In a companion paper, Hamano (2016) describes the risk-sharing properties and implications for relative prices nested in this paper's model.

2 The model

We present a two-country DSGE model with heterogeneous firms that can determine the quality level of their product. By assuming that the production of goods of higher quality requires more labor input, firms with high specific productivity levels produce goods of a high quality. The quality of goods produced is thus endogenously determined. Other characteristics of the model are isomorphic to those in Ghironi and Melitz (2005, 2007).

2.1 Household Preferences and Intratemporal Choices

The world consists of two countries, Home and Foreign. Foreign variables are denoted with an asterisk (*). Each country is populated by one unit mass of atomic households. We discuss the representative household in Home. Similar arguments hold for households in Foreign. The Home representative household maximizes its expected intertemporal utility, $E_t \sum_{s=t}^{\infty} \beta^{s-t} U_t$, where β $(0 < \beta < 1)$ is an exogenous discount factor. The utility at time t depends on consumption and labor supply as follows

$$U_t = \frac{C_t^{1-\gamma}}{1-\gamma} - \chi \frac{L_t^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}}.$$

In the above expression, γ (≥ 1) denotes risk aversion, χ (> 0) represents the degree of non-satisfaction from supplying labor L_t, and φ (≥ 0) denotes the Frisch elasticity of labor supply.

The basket of goods C_t is defined as

$$C_t = \left[C_{H,t}^{1-\frac{1}{\omega}} + C_{F,t}^{1-\frac{1}{\omega}} \right]^{\frac{1}{1-\frac{1}{\omega}}},$$

where α $(> 1/2)$ is the home bias in consumption attached to the bundle of goods produced locally $C_{H,t}$, ω (> 0) denotes the elasticity of substitution between local $(C_{H,t})$ and imported goods $(C_{F,t})$, and$C_{H,t}$ and $C_{F,t}$ are defined over a continuum of goods Ω as follows

$$C_{H,t} = V_{H,t} \left(\int_{\zeta \in \Omega} q_{D,t}(\zeta) \, c_{D,t}(\zeta)^{1-\frac{1}{\sigma}} \, d\zeta \right)^{\frac{1}{1-\frac{1}{\sigma}}}, \quad C_{F,t} = V_{F,t}^{*} \left(\int_{\vartheta \in \Omega} q_{X,t}^{*}(\vartheta) \, c_{X,t}(\vartheta)^{1-\frac{1}{\sigma}} \, d\vartheta \right)^{\frac{v1}{1-\frac{1}{\sigma}}},$$

where $V_{H,t} \equiv N_{D,t}^{\psi-\frac{1}{\sigma-1}}$, $V_{F,t}^{*} \equiv N_{X,t}^{*\psi-\frac{1}{\sigma-1}}$, and $N_{D,t}$ and $N_{X,t}^{*}$ stand for the number of domestic and imported product varieties, respectively. Here, ψ (≥ 0) represents the marginal utility that stems from an additional increase in the number of varieties in each basket (Benassy 1996). Specifically, the preferences become Dixit and Stiglitz (1977) preferences when $\psi = \frac{1}{\sigma-1}$. At any given time t, only a subset of goods Ω_t is available from total universe of goods Ω. $c_{D,t}(\zeta)$ and $c_{X,t}(\vartheta)$ represent the demand addressed for individual product variety indexed by ζ and ϑ, which are produced domestically and imported, respectively. $q_{D,t}(\zeta)$ and $q_{X,t}(\vartheta)$ indicate the quality attributed to these product varieties, and σ (> 1) denotes the elasticity of substitution between varieties. We assume conventionally $\sigma \geq \omega$.

The optimal consumption for each domestic basket, imported basket and individual product variety is found to be

$$C_{H,t} = \left(\frac{P_{H,t}}{P_t}\right)^{-\omega} C_t, \qquad C_{F,t} = \left(\frac{P_{F,t}}{P_t}\right)^{-\omega} C_t.$$

$$c_{D,t}(\zeta) = \left(V_{H,t} q_{D,t}(\zeta)\right)^{\sigma-1} \left(\frac{p_{D,t}(\zeta)}{P_{H,t}}\right)^{-\sigma} C_{H,t}, \qquad c_{X,t}(\vartheta) = \left(V_{F,t}^{*} q_{X,t}^{*}(\vartheta)\right)^{\sigma-1} \left(\frac{p_{X,t}^{*}(\vartheta)}{P_{F,t}}\right)^{-\sigma} C_{F,t}.$$

In particular, $p_{X,t}^{*}(\vartheta)$ denotes the price of exported goods from Foreign.

Price indices that minimize expenditures on each consumption basket are given by

$$P_t = \left[P_{H,t}^{1-\omega} + P_{F,t}^{1-\omega}\right]^{\frac{1}{1-\omega}},$$

$$P_{H,t} = \frac{1}{V_{H,t}} \left(\int_{\zeta\in\Omega_t} \left(\frac{p_{D,t}(\zeta)}{q_{D,t}(\zeta)}\right)^{1-\sigma} d\zeta\right)^{\frac{1}{1-\sigma}}, \qquad P_{F,t} = \frac{1}{V_{F,t}^{*}} \left(\int_{\vartheta\in\Omega_t} \left(\frac{p_{X,t}^{*}(\vartheta)}{q_{X,t}^{*}(\vartheta)}\right)^{1-\sigma} d\vartheta\right)^{\frac{1}{1-\sigma}}.$$

Observe that the price indices defined on a welfare basis depend on both the number of varieties and the product quality. Finally, we choose the welfare-based consumer price index, P_t, as the numéraire for Home and define the real prices as $\rho_{H,t} \equiv \frac{P_{H,t}}{P_t}$, $\rho_{F,t} \equiv \frac{P_{F,t}}{P_t}$, $\rho_{D,t}(\zeta) \equiv \frac{p_{D,t}(\zeta)}{P_t}$ and $\rho_{X,t}^{*}(\vartheta) \equiv \frac{p_{X,t}^{*}(\vartheta)}{P_t}$.

Similar expressions hold for Foreign. Crucially, the subset of goods available to Foreign during period t, $\Omega_t^* \in \Omega$, can be different from the subset of goods and quality available to Home.

2.2 Production, Pricing and the Export Decision

In every period, there is a mass of $N_{E,t}$ entrants. Prior to entry, these new entrants are identical and face a sunk entry cost of f_E, which is defined as follows

$$f_E = Z_{E,t} l_{E,t},$$

where $Z_{E,t}$ denotes the labor productivity level that is specific to firm setup and common to all firms, and $l_{E,t}$ is the demand for labor in firm setup. Upon entry, firms draw their productivity level z from a distribution $G(z)$ on $[z_{\min}, \infty)$. Since there are no fixed production costs, all firms produce unless they are hit by an exogenous depreciation shock, which takes place with probability $\delta \in (0, 1)$ in every period. This exit-inducing shock is independent of firm-specific productivity and assumed to take place at the very end of the period. Thus, $G(z)$ also represents the productivity distribution of all producing firms.

Exporting requires an operational fixed cost f_X in every period. Specifically, f_X is defined as

$$f_X = Z_t l_{f_X,t},$$

where Z_t denotes the labor productivity level in production that is common to all firms, and $l_{f_X,t}$ is the demand for labor required to produce an amount f_X of fixed costs. Only a subset of firms whose productivity level z is above the cutoff level $z_{X,t}$ exports by charging sufficiently lower quality-adjusted prices and earning positive profits despite the existence of a fixed export cost f_X. Thus, non-tradeness in the economy appears endogenously with changes in the productivity cutoff.

The firm faces a residual demand curve with constant elasticity σ. The production scale is thus determined by the demand addressed to the firm. Firm profit maximization under flexible prices yields the following optimal real prices

$$\rho_{D,t}(z) = \frac{\sigma}{\sigma - 1} mc_t(z),$$

where $mc_t(z)$ is the real marginal cost. We assume that producing higher-quality goods requires a higher marginal cost $mc_t(z)$ such that

$$mc_t(z) = \left(1 + \frac{q(z)^{\frac{1}{\phi}}}{z}\right) \frac{w_t}{Z_t z},$$

where ϕ $(0 \leq \phi < 1)$ is a parameter that determines the quality ladder, and w_t denotes the real wage. Given a firm-specific productivity level z, the firm endogenously chooses its specific quality level $q(z)$. Specifically, the firm minimizes the quality-adjusted marginal cost $mc_t(z)/q(z)$. As a result, the optimal quality is given by

$$q(z) = \left(\frac{\phi}{1 - \phi} z\right)^{\phi}.$$

Given $\phi > 0$, a highly productive firm produces high-quality goods. Observe that when there is no quality ladder ($\phi = 0$), firms produce goods of identical quality, irrespective of their specific productivity levels. In such a case, the model becomes similar to that of Ghironi and Melitz (2005) in which there is no choice of quality.

Due to a fixed operational export cost f_X, firm z may not export. If the firm exports, its export price is $\rho_{X,t}(z) = \tau_t \rho_{D,t}(z) Q_t^{-1}$, τ_t is the iceberg trade cost, and Q_t is the real exchange rate defined as the price of foreign consumption goods in terms of home consumption goods. Thus, $Q_t \equiv P_t^*/P_t$. $\rho_{X,t}(z)$ is denominated in terms of the price of the consumption basket in the export market.

Total firm profits $d_t(z)$ can be decomposed into those from domestic sales $d_{D,t}(z)$ and those from exporting sales $d_{X,t}(z)$: $d_t(z) = d_{D,t}(z) + d_{X,t}(z)$. Using the demand functions found previously, we can write the profits from each market as

$$d_{D,t}(z) = \frac{1}{\sigma} N_{D,t}^{\psi(\omega-1)-1} \left(\frac{\rho_{D,t}(z)}{q(z)}\right)^{1-\omega} C_t,$$

$$d_{X,t}(z) = \frac{Q_t}{\sigma} N_{X,t}^{\psi(\omega-1)-1} \left(\frac{\rho_{X,t}(z)}{q(z)}\right)^{1-\omega} C_t^* - \frac{w_t f_X}{Z_t}, \text{ if firm } z \text{ exports.}$$

2.3 Firm Averages

Given a distribution $G(z)$, a mass of $N_{D,t}$ of domestically producing firms has a distribution of productivity levels over $[z_{\min},\ \infty)$. Among these firms, there are $N_{X,t} = [1 - G(z_{X,t})] N_{D,t}$ exporters in Home. Following Melitz (2003), we define two average productivity levels, $\widetilde{z}_D$ for domestically producing firms and $\widetilde{z}_{X,t}$ for exporters as follows

$$\widetilde{z}_D \equiv \left[\int_{z_{\min}}^{\infty} z^{\sigma-1} dG(z) \right]^{\frac{1}{\sigma-1}}, \qquad \widetilde{z}_{X,t} \equiv \left[\frac{1}{1-G(z_{X,t})} \int_{z_{X,t}}^{\infty} z^{\sigma-1} dG(z) \right]^{\frac{1}{\sigma-1}}.$$

These average productivity levels summarize all the information about the distribution of productivities. Given these averages, we define the average real domestic and exporting price as $\widetilde{\rho}_{D,t} \equiv \rho_{D,t}(\widetilde{z}_D)$ and $\widetilde{\rho}_{X,t} \equiv \rho_{X,t}(\widetilde{z}_{X,t})$, respectively. Similarly, the average domestic and average exporting quality (AEQ) are provided by $\widetilde{q}_D \equiv q_D(\widetilde{z}_D)$ and $\widetilde{q}_{X,t} \equiv q_{X,t}(\widetilde{z}_{X,t})$, respectively. We also define average real profits from domestic sales and exporting sales as $\widetilde{d}_{D,t} \equiv d_{D,t}(\widetilde{z}_D)$ and $\widetilde{d}_{X,t} \equiv d_{X,t}(\widetilde{z}_{X,t})$. Finally, average real profits among all Home producers is given by $\widetilde{d}_t^s = \widetilde{d}_{D,t} + (N_{X,t}/N_{D,t})\, \widetilde{d}_{X,t}$.

2.4 Firm Entry and Exit

We assume that entrants at time t only start producing at time $t+1$. These entrants discount the stream of their expected profits $\left\{ \widetilde{d}_i^s \right\}_{i=t+1}^{\infty}$ using the household's discount factor adjusted by exogenous exit inducing shock δ. Thus, their expected post-entry value is

$$\widetilde{v}_t^s = E_t \sum_{i=t+1}^{\infty} \beta^{i-t} \left(\frac{C_i}{C_t} \right)^{-\gamma} (1-\delta)^{s-t}\, \widetilde{d}_i^s.$$

This firm value is the price of Home equities, the value of mutual funds of heterogenous firms of Home origin. Entry occurs until this expected firm value is equalized with the entry cost, leading to the following free entry condition

$$\widetilde{v}_t^s = \frac{w_t}{Z_{E,t}} f_E.$$

The timing of entry and production implies that the number of domestically producing firms evolves according to $N_{D,t} = (1-\delta)(N_{D,t-1} + N_{E,t-1})$.

2.5 Parametrization of Productivity Draws

We assume the following Pareto distribution for $G(z)$

$$G(z) = 1 - \left(\frac{z_{\min}}{z}\right)^k,$$

where $z_{\min}$ is the minimum productivity level, and k $(> \sigma - 1)$ is a shape parameter. With this parametrization, we have

$$\widetilde{z}_D = z_{\min}\left[\frac{k}{k-(\sigma-1)}\right]^{\frac{1}{\sigma-1}}, \quad \widetilde{z}_{X,t} = z_{X,t}\left[\frac{k}{k-(\sigma-1)}\right]^{\frac{1}{\sigma-1}}.$$

The share of exporters in the total number of domestic firms is then given by

$$\frac{N_{X,t}}{N_{D,t}} = z_{\min}^k \left(\widetilde{z}_{X,t}\right)^{-k}\left[\frac{k}{k-(\sigma-1)}\right]^{\frac{k}{\sigma-1}}.$$

Finally, there exists a firm with a specific productivity cutoff $z_{X,t}$ that earns zero profits from exporting, as $d_{X,t}(z_{X,t}) = 0$. With the above Pareto distribution, this implies that

$$\widetilde{d}_{X,t} = \frac{w_t f_X}{Z_t}\frac{\sigma-1}{k-(\sigma-1)}.$$

2.6 Household Budget Constraint and Intertemporal Choices

There are two types of financial assets, equities and bonds. Equities are held only domestically, while bonds are traded internationally, allowing international borrowing and lending.

The Home representative household finances the entry cost of new entrants $N_{E,t}$ and all producing firms $N_{D,t}$ in Home at time t by purchasing a share of Home equities $s_{h,t+1}$. The gross returns of Home and Foreign equities between t and $t+1$ (in units of Home consumption) are given by

$$R^s_{h,t+1} \equiv (1-\delta)\frac{\widetilde{v}^s_{t+1}+\widetilde{d}^s_{t+1}}{\widetilde{v}^s_t}.$$

These returns on equities are adjusted by $1-\delta = N_{D,t+1}/\left(N_{D,t}+N_{E,t}\right)$, the surviving rate of producing firms and entrants between the two time periods. In addition to equities, the household holds bonds defined in terms of the domestic consumption basket. Letting $b_{h,t+1}$ and $b_{f,t+1}$ be holdings for bonds issued in Home and Foreign, respectively, and the returns on each bond be r_{t+1} and r^*_{t+1} from time t to $t+1$, they give the following gross returns between t and $t+1$ (in units of Home consumption)

$$R^b_{h,t+1} \equiv 1+r_{t+1}, \quad R^b_{f,t+1} \equiv \left(1+r^*_{t+1}\right)\frac{Q_{t+1}}{Q_t}.$$

Using the above-mentioned notation, the budget constraint of the Home representative households is thus given by

$$\begin{aligned} C_t + \widetilde{v}^s_t\left(N_{D,t}+N_{E,t}\right)s_{h,t+1} + b_{h,t+1} + Q_t b_{f,t+1} + \frac{\vartheta}{2}b^2_{h,t+1} + \frac{\vartheta}{2}Q_t b^2_{f,t+1} \\ = w_t L_t + R^s_{h,t}\widetilde{v}^s_{t-1}\left(N_{D,t-1}+N_{E,t-1}\right)s_{h,t} + R^b_{h,t}b_{h,t} + R^b_{f,t}Q_{t-1}b_{f,t} + T^f_t. \quad (1) \end{aligned}$$

With bond-holdings in the budget constraint, indeterminacy of the equilibrium portfolio position and non-stationarity arise when using a linear approximation. We overcome this problem by introducing quadratic adjusting costs for bond holdings ϑ, which guarantee a locally unique symmetric steady state with zero bond holdings and stationarity of the model. T^f_t is a free rebate of adjusting costs. The representative household maximizes the expected intertemporal utility with respect to $s_{h,t+1}$, $b_{h,t+1}$, $b_{f,t+1}$, L_t and C_t subject to (1) for all periods. As a result, the Euler equation for share holdings can be derived as

$$1 = \beta E_t\left[\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma} R^s_{h,t+1}\right],$$

The Euler equations for domestic and Foreign bond holdings are given by

$$1+\vartheta b_{h,t+1} = \beta E_t\left[\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma} R^b_{h,t+1}\right], \quad 1+\vartheta b_{f,t+1} = \beta E_t\left[\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma} R^b_{f,t+1}\right].$$

Finally, the optimal labor supply is found to be

$$\chi (L_t)^{\frac{1}{\varphi}} = w_t C_t^{-\gamma}.$$

In particular, when $\varphi = 0$, marginal disutility becomes infinite, and the labor supply becomes inelastic. Similar expressions hold for Foreign.

2.7 General Equilibrium and Net Foreign Asset Dynamics

Here, we discuss the conditions that hold in general equilibrium. Supplied labor units L_t are demanded for fixed costs of exporting and firm creation as well as for production of domestic and tradable goods. This implies that

$$L_t = \frac{N_{E,t}\widetilde{v}_t^s}{w_t} + \frac{(\sigma - 1) N_{D,t}\widetilde{d}_t}{w_t} + \frac{\sigma N_{X,t} f_X}{Z_t}.$$

In addition, net foreign assets (in Home consumption units) at the end of period t is defined as

$$NFA_{t+1} \equiv b_{f,t+1} Q_t - b_{h,t+1}^*.$$

where $b_{h,t+1}^*$ stands for the bonds issued in Home but held by Foreign households. Since there are no cross-border equity holdings by assumption, only cross-border bond holdings appear in the definition. With the above expression of net foreign assets, the budget constraint (1) can be rewritten and provides the following net foreign asset dynamics:

$$NFA_{t+1} = NX_t + NFA_t R_{h,t}^b + \xi_{h,t},$$

where NX_t denotes net exports, and ξ_t stands for "excess returns" between $t-1$ and t relative to returns on Home bonds $R_{h,t}^b$. Precisely, NX_t and ξ_t are given by

$$NX_t = \frac{1}{2}\left[w_t L_t + N_{D,t}\widetilde{d}_t - Q_t\left(w_t^* L_t^* + N_{D,t}^*\widetilde{d}_t^*\right)\right] - \frac{1}{2}\left[(C_t - N_{E,t}\widetilde{v}_t^s) - Q_t\left(C_t^* - N_{E,t}^*\widetilde{v}_t^{s*}\right)\right],$$

and

$$\xi_t \equiv b_{f,t} Q_t \left(R^b_{f,t} - R^b_{h,t} \right).$$

Note that excess returns are zero in the first-order dynamics because of zero bond holdings due to adjustment costs in the steady state. Finally, asset markets clear in all periods as

$$b_{h,t+1} + b^*_{h,t+1} = b_{f,t+1} + b^*_{f,t+1} = 0,$$

where $b^*_{f,t+1}$ represents the holdings of bonds issued in Foreign and held by Foreign households. The whole system is summarized in Table 1. The details of the steady state are provided in Appendix A.

3 Macroeconomic Dynamics

In this section, we calibrate the benchmark model and explore its implications. First, impulse response functions with respect to a permanent reduction in the iceberg and fixed export costs are presented. Second, we specify a transitory aggregate labor productivity process and investigate its implications with impulse response functions. We also document the second moments implied by the theoretical model. Finally, cross-correlations regarding trade dynamics are presented and compared with actual US data.

3.1 Calibration

We calibrate the theoretical models using the parameter values in Table 2. The calibration is conducted on a quarterly basis. The value of constant risk aversion (γ), steady state discount factor (β), the Frisch elasticity of labor supply (φ), the elasticity of substitution between local goods and imported goods (ω) are taken from the IRBC literature. The value of a death shock (δ), the elasticity of substitution among product varieties (σ), the preference for variety (ψ), the fixed export costs (f_X) and the shape of the Pareto distribution (k) follow Ghironi and Melitz (2005). We also normalize the steady state value of sunk entry costs and the minimum level of firm-specific productivity to $f_E = z_{\min} = 1$. The parameter that determines the quality ladder in the economy (ϕ) is taken

Table 1: The model

Price indices	$\rho_{H,t}^{1-\omega}+\rho_{F,t}^{1-\omega}=1,\ \rho_{H,t}=N_{D,t}^{-\psi}\frac{\widetilde{\rho}_{D,t}}{\widetilde{q}_D},\ \rho_{F,t}=N_{X,t}^{*-\psi}\frac{\widetilde{\rho}^*_{X,t}}{\widetilde{q}^*_{X,t}}$
	$\rho_{F,t}^{*1-\omega}+\rho_{H,t}^{*1-\omega}=1,\ \rho^*_{F,t}=N_{D,t}^{*-\psi}\frac{\widetilde{\rho}^*_{D,t}}{\widetilde{q}^*_D},\ \rho^*_{H,t}=N_{X,t}^{-\psi}\frac{\widetilde{\rho}_{X,t}}{\widetilde{q}_{X,t}}$
Pricing	$\widetilde{\rho}_{D,t}=\frac{\sigma}{\sigma-1}\frac{1}{1-\phi}\frac{w_t}{Z_t\widetilde{z}_D},\quad \widetilde{\rho}_{X,t}=\tau_t\frac{\sigma}{\sigma-1}\frac{1}{1-\phi}\frac{w_t}{Z_t\widetilde{z}_{X,t}}Q_t^{-1},$
	$\widetilde{\rho}^*_{D,t}=\frac{\sigma}{\sigma-1}\frac{1}{1-\phi}\frac{w_t}{Z^*_t\widetilde{z}^*_D},\quad \widetilde{\rho}^*_{X,t}=\tau_t\frac{\sigma}{\sigma-1}\frac{1}{1-\phi}\frac{w^*_t}{Z^*_t\widetilde{z}^*_{X,t}}Q_t$
Profits	$\widetilde{d}_t=\widetilde{d}_{D,t}+\frac{N_{X,t}}{N_{D,t}}\widetilde{d}_{X,t},\quad \widetilde{d}_{D,t}=\frac{1}{\sigma}N_{D,t}^{\psi(\omega-1)-1}\left(\frac{\widetilde{\rho}_{D,t}}{\widetilde{q}_D}\right)^{1-\omega}C_t$
	$\widetilde{d}_{X,t}=\frac{Q_t}{\sigma}N_{X,t}^{\psi(\omega-1)-1}\left(\frac{\widetilde{\rho}_{X,t}}{\widetilde{q}_{X,t}}\right)^{1-\omega}C^*_t-\frac{w_tf_X}{Z_t}$
	$\widetilde{d}^*_t=\widetilde{d}^*_{D,t}+\frac{N^*_{X,t}}{N^*_{D,t}}\widetilde{d}^*_{X,t},\quad \widetilde{d}^*_{D,t}=\frac{1}{\sigma}N_{D,t}^{*\psi(\omega-1)-1}\left(\frac{\widetilde{\rho}^*_{D,t}}{\widetilde{q}^*_D}\right)^{1-\omega}C^*_t$
	$\widetilde{d}^*_{X,t}=\frac{Q_t^{-1}}{\sigma}N_{X,t}^{*\psi(\omega-1)-1}\left(\frac{\widetilde{\rho}^*_{X,t}}{\widetilde{q}^*_{X,t}}\right)^{1-\omega}C_t-\frac{w^*_tf^*_X}{Z^*_t}$
Free entry	$\widetilde{v}^s_t=\frac{w_t}{Z_{E,t}}f_E,\qquad \widetilde{v}^{s*}_t=\frac{w^*_t}{Z^*_{E,t}}f^*_E$
LMC	$w_tL_t=N_{E,t}\widetilde{v}^s_t+(\sigma-1)N_{D,t}\widetilde{d}_t++\sigma N_{X,t}\frac{w_tf_X}{Z_t}$
	$w^*_tL^*_t=N^*_{E,t}\widetilde{v}^{s*}_t+(\sigma-1)N^*_{D,t}\widetilde{d}^*_t+\sigma N^*_{X,t}\frac{w^*_tf^*_X}{Z^*_t}$
Export share	$\frac{N_{X,t}}{N_{D.t}}=z_{\min}^k\left(\widetilde{z}_{X,t}\right)^{-k}\left[\frac{k}{k-(\sigma-1)}\right]^{\frac{k}{\sigma-1}},\quad \frac{N^*_{X,t}}{N^*_{D,t}}=z_{\min}^k\left(\widetilde{z}^*_{X,t}\right)^{-k}\left[\frac{k}{k-(\sigma-1)}\right]^{\frac{k}{\sigma-1}}$
ZCP	$\widetilde{d}_{X,t}=\frac{w_tf_X}{Z_t}\frac{\sigma-1}{k-(\sigma-1)},\quad \widetilde{d}^*_{X,t}=\frac{w^*_tf^*_X}{Z^*_t}\frac{\sigma-1}{k-(\sigma-1)}$
AEQ	$\widetilde{q}_{X,t}=\left(\frac{\phi}{1-\phi}\widetilde{z}_{X,t}\right)^{\phi},\quad \widetilde{q}^*_{X,t}=\left(\frac{\phi}{1-\phi}\widetilde{z}^*_{X,t}\right)^{\phi}$
Number of firms	$N_{D,t+1}=(1-\delta)(N_{D,t}+N_{E,t}),\quad N^*_{D,t+1}=(1-\delta)\left(N^*_{D,t}+N^*_{E,t}\right)$
Euler shares	$1=\beta E_t\left[\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}R^s_{h,t+1}\right]$
	$1=\beta E_t\left[\left(\frac{C^*_{t+1}}{C^*_t}\right)^{-\gamma}R^s_{f,t+1}\frac{Q_t}{Q_{t+1}}\right]$
Euler bonds	$1+\vartheta b_{h,t+1}=\beta E_t\left[\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}R^b_{h,t+1}\right]$
	$1+\vartheta b_{f,t+1}=\beta E_t\left[\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}R^b_{f,t+1}\right]$
	$1+\vartheta b^*_{f,t+1}=\beta E_t\left[\left(\frac{C^*_{t+1}}{C^*_t}\right)^{-\gamma}R^b_{f,t+1}\frac{Q_t}{Q_{t+1}}\right]$
	$1+\vartheta b^*_{h,t+1}=\beta E_t\left[\left(\frac{C^*_{t+1}}{C^*_t}\right)^{-\gamma}R^b_{h,t+1}\frac{Q_t}{Q_{t+1}}\right]$
BMC	$b_{h,t+1}+b^*_{h,t+1}=0,\quad b_{f,t+1}+b^*_{f,t+1}=0.$
Net foreign assets	$NFA_{t+1}=NX_t+NFA_t(1+r_{t+1})+\xi_t$
Net exports	$NX_t=\frac{1}{2}\left[w_tL_t+N_{D,t}\widetilde{d}_t-Q_t\left(w^*_tL^*_t+N^*_{D,t}\widetilde{d}^*_t\right)\right]$
	$-\frac{1}{2}\left[(C_t+N_{E,t}\widetilde{v}^s_t)-Q_t\left(C^*_t+N^*_{E,t}\widetilde{v}^{s*}_t\right)\right]$
Excess returns	$\xi_t=Q_tB_{*,t}\left(r^*_{t+1}-r_{t+1}\right)$

Table 2: Baseline parameter values

γ	constant risk aversion	2
β	discount factor	0.99
φ	Frisch elasticity of labor supply	2
σ	elasticity of substitution among varieties	3.8
ω	between Home and Foreign goods	2
τ	steady state trade cost	1.3
δ	death shock	0.025
k	Pareto distribution	3.34
ψ	preference for variety	Dixit-Stiglitz
ϕ	quality ladder	0.61

from Feenstra and Romalis (2014) in which they estimate the elasticity of firm-specific quality with respect to firm-specific productivity using world trade data. Specifically with $\phi = 0.61$, the fixed export cost f_X is equal to 44 percent of the per-period, amortized flow value of the entry costs. Exporters are, on average, 58.3 percent more productive and produce goods of 32.3 percent higher quality than non-exporters. With these values, the steady state share of domestic goods is 0.52. Note that by setting $\phi = 0$, the steady state becomes similar to those analyzed in Ghironi and Melitz (2005, 2007).

3.2 Trade liberalization

Figure 2 documents the impulse response functions (IRFs) obtained with a worldwide permanent one-percent reduction in iceberg trade costs, τ_t and τ_t^*. We document only variables for the Home country due to symmetry across countries. The bold lines show the IRFs for the benchmark model, while the dashed lines show the model without quality ladder ($\phi = 0$).

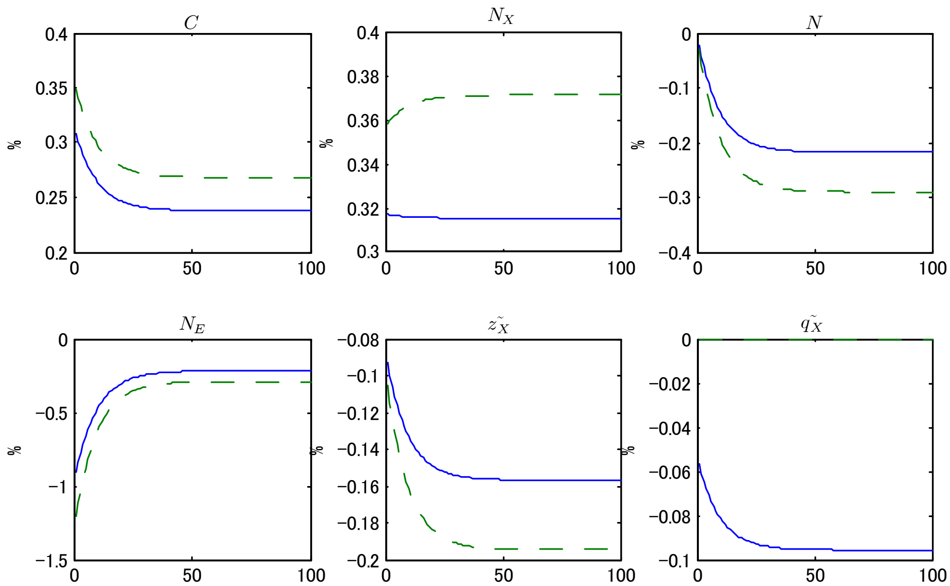

Figure 2: Response to Permanent 1% Reduction in $\tau = \tau^*$.

The IRFs of consumption C_t, the number of exporters $N_{X,t}$, the number of domestic firms N_t, the number of new entrants $N_{E,t}$ and the productivity cutoff $\widetilde{z}_{X,t}$ of the benchmark model (solid lines) are qualitatively similar to those obtained with the no-quality-ladder model (dotted lines). A permanent reduction in trade costs induces the entry of exporters with a relatively low productivity level (rise in $N_{X,t}$ and decrease in $\widetilde{z}_{X,t}$ in the new steady state). Although the congestion created by these new exporters results in smaller numbers of domestic firms N_t and new entrants $N_{E,t}$, the total number of available product varieties (domestic N_t plus imported number of varieties $N_{X,t}$) expands, contributing a higher level of welfare-consistent consumption C_t. With the quality ladder model, however, the boosting effect of trade liberalization on the number of exporters is limited, and these less efficient new exporters produce goods of lower quality (decrease in $\widetilde{q}_{X,t}$). The welfare gains from trade liberalization are thus constrained compared to the model without the quality ladder.

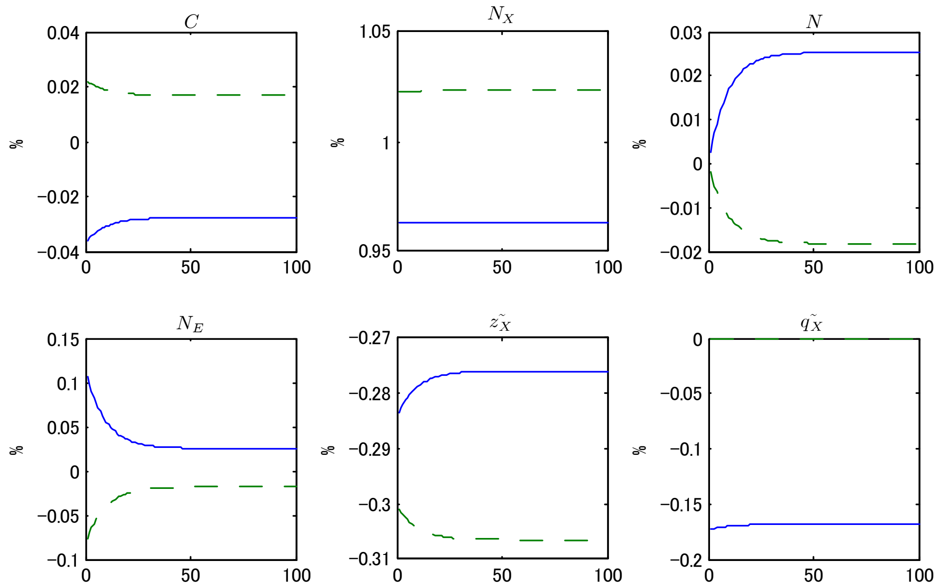

Figure 3: Response to Permanent 1% Reducion in $f_X = f_X^*$.

Figure 3 reports the IRFs for a permanent one-percent reduction in the fixed export costs, $f_{X,t}$ and $f_{X,t}^*$. In both models, the number of exporters $N_{X,t}$ increases and the productivity cutoff $\widetilde{z}_{X,t}$ decreases in the new steady state. Changes in the number of domestic firms N_t and new entrants $N_{E,t}$ are very limited in both models. Comparing the benchmark model and the model without the quality ladder, however, increases the number of exporters and reduces the productivity cutoff to a lesser extent than in our benchmark model when a downward adjustment in quality $\widetilde{q}_{X,t}$ simultaneously takes place. What is striking is that steady consumption C_t is lower in the benchmark quality ladder model following such a reduction in fixed export costs. Welfare losses stemming from lower-quality imported goods outweigh the welfare gains of a small increase in the number of imported varieties.

Reductions in both iceberg and fixed export costs induce a higher number of less-efficient exporters producing lower-quality goods Our simulation results thus highlight a probable overestimation of gains arising from trade liberalization by omitting adjustments in product quality.

3.3 International Transmission of a Transitory Productivity Shock

In the following section, we explore the implication of a transitory productivity shock. For simplicity, the marginal costs of production Z_t and firm creation $Z_{E,t}$ are assumed to be perfectly correlated such that $Z_t = Z_{E,t}$ and $Z_t^* = Z_{E,t}^*$. The productivity process is selected from Backus et al. (1992) such that $\mathsf{Z}_{t+1}{=}\Omega \mathsf{Z}_t{+}\xi_t$, where $\mathsf{Z}_t = \begin{bmatrix} Z_t, & Z_t^* \end{bmatrix}'$, $\xi_t = \begin{bmatrix} \xi_t, & \xi_t^* \end{bmatrix}'$ and

$$\Omega = \begin{bmatrix} 0.906 & 0.088 \\ 0.088 & 0.906 \end{bmatrix} \text{ and } V(\xi) = \begin{bmatrix} 0.73 & 0.19 \\ 0.19 & 0.73 \end{bmatrix}.$$

Here, ξ_t is assumed to be zero mean i.i.d. Finally, as outlined in Ghironi and Melitz (2005), we define the data-consistent variables by deflating them by the observed price index, $\widehat{P}_t$. Therefore, any real variable X_t measured in welfare-based CPI P_t is transformed to $X_{R,t}$ deflated by the empirically based CPI $\widehat{P}_t$ through the following operation: $X_{R,t} \equiv P_t X_t / \widehat{P}_t$. With such a definition, the empirically consistent measures underestimate fluctuations in the number of product quality as well as product quality in a similar magnitude.

3.3.1 Impulse Response Functions

Figure 4 reports the IRFs following a one-percent rise in aggregate Home labor productivity Z_t. For the sake of simplicity, we set the spillover coefficients across countries in the shock process to zero. The first row in Figure 4 shows the data-consistent trade balance to output ratio, exports, imports, and the outputs of Home and Foreign. The IRFs with the benchmark model (solid lines) are very similar to those for the no-quality-ladder model (dotted lines). The trade balance to output ratio is worse for the Home country on impact. Such a movement in the trade balance reflects higher borrowing in the international financial market in order to finance the higher investments that allow the creation of new firms. This is in line with the literature, as first documented in Backus et al. (1992) with an IRBC model that embeds capital accumulation instead of firm creation. Simultaneously, on impact of the shock, the terms of labor ($TOL_t \equiv (w_t^*/Z_t)\,/\,(w_t/Z_t)$) depreciate and then appreciate in transitory dynamics reflecting wage appreciation. Since the real fixed export costs appreciate as well, the productivity cutoff for Home $\widetilde{z}_{X,t}$ increases gradually and peaks after several quarters, yielding a hump-shaped pattern. Under such harsh selection, the number of exporters $N_{X,t}$ (which increases on impact and for several quarters thereafter due to higher investments) decreases in transitory dynamics. On the other hand, as highly productive exporters produce goods of high quality, product quality $\widetilde{q}_{X,t}$ increases, yielding a similar hump-shaped pattern as that of the productivity cutoff in the benchmark model. As a result, stronger welfare transmission is driven by the higher quality of exported varieties: Foreign consumption increases sharply on impact and in transitory dynamics compared to under the model with no quality ladder.[6] The intuition arising from the IRFs following a transitory productivity shock is key to understanding the negative correlation between aggregate exports and product quality, as we detail in the following section.

We also document the second moments of the theoretical models compared with the US data.[7] Qualitatively, the second moments implied by the benchmark model and the model without the quality ladder are very similar. In particular, both theoretical models replicate the observed volatility of output, consumption, investments and labor supply well. This is also the case for their contemporaneous correlations with output. These results are in line with those obtained with the standard IRBC model, as in Heathcote and Perri (2002) in which the current account fluctuates due to internationally held state-non-contingent bonds.

[6] For international relative prices and the resulting risk-sharing properties of the model, see the detailed discussion in Hamano (2016)

[7] The nominal values of GDP, exports, imports and GDP deflator are drawn from the IMF International Financial Statistics. Consumption (private plus public) and investments (fixed capital formation) are taken from the Bureau of Economic Analysis. Labor supply (the number of employees multiplied by hours worked) is drawn from the Bureau of Labor Statistics. The data range from 1984 Q1 to 2011Q1. All series are logged and HP filtered with a smoothing parameter, λ =1600, with the exception of the trade balance to GDP ratio, which is not logged. The second moments of the theoretical models are computed using the frequency domain techniques proposed by Uhlig (1998). All series are transformed into empirically consistent series as in Ghironi and Melitz (2005) and HP filtered using the same smoothing parameter. GDP and investments are defined as $Y_{R,t} \equiv w_{R,t}L_t + N_t\widetilde{d}_t$ and $I_{R,t} \equiv \widetilde{v}_{R,t}N_{Et}$, respectively.

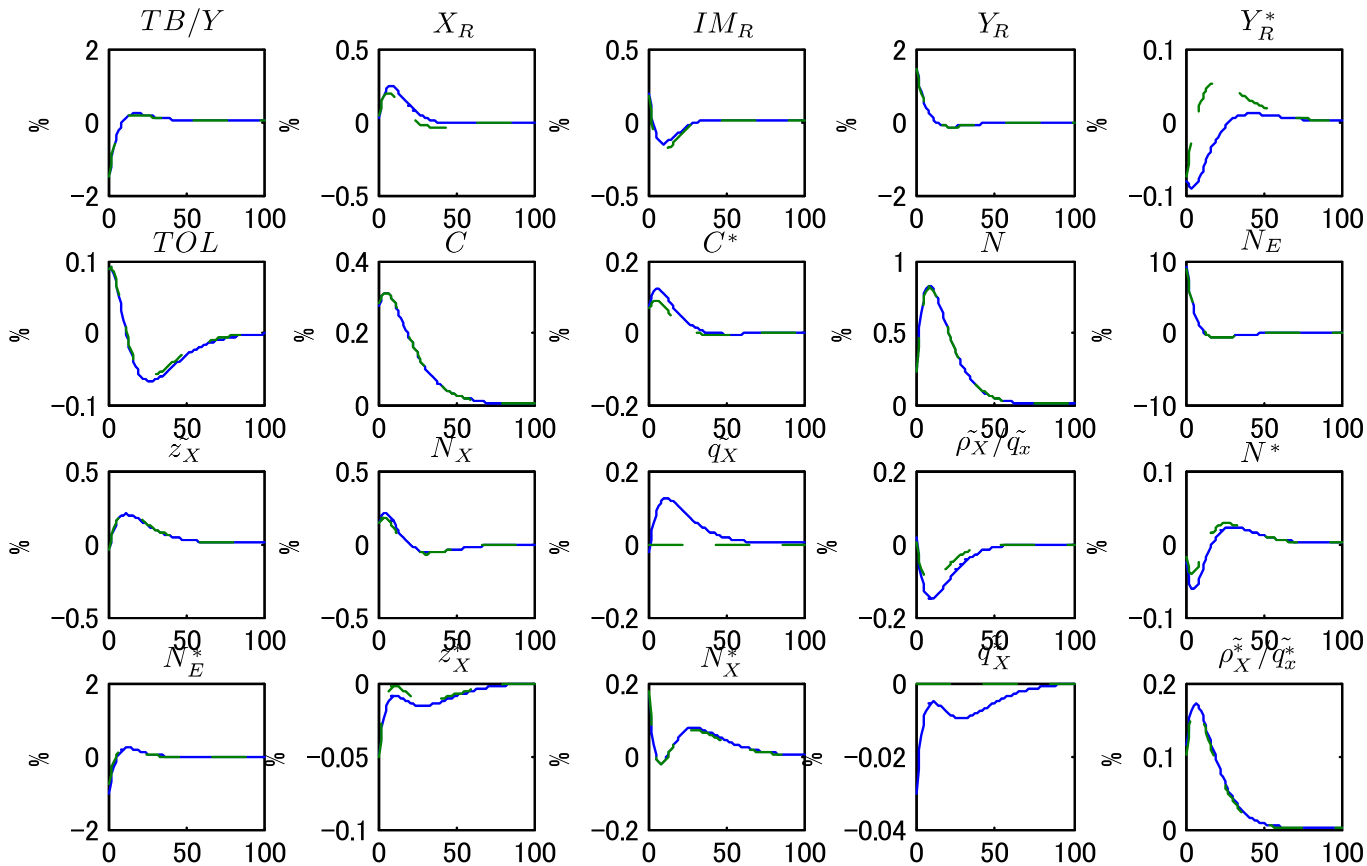

Figure 4: Transitory Aggregate Productivity Shock

With respect to trade statistics, both theoretical models fail to reproduce the observed high volatility of exports and imports and the relatively low volatility of the trade balance to GDP ratio. Specifically, exports and imports are approximately six times as volatile as GDP in the data.[8] In order to match these moments, another theoretical device would be required, such as an uncovered interest parity shock or recursive preferences, as argued in Kollmann (2004, 2016). The theoretical models, however, perform remarkably well in replicating the observed pro-cyclicality of both exports and imports and the counter-cyclicality of the trade balance. The correlation between exports and imports are also positive in both theoretical models, as in the data. In the next section, we look more deeply at the cyclicality of trade together with the cyclicality of the extensive margins of trade and product quality.

Table 3: Second moments

% std.dev.	Y_R	C_R	I_R	L_R	X_R	IM_R	TB_R/Y_R	
US Data	1.14	1.01	4.61	1.66	6.59	7.33	0.14	
The Benchmark	1.04	0.35	5.98	0.64	1.29	0.43	1.08	
No Quality Ladder	1.01	0.29	5.79	0.63	1.66	0.82	1.01	
Std.dev. relative to Y_R								
US Data	1.00	0.88	4.04	1.46	5.77	6.42	0.12	
The Benchmark	1.00	0.33	5.72	0.61	1.24	0.41	1.04	
No Quality Ladder	1.00	0.29	5.74	0.63	1.64	0.81	1.00	
Correlation with Y_R								$Corr(X_R, IM_R)$
US Data	1.00	0.82	0.90	0.89	0.68	0.78	-0.57	0.74
The Benchmark	1.00	0.71	0.96	0.75	0.26	0.55	-0.93	0.78
No Quality Ladder	1.00	0.72	0.98	0.76	0.32	0.51	-0.97	0.82

[8] As documented in Engel and Wang (2011), these are common features among advanced economies. They find that exports and imports are approximately three times as volatile as GDP.

3.4 Correlation

In this section, we explore the cyclical properties of the extensive margins and the quality of trade with trade dynamics. For that purpose, we compare the actual US data with the model generated correlations using different leads and lags.[9] Figures 5 to 7 show the cross-correlations computed from the data, indicated by squares, while the model-generated cross-correlations are shown with crosses. The first panel in Figure 5 shows the correlation between the trade balance to GDP ratio and current period GDP with different leads and lags. The data show a wedge-shaped pattern, and the model replicates it remarkably well. A good fit with the data is also observed for cross-correlations between exports and GDP (the second panel in Figure 5) and those between imports and GDP (third panel in Figure 5). As explained in Ghironi and Melitz (2007), such a counter-cyclical pattern arises in the trade balance due to international borrowing and lending, thus resulting investments in terms of new firms/products creation.

[9] We use the data presented in the introduction for the US time series. The annual average US export varieties and quality of US exports are 762.9 and 1.1799, respectively, for four-digit SITC codes; the corresponding values for imports are 761.4 and 1.032, respectively. Quarterly series are interpolated from annual series as in Ghironi and Melitz (2007).

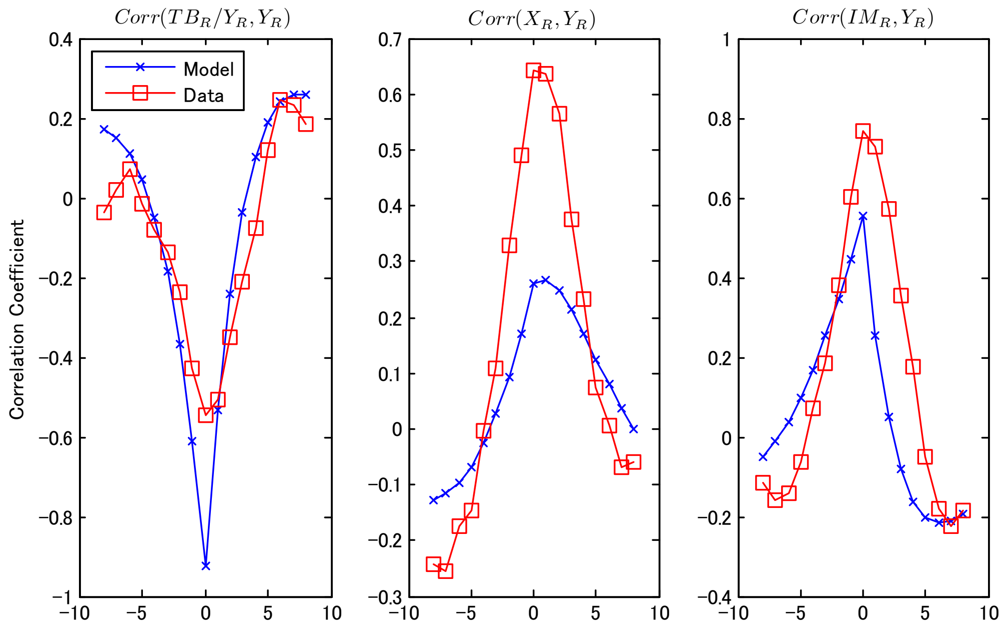

Figure 5: The Cyclicality of Trade Balance, Exports and Imports

The first panel in Figure 6 presents cross-correlations with exports and the number of export varieties. Cross-correlations between export varieties and exports show a mirrored S-shaped pattern in our data, while the model generates a tent-shaped pattern. This is also the case for imports (the first panel in Figure 6).[10] We conjecture that such a pattern in data may be related to the forward-looking behavior of exporters who anticipate a future rise in demand and enter the export market before the boom occurs. On the contrary, since the selection of export markets occurs instantaneously in the theoretical model, we see the a tent-shaped pattern of cross-correlations. The tent-shaped patterns for the number of export and import varieties implied by the theoretical model are similar to those in Ghironi and Melitz (2007). An important message from the data is that the rise in amount of exports and imports is driven by an increase in the number of export and import varieties. Our theoretical model qualitatively mimics such a pattern.

[10]Ghironi and Melitz (2007) use data for different periods at the six-digit SITC level and find a tent-shaped pattern for exports and a flat pattern for imports, however.

What are the cyclical properties of quality of exports and imports in trade dynamics? The second panel in Figure 6 shows cross-correlations between the quality of exports and the aggregate value of exports. Similarly, the second panel in Figure 7 presents those between quality of imports and the aggregate value of imports. The data show a wedge-shaped pattern for these cross-correlations in both exports and imports. A rise in exports and imports involves simultaneous degradation in terms of export and import product quality. The theoretical model replicates such a pattern quite well, although the contemporaneous correlations are too strong for both exports and imports. As the IRFs following a transitory positive productivity shock in the previous section reveals, when quality increases, the number of exporters decreases in transitory dynamics due to a rising productivity cutoff induced by the terms of labor appreciation. Since exports (imports) co-move with the number of exported (imported) varieties, there is a positive correlation between exports (imports) and the number of exported (imported) varieties with different leads and lags. On the other hand, following the same positive shock, such a smaller set of productive exporters produces high-quality goods in transitory dynamics, which results in a negative correlation between exports (imports) and exported (imported) quality. The observed milder contemporaneous correlations in the data may be due to a more elaborate mechanism of product quality upgrading, such as the existence of sunk costs, than that in our model. The third panel in Figure 6 presents the cross-correlation between the number of export varieties and export quality. Similarly, the third panel in Figure 7 shows those for quality of imports and imports. These last two panels summarize the previous two panels' cross-correlations in the same Figures. The theoretical model qualitatively reproduces this pattern.

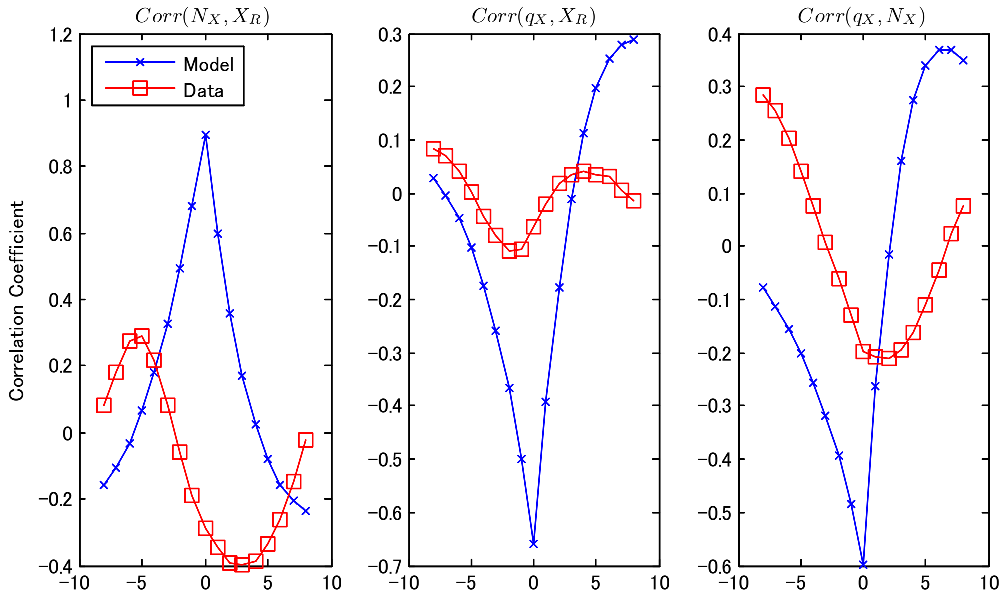

Figure 6: The Cyclicality of Export Variety and Export Quality

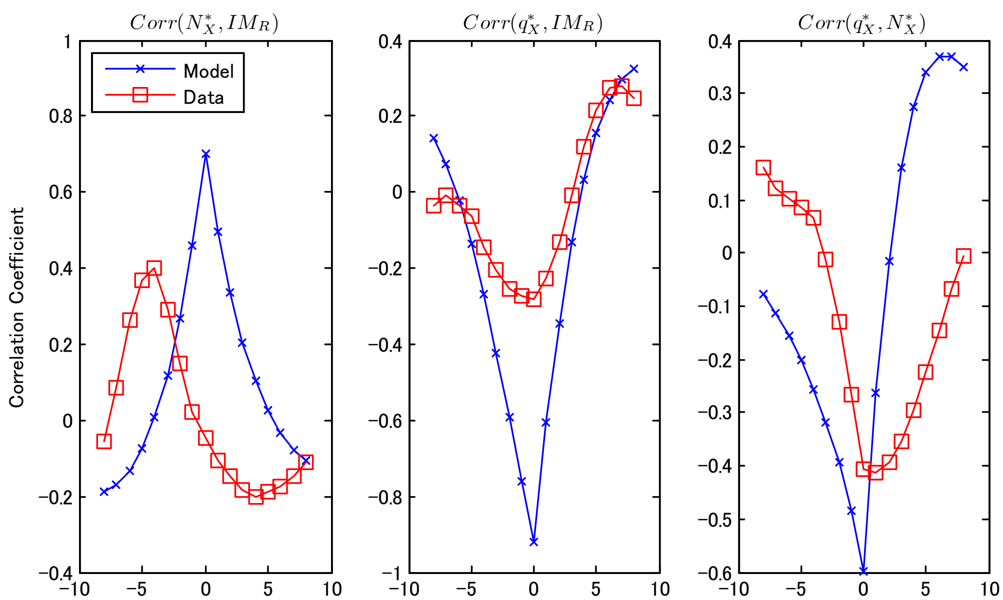

Figure 7: The Cyclicality of Import Variety and Import Quality

4 Conclusion

The paper presents a simple two-country DSGE model that embeds endogenous determination of both the number of traded varieties and traded product quality. By introducing product quality explicitly, the theoretical model departs from Ghironi and Melitz (2005, 2007) and sheds light on the cyclical properties of quality dynamics in international trade. The product quality of exports and imports are both negatively correlated with aggregate real exports and imports in the data. Our model can quantitatively replicate the wedge-shaped pattern of cross-correlations together with other statistics for the international real business cycle. The impulse response functions following a transitory productivity shock obtained with our theoretical model shed light on the underlying mechanism. Quality upgrading of the goods produced by a smaller number of more productive exporters is key.

We also highlight the welfare implications arising from trade liberalization. Since reductions in trade costs or fixed export costs, simultaneously induce quality downgrading of exported goods, not accounting explicitly for these welfare losses implies that the gains from trade liberalization are overestimated. Our numerical analysis with permanent trade liberalization shocks notes this possibility. Exploring the international transmission mechanisms for product variety and quality of changes in international relative prices would be a promising line of future research.

References

Antoniades, A. (2012). Heterogeneous frims, quality, and trade,. Technical report, Princeton University and Georgetown University,.

Arespa, M. and D. Gruber (2016). Product Quality and International Price Dynamics. UB Economics Working Papers 2016/340, Universitat de Barcelona, Facultat d'Economia i Empresa, UB Economics.

Backus, D. K., P. J. Kehoe, and F. E. Kydland (1992, August). International real business cycles. *Journal of Political Economy 100*(4), 745–75.

Baldwin, R. and J. Harrigan (2011, May). Zeros, quality, and space: Trade theory and trade evidence. *American Economic Journal: Microeconomics 3*(2), 60–88.

Benassy, J.-P. (1996, July). Taste for variety and optimum production patterns in monopolistic competition. *Economics Letters 52*(1), 41–47.

Broda, C. and D. E. Weinstein (2006, May). Globalization and the gains from variety. *The Quarterly Journal of Economics 121*(2), 541–585.

Broda, C. and D. W. Weinstein (2004, May). Variety growth and world welfare. *American Economic Review 94*(2), 139–144.

Dixit, A. K. and J. E. Stiglitz (1977, June). Monopolistic competition and optimum product diversity. *American Economic Review 67*(3), 297–308.

Engel, C. and J. Wang (2011, January). International trade in durable goods: Understanding volatility, cyclicality, and elasticities. *Journal of International Economics 83*(1), 37–52.

Feenstra, R. C. and J. Romalis (2014). International Prices and Endogenous Quality. *The Quarterly Journal of Economics 129*(2), 477–527.

Ghironi, F. and M. J. Melitz (2005, August). International trade and macroeconomic dynamics with heterogeneous firms. *The Quarterly Journal of Economics 120*(3), 865–915.

Ghironi, F. and M. J. Melitz (2007, May). Trade Flow Dynamics with Heterogeneous Firms. *American Economic Review 97*(2), 356–361.

Hallak, J. C. (2006, January). Product quality and the direction of trade. *Journal of International Economics 68*(1), 238–265.

Hallak, J. C. and P. K. Schott (2011). Estimating cross-country differences in product quality. *The Quarterly Journal of Economics 126*(1), 417–474.

Hamano, M. (2016). International risk sharing with product variety and quality. Technical report, Sophia University, mimeo.

Heathcote, J. and F. Perri (2002, April). Financial autarky and international business cycles. *Journal of Monetary Economics 49*(3), 601–627.

Hummels, D. and P. J. Klenow (2005, June). The variety and quality of a nation's exports. *American Economic Review 95*(3), 704–723.

Johnson, R. C. (2012). Trade and prices with heterogeneous firms. *Journal of International Economics 86*(1), 43–56.

Khandelwal, A. (2010, October). The long and short (of) quality ladders. *Review of Economic Studies 77*(4), 1450–1476.

Kollmann, R. (2004, 04/05). Welfare Effects of a Monetary Union: The Role of Trade Openness. *Journal of the European Economic Association 2*(2-3), 289–301.

Kollmann, R. (2015, April). Exchange Rates Dynamics with Long-Run Risk and Recursive Preferences. *Open Economies Review 26*(2), 175–196.

Kugler, M. and E. Verhoogen (2012, January). Prices, plant size and product quality. *Review of Economic Studies 79*(1), 307–339.

Manova, K. and Z. Zhang (2012). Export Prices Across Firms and Destinations. *The Quarterly Journal of Economics 127*(1), 379–436.

Melitz, M. J. (2003, November). The impact of trade on intra-industry reallocations and aggregate industry productivity. *Econometrica 71*(6), 1695–1725.

Picard, P. (2015). Trade, economic geography and the choice of product quality. *Regional Science and Urban Economics 54*(C), 18–27.

Rodrigue, J. and Y. Tan (2015). Price and quality dynamics in export markets. Technical report, Department of Economics Vanderbilt University, mimeo.

Schott, P. K. (2004, May). Across-product versus within-product specialization in international trade. *The Quarterly Journal of Economics 119*(2), 646–677.

Uhlig, H. (1998). A toolkit for analysing nonlinear dynamic stochastic models easily. QM&RBC Codes, Quantitative Macroeconomics & Real Business Cycles.

Verhoogen, E. A. (2008, 05). Trade, quality upgrading, and wage inequality in the mexican manufacturing sector. *The Quarterly Journal of Economics 123*(2), 489–530.

A Steady State

In the symmetric steady state, we assume without loss of generality that $Z = Z^* = f_E = f_E^* = z_{\min} = z_{\min}^* = 1$ and drop the asterisks that denote Foreign variables and time indices. Note that $NFA = NX = 0$ and $Q = 1$ in the symmetric steady state. We choose the parameter χ so that the steady state labor supply is unity $L = 1$.

First, we solve for the value of f_X so that it matches the empirical findings on the share of exporters. The free entry condition gives $\widetilde{v}^s = w$. Thus, using the steady state Euler equation for share holdings, we have

$$\widetilde{d} = \frac{1 - \beta (1 - \delta)}{\beta (1 - \delta)} w. \tag{2}$$

Therefore, by definition of $\widetilde{d}$, we obtain

$$\widetilde{d}_D + \frac{N_X}{N_D} \widetilde{d}_X = \frac{1 - \beta (1 - \delta)}{\beta (1 - \delta)} w. \tag{3}$$

Now, we rewrite $\widetilde{d}_D$ and $\widetilde{d}_X$ in the above expression. From the zero-profit export cutoff condition we have

$$\widetilde{d}_X = w f_X \frac{\sigma - 1}{k - (\sigma - 1)}. \tag{4}$$

With the above expression and the steady state average domestic and exporting profits $\widetilde{d}_D$ and $\widetilde{d}_X$, respectively, $\widetilde{d}_D$ can be rewritten as

$$\widetilde{d}_D = \frac{1}{\tau^{1-\omega}} \left(\frac{N_X}{N_D}\right)^{1-\psi(\omega-1)} \left(\frac{\widetilde{z}_X}{\widetilde{z}_D}\right)^{(1-\omega)(1+\phi)} \left[\frac{\sigma-1}{k-(\sigma-1)}+1\right] w f_X, \tag{5}$$

where we have used the fact that $\widetilde{\rho}_D/\widetilde{q}_D = \frac{\sigma}{\sigma-1}\frac{1}{1-\phi}\frac{w}{\widetilde{q}_D \widetilde{z}_D}$, $\widetilde{\rho}_X/\widetilde{q}_X = \frac{\sigma}{\sigma-1}\tau\frac{1}{1-\phi}\frac{w}{\widetilde{q}_X \widetilde{z}_X}$ and $\widetilde{q}_D = \left(\frac{\phi}{1-\phi}\widetilde{z}_D\right)^{\phi}$, $\widetilde{q}_X = \left(\frac{\phi}{1-\phi}\widetilde{z}_X\right)^{\phi}$.

Plugging (5) and (4) into (3), we obtain

$$\left[\frac{1}{\tau^{1-\omega}} \left(\frac{N_X}{N_D}\right)^{1-\psi(\omega-1)} \left(\frac{\widetilde{z}_X}{\widetilde{z}_D}\right)^{(1-\omega)(1+\phi)} \frac{k}{k-(\sigma-1)} + \frac{N_X}{N_D}\frac{\sigma-1}{k-(\sigma-1)}\right] f_X = \frac{1-\beta(1-\delta)}{\beta(1-\delta)}. \tag{6}$$

In the above expression, $\widetilde{z}_D$ is given by the Pareto distribution. $\frac{N_X}{N_D}$ is set to 0.21. Given this value and the Pareto distribution, this requires that $\widetilde{z}_X = 2.9425$, with the values of the parameters in the benchmark calibration. By plugging these values into the above equation, f_X can be solved.

Given this subsidy, the steady state labor supply is set to unity by controlling χ. Thus, the labor market clearing condition at the steady state gives

$$w = \left[N_E \widetilde{v}^s + (\sigma-1) N_D \widetilde{d} + +\sigma N_X w f_X\right].$$

The equation about the motion of firms gives $N_E = \frac{\delta}{1-\delta} N_D$. Using (2) and replacing $\widetilde{v}^s$ as previously, the above expression can be rewritten as

$$N_D = \frac{1}{\frac{\delta}{1-\delta} + (\sigma-1)\frac{1-\beta(1-\delta)}{\beta(1-\delta)} + \sigma\frac{N_X}{N_D} f_X}. \tag{7}$$

This is the solution for N_D.

Finally, the second equation can be obtained using the steady state price index

$$\left(\frac{\widetilde{z}_X}{\widetilde{z}_D}\right)^{(1-\omega)(1+\phi)} + \tau^{1-\omega}\left(\frac{N_X}{N_D}\right)^{-\psi(1-\omega)} = \left(\frac{N_D^{\psi}}{\frac{\sigma}{\sigma-1}\frac{1}{1-\phi}\frac{w}{\widetilde{q}_X \widetilde{z}_X}}\right)^{1-\omega} \tag{8}$$

By rearranging, we obtain the solution for w:

$$w = \left\{ \left(N_D^{\psi} \frac{\sigma}{\sigma - 1} \frac{1}{1-\phi} \frac{1}{\widetilde{q}_X \widetilde{z}_X} \right)^{1-\omega} \left[\left(\frac{\widetilde{z}_X}{\widetilde{z}_D} \right)^{(1-\omega)(1+\phi)} + \tau^{1-\omega} \left(\frac{N_X}{N_D} \right)^{-\psi(1-\omega)} \right] \right\}^{\frac{1}{-(1-\omega)}}.$$

Once found w, N_D can be found from (7). The steady state values of the other variables are relatively easy to find. In particular, the value of the parameter χ is $\chi = wC^{-\gamma}$ so that $L = 1$. Finally, the shares of domestic and imported goods of total expenditures are

$$S_{ED} \equiv \rho_H^{1-\omega} \text{ and } 1 - S_{ED} \equiv \rho_F^{1-\omega}.$$